Low-Frequency Oscillation Analysis of VSMs-Based VSC-HVDC Systems Based on the Five-Dimension Impedance Stability Criterion

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Abstract—Virtual Synchronous Machines (VSMs)-based high voltage DC (HVDC) systems enhance the inertia of the power system. However, the dynamic interactions between the VSM-based rectifier station, the inverter station, and the grid could induce the system oscillation, which has been investigated in this paper. At first, the hybrid AC/DC impedance models of the VSMs considering the coupling between the AC and DC dynamics are established. Then, the relationships between the DC impedance, dq-frame impedance, and the hybrid AC/DC impedance are presented. It is found that the DC impedance of the VSM-based inverter station and the d-d channel impedance of the dq-frame impedance of the rectifier station behave as negative resistors in the low-frequency range. Moreover, a five-dimension impedance stability criterion based on the hybrid AC/DC impedance and generalized inverse Nyquist criterion is proposed to assess the system stability. The analysis results show that the low-frequency oscillation occurs when the grid SCR of the rectifier station is small. Finally, the simulation and experimental results verify the impedance models and the stability criterion.

Index Terms—Impedance stability criterion; Virtual synchronous machine; VSC-HVDC system; Hybrid AC/DC impedance model

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{abc}$</td>
<td>Output current of the HVDC inverter</td>
</tr>
<tr>
<td>$e_{abc}$</td>
<td>Internal electric potential of VSM</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Cutoff angular frequency of LPF of current sampling (voltage sampling)</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Fundamental (switch) frequency</td>
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<tr>
<td>$T_d$</td>
<td>Control delay of VSM</td>
</tr>
<tr>
<td>$J$</td>
<td>Virtual inertia of VSM</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Reactive (Active) damping coefficient of VSM</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Inertia gain of the reactive power loop of VSM</td>
</tr>
<tr>
<td>$L_v$</td>
<td>Virtual inductor (resistor) of VSM</td>
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<td>$\theta$</td>
<td>Phase angle of the VSM</td>
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<td>$k_p$</td>
<td>Proportional gain of voltage controllers of VSM</td>
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<td>$k_i$</td>
<td>Integral gain of voltage controllers of VSM</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Rated angular frequency of the grid</td>
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<tr>
<td>$\omega_v$</td>
<td>Angular frequency of the VSM</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Cutoff frequency of PR controller</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Phase difference between the VSM and the PCC</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Steady-state value of the $\delta$</td>
</tr>
<tr>
<td>$D_{s0}$</td>
<td>Steady-state values of duty ratio</td>
</tr>
<tr>
<td>$U_{s0}$</td>
<td>Steady-state values of voltages</td>
</tr>
<tr>
<td>$I_{s0}$</td>
<td>Steady-state values of currents</td>
</tr>
<tr>
<td>$U_{dc0}$</td>
<td>Steady-state value of dc-link voltage of the VSM</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Output voltage magnitude of VSM</td>
</tr>
<tr>
<td>$u_m$</td>
<td>Voltage magnitude of the PCC</td>
</tr>
<tr>
<td>SCRi (SCRr)</td>
<td>Short circuit ratio at the inverter (rectifier) side</td>
</tr>
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</table>

I. INTRODUCTION

VOLTAGE source converter (VSC)-based high voltage DC (HVDC) systems have been generally utilized as transmission media to transfer power over a long distance [1]-
[2]. Due to the lack of inertia, the application of large quantities of the VSC-HVDC links changes the characteristics and threatens the stable operation of the power grid dominated by synchronous generators (SGs). Thus, the virtual synchronous machine (VSM)-based HVDC [3]-[6] system, which enables the rectifier station and the inverter station to simultaneously simulate the dynamic characteristics of synchronous generator, has been generally studied to provide the inertia and damping for the power system. It can be connected to the power grid friendly. However, the interactions between the VSM-based rectifier station, the inverter station, and the grid could cause the system oscillation, which needs to be further explored.

There are several tools widely used for the small-signal stability analysis of the HVDC system: the damping torque analysis method [6]-[7], the time-domain state-space method [8], and the impedance-based method [9]-[26]. The first and second methods need full knowledge of the hardware and control design of the converter, which are difficult to obtain due to commercial secrecy. By contrast, the impedance that can be measured is more suitable for stability analysis.

The impedance-based analysis methods can be divided into three categories: the DC impedance-based method [9]-[14], the AC impedance-based method [16]-[10], and the hybrid AC/DC impedance-based method [21]-[23]. Their common principle is to divide the system into two independent subsystems according to the source and load parts, and then apply the Nyquist stability criterion to the impedance ratio of the two subsystems. The DC impedance-based method is proposed in [9] for the first time to investigate the interactions of DC systems. Then, it is further utilized in the VSC-HVDC system [10]-[13]. Afterward, the detailed DC impedances of the VSC-HVDC system are developed with considering the control modes, and the phase-locked loop (PLL) [12], [14], etc. However, to the author’s knowledge, the DC impedance modeling of the VSM-HVDC system has not been reported.

Originated from DC impedances, the AC impedance-based method has been extensively researched for power electronic converters-based systems. They are systematically classified based on the different frames, including the sequence impedance [15], the dq-frame impedance [16], the αβ-frame, etc. The dq-frame impedance [17] and the sequence impedance [18] are the most widely used for the VSC-HVDC system. They are essentially the same after considering the frequency coupling. For example, based on sequence impedances, studies in [18] show that the VSM without inner loops behaves inductive and can run stably in a weak grid. Based on dq-frame impedances, the subsynchronous oscillation of the VSM with the DC voltage loop and inner loops is studied in [19]. However, the DC dynamics effects exerting on AC impedances remain unknown when the VSC-HVDC system operates as VSMs.

The above stability analyses ignore the coupling between the AC and DC dynamics, and the stability analysis of the VSC-HVDC system by using the DC or AC impedance is difficult to deal with the right half-plane (RHP) poles. To solve these issues, the hybrid AC/DC impedance is first proposed in [21], and the promising stability criteria of the VSC-HVDC system based on the hybrid AC/DC impedance are proposed in [21]-[24]. A stability analysis method of the modular multilevel converter (MMC)-HVDC systems for the wind farm integration based on the three-by-three ratio matrix is proposed in [22], but the RHP poles of the DC impedance of grid-side MMC is still difficult to deal with. Thus, a six-dimension stability criterion is proposed in [23] to solve the issues of the RHP poles. However, the calculation of the six characteristic loci is quite complicated. Besides, the hybrid AC/DC impedance model in [23] contains no DC-link capacitor filters. If the DC-link capacitor filters are considered in the modeling, the stability criterion is unavailable because (47) in [23] has no inverse transformation.

To address the aforementioned issues, this paper investigates the stability of the VSM-based HVDC system by using the hybrid AC/DC impedances and the proposed five-dimension impedance stability criterion. The main contributions are:
1) The hybrid AC/DC impedance models of the VSM-based rectifier station and inverter station are established and verified.
2) The DC and dq-frame impedance models of VSMs are established based on the hybrid AC/DC impedance models, and their impedance characteristics are compared.
3) The five-dimension impedance stability criterion based on the hybrid AC/DC impedance models and the generalized inverse Nyquist criterion (GINC) is proposed to analyze the stability of the VSM-based HVDC systems, and the low-frequency oscillation of the system is revealed.

The rest of the paper is organized as follows: Section II presents the description of the HVDC system based on VSMs. Section III establishes the hybrid AC/DC impedance models of the VSMs. Section IV presents the transformation between the hybrid AC/DC impedance, dq-frame impedance, and DC impedance. Section V contains the proposed five-dimension impedance stability criterion, the stability analysis, and the experimental results. Section VI presents the brief conclusions.

II. SYSTEM DESCRIPTION

Figure 1(a) illustrates the VSMs-based VSC-HVDC systems, where variables with an asterisk (*) correspond to reference signals. The system parameters of the rectifier station and inverter station are the same, as shown in Table I of section IV. Since this paper focuses on the control method effects exerting on the system stability, a two-level VSC-HVDC system is investigated. One key feature of VSMs is that the rotor swing equation is used for the synchronization and inertia emulation so that the converter can share similar dynamics to SGs [6]. The reactive power controller simulates primary voltage regulations of synchronous machines, and the active power controller emulates the inertia and primary frequency regulation of synchronous machines:

\[ a_n - a_s = \frac{p^* - p}{(J_s + D_p)a_n} \]  

\[ E_m = (D_q(u_{q}^* - u_{q}) + Q^* - Q)/(K_s) \]

The ac voltage controllers of the VSM adopt the virtual impedance to emulate the electrical part of synchronous machines [27], and \( L_v \) and \( R_v \) are the virtual resistor and inductor, respectively. Besides, the current controllers adopt PR regulators in the αβ frame.

The only difference between the VSM rectifier and inverter is the dc-link voltage controller:

\[ P^* = -(u_{dc}^* - u_{dc})k_{pu} + k_{pu}^*/s \]

The output voltage magnitude \( (u_m) \), the active power \( (P) \), and
the reactive power ($Q$) of the VSM are expressed as follows:

$$
\begin{align*}
\Delta u_{\alpha} &= \frac{1}{2}(u_{m}^2 + u_{p}^2) \\
P &= \frac{3}{2}(u_{\alpha}u_{\alpha} + u_{\beta}u_{\beta}) \\
Q &= \frac{3}{2}(u_{\alpha}u_{\beta} - u_{\beta}u_{\alpha})
\end{align*}
$$

Figure 1(b) illustrates the equivalent circuit model of the VSC-HVDC system, where $D$ impedances of the rectifier and inverter ($Z_{\text{drec}}$, $Z_{\text{dinv}}$), dq-frame impedances of the rectifier and inverter ($Z_{\text{drec}}$, $Z_{\text{dinv}}$), and the hybrid AC/DC impedances of the rectifier and inverter ($Z_{\text{vsm-rec}}$, $Z_{\text{vsm-inv}}$) will be used later.

## III. HYBRID AC/DC IMPEDANCE MODELING OF THE VSMs

### A. Small-Signal Model of Main Circuit

The main circuit of the converter as shown in Fig. 1(a) can be modeled as follows:

$$
\begin{align*}
L_{d}\frac{d i_{d}}{dt} &= \omega_{a}L_{d}i_{q} + i_{d}R_{q} = d_{u}u_{dc} / 2 - u_{d} \\
L_{q}\frac{d i_{q}}{dt} &= \omega_{a}L_{q}i_{d} + i_{q}R_{u} = d_{u}u_{dc} / 2 - u_{q} \\
C_{dc}\frac{du_{dc}}{dt} &= -\frac{3}{4}(d_{d}i_{d} + d_{q}i_{q}) - i_{dc}
\end{align*}
$$

The small-signal model of (5) can be expressed as follows:

$$
\begin{bmatrix}
D_{d0} & \Delta u_{dc} \\
D_{q0} & \Delta u_{dc}
\end{bmatrix} = \frac{1}{2}
\begin{bmatrix}
\Delta i_{d} \\
\Delta i_{q}
\end{bmatrix}
$$

where “\Delta” denotes the small-signal perturbation of a variable.

Finally, the small-signal model of the main circuit according to (7) and (8) is expressed as follows:

$$
\begin{bmatrix}
\Delta i_{d} \\
\Delta i_{q} \\
\Delta i_{dc}
\end{bmatrix} = F_{1}^{-1} \begin{bmatrix}
\Delta u_{d} \\
\Delta u_{q} \\
\Delta u_{dc}
\end{bmatrix}
$$

where $F_{1}$ and $F_{2}$ are represented as follows:

$$
F_{1} = \begin{bmatrix} G_{1} & G_{3} \\
3D_{d0} & 3D_{q0} \end{bmatrix}
$$

$$
F_{2} = \begin{bmatrix} G_{2} & 0 \\
-3I_{d0} & -3I_{q0} \end{bmatrix}
$$

### B. Hybrid AC/DC Impedance Modeling of the VSM

The controllers, the control delay, and voltage and current sampling filters, which are all in static coordinates, need to be...
transformed to the synchronous frame. Assuming that the transfer function in static coordinates is \(H(s)\), the expression converted to a \(dq\) frame given as follows [29]:

\[
\begin{bmatrix}
H_{dq} &=& \frac{1}{j} \begin{bmatrix}
A + B & -j(A - B)
\end{bmatrix}
\end{bmatrix}
\]

\[
A = H_1(s + j\omega_n), \quad B = H_1(s - j\omega_n)
\]

The three by three matrix of the control delay based on (13) and the Euler formula is obtained as follows:

\[
G_{d\delta} = e^{-T_s s} \begin{bmatrix}
\cos(\omega_n T_s) & \sin(\omega_n T_s) & 0 \\
-sin(\omega_n T_s) & \cos(\omega_n T_s) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Similarly, the first-order low-pass filters for voltage or current signals are expressed as follows:

\[
K_x = \begin{bmatrix}
1 + T_s s & 0 & 0 \\
\omega_n T_s & 1 + T_s s & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Besides, the current controller can be expressed as follows:

\[
\begin{align*}
G_{lc} &= \frac{1}{C} \begin{bmatrix}
C_1 & C_2 & 0 \\
0 & C_1 & 0 \\
0 & 0 & 0
\end{bmatrix} \\
C_1 &= k_p + 2k_w (2s^3 + 4\omega_n s^2 + 4\omega_n s + 4\omega_n^2) \\
C_2 &= 2k_w (2s^3 + 4\omega_n s^2 + 4\omega_n s + 4\omega_n^2)
\end{align*}
\]

In addition, the equivalent three-by-three matrix of the voltage controller is derived as follows:

\[
G_{vc} = \begin{bmatrix}
0 & 0 & Z_n^{-1} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

where \(Z_n\) is the \(dq\) impedance of the virtual inductor.

As shown in Fig. 2, \(G_{dc}\) representing the dc-link voltage controller is denoted by a three-by-three transfer matrix:

\[
G_{dc} = \begin{bmatrix}
0 & 0 & \frac{0}{s} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

reactive power controllers:

\[
\begin{align*}
\Delta\theta &= \begin{bmatrix}
\frac{1}{(Js^2 + D_p\omega_n^2)} & 0 \\
0 & \frac{1}{K_s}
\end{bmatrix} \begin{bmatrix}
\Delta\phi \Delta\phi_m \\
\Delta\phi_m
\end{bmatrix}
\end{align*}
\]

Then, \(M_{dq}\) and \(M\) are defined as follows:

\[
M_{dq} = \begin{bmatrix}
1/(J\omega_n^2 + D_p\omega_n^2) & 0 \\
0 & 0
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Adding the \(dq\)-frame small-signal disturbances to the instantaneous active and reactive power calculation, \(F_1\) and \(F_u\) are defined as follows:

\[
\begin{align*}
F_1 &= \frac{3}{2} \begin{bmatrix}
U_{d0} & U_{q0} \\
0 & 0
\end{bmatrix} \\
F_u &= \frac{3}{2} \begin{bmatrix}
I_{d0} & I_{q0} \\
0 & 0
\end{bmatrix}
\end{align*}
\]

The \(dq\)-frame command voltage is expressed as follows:

\[
\begin{bmatrix}
e_d \\
e_q
\end{bmatrix} = \begin{bmatrix}
\cos(\delta) & -\sin(\delta) \\
\sin(\delta) & \cos(\delta)
\end{bmatrix} F_{em}
\]

Add the \(dq\)-frame small-signal disturbances to (24) and eliminate the steady-state value and the secondary disturbance:

\[
\begin{bmatrix}
\Delta\phi_d \\
\Delta\phi_q \\
\Delta\phi_m
\end{bmatrix} = \begin{bmatrix}
-E_m \sin(\delta_0) & \cos(\delta_0) & \Delta\phi_m \\
E_m \cos(\delta_0) & \sin(\delta_0) & \Delta\phi_m \\
0 & 0 & 0
\end{bmatrix}
\]

Then, \(T_1\) is defined as follows:

\[
T_1 = \begin{bmatrix}
-E_m \sin(\delta_0) & \cos(\delta_0) & 0 \\
E_m \cos(\delta_0) & \sin(\delta_0) & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The relationship between the input voltage and the output current can be expressed as follows:

\[
\begin{bmatrix}
\Delta u_d \\
\Delta u_q \\
\Delta u_{dc}
\end{bmatrix} = -Z \begin{bmatrix}
\Delta i_d \\
\Delta i_q \\
\Delta i_{dc}
\end{bmatrix}
\]

Fig. 2 The wideband \(dq\)-frame small-signal model of the VSM.
Solving the equations represented by Fig. 2, the hybrid AC/DC impedance of the VSM rectifier is derived as follows:

\[
Z_{\text{vsm-rec}} = \left( F_2 G_{\text{dc}} G_{\text{lc}} G_{\text{uc}} \left( I + T_1 \left( M + F_{\text{pq}} (F_u + G_{\text{dc}}) \right) \right) K_x + I \right)^{-1} \cdot \left( F_1 + F_2 G_{\text{dc}} G_{\text{lc}} \left( I + G_{\text{uc}} T_1 F_{\text{pq}} F_1 \right) K_x \right).
\]

(28)

The hybrid AC/DC impedance model of the VSM-based inverter by removing \( G_{\text{dc}} \) of (28) is derived as follows:

\[
Z_{\text{vsm-inv}} = \left( F_2 G_{\text{dc}} G_{\text{lc}} G_{\text{uc}} \left( I + T_1 \left( M + F_{\text{pq}} F_u \right) \right) K_x + I \right)^{-1} \cdot \left( F_1 + F_2 G_{\text{dc}} G_{\text{lc}} \left( I + G_{\text{uc}} T_1 F_{\text{pq}} F_1 \right) K_x \right).
\]

(29)

IV. IMPEDANCE COMPARATIVE ANALYSIS OF VSM-BASED RECTIFIER STATION AND VSM-BASED INVERTER STATION

A. Verification and Characteristics Analysis of Hybrid AC/DC Impedance Model of VSMs

The impedance measurement is the most direct way to verify the impedance models. The simulation measurement results are obtained by MATLAB/Simulink, and the experimental results are obtained by RT_LAB. The measurement method is given in Appendix. When the impedance measurements are carried out by RT_LAB, the serious disturbance voltages adopt ideal voltage sources, and only the controllers of the HVDC system are real. The parameter design of VSMs refers to [4], as shown in Table I. The bandwidth of the current loop, the dc voltage loop, and the synchronization loop are set as 230 Hz, 8 Hz, and 16 Hz, respectively. The simulation and experimental measurement results verify the impedance models (Fig. 3 and Fig. 4). Besides, the impedance characteristics of the VSM-based rectifier and inverter are compared as follows:

1) There exists AC/DC coupling in both \( Z_{\text{vsm-rec}} \) and \( Z_{\text{vsm-inv}} \), especially when the dc-link capacitor is small, and the AC/DC

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( U_{dc} )</td>
<td>DC-link voltage</td>
<td>±100 kV</td>
</tr>
<tr>
<td>( U_r )</td>
<td>Grid voltage</td>
<td>90 kV</td>
</tr>
<tr>
<td>( L_g )</td>
<td>Grid-side inductor of the rectifier</td>
<td>5 mH</td>
</tr>
<tr>
<td>( L_s )</td>
<td>Grid-side inductor of the inverter station</td>
<td>5 mH</td>
</tr>
<tr>
<td>( S_n )</td>
<td>Rated capacity</td>
<td>200 MVA</td>
</tr>
<tr>
<td>( L_{dc} )</td>
<td>Inductance of the DC line</td>
<td>7.9 mH</td>
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<tr>
<td>( R_{dc} )</td>
<td>Resistance of the DC line</td>
<td>0.6950 ( \Omega )</td>
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<td>( \omega_{dc} )</td>
<td>Cut-off frequency of sampling filter</td>
<td>10 kHz</td>
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<tr>
<td>( \omega_{uc} )</td>
<td>Cut-off frequency of voltage sampling</td>
<td>10 kHz</td>
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<td>( C_{dc} )</td>
<td>DC-link capacitor of the VSM</td>
<td>5 mF</td>
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<td>( f_0 )</td>
<td>Grid frequency</td>
<td>50 Hz</td>
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<td>( f_s )</td>
<td>Switch frequency</td>
<td>2 kHz</td>
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<td>( J_t )</td>
<td>Virtual moment of inertia of rectifier</td>
<td>1089.8 kg.m²</td>
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<td>( D_{pr} )</td>
<td>Active damping coefficient of rectifier</td>
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<td>Virtual inductance</td>
<td>5 mH</td>
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<td>( R_r )</td>
<td>Virtual resistance</td>
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<td>( K )</td>
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<td>( J_r )</td>
<td>Virtual moment of inertia of the rectifier</td>
<td>5000 kg.m²</td>
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<td>( D_{pi} )</td>
<td>Active damping gain of the rectifier</td>
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<td>( D_k )</td>
<td>Reactive damping gain</td>
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<td>( k_{pu} )</td>
<td>Proportional gain of voltage controller</td>
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<td>( k_{iu} )</td>
<td>Integrator gain of voltage controller</td>
<td>298</td>
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<td>( k_{pr} )</td>
<td>Proportional gain of current controller</td>
<td>4.6250e-05</td>
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<td>( \omega_{pr} )</td>
<td>Cutoff frequency of PR controller</td>
<td>6\pi</td>
</tr>
<tr>
<td>( k_r )</td>
<td>Resonance gain of current controller</td>
<td>0.001</td>
</tr>
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TABLE I

SYSTEM PARAMETERS OF THE INVERTER STATION

Fig. 3 Hybrid AC/DC impedance of the VSM-based rectifier.

Fig. 4 Hybrid AC/DC impedance of the VSM-based inverter.
coupling of $Z_{\text{vsm-rec}}$ is stronger than that of $Z_{\text{vsm-inv}}$ in the low-frequency range. It implies that dc-link voltage controllers enhance the AC/DC coupling of the VSMs. Besides, increasing the dc-link capacitor can weaken the coupling.

2) $Z_{\text{dc}}$ of the VSM-based rectifier station behaves as a negative resistor, and its impedance magnitude is V-shaped in the low-frequency range. Besides, $Z_{\text{dc}}$ of the VSM-based inverter also behaves as a negative resistor below 2Hz.

**B. The Relationship between the Hybrid AC/DC Impedance, DQ-Frame Impedance, and DC Impedance**

Both the DC impedance and dq-frame impedance can be obtained from the hybrid AC/DC impedance. The hybrid AC/DC impedance of the converter is given by:

$$
\begin{bmatrix}
\Delta u_d \\
\Delta u_q \\
\Delta u_{dc}
\end{bmatrix} =
\begin{bmatrix}
Z_{dd} & Z_{dq} & Z_{dc} \\
Z_{qd} & Z_{qq} & Z_{qc} \\
Z_{dc} & Z_{dq} & Z_{dc}
\end{bmatrix}
\begin{bmatrix}
\Delta i_d \\
\Delta i_q \\
\Delta i_{dc}
\end{bmatrix}
$$

(30)

The grid-side small-signal dynamics are expressed as

$$
\begin{bmatrix}
\Delta u_d \\
\Delta u_q \\
\Delta u_{dc}
\end{bmatrix} = -Z_{\text{g}}
\begin{bmatrix}
\Delta i_d \\
\Delta i_q \\
\Delta i_{dc}
\end{bmatrix}
$$

(31)

Embedding (30) and (31), the relationship between the DC impedance and the hybrid AC/DC impedance is

$$
Z_{\text{dc}} = -\left[Z_{\text{dc}} + Z_{\text{dq}}\right] + \frac{1}{Z_{\text{dc}} + Z_{\text{dc}}}
$$

(32)

The DC-side small-signal dynamic in the latter stage is

$$
\Delta u_{dc} = -Z_{\text{dc}}\Delta i_{dc}
$$

(33)

where $Z_{\text{dc}}$ is the DC impedance of the later-stage converter.

**C. Comparative Analysis of DQ-Frame Impedances**

The dq-frame impedance of the VSM-based rectifier station according to (28) and (34) is expressed as follows:

$$
Z_{\text{dq} \text{rec}} =
\begin{bmatrix}
Z_{ddr} & Z_{dq r} & Z_{dcr} \\
Z_{qdr} & Z_{qq r} & Z_{qcr} \\
Z_{dcr} & Z_{dqr} & Z_{dcr}
\end{bmatrix}
\begin{bmatrix}
\Delta i_{dd} & \Delta i_{dq} & \Delta i_{dc} \\
\Delta i_{qd} & \Delta i_{qq} & \Delta i_{qc} \\
\Delta i_{dc} & \Delta i_{dq} & \Delta i_{dc}
\end{bmatrix}
$$

(35)

The dq-frame impedance of the VSM-based inverter station according to (29) and (34) is expressed as follows:

$$
Z_{\text{dq} \text{inv}} =
\begin{bmatrix}
Z_{ddi} & Z_{dq i} & Z_{dci} \\
Z_{qdi} & Z_{qq i} & Z_{qci} \\
Z_{dci} & Z_{dqi} & Z_{dci}
\end{bmatrix}
\begin{bmatrix}
\Delta i_{dd} & \Delta i_{dq} & \Delta i_{dc} \\
\Delta i_{qd} & \Delta i_{qq} & \Delta i_{qc} \\
\Delta i_{dc} & \Delta i_{dq} & \Delta i_{dc}
\end{bmatrix}
$$

(36)

$Z_{\text{dq} \text{rec}}$ denotes the dq-frame impedance of the VSM-based rectifier regarding the dc side as a constant resistor. $Z_{\text{dq} \text{inv}}$ denotes the dq-frame impedance of the VSM-based inverter regarding the dc side as a constant value. The sweep results verify the accuracy of the theoretical $Z_{\text{dq} \text{rec}}$ and $Z_{\text{dq} \text{inv}}$ (Fig. 5 and Fig. 6). Figure 5 shows that the dc-link dynamics of the inverter station have great impacts on the dq-frame impedance of the rectifier station in the low-frequency range. Figure 6 shows that the dc-link dynamics of the rectifier also have impacts on the dq-frame impedance of the inverter especially when the dc-link capacitor is small.

**D. Comparative Analysis of DC Impedances**

As shown in Fig. 7, the magnitude of the hybrid DC impedance (green line) is higher than that of the DC impedance of the later-stage converter (red line). This is because the dc-link capacitor weakens the coupling of the VSMs. Besides, increasing the dc-link capacitor also behaves as a negative resistor below 2Hz.

**Fig. 5 The dq-frame impedance of the VSM-based rectifier station.**

**Fig. 6 The dq-frame impedance of the VSM-based inverter station.**

**Fig. 7 DC impedance of the VSM-based rectifier station and inverter station.**
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Z_{drec} that denotes the DC impedance of the VSM-based rectifier station is obtained according to (28) and (34). Z_{dinv} that denotes the DC impedance of the VSM-based inverter station is obtained according to (29) and (34). The measurement results are consistent with the theoretical models, which verifies the accuracy of the models (Fig. 7).

Z_{drec} shows negative resistance characteristics and is easy to intersect with Z_{drec} in the low-frequency range. It means that the low-frequency oscillation could occur due to the DC-side interaction between the VSM-based rectifier station and the inverter station. As compared in Table II, the DC impedance of the converter contains the information of the grid, and the dq-frame impedance contains the information of the converter in the later stage. By comparison, the hybrid AC/DC impedance only includes the information of the converter. Thus, the stability analysis based on the DC or dq-frame impedances needs to calculate RHP poles or zeros that cannot be obtained in the actual system, which makes the analysis results incorrect.

**V. SYSTEM STABILITY ANALYSIS BASED ON THE FIVEDIMENSION IMPEDANCE STABILITY CRITERION**

**A. Five-Dimension Impedance Stability Criterion**

The five dimension stability criterion based on the hybrid AC/DC impedances is derived below.

The equivalent model of the HVDC system is shown in Fig. 1(b), and two AC grid-side circuits are described as follows:

$$\begin{bmatrix}
\Delta i_{d1} \\
\Delta i_{q1} \\
\Delta i_{d2} \\
\Delta i_{q2}
\end{bmatrix} = Z^{-1}_{d \rightarrow q} \begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix}$$

(37)

$$\begin{bmatrix}
\Delta i_{d1} \\
\Delta i_{q1} \\
\Delta i_{d2} \\
\Delta i_{q2}
\end{bmatrix} = Z^{-1}_{q \rightarrow d} \begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix}$$

(38)

where Z_{d \rightarrow q} and Z_{q \rightarrow d} represent the grid impedance of the rectifier station and inverter station in the dq frame, respectively.

The DC-side passive network can be expressed as follows:

$$\Delta i_{dc1} = -\Delta i_{dc2} = \frac{\Delta u_{dc1} - \Delta u_{dc2}}{2(L_{dc} s + R_{dc})}$$

(39)

Based on (37)-(39), the variables of the AC grids and DC parts can be modeled together by

$$\begin{bmatrix}
\Delta i_{d1} \\
\Delta i_{q1} \\
\Delta i_{d1} \\
\Delta i_{q2}
\end{bmatrix} = Y_{acdcnet} \begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix}$$

(40)

where \(Y_{acdcnet}\) is defined as follows:

$$\begin{bmatrix}
\frac{1}{Z_{g1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

(41)

Meantime, according to (30) and (39), the small-signal voltages and currents of the AC/DC ports of the HVDC rectifier and inverter station can be expressed as follows:

$$\begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{dc1} - \Delta u_{dc2} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix} = Z_{vsmacdc} \begin{bmatrix}
\Delta i_{d1} \\
\Delta i_{q1} \\
\Delta i_{dc1} \\
\Delta i_{d2} \\
\Delta i_{q2}
\end{bmatrix}$$

(42)

where \(Z_{vsmacdc}\) is defined as follows:

$$\begin{bmatrix}
\Delta i_{d1} \\
\Delta i_{q1} \\
\Delta i_{dc1} \\
\Delta i_{d2} \\
\Delta i_{q2}
\end{bmatrix} = (I + Y_{acdcnet} Z_{vsmacdc})^{-1} Y_{acdcnet} \begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix}$$

(43)

The DC and AC-side currents of the VSC-HVDC system are derived according to (40) and (42) as follows:

$$\begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{dc1} - \Delta u_{dc2} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix} = (I + Y_{acdcnet} Z_{vsmacdc})^{-1} Y_{acdcnet} \begin{bmatrix}
\Delta u_{d1} \\
\Delta u_{q1} \\
\Delta u_{d2} \\
\Delta u_{q2}
\end{bmatrix}$$

(44)

The system minor loop gain based on (44) is defined as follows:

$$L(s) = Y_{acdcnet} Z_{vsmacdc}$$

(45)

Based on GINC, the system stability depends on the Nyquist diagram and the number of the RHP zeros [25]. Since both converters are designed to be stable when they operate standalone, \(Z_{vsmacdc}\) and \(Y_{acdcnet}\) contain no RHP zeros. Thus, the stability of the HVDC system is assessed by judging whether the inverse characteristic loci of \(L(s)\) encircles (-1, 0) point or not. Besides, the frequency where the inverse characteristic loci intersect the unit circle is the predictive oscillation frequency of the system in the dq frame. The advantages of the proposed criterion are that RHP poles or zeros of the system impedance...
Fig. 8 Inverse characteristic loci of $L(s)$, SCRi=4. (a) SCRr=3, $k_{ui}=288$, $D_p=326929$, $C_{dc}=5$ mF; (b) SCRr=2, $k_{ui}=288$, $D_p=326929$, $C_{dc}=5$ mF; (c) SCRi=3, $k_{ui}=288$, $D_p=490390$, $C_{dc}=5$ mF; (d) SCRr=3, $k_{ui}=288$, $D_p=490390$, $C_{dc}=0.5$ mF; (f) SCRr=3, $k_{ui}=288$, $D_p=523090$, $C_{dc}=5$ mF.

Fig. 9 Experimental results based on the RT-LAB corresponding to the stability analyses in Fig. 8, SCRi=4.
ratio don’t need to be calculated, and it has a relatively small computation and a wider range of applications. The shortcoming is that the instability mechanism cannot be directly observed by the impedance bode plot, due to the strong coupling among the d, q, and dc frames.

B. Stability Analysis and Experimental Results

To further verify the correctness of the theoretical analysis, experiments have been carried out on the hardware-in-the-loop experimental platform, as shown in Fig. 10. The experimental parameters are the same as those of the previous simulation. The VSM-based rectifier station and inverter station are controlled by TI DSP TMS320F28335 through the I/O interface of the RT_LAB. The other parts of the HVDC system such as the main circuit of the rectifier station and the inverter station are simulated in FPGA and the program is implemented in CPU.

All characteristic loci of \( L(s) \) are obtained by MATLAB, and the reciprocals of characteristic loci are plotted in Fig. 8 after the implementation of the sort program. Each Nyquist diagram shown in Fig. 8 is supposed to have five curves.

As shown in Fig.8 (a), all inverse characteristic loci do not encircle (-1, 0) when SCR=3, meaning that the HVDC system is stable. It is verified by the experimental results in Fig. 9 (a). Compared with Fig. 8(a), Fig. 8 (b) shows that one inverse characteristic locus encircles (-1, 0) when SCR=2, and it intersects with the unit cycle at 0.96 Hz. It means that the oscillation with 0.96 Hz will occur in the dq frame, corresponding to 50 Hz±0.96 Hz in the abc frame. It is verified by experimental results in Fig. 9 (b), where the DFT analysis of \( i_{\text{rec}} \) shows two main resonant components at 50±1 Hz. Compared with Fig. 8 (b), Fig. 8(c) shows that all inverse characteristic loci do not encircle (-1, 0) only when the integral coefficient \( (k_p) \) of the VSM rectifier decreases from 288 to 144, meaning that decreasing the \( k_p \) enhances the stability of HVDC system. It is verified by the experimental results in Fig. 9 (c).

Compared with Fig.8 (d), Fig.8 (e) shows that one inverse characteristic locus encircles (-1, 0) with decreasing the dc-link capacitor \( (C_d) \) of the VSM, and it intersects with the unit cycle at 3.3Hz, meaning that decreasing \( C_d \) destabilizes the system. It is verified by experimental results in Fig. 9 (d) and Fig. 9 (e), where the system becomes unstable after only changing \( C_d \) from 5mH to 0.5mH. Besides, Fig. 9 (e) shows that the DFT analysis of \( i_{\text{rec}} \) shows two main resonant components at 50±3Hz, which is approximately consistent with the predictive value (50±3.3Hz).

Compared with Fig. 8(d), Fig. 8(f) shows that one inverse characteristic locus gradually encircles (-1, 0) with increasing the damping coefficient \( (D_p) \) of the VSM-based rectifier, meaning that increasing \( D_p \) destabilizes the system. It is verified by the experimental results in Fig. 9(d) and Fig. 9(e), where the system becomes unstable after only increasing \( D_p \). Besides, the DFT analysis of \( i_{\text{rec}} \) shows two main resonant components at 50±1Hz, which is consistent with the predictive value (50±1.03 Hz). The experimental results in Fig. 9 further verify the effectiveness of the stability criterion. Besides, It is found that increasing the integral gain of the dc-link voltage controller and the damping coefficient or decreasing the dc-link capacitor of the VSM rectifier destabilizes the system.

VI. CONCLUSIONS

This paper aims to assess the stability of the VSM-based HVDC systems, and the established hybrid AC/DC impedance models of the VSMs and the five-dimensional impedance stability criterion has been verified by experimental results. Some conclusions were drawn:

1) The accurate hybrid AC/DC impedance models of the VSM-based rectifier station and inverter station have been established and verified by the improved measurement method.

2) The DC impedance of the VSM-based inverter station behaves as a negative resistor in the low-frequency range.

3) A five-dimension impedance stability criterion based on the GINC and the hybrid AC/DC impedance has been proposed to accurately analyze the point-to-point VSC-HVDC system.

4) The d-d channel negative resistor behavior of the VSM-based rectifier station could induce low-frequency oscillations when the grid SCR is small. Decreasing the integral coefficient of the dc-link voltage controller and the damping coefficient, and increasing the dc-link capacitor of the VSM rectifier can enhance the system stability.

APPENDIX

The hybrid AC/DC impedance, dc impedance, and dq-frame impedance of the three-phase converter can be measured together, as shown in Fig. 11. At first, three linear independent series voltages are injected. Then, the dq-frame and dc-link voltages and currents are collected. Applying FFT to the sample signals, the hybrid AC/DC impedance is calculated as follows:

\[
Z_{\text{ad}} = \begin{bmatrix} U_{\text{d1}} & U_{\text{d2}} & U_{\text{d3}} & I_{\text{d1}} & I_{\text{d2}} & I_{\text{d3}} \\ U_{\text{q1}} & U_{\text{q2}} & U_{\text{q3}} & I_{\text{q1}} & I_{\text{q2}} & I_{\text{q3}} \\ U_{\text{dc1}} & U_{\text{dc2}} & U_{\text{dc3}} & I_{\text{dc1}} & I_{\text{dc2}} & I_{\text{dc3}} \end{bmatrix}^{-1} \tag{46}
\]

The PLL used for the impedance measurement has great impacts on the hybrid AC/DC impedance measurement, especially during the PLL bandwidth. Thus, after mitigating the PLL dynamics, the actual impedance \( Z_{\text{acde}} \) is

\[
Z_{\text{acde}} = Z_{\text{ad}}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -U_{\text{d0}} G_{\text{pll}} & -I_{\text{q0}} G_{\text{pll}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{47}
\]

\( G_{\text{pll}} \) is expressed as follows:

\[
G_{\text{pll}} = (k_{\text{qpl}} + k_{\text{qpl}}/s)(s+U_{\text{d0}}(k_{\text{qpl}} + k_{\text{qpl}}/s)) \tag{48}
\]

where \( k_{\text{qpl}} \) and \( k_{\text{qpl}} \) are PI gains of the PLL, respectively.
Likewise, the dq-frame impedance is calculated as [28]:

\[
Z_{dq} = \left( \begin{array}{cc}
U_{d1} & U_{d2} \\
U_{q1} & U_{q2}
\end{array} \right) \left( \begin{array}{cc}
I_{d1} & I_{d2} \\
I_{q1} & I_{q2}
\end{array} \right)^{-1}, \tag{49}
\]

\[
Z_{dqm} = \left( \begin{array}{cc}
1 & U_{q0}G_{pil} \\
0 & 1-U_{d0}G_{pil}
\end{array} \right)^{-1} \left( \begin{array}{cc}
1 & I_{q0}G_{pil} \\
0 & -I_{d0}G_{pil}
\end{array} \right) \tag{50}
\]

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