Abstract—This paper investigates power sharing and power quality improvement issues of islanded single-/three-phase microgrids (S/T-MGs) where both sources and loads are unbalanced. A hierarchical distributed control approach is proposed, which consists of 1) a phase-independent virtual synchronous generator (P-VSG) control used for primary control of distributed generators (DGs), 2) a distributed secondary power flow regulator used for power sharing control among DGs and among phases, and 3) a distributed secondary voltage regulator used for voltage restoration and power quality improvement. Compared with conventional methods, the proposed control has several salient features: 1) the P-VSG control allows for independent and flexible power control and voltage regulation for each phase and accurate phase shifts; 2) distributed containment control proposed in the secondary power control and voltage regulation layer guarantees admissible output phase powers, voltage profiles and power quality; 3) the constraint operator developed for the secondary controllers makes charging/discharging power of the energy storage system (ESS) within permitted values; 4) communication delays are also considered in the proposed distributed approach. Simulation results are presented to demonstrate the proposed control method.

Index Terms—single-/three-phase microgrids, power sharing, power quality, distributed control, phase-independent virtual synchronous generator.

I. INTRODUCTION

MICROGRID (MG) has been regarded as a promising solution to integrate renewable energy sources (RESS) as well as distributed energy storage systems (ESSs), and has been widely studied including AC MGs [1], [3], DC MGs [2], and hybrid AC/DC MGs [4], being mainly focused on balanced MGs. However, the MG system is usually characterized by unbalance [5]–[7] due to the integration of single-phase distributed generators (SDGs)/loads and the occurrence of asymmetrical faults, leading to significant challenges for the secure and reliable operation of the MG. Such unbalanced systems could be found in different countries like Australia, Sweden and Germany [8], [9]. Take a real 415V low-voltage (LV) distribution system with 101 customers in Australia [8] as an example, 3 customers and 38 customers with photovoltaic (PV) generators are connected to the LV grid through three-phase and single-phase PV inverters, respectively.

Load power sharing and voltage regulation (including voltage quality enhancement) are the most important issues for microgrid operation, which is a challenging work in unbalanced S/T-MGs. Current research works in unbalanced MGs mainly include three aspects: i) power sharing control, ii) power quality control, and iii) simultaneous power sharing and power quality control. Approaches reported in literatures related to these research aspects can be divided into three categories: centralized, decentralized and distributed approaches.

For the first aspect, power sharing control in unbalanced MGs, various research works have been reported in literature [5], [10]–[15] and therein. These approaches are primarily based on droop control and virtual impedance control. For instance, He et al. [10] proposed a centralized approach to realize reactive power, imbalance power and harmonic power sharing based on virtual impedance regulation and droop control. Decentralized schemes have also been developed in literatures [11], [12], where small-ac-signal injection method and single-phase droop control was proposed, respectively. To overcome the drawbacks of centralized and decentralized methods, authors in [5] proposed a distributed method to realize the above power sharing control objective. However, the above works mainly consider the unbalanced loads connected to the common bus. Unbalanced sources (Hybrid SDGs and three-phase DGs (TDGs)) are not considered while SDGs can be randomly integrated into the system besides TDGs in practice. Regarding this scenario, our previous work [13] proposed a power sharing unit (PSU) to manage the power flow between phases. The PSU is composed of three single-phase back-to-back (BTB) converters connected in a Δ-structure. Each BTB converter is connected between two phases. Thus, we can navigate the power flow between phases.
to enhance the power supply reliability and RES utilization by controlling the PSU. A similar method was also proposed in [14], where BTB converters are also utilized between phases, and multi-segment droop method, intra- and inter-phase power control and management scenarios are considered in order to maintain desired voltage and frequency profiles. However, installing extra converters is required in [13] and [14], which could result in high costs. To address this issue, Karimi et al. [15] firstly developed decentralized modified $P - f$ droop functions to automatically perform the power flow among different phases through bidirectional four-leg TDGs in hybrid single-/three-phase microgrids (S/T-MG), where hybrid source PV/battery units were considered. However, better performance could be achieved via coordination between DGs instead of this decentralized method and how to determine the power of each phase is not discussed in detail. In unbalanced MGs, overloading of a phase could result in unnecessary DG tripping, load shedding, and reduce the overall system operation security and reliability. In [16], a dynamic power routing based optimal power flow method among phases was proposed to maximize the loadability of the hybrid AC/DC microgrids by using interlinking converters connected to the AC and DC subgrids, where all DGs’ information is required for this centralized method. An event-based distributed method was proposed in [17] to balance the output power of TDGs. The basic idea of [16] and [17] is similar, but similar to [13], [14], both of them require extra equipments (ICs). Moreover, there is a lack of adequate coordination between TDGs and SDGs, and voltage quality also needs to be considered.

Although the above technologies can provide satisfactory power sharing performance, power quality control is another equally important issue in unbalanced MGs. The aim of the works discussed in [18]–[22] is predominantly to achieve unbalance voltage compensation among DGs. In [18], a customized power quality method using optimization was studied for different areas on the customer side. Similarly, a real-time supervisory control approach based on a scheduling framework was developed for voltage unbalance/harmonic improvement of multi-area MGs in [19]. Unlike these centralized methods, Li et al. [20] first proposed a standard data-driven controller for DGs via decentralized deep reinforcement learning with satisfactory power quality and Meng et al. [21] proposed a distributed method for voltage unbalance compensation. In [22], a three-phase electric spring was developed for voltage regulation and source current balancing in the unbalanced system. But, many electric springs shall be installed if used in a large-scale system. Moreover, only unbalanced loads are considered in the MGs in these works. Consequently, centralization approaches [23], [24] and master-slave method [25] were, respectively, developed to improve the voltage quality only using SDGs without coordination with TDGs. Also, it should be pointed out that power sharing control and power quality control should be simultaneously considered. The voltage magnitudes and voltage unbalance factors (VUF) should be regulated to fulfill the standard, e.g., IEEE 1547 standard [27] while designing power-sharing methods. Currently, only very few works explored this topic. For example, C. Burgos-Mellado et al. [28] proposed a cooperative control scheme based on the conservative power theory to share the unbalanced and distorted components of the currents and powers. And a secondary control loop was implemented to regulate the maximum voltage imbalance/distortion at the PCC. After that, a distributed version was developed in [29]. Nevertheless, these works still only discussed the unbalanced loads scenario.

Despite the significant research progress in the above aspects, there are still some obvious research gaps. i) Only unbalanced loads considered in most of the existing works. Unbalanced sources with SDGs and TDGs are usually not considered except [13]–[16], where, however, they mainly focus on either power quality improvement or power sharing. Moreover, in [13], [14], [16], extra power converters or systems are required. ii) Coordination between SDGs and TDGs is not explored in most of the existing works like [15]. To the best of the authors’ knowledge, only [26] considered this cooperation for unbalance compensation and balance operation of TDGs. More exploration is necessary for simultaneous power sharing and power quality improvement. iii) When considering unbalanced sources, conventional droop control [1] and VSG control [30] are not conductive to flexible regulation of power and voltage of each phase. Moreover, conventional secondary controllers, e.g., [2], [3], [5], [21], designed for accurate power sharing among DGs and voltage regulation could lead to TDG’s phase of heavy load more easily overloaded and cannot guarantee voltage quality at all nodes fulling requirements.

With these motivations mentioned above, this paper focuses on power sharing and power quality issues of S/T-MGs. The main contributions can be summarized as follows:

1) Power sharing and power quality improvement of islanded S/T-MGs are investigated, where both DGs and loads are unbalanced, and hybrid RESs/ESSs are also considered. This is different from existing works, where only unbalanced loads and ideal dc sources are considered, or coordination between SDGs and TDGs is inadequately explored.

2) Different from conventional control approaches [1], [11], [30], [31], a phase-independent virtual synchronous generator (P-VSG) control is proposed for the primary control of DGs, which allows for independent and flexible power control and voltage regulation for each phase, and as well as accurately balanced phase shifts that make phase shifts balancing requirement and some negative effects avoided.

3) Distributed secondary containment controllers with communication delays are proposed for power sharing, voltage restoration and voltage quality control of SDGs/TDGs. Different from previous works like [2], [3], [5], [21], the proposed method with containment control and constraint operator can guarantee secure output phase powers, admissible voltage profiles, voltage quality, and charging/discharging power of ESSs, resulting in more secure and reliable operation.

The remainder of this paper is structured as follows. The notations, preliminaries and assumptions are briefly introduced in Section II. The S/T-MG system structure and problem formulation are presented in Section III. Section IV discusses the proposed control strategy and as well as the stability analysis. Simulation results are provided to validate our method in Section V. Conclusions are finally drawn in Section VI.
MGs is typically an unbalanced system, which means that not only the load but also the DG units are unbalanced. Under this scenario, the output power of each phase of TDGs can be different from each other, which in turn affects the MG loadability and reliability. All the DG units in the MG should cooperate with each other to provide reliable power supply for the loads and guarantee proper load power sharing, admissible voltages, power quality and as well as ESS constraints. We will primarily focus on these issues in this work. And frequency regulation and as well as the zero sequence issue will not be discussed in the secondary control layer throughout the following paper. But it could be included if necessary.

B. Problem Formulation

As mentioned above, coordinated power sharing among different DGs including ESSs and among different phases of the TDGs in the S/T-MGs is very crucial and challenging due to the fast proliferation of different types of DG units and loads. The control objective of the S/T-MG includes the following three aspects:

1) Proper Power Sharing: The first objective of these DGs is to achieve proper real and reactive power sharing among them, which can be realized at the steady state if all units are ideally and properly controlled, i.e.,

$$\sum_{k} k_{TDG}^{i} p_{TDG}^{i} = \sum_{k} k_{SDG}^{i} p_{SDG}^{i}$$

(1)

3) Load Sharing: The second objective of these DGs is to achieve proper load sharing among them, which can be realized at steady state if all units are ideally and properly controlled, i.e.,

$$\sum_{k} f_{TDG}^{i} = \sum_{k} f_{SDG}^{i}$$

(2)

4) Allowable Power Sharing: The third objective of these DGs is to achieve proper power sharing among them, which can be realized at steady state if all units are ideally and properly controlled, i.e.,

$$\sum_{k} k_{TDG}^{i} q_{TDG}^{i} = \sum_{k} k_{SDG}^{i} q_{SDG}^{i}$$

(3)

$$\sum_{k} k_{TDG}^{i} q_{TDG}^{i} = \sum_{k} k_{SDG}^{i} q_{SDG}^{i}$$

(4)

Therefore, from the above two aspects, the power balancing sharing objective under ideal conditions can be summarized as

$$3k_{TDG}^{i} p_{TDG}^{i} = 3k_{SDG}^{i} p_{SDG}^{i}$$

(5)

$$3k_{TDG}^{i} q_{TDG}^{i} = 3k_{SDG}^{i} q_{SDG}^{i}$$

(6)

where $i \neq j = 1, \ldots, N_{TDG}$, $r = 1, \ldots, N_{SDG}$, $a = a, b, c$.

However, in the real world, it may be difficult and unnecessary to accurately realize the above control goals especially for the S/T-MGs. In the S/T-MGs, the output power of each phase of the TDG is most likely to be different from each other, which can deteriorate the operating reliability, security
and power quality. Therefore, the balancing of output phase powers of TDG units should be considered besides achieving proper power sharing among DGs. The output phase power of each TDG unit and as well that of SDG unit should be kept within the permitted maximum value,
\[
0 \leq k_{p,i,b}^b \frac{P_{i,b}}{P_{i,b}} < k_{q,i,b} \frac{Q_{i,b}}{Q_{i,b}} \leq k_{q,i,b}^b
\]
where the superscript \( ^\dagger \) = TDG or SDG, \( P_{i,b}^{\dagger,\text{max}} = S_{i,b}^b \), \( Q_{i,b}^{\dagger,\text{max}} = S_{i,b}^b \), \( S_{i,b}^b \) is the apparent power. For \( \dagger = \text{TDG} \), \( P_{i,b} \) = 3\( P_{i,b} \) and \( k_{TDG}^{q,i,b} = 3k_{q,i,b} \). For better readability and simplicity, “\( \dagger \)” is omitted throughout the following paper.

With this consideration, the accuracy of power sharing among DGs could be compromised. Therefore, the power sharing control performance, in this paper, is designed as
\[
\lim_{k \to +\infty} \left\| x_{p,i,b}(k) - P_{Y_T}(x_{p,i,b}(k)) \right\| = 0
\]
\[
\lim_{k \to +\infty} \left\| x_{q,i,b}(k) - Q_{Y_T}(x_{q,i,b}(k)) \right\| = 0
\]
where \( x_{p,i,b}(k) = k_{p,i,b} P_{i,b}(k) \), \( x_{q,i,b}(k) = k_{q,i,b} Q_{i,b}(k) \), \( P_{Y_T} = \{P_{i,b}^l, P_{i,b}^u\} \), \( Q_{Y_T} = \{Q_{i,b}^l, Q_{i,b}^u\} \), \( P_{i,b}^l = 0 \), \( Q_{i,b}^l = 0 \). This means that we make the power sharing among units asymptotically converge to the convex hull rather than mandate accurate power sharing among DG units, which could maintain the system operation reliability and security.

2) Admissible Voltage Profiles and Power Quality: Another important performance criterion in a S/T-MG is to maintain acceptable output voltage profiles. Firstly, node voltages need to be regulated to be close to the rated values, \( E^* - \varepsilon^\text{max} \leq E_{i,b} \leq E^* + \varepsilon^\text{max} \) where \( E^* \) is the desired DG voltage magnitude and \( \varepsilon^\text{max} \) is the maximum permitted voltage regulation requirement (usually 3% of the rated voltage from the IEEE 1547 standard [27]). That is to say, voltages should converge to the convex hull spanned by voltage reference leaders
\[
\lim_{k \to +\infty} \left\| E_{i,b}(k) - P_{Y_E}(E_{i,b}(k)) \right\| = 0
\]
where \( Y_E = \{[E_{i,b}^l, E_{i,b}^u]\} \), \( E_{i,b}^l = E^* - \varepsilon^\text{max} \), \( E_{i,b}^u = E^* + \varepsilon^\text{max} \).

Secondly, keeping acceptable voltage quality in the S/T-MG is also important, the VUF at each TDG node and PCC should not be larger than the allowed maximum VUF\(^{\text{max}}\) (usually 2% defined by IEEE 1547 standard [27]), i.e.,
\[
\lim_{k \to +\infty} \left\| \text{VUF}_{TDG}(k) - P_{Y_{\text{VUF}}}(\text{VUF}_{TDG}(k)) \right\| = 0
\]
where \( \text{VUF}_{TDG} = \{[\text{VUF}_{TDG}^l, \text{VUF}_{TDG}^u]\} \), \( \text{VUF}_{TDG}^l = 0 \), \( \text{VUF}_{TDG}^u = \text{VUF}_{TDG}^{\text{max}} \). \( \text{VUF}_{TDG}^{\text{max}} = 100\% \frac{E_{i,b}^{\text{max}}}{E_{i,b}^*}, \) where \( E_{i,b}^{\text{max}}, = \left| e_{i,b}^\text{TDG,}, \right| \) and \( E_{i,b}^{\text{TDG,}} = \left| e_{i,b}^\text{TDG,} \right| \) are respectively the voltage negative and positive sequence magnitude derived by symmetrical component analysis, i.e., \( e_{i,b}^\text{TDG,} = \frac{1}{3} \left( e_{i,b}^\text{TDG,} + \frac{a^2 e_{i,b}^\text{TDG,} + a e_{i,b}^\text{TDG,}}{3} \right) \), \( e_{i,b}^{\text{TDG,}} = \frac{1}{3} \left( e_{i,b}^{\text{TDG,}} + a^2 e_{i,b}^{\text{TDG,}} + a e_{i,b}^{\text{TDG,}} \right) \), \( a = 1/\sqrt{3} \). \( e_{i,b}^\text{TDG,} = a, b, c \) are the measured three-phase voltages.

3) Meeting ESS Constraints: ESSs are considered in our research work. The SoC and charging/discharging power of the ESSs should be maintained within permitted values for the sake of security and reliability, i.e.,
\[
-P_{\text{dsch,max}}^{\text{ESS,},i} < P_{\text{ess,},i} < P_{\text{dch,max}}^{\text{ESS,},i}
\]
\[
S_{\text{OC,},i}^{\text{min}} < S_{\text{OC,},i} < S_{\text{OC,},i}^{\text{max}}
\]
where \( S_{\text{OC,},i} \) and \( P_{\text{ess,},i} \) are, respectively, the SoC and charging and discharging power of ESSs; \( P_{\text{dch,max}}^{\text{ESS,},i} \) and \( P_{\text{dsch,max}}^{\text{ESS,},i} \) are, respectively, the permitted maximum charging and discharging power of ESSs; \( S_{\text{OC,},i}^{\text{min}} \) and \( S_{\text{OC,},i}^{\text{max}} \) are, respectively, the minimum and maximum SoC of ESSs. The discrete dynamic of the SoC can be described as [33]
\[
S_{\text{OC,},i}(k+1) = (1 - \gamma_i) S_{\text{OC,},i}(k) + \eta_{\text{ess,}i} C_{\text{ess,}i}^{\text{max}} T P_{\text{ess,},i}(k)
\]
where \( \eta_{\text{ess,}i} \) and \( C_{\text{ess,}i} \) are the charging/discharging efficiency and capacity of the ESSs, respectively; \( \gamma_i \) and \( T \) are self-discharging coefficient and time interval, respectively.

IV. PROPOSED CONTROL STRATEGY FOR THE S/T-MG
In this section, we present the proposed control strategy in detail, which includes 1) P-VSG controller and 2) secondary power sharing and voltage regulators. Then, we give the stability analysis and the proof.

A. P-VSG Control Used for Primary Control of DGs
In order to achieve flexible control and operation for the S/T-MG, we firstly propose a modified VSG control approach used for the primary control of DG units, which is given by
\[
\dot{\theta}_{i,b} = \omega_{i,b} = \omega^* + \sum_{b = a, b, c} \Delta \omega_{i,b}
\]
\[
M_{i} \Delta \omega_{i,b} = P_{\text{RES,},i+b} + P_{\text{ess,},i+b} - P_{i,b} - D_{p,i} \Delta \omega_{i,b}
\]
\[
K_i \dot{E}_{i,b} = Q_{\text{set,},i+b} + \Delta Q_{i,b} - Q_{i,b} - D_{q,i} \left( E_{i,b} - E^* - \Delta E_{i,b} \right)
\]
where \( \omega^* \) is the desired angular frequency of the DG units, respectively; \( \omega_{i,b} \) and \( E_{i,b} \) are the output angular frequency and phase voltage magnitude of the DG units, respectively; \( P_{i,b} \) and \( Q_{i,b} \) are the active output and reactive powers of each phase, respectively; \( P_{\text{RES,},i+b} \) and \( P_{\text{ess,},i+b} \) are RES output power and ESS output power used for phase-b, respectively; \( Q_{\text{set,},i+b} \) is reactive power set point of phase-b; \( \Delta Q_{i,b} \) and \( \Delta E_{i,b} \) are regulation terms of the reactive power and voltage magnitude of phase-b, respectively, which will be determined by the secondary controllers; \( M_i \) and \( D_{p,i} \) are the virtual inertia and damping constants, respectively; \( K_i \) and \( D_{q,i} \) are the integrator gain to regulate the field excitation and the voltage droop coefficient, respectively. Note that for SDG units, there is no sum of frequency deviations, and (15) can be rewritten as
\[
\dot{\theta}_{i,b} = \omega^* + \Delta \omega_{i,b}
\]
Then, from (15)-(17), the reference voltage of TDG units, \( e_{i,b}^{TDG} = (e_{i,b}^{TDG} + e_{i,b}^{TDG}, e_{i,b}^{TDG}) \), can be generated by
\[
e_{i,b}^{TDG} = \frac{F_{TDG}^{\text{max}}}{\pi} \sin \left( \omega^* + \sum_{b = a, b, c} \Delta \omega_{i,b} \right)
\]
\[
e_{i,b}^{TDG} = \frac{F_{TDG}^{\text{max}}}{\pi} \sin \left( \omega^* + \sum_{b = a, b, c} \Delta \omega_{i,b} \right)
\]
where \( b = a, b, c \). And the reference voltage of SDG units connected to phase-\( b \) is given by
\[
e_{\text{ref}}^{\text{b}} = E_{\text{ref}}^{\text{b}} \sin \left( \omega^* + \Delta \omega_{\text{ref}}^{\text{b}} \right) t + \varphi_{\text{b}}
\]
where \( \varphi_{\text{b}} \) is set as \( 0 \), \( -\frac{2\pi}{3} \), and \( \frac{2\pi}{3} \) for SDGs in phase-\( a \), phase-\( b \) and phase-\( c \), respectively. Note that \( \varphi_{\text{b}} \) in (20) could be not a required term, but in order to keep a good voltage transient performance and make it easy to be understood, this term is remained here.

Remark 1: Compared with conventional VSG control like [30], [31], the proposed P-VSG control allows for independent and flexible power and voltage control for each phase. Specifically, i) for TDGs, the active and reactive power of each phase can be independently controlled by adjusting \( P_{\text{ess},i,b} \) and \( Q_{\text{set},i,b} \) to regulate \( \Delta \omega_{\text{ref}}^{i,b} \) and \( E_{\text{ref}}^{i,b} \), respectively, which facilitates the secondary controller design in the S/T-MG; ii) the voltage magnitude of each phase can also be independently controlled by regulating \( E^* \) thereby flexibly controlling the output voltage waveform if necessary for power quality control; iii) for TDGs, the sum of the frequency deviations among phases in (15), (19) and (21) can accurately guarantee balanced phase shifts of \( \frac{2\pi}{3} \) thereby phase shifts balancing strategy and some negative effects can be avoided, which is also evaluated by the simulation results in Section V.

B. Distributed Secondary Control for Power Sharing and Voltage Regulation

The simplified control diagram of the proposed P-VSG controlled DG unit is shown in Fig. 2, where the dynamics of the LC filter, the RL output converter, and the voltage and current control loops are not considered since it is much faster than that of the P-VSG control loop. The active and reactive powers are usually processed through a low-pass filter and then fed to the control system. From Fig. 2, we have
\[
P_{\text{ess},i,b} = -\tau_1^{-1} P_{\text{ess},i,b} + \tau_1^{-1} P_{\text{max},i,b} (\theta_{i,b} - \theta_{\text{pcc},b}) \tag{21}
\]
\[
Q_{\text{set},i,b} = -\tau_1^{-1} Q_{\text{set},i,b} + \tau_1^{-1} Q_{\text{max},i,b} (E_{\text{ref}}^{i,b} - V_{\text{pcc}}) \tag{22}
\]
where \( \tau_1 \) is time constant of the low-pass filter; \( P_{\text{max},i,b} = \frac{E^*}{X_{\text{line},i}} \) and \( Q_{\text{max},i,b} = \frac{E^*}{X_{\text{line},i}} \) are, respectively, the maximum active and reactive powers of each phase that can be delivered by the DG unit, and rated DG voltage and PCC voltage are adopted for simplicity. \( X_{\text{line},i} \) is the coupling line parameter. Note that the model (21) and (22) is obtained based on the star-connected circuit but it is also applicable to the mesh-connected circuit via equivalent transformation operation.

Define variables
\[
P_{\text{ess},i,b} = u_{i,b}, \quad \Delta Q_{i,b} = u_{i,b}^Q \tag{23}
\]
\[
\Delta E_{\text{REF}}^{i,b} = u_{i,b}^{\text{REF}} \tag{24}
\]
and \( \pi_i = \left( P_{i,b}, \theta_{i,b}, \Delta \omega_{i,b}, Q_{i,b}, E_{\text{ref}}^{i,b} \right)^T, \ x_i = \left( \pi_i, S_{\text{MG}}^{i,c} \right)^T, \ v_i = \left( P_{\text{ess},i,b}, \Delta Q_{i,b}, \Delta E_{\text{REF}}^{i,b} \right)^T, \ u_i = \left( u_{i,b}^P, u_{i,b}^Q, u_{i,b}^{\text{REF}} \right)^T, \ d_i = \left( \theta_{\text{pcc},b} \sum_{i,j \neq b} \Delta \omega_{i,j} + \omega^*, P_{\text{RES},i,b}, V_{\text{pcc}}, Q_{\text{set},i,b}, E^* \right)^T \]
then, from (15)-(17) and (21)-(24), we have
\[
\begin{align*}
\pi_i (t) & = A_i \pi_i (t) + B_i v_i (t) + D_i d_i (t) \\
v_i (t) & = v_i (t) + T u_i (t)
\end{align*}
\]
In order to facilitate practical implementation of the proposed method, we discretize the above dynamic system
\[
\begin{align*}
\pi_i (k + 1) & = G_i \pi_i (k) + H_i v_i (k) + F_i d_i (k) \\
v_i (k + 1) & = v_i (k) + T u_i (k) \\
y_i (k) & = C_i x_i (k)
\end{align*}
\]
where \( T \) is the sampling period. Then, combine this discrete-time dynamics and the dynamics (14), we can have the dynamics of the the DG unit in a compact discrete-time form
\[
\begin{align*}
x_i (k + 1) & = A_i x_i (k) + B_i v_i (k) + D_i d_i (k) \\
v_i (k + 1) & = v_i (k) + T u_i (k) \\
y_i (k) & = C_i x_i (k)
\end{align*}
\]
where \( y_i (k) \) is the output variable; \( A_i, B_i, C_i, D_i \) are given in Appendix.

Therefore, the objective of this section is to design the secondary control schemes \( u_i (k) \) such that the S/T-MG achieves the power sharing (8)-(9) and voltage regulation (10)-(11) goals while keeping ESSs’ constraints (12)-(13) satisfied in a distributed framework. The overall schematic diagram of the proposed control policy for DG \( i \) is shown in Fig. 3, in which the controller consists of four separate modules: primary P-VSG controller, active power balancing regulator, reactive power balancing regulator, and voltage regulator.

1) Active Power Sharing Regulator: In order to balance the phase powers of TDGs while simultaneously maintaining proper power sharing among DG units and admissible SoC and charging/discharging power constraints, the phase active power \( P_{\text{TDG}}^{i} \) of TDGs, the active power \( P_{\text{ess}}^{i,b} \) of SDGs and as well as \( P_{\text{ess},i,b} \) are utilized to construct the controller
\[
\begin{align*}
u_{i,b}^P (k) & = S_{V_i} \left[ P_{\text{ess},i,b} (k) - p_i P_{\text{ess},i,b} (k) + \pi_{i,b} (k) \right] \\
\pi_{i,b} (k) & = \sum_{i,j \neq i} \frac{a_{ij}}{H_i} \left[ x_{p,j,b} (k) - x_{i,b} + x_{p,i,b} (k) \right] - c \left[ x_{p,i,b} (k) - P_{\text{Yf}} (x_{p,i,b} (k)) \right]
\end{align*}
\]
where \( \pi_{ij} < \pi_{max} \) is the communication time delay from agent \( j \) to agent \( i; p_i \) is the feedback damping gain of follower \( i; a_{ij} \) is the edge weight of the communication edge \((j, i)\) and \( c \) is the pinning gain. If agent \( i \) can receive information directly from one or more leaders at time \( k \), then \( c > 0 \) is required.
a positive constant; otherwise, \( c = 0 \). This guarantees proper active power sharing among DG units.

2) Reactive Power Sharing Regulator: The reactive power sharing strategy is similar to that of the active power sharing strategy. The phase reactive power \( Q_{i,TDG}^{TDG} \) of TDGs, the reactive power \( Q_{i,SDG}^{SDG} \) of SDGs are used to design the controller

\[
u_{i,b}^Q(k) = \sum_{j \in \mathcal{N}_i} a_{ij} \left[ x_{q,i,j}(k - \tau_{ij}) - x_{q,i,b}(k) \right] - c \left[ x_{q,i,b}(k) - \mathcal{P}_{YQ} \left( x_{q,i,b}(k) \right) \right].
\]

3) Voltage Regulator: The voltage regulator consists of two parts: a secondary voltage regulator and a voltage unbalance factor (VUF) regulator. In this part, the secondary voltage regulator is firstly designed to control the voltage of each DG unit close to rated voltage and within the admissible range (3% of the rated voltage from the IEEE 1547 standard [27]). Secondly, the VUF regulator is also designed for the TDGs to regulate the VUF to an acceptable value since admissible voltage range does not guarantee satisfied VUF. It could be more complicated if VUF is directly used as the feedback information. So, we use the negative component of the voltage, \( E_{i,TDG}^{TDG} \), as the feedback signal in this work.

The secondary voltage regulator \( u_{i,b}^E(k) \) and the VUF regulator \( u_{i,b}^{VUF}(k) \) are, respectively, designed as

\[
u_{i,b}^E(k) = \sum_{j \in \mathcal{N}_i} a_{ij} \left[ E_{j,TDG}(k - \tau_{ij}) - E_{i,b}(k) \right] - c \left[ E_{i,b}(k) - \mathcal{P}_{YE} \left( E_{i,b}(k) \right) \right]
\]

(31)

\[
u_{i,b}^{VUF}(k) = \sum_{j \in \mathcal{N}_i} a_{ij} \left[ x_{V,i,j}(k - \tau_{ij}) - x_{V,i,b}(k) \right] - c \left[ x_{V,i,b}(k) - \mathcal{P}_{YV} \left( x_{V,i,b}(k) \right) \right]
\]

(32)

where \( x_{V,i,b}(k) = E_{i,TDG}^{TDG} - |e_{i,TDG}^{TDG}| \) and \( e_{i,a}^{TDG} = e_{i,b}^{TDG} = e_{i,c}^{TDG} = 0 \), \( a = a, b, c \).

Remark 2: Compared with existing literature, several differences of the proposed distributed control can be summarized as follows. i) For the secondary controller design, the system is formulated as an output containment control problem of multiple agents (25)-(27) with uncertain disturbances from RESs and constraints of ESSs. This is different from many existing literature where the disturbance and constraints are not considered. ii) Although accuracy of power sharing among DG units may be compromised, the distributed containment controllers (28)-(30) can guarantee that the output power of each phase is less than the maximum permitted value and that the unbalance of three-phase power of TDGs is improved, and controllers (31)-(32) make admissible voltage profiles and VUF satisfied, which is different from those of literature [2], [3], [5], [21]. iii) The constraint operator developed in the controller (28) makes charging/discharging power of the ESSs within permitted values, which is not considered in many studies where ideal sources are usually assumed. iv) Although the VUF regulator (32) is primarily designed for the TDGs in this work, it could be extended to be applicable to both TDGs and SDGs by assuming a virtual TDG for the SDG with virtual balanced three phase voltage that could be obtained by using a similar approach mentioned in Remark 3. Then, SDGs could coordinate with TDGs to regulate the VUF. v) Grid-following DGs usually regulate their output active and reactive power injected into the grid by measuring/estimating the angle and frequency of the bus voltage using a phase-locked loop (PLL) [40] and their power response from the reference power can be simply modeled as a first-order system [41]. Then the model similar to (25)-(27) can be obtained. It is possible to extend the proposed approach to the scenario that grid-forming DGs and grid-following DGs coexist but still require deeper analysis and even validation through simulation and experiment. We leave this issue in the future work considering the page limits.

Remark 3: The average (DC component) active and reactive power of each phase of TDGs and as well as that of SDGs, which are utilized in (28)-(30), can be calculated by

\[
P_{i,b} = 0.5E_{i,b}I_{i,b} \cos(\psi)
\]

(33)

\[
Q_{i,b} = 0.5E_{i,b}I_{i,b} \sin(\psi)
\]

(34)

where \( E_{i,b} \) and \( I_{i,b} \) are, respectively, the output phase voltage and current magnitudes of the DGs; \( \psi \) is the phase deviation between the voltage and current. These parameters can be obtained by using the Fourier analyser block in Matlab/Simulink toolbox or the All-Pass-Filter-Based PLL Systems proposed in [34]. Alternatively, the active and reactive power can also be calculated based on the \( a_{i}\beta \) transformation and second-order generalized integrator (SOGI) [35]

\[
P_{i,b} = e_{i,\alpha}I_{i,\alpha} + e_{i,\beta}I_{i,\beta}
\]

(35)

\[
Q_{i,b} = e_{i,\alpha}I_{i,\alpha} - e_{i,\beta}I_{i,\beta}
\]

(36)

where \( e_{i,\alpha}, e_{i,\beta} \) and \( I_{i,\alpha}, I_{i,\beta} \) are given by

\[
(i_{i,\alpha}, i_{i,\beta})^T = T (\hat{i}_{i,\alpha}, \hat{i}_{i,\beta}, \hat{i}_{i,c})^T
\]

(37)

\[
(i_{i,\alpha}, i_{i,\beta})^T = T (\hat{i}_{i,\alpha}, \hat{i}_{i,\beta}, \hat{i}_{i,c})^T
\]

(38)

where \( T = \frac{3}{2} \left[ 1 - \frac{1}{2} - \frac{1}{2} \right] \sqrt{\frac{3}{2}} \). For TDGs, we set \( \hat{i}_{i,\alpha} = e_{i,TDG}^{TDG}, \hat{i}_{i,\beta} = e_{i,TDG}^{TDG}, \hat{i}_{i,\beta} = 0, \quad b = a, b, c, \nu \neq b \), where \( e_{i,TDG}^{TDG} \) and \( e_{i,TDG}^{TDG} \) are the measured phase voltage and current. For SDGs, we could regard
\((c_{i,a}, c_{i,b}, c_{i,c})^T\) and \((\tilde{c}_{i,a}, \tilde{c}_{i,b}, \tilde{c}_{i,c})^T\) as virtual three-phase voltage and current. Specifically, for SDGs connected into phase-\(b\), we set \(\tilde{c}_{i,b} = c_{i,b} + \tilde{c}_{i,b}\), \(\tilde{c}_{i,b} = i_{i,b} + \tilde{i}_{i,b}\), and \(\tilde{i}_{i,b} = 0\), \(b = a, b, c, \nu \neq b\), and the other two virtual phase voltages \(\tilde{c}_{i,\nu}\) based on the voltage \(c_{i,\nu}\) and current \(\tilde{i}_{i,\nu}\). Therefore, the active and reactive power \(E_{i} + V_{SDG}\) on the measured voltage \(\tilde{c}_{i,a}\) based on \(\tilde{i}_{i,a}\) for the S/T-MG units in phase-\(a\) as an example, we can set \(\tilde{c}_{i,a} = E_{i,a} + V_{SDG}\) and \(\tilde{i}_{i,a} = i_{i,a} + \tilde{i}_{i,a}\). Take SDG (SDG units) as an example, we can set \(\tilde{c}_{i,a} = E_{i,a} + V_{SDG}\) and \(\tilde{i}_{i,a} = i_{i,a} + \tilde{i}_{i,a}\). Therefore, the active and reactive power can be obtained using (35)-(38). But the active and reactive power calculated by using this approach has the oscillation component at the double fundamental frequency. The third method is based on the measured phase voltage and current of DGs, self-tuned notch filter and low pass filter. More details about this method can be found in [11]. In this paper, the first approach is adopted.

C. Stability Analysis

In order to analyze the convergence of the S/T-MG system, we first make the following model transformation.

Define \(\beta_i = \frac{S_{V_i}(v_i(k) - p_i v_i(k) + \sigma_i)}{\|v_i(k) - p_i v_i(k) + \sigma_i\|}\) for all \(k > 0\).

Obviously, \(0 < \beta_i \leq 1\). From the definition of the constraint operator \(S_{V_i}()\) and the controller \(u_i(k)\) given by (28)-(32), we have

\[
S_{V_i}[v_i(k) - p_i v_i(k) + \sigma_i] = \beta_i [v_i(k) - p_i v_i(k)] + \beta_i \sigma_i(k)
\]

where \(\sigma_i(k) = \sum_{j \in N_i} a_{ij} [C_j x_j(k) - C_j y_j(k - \tau_{ij})] - \sigma_i(C_j x_j(k) - P_{Y_j}(C_j x_j(k)))\). We define \(\phi_i(k) = \left[\begin{array}{c} x_i^T(k) \\ v_i^T(k) \end{array}\right]\), then, the DG system (25)-(26) with the corresponding controllers (28)-(32) can be written as the following closed-loop system form:

\[
\phi_i(k + 1) = A_i \phi_i(k) + \sum_{j \in N_i} a_{ij} A_j \phi_j(k - \tau_{ij}) + B_i P_{Y_j}(C_i \phi_i(k)) + D_i d_i(k)
\]

where the matrix parameters are given in Appendix. Then, we have the following Theorem.

**Theorem 1:** Assume that the communication graph \(G\) is connected and there exists at least one DG that can achieve the leader's reference information. Under Assumption 1, the DG agent system (25)-(27) of the S/T-MG can solve the containment active/reactive power sharing (8)-(9), voltage regulation (10) and power quality improvement (11) control problems by using the proposed distributed controllers (28)-(32) while keeping ESSs constraints (12)-(13) satisfied.

**Proof:** The proof is presented in Appendix.

D. Tuning of Controller Parameters

1) Tuning of P-VSG Parameters: In the primary P-VSG controller (15)-(17), the time constant of the frequency loop \(\tau_f\) and the time constant of the field excitation loop \(\tau_e\) are, respectively, given as \(\tau_f = \frac{M_i}{\omega J\tau_{D,\nu}}\) and \(\tau_e = \frac{K_i}{\omega^2 J\tau_{D,\nu}}\) [30]. Hence, the virtual inertia \(M_i\) and the gain \(K_i\) can be determined by \(M_i = D_{q,i} \tau_f\) and \(K_i = D_{q,i} \tau_e\), if we have decided the constant \(\tau_f\) that can be chosen similar or much smaller compared to the case of a physical synchronous generator and the constant \(\tau_e\) that is often chosen much larger than \(\tau_f\) [36]. Initially, \(D_{q,i}\) can be chosen such that a frequency drop of 0.5% causes the torque to increase by 100% from its nominal value, \(D_{q,i}\) can be chosen such that a voltage drop of 5% causes the reactive power to increase by 100% [37]. However, these parameters may not be directly used or optimized values, and a specific method presented in [38] could be used to improve the parameter tuning.

2) Tuning of Secondary Controller Parameters: For the secondary controllers, two parameters, i.e., \(p_i\) and \(c\), need to be determined. According to the literature [32], the controller parameter \(p_i\) could be chosen to satisfy \(0 < b_i(k) \leq p_i(k + 1) < \frac{1}{T} \) for all \(k \geq 0\), where \(b_i(k)\) is defined as \(b_i(k) = \frac{1 - \beta_i(k - 1) - p_i(k)T}{T}\). With regard to the pinning gain \(c\), it is set as \(c = 1\) for all the agents when they can receive the information directly from the leaders.

V. SIMULATION RESULTS

To validate the performance of the proposed control scheme for the S/T-MG under various conditions, the S/T-MG depicted in Fig. 1 is simulated in MATLAB/Simulink environment. The solver used is ode23tb with a relative tolerance of \(10^{-3}\), and the system sampling time is 0.1 ms. In Fig. 1, two TDGs, three SDGs connected to phase-A, phase-B and phase-C, and unbalanced loads are considered. A three H-bridge converter based three-phase four-wire DC-AC inverter with LC filters is adopted to interface the TDGs while a single-phase H-bridge inverter is used for the SDGs. The ratios of power ratings of DG units are considered as T-DG1:TDG2:SDG1:SDG2:SDG3 = 2:2:1:1:1. Parameters of the test microgrid system and the controllers are listed in Table I. The time constants of frequency and voltage loops are chosen to be \(\tau_f = 0.002\) s and \(\tau_e = 0.02\) s. For simplicity, the secondary controller parameter \(p_i\) is taken as \(p_i = 10\) for all the agents. The agents’ communication topology is shown in Fig. 4 in detail, in which each phase of the DG units including TDGs and SDGs has a dedicated control agent. These agents of the TDGs and SDGs that have the same phases are connected together forming a ring-shape communication topology for data exchange. In order to further validate the proposal, a five-busbar S/T-MG depicted in Fig. 5 is simulated in the last case, where balanced loads (L1 and L2) are connected at the bus of B1 and B2, respectively. And the loads La and Lc are, respectively, connected at Ba and Bc. There is no load connected at Bb.

A. Case 1: Performance under Load Change and PnP

This case aims at validating the performance of proposed control approach under load change and plug-and-play (PnP) by comparing with the conventional method presented in [3] and [30]. At the beginning, the system is experiencing a balanced active and reactive power load demand. But the supply
is unbalanced, i.e., TDG1, TDG2 and SDGa provide power for the load together. And then, at \( t = 1.5s \), the loads connected to phase-A and phase-B are increased. Consequently, the loads are also unbalanced. Finally, at \( t = 5.5s \) and \( t = 9s \), the DG units, SDGb and SDGc are, respectively, plugged into the microgrid, providing power supply for the system load demand together with TDG1, TDG2 and SDGa. The details are as follows.

1) **Accurate Power Sharing Among DG Units (Conventional Approach):** Fig. 6(a), Fig. 7(a) and Fig. 8(a) show the performance of the conventional VSG control method in [30] and the distributed method in [3], where the steady state values of active and reactive powers are shown in Fig. 10. From \( t = 3 s \), the conventional distributed method [3] designed for accurate power sharing among DGs (SDGs and TDGs) is activated. From the results, it can be obviously observed that proportional power sharing cannot be realized only using conventional VSG method due to line impedance, and that although the active and reactive power is proportionally shared among these DGs (The power sharing ratio is about 2:1:0:1:0:1.0 (TDG1:TDG2:SDGa:SDGb:SDGc)), the admissible voltage profile cannot be guaranteed, some voltages (SDGa, SDGb and SDGc) are beyond the boundary (300V-320V, 3% of rated voltage) during the time of \( t = 3 - 5.5s, t = 5.5 - 9s \) and \( t = 9 - 12s \), respectively. More over, the discharging powers of SDGb and SDGc (\( P_{SDG_b}^{max} = 2.912 kW, P_{SDG_c}^{max} = 2.596 kW \)) are larger than maximum permitted value 2.5 kW, which could be harmful to the operation of ESS system. Additionally, the VUF at the PCC is higher than 2% during \( t = 3 - 5.5s \) and \( t = 9 - 12s \), which may not be acceptable according to the IEEE 1547 standard [27]. The results shown in Fig. 8(a) demonstrate that PhP, stable frequency response, an accurate phase shift of 120° of the output voltage can be achieved with conventional method. The maximum phase difference (The definition of the index, phase difference, is the same as that proposed in [11].) at steady state is about 0.36°. 

2) **Control Performance with Proposed Approach:** The proposed control approach is activated in this test where load condition is the same as that of the above test, and the results are presented in Fig. 6(b), Fig. 7(b) and Fig. 8(b), where the steady state values of active and reactive powers are shown in Fig. 10. Before the time of \( t = 3 s \), only the proposed P-VSG method is implemented. It can be observed that the control performance of the proposed method is comparable to that of the traditional method, which validates the effectiveness of the P-VSG. After \( t = 3 s \), the proposed distributed containment controllers (28), (29), (30), (31) and (32) are activated for active and reactive power sharing, voltage

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Nominal voltage</td>
<td>( E^* )</td>
<td>311 V</td>
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</tr>
<tr>
<td>Nominal frequency</td>
<td>( \omega^* )</td>
<td>( 2\pi \times 50 )</td>
<td>rad</td>
</tr>
<tr>
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<tr>
<td>DC capacitor</td>
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<td>Filter inductance</td>
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</tr>
<tr>
<td>Filter capacitor</td>
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<td>( \mu F )</td>
</tr>
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<td>Power rating ( P_{T^1D^1max} )</td>
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</tr>
<tr>
<td>Power rating ( P_{SDG_b}^{max} )</td>
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</tr>
<tr>
<td>Power rating ( P_{SDG_c}^{max} )</td>
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</tr>
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<td>( Z_{line,a} )</td>
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<td>( \Omega )</td>
</tr>
<tr>
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<td>Droop coefficient ( D_{q,i} )</td>
<td>( D_{q,i} )</td>
<td>6( ^1 )</td>
<td>kVar/V</td>
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<td>Sampling time ( T )</td>
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<td>Feedback damping gains ( p_i )</td>
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<tr>
<td>Pinning gains ( c )</td>
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<td>-</td>
</tr>
</tbody>
</table>

\( ^1 \) TDGs; \(^1\) SDGs.
regulation and VUF control, respectively. Compared to the conventional method, some significant improvements can be easily observed once the proposed method is implemented. 

i) Although the accuracy of power sharing among DGs is compromised, the balancing performance of the TDGs’ output three-phase power is improved, which can also be observed from Fig. 6(b), Fig. 7(b), Fig. 9 and Fig. 10. 

ii) Moreover, the output powers of each phase of TDGs and SDGs are more stable and resilient, which means that load change of one phase has little effect on the output powers of other phases (see Fig. 9) since DGs are inclined to share the load power according to the power ratings of each phase but not the ratings of DGs. 

iii) The voltages and the corresponding VUFs (<2%) are all regulated to fulfill the admissible requirements of the standard. The neutral current of TDGs is also reduced compared to that with conventional method (see Fig. 8). 

iv) The charging/discharging power of the ESSs ($P_{\text{ess},b}$) is controlled to less than the maximum permitted value (2.5 kW) with the help of constraint operator. 

v) Seen from Fig. 8(b), an accurate phase shift of 120° of the output voltage of the three-phase converters is approximately achieved with the proposed P-VSG method. The maximum phase difference at steady state is about 0.35°. This validates the effectiveness of the proposed approach and as well as PnP capability. Note that some differences about the behaviour of phase shift could be observed compared to those obtained by using the conventional method, which is mainly caused by the differences from the primary and secondary controllers. The proposed secondary controllers along with the primary P-VSG controller will make \( \Delta \omega_{b}^{T} \) different from that obtained from the conventional method, which means that rate of change of phase shift is different and so is the behavior of phase shift.

**B. Case 2: Communication Time-Delay and Link Failure**

In this case, the effects of the communication time-delays are considered due to its ubiquitous existence in practical engineering applications and possible deterioration and instability of the system operation. The time-delay is set to be 20 ms, 40 ms, and 200 ms, respectively. Fig. 11 (a), (b) and (c) show the responses of DGs within the S/T-MG when the proposed controllers are applied at $t = 2$ s. It can be seen that the TDGs and SDGs can properly share the load power (see Fig. 11(a) and (b)) for the case of $\tau = 20$ ms and $\tau = 40$ ms. The system can converge to the steady state. But the longer the time delay, the worse the control performance. The control performance of output power and voltages of SDGs is much more seriously deteriorated. For the case of $\tau = 200$ ms, neither the admissible voltages nor the admissible VUF can be achieved (Fig. 11(c)). The convergence rate is very slow. It will take much longer...
time for the system to converge to the steady state. But it still can be stabilized, which could satisfy the requirement of 100 ms time delay as an admissible value for control information in microgrids according to the IEC 61850 standard [39].

The scenario of communication link failures is studied in this case. A ring-shape topology is adopted for the communication of agents shown in Fig. 4, where the link failures are set as \( t_1 = 3 \) s, \( t_2 = 3.5 \) s, \( t_3 = 5 \) s, and \( t_4 = 6.5 \) s. Finally, at \( t = 8 \) s, the failed links are retrieved. Additional single-phase load is connected to phase-\(a\) and phase-\(c\) at \( t = 3.5 \) s and \( t = 5.5 \) s, respectively, and disconnected at \( t = 7 \) s. From the results shown in Fig. 12, the connectivity of the communication is maintained before \( t_3 = 6.5 \) s, so DGs can still share the load power as expected. Although connectivity in the graph is lost during \( 6.5 \) s to \( 8 \) s, the output powers of DGs are maintained unchanged until the load is changed at \( t = 7 \) s due to the application of integrator. When the connectivity of the communication graph is established again after \( t = 8 \) s, the system can be brought back to the normal operation state.

**C. Case 3: The Scenario of All SDGs Connected to Phase-\(a\)**

In this case, we consider a special scenario that all the SDGs are connected to phase-\(a\). Two TDGs and two SDGs connected to phase-\(a\) are considered in the microgrid (see Fig. 5). During the time \( t = 1 \sim 3.5 \) s, conventional distributed method is utilized for active power sharing while during the time \( t = 3.5 \) s to \( t = 8 \) s, the proposed distributed method for active power sharing is utilized. Finally, after \( t = 8 \) s, the conventional method is used again. At time \( t = 6 \) s, the generation unit, TDG1, is plugged out. From the results shown in Fig. 13, it can be observed that during the time \( t = 3.5 \sim 6 \) s, the unbalance is obviously improved by using the proposed approach compared to the response before the time \( t = 3.5 \) s by using the conventional approach. Moreover, the proposed approach doesn’t result in reverse power flow on phase-\(a\) of TDG2 when TDG1 is plugged out at \( t = 6 \) s. However, the conventional method results in reverse power flow on this phase, which may cause adverse effects on the system.
Fig. 8. Simulation results under case 1. (a) Results with conventional method. (b) Results with proposed method.

Fig. 11. System performance under different time-delays in case 2. (a) 20 ms time-delay. (b) 40 ms time-delay. (c) 200 ms time-delay.
could be interpreted as that generation units on each phase share the load power based on their phase ratings instead of their total ratings. Using this way, phase-a of the three phase inverters and all the single phase inverters connected to this phase will share the load power on this phase based on their phase ratings. Therefore, it will create a positive output power of phase-a of the three phase inverter but not reverse power flow on it. This is different from the conventional method.

**D. Case 4: Performance Validation in A Five-Busbar MG**

A five-busbar S/T-MG depicted in Fig. 5 is tested in this case. Before the time $t = 4.5$ s, the conventional VSG method and the distributed power sharing method (activated at $t = 1.5$ s) are used in the simulation while the proposed P-VSG method and distributed secondary control method are utilized after $t = 4.5$ s. The simulation results are shown in Fig. 14. From the results, it can be observed that, when the conventional approach is implemented, accurate active and reactive power sharing is asymptotically achieved among DGs based on their ratings. The output active and reactive power of TDGs (about 8.58 kW, 7.01 kVar) is almost twice that of SDGs (about 4.28 kW, 3.46 kVar). However, the voltage magnitude at some buses is beyond the accepted range. Some voltages are higher than the upper limit (320 V) while some are below the lower limit (300 V). Additionally, the VUF at B1 and B2 (about 2.56%) fails to fulfill the requirement of the IEEE 1547 standard. These problems are solved by utilizing the proposed approach after $t = 4.5$ s. It can be observed that, although the active and reactive power cannot accurately be shared among DGs like that of using conventional approach, each phase of the generation units (TDGs and SDGs) can properly share the load power on this phase based on the phase ratings, and the voltages and VUF are also regulated to fulfill the power quality requirements. Compared to the conventional approach, the neutral current of TDG1 (TDG2 is not presented due to page limits.) and the VUF are obviously reduced. VUF at B1 and B2 is reduced by about 31.5% (from 2.56% to 1.75%). At the same time, the accurate phase shift of 120° of the output voltage of the three-phase converters is achieved with the proposed P-VSG method, which is even better than that of the conventional method. The maximum

![Fig. 13. System performance under case 3.](image-url)
phase difference at steady state is about 0.00142 (0.17°), which is reduced by approximately 34.6% compared to that of using conventional VSG method (about 0.00215 (0.26°)). This validates the effectiveness of the proposed approach.

Finally, we consider the performance of the proposed control system under the scenario that ESS is full charged and not available. We assume that the total load is less than the source power. Specifically, the ESS in SDGa cannot continue to discharge and is disconnected to prevent damage at \( t = 4 \) s since the SoC has reached to its minimum value. At \( t = 5.5 \) s, the ESS in TDG1 is full charged and cannot absorb the energy anymore. The simulation results are shown in Fig. 15, from which it can be observed that the output active power of SDGa is decreased since its ESS is not available. This power deficiency is supplied by TDG1 and TDG2. Consequently, the output power of TDG1 and TDG2 is increased. After \( t = 5.5 \) s, TDG1 outputs more active power since its ESS has been full charged and the wind power is larger than the output power before \( t = 5.5 \) s. But the most important thing is that although the output powers of DGs have changed due to the charge and discharge events, the output power can still be kept within the permitted ranges with the proposed approach. We just present the active power results and other results omitted due to the limited space.

VI. CONCLUSION

This paper investigates the power sharing and power quality improvement issues of islanded S/T-MGs with both unbalanced sources and loads and as well as the hybrid RESs/ESSs sources. The proposed control approach includes 1) a P-VSG control used for primary control of DGs, 2) four distributed secondary containment controllers with communication delays used for power sharing among DGs and among phases, voltage restoration and power quality improvement. The proposed P-VSG control allows for independent and flexible power and voltage control for each phase and accurate phase shifts. The distributed secondary controllers based on containment control and constraint operator guarantee admissible output phase powers, voltage profiles, VUF, and permitted values of charging/discharging power of the ESSs, resulting in better operation security and reliability. Simulation results verify the proposed approach and show that unbalance of three-phase power of TDGs is obviously improved and that each phase power is more stable and resilient to load changes.

However, in a real microgrid, different inverters may be manufactured by different manufacturers and these inverters could have different topologies. Therefore, their control methods and operation modes may be different. Some of them may be VSG controlled, some of them may be droop controlled, some of them may be P-VSG controlled, and others may be MPPT controlled. Some DGs may be grid-forming and some others may be grid-following. On the other hand, ESSs are considered and placed at the same power point.
electronics converter in this work. However, ESS may be not available in some areas and some DG units could not have ESS in reality. Information of these DG units could be used to optimize and generate proper leader information for the control system to control the DG units that have available ESS to regulate their outputs to realize the control goal. Moreover, some DGs, like PV and wind power generation units, are not dispatchable. They usually operate in MPPT mode. These factors could affect the performance and practical implementation of the proposed approach. In the future research, we will be committed to studying the issues mentioned above and the practical applications of the proposed approach in actual controllers, and providing hardware implementation and experimental results using the microgrid test system that is being built now to further investigate the performance of this approach and guarantee the implementation feasibility.

APPENDIX

A. Proof of Theorem 1

Proof: Consider the Lyapunov function

$$V(k) = \max_{i,m \leq \tau_{max}} \{ ||x_i(k-m) - P_Y(x_i(k-m))|| \} \quad (41)$$

From (40) and use similar analysis method in [32], we have

$$||x_i(k+1) - P_Y(x_i(k+1))|| \leq (1 - b_i T) V(k), \quad (42)$$

where $0 < (1 - b_i T) = \beta_i (1 - p_i T) < 1$. Thus, $V(k+1) \leq V(k)$, which means that $V(k)$ is nonincreasing with respect time $k$. Recall that $V(k) \geq 0$, hence, the limit of $V(k)$ exists.

Suppose that at time $kT$, there exist two set sequences

$$\{ F_{P(z)} \} : z = k, k+1, k+2, \ldots \}$$

and

$$\{ D_{D(z)} : z = k, k+1, k+2, \ldots \}$$

such that for all $z \geq k$, $F_{P(z)} \cup D_{D(z)} = \mathcal{F}$, $F_{P(z)} \cap D_{D(z)} = \emptyset$, $x_p(z) - P_Y(x_p(z)) = V(k)$ holds for follower $p \in F_{P(z)}$, and $||x_d(z) - P_Y(x_d(z))|| \leq (1 - \delta)V(k)$ holds for follower $d \in D_{D(z)}$ where $0 < \delta < 1$ is a constant. By using similar analysis, we have that $||x_i(k+C) - P_Y(x_i(k+C))|| < V(k)$ holds for all $C \in \mathbb{Z}_+$ which means that the agent number in $F_{P(k+1)}$ is no more than that in $F_{P(k)}$. Further, the agent number in $F_{P(k+\tau)}$ will be zero.

Based on the assumption that there must exist some followers $i_b \in F$ such that $b_i \geq \mu > 0,$

$$||x_{i_b}(k+1) - P_Y(x_{i_b}(k+1))|| \leq (1 - \mu T) V(k).$$

If a follower $i \in F$ can receive information from a follower $j_b$ at time $(k + 1)T$, we have

$$||x_i(k+2 + \tau_{j_b}) - P_Y(x_i(k+2 + \tau_{j_b}))|| \leq (1 - \mu T^2) V(k).$$

By iterations, we further have

$$||x_i(k+k \tau_{j_b} + \tau_{j_b}) - P_Y(x_i(k+k \tau_{j_b} + \tau_{j_b}))|| \leq (1 - \delta_i^k) V(k).$$

Clearly, all other followers will receive the information from $j_b$, directly or indirectly in finite time. Since there are at most $n$ following DGs, we can conclude that there exists a finite positive integer $N > 0$ such that

$$\max_{i \in \mathcal{F}} ||x_i(k+N) - P_Y(x_i(k+N))|| \leq (1 - \delta_N) V(k)$$

where $0 < \delta_N < 1$ is a constant. By similar analysis, there exists $0 < \delta_{\mathcal{F}} < 1$ such that $V(k+N+\tau_{max}) \leq (1 - \delta_{\mathcal{F}}) V(k)$. It follows that $\lim_{k \to +\infty} V(k) = 0$ and hence, $\lim_{k \to +\infty} ||x_i(k) - P_Y(x_i(k))|| = 0$. Therefore, proper power sharing among DGs, admissible voltage regulation, VUF and ESSs constraints can be realized. The proof is completed.

B. Matrix Parameters in (25) and (26)

$$\bar{A}_i = \begin{bmatrix} \tau_{i}^{-1} P_{\text{max},i,b} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{B}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & M_{i-1} & 0 & 0 & 0 \\ 0 & 0 & K_{i-1} & D_{i,q}K_{i-1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{D}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} - \frac{Q_{\text{max},i,b}}{\tau_{i}},$$

$$A_i = \begin{bmatrix} e^{\mathcal{A}_i T} & 0 \\ 0 & 1 - \gamma_i \end{bmatrix},$$

$$B_i = \begin{bmatrix} \int_0^T e^{\mathcal{A}_i t} d\bar{B}_i \\ \int_0^{\min(T, T_{\text{comm}})} e^{\mathcal{A}_i t} d\bar{B}_i \end{bmatrix},$$

$$C_i = \begin{bmatrix} k_{p,i} & 0 & 0 & 0 & 0 \\ 0 & k_{q,i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{n_{\text{comm}}}{\tau_{i}} & 0 & 0 & 0 \end{bmatrix},$$

$$F_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\sigma_{i} = \left( \sum_{j \in \mathcal{N}_i} a_{ij} + c \right),$$

$$\bar{B}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{C}_i = \begin{bmatrix} C_i & O \end{bmatrix},$$

$$\mathcal{D}_i = \begin{bmatrix} D & O \end{bmatrix}.$$
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