Sparse Representations in Unions of Bases

The purpose of this correspondence is to generalize a result by Donoho and Huo and Elad and Bruckstein on sparse representations of signals in a union of two orthonormal bases for $\mathbb{R}^N$. We consider general (redundant) dictionaries for $\mathbb{R}^N$, and derive sufficient conditions for having unique sparse representations of signals in such dictionaries. The special case where the dictionary is given by the union of $L \geq 2$ orthonormal bases for $\mathbb{R}^N$ is studied in more detail. In particular, it is proved that the result of Donoho and Huo, concerning the replacement of the ℓ^0 optimization problem with a linear programming problem when searchin for sparse representations, has an analog for dictionaries that may be highly redundant.