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Symbolic Power and Mathematics

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Abstract

Symbolic power will be discussed with reference to mathematics. Two distinctions are pointed out as crucial for exercising such power: one between appearance and reality, and one between sense and reference. These distinctions include a nomination of what to consider primary and what to consider secondary. They establish the grammatical format of a mechanical and formal world view. Through an imposition of such world views symbolic power is exercised through mathematics.

This power is further investigated through different dimensions of mathematics in action: (1) Technological imagination which refers to the possibility of formulating technical possibilities. (2) Hypothetical reasoning which addresses consequences of not-yet realised technological initiatives. (3) Legitimation or justification which refers to possible validations of technological actions. (4) Realisation which signifies that mathematics itself comes to constitute part of reality. And (5) dissolution of responsibility, which may occur when issues of responsibility are eliminated from the discourse about technological initiatives and their implications. Finally, it is emphasised that whatever form symbolic power may take, it cannot be addressed along a single good-evil axis.

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1. Symbolic Power

Symbolic power is not a well-defined notion, yet it has been used in many contexts[1]. Symbolic power can be exercised through discourses which impose a range of priorities and implicit notions on that which is being addressed[2].

Symbolic power can be exercised by way of labelling, for instance by singling out particular groups of people. One can refer to immigrants when trying to shed light on street violence; blacks when addressing poverty in Africa; slow learners when trying to explain certain educational problems. In fact there is a close correlation between designating and an imposition of stereotypes. A language can operate as an instrument of simplification; one may think of the language developed around production efficiency. Such a language may refer to workers, but stripped of their human relationships, instead highlighting them as more of less efficient elements of a production machinery. Symbolic power can be exercised through concepts like “soul”, “God”, “salvation” just to mention some designations that includes layers of metaphysical assumptions. The discussion of symbolic power can refer to any form of discourse and to any form of language.

Rudolf Carnap found that one could get rid of all the misunderstandings and preconceptions that have been instilled in natural language by constructing formal languages[3]. In this way science would have a true universal formal format. Thus Carnap envisioned formal languages as liberators from the illegitimate power exercised by natural language.

2. Mathematics and Symbolic Power

A formal language is also a language, and as such it may exercise symbolic power. One can discuss, then, to what extent the symbolic power connected to formal languages is benevolent, or if it might be questionable, illegitimate, and suspicious. In fact a formal language might not be a liberator as assumed by Carnap. It might be the bearer of a power that is in need of being both identified and criticize; it might bring along with it heavy loads of metaphysical assumptions. In fact, it is possible for symbolic power to have the same huge range of qualities that can be associated to power in general.[4] It could be problematic, unfair, blind, helpful, ruthless, benevolent, etc.

I do not assume that the notion of mathematics can be captured in any single definition. Instead I find that mathematics can take many different forms,

[1]See, for instance, Bourdieu (1991). For a discussion of knowledge and power, see Foucault (1989, 1994, 2000).

[2]Several philosophers, from Nietzsche (1998) via Carnap (1959) to iek (2008) has formulated a heavy critique of the power exercised by language.

[3]See Carnap (1937).

[4]I use the notion of “quality” in a classic philosophical way as referring to “property”, and not to some degree of desirability.

such as making a budget, calculating a salary, making an investment, reading a map, completing a design, solving school mathematics exercises, solving an engineering problem, not to forget doing mathematical research. One can see mathematics as a language, as a discourse. In fact one can see it as an extended family of discourses that involve different degrees of formalism.

I will discuss symbolic power with reference to this extended family of languages in two steps: (1) One can see mathematics as a descriptive tool. However, there are no neutral descriptions. Any description includes priorities with respect to what to include and what to exclude. Also mathematics-based descriptions exercise symbolic power by nominating what to call primary and what to call secondary. (2) One can see mathematics as making part of actions, and I will explore this dimension of symbolic power by addressing different features of mathematics in action. Together the discussion of (1) and (2) will illustrate how I associate symbolic power to mathematics.[5]

3. What Is Primary and what Is Secondary?

We can imagine that symbolic power can be exercised through the invention of something that is not already the case. It appears that by applying mathematics one can invent measures, norms, and standards that were not really there before the mathematical discourse nominated the entities to be addressed. One can also assume that symbolic power manifests as the systematic overlooking of particular groups of phenomena. Thus, we can search for symbolic power by examining priorities for both "seeing" and "overlooking". Such priorities can be imprinted in the grammar (or the structure) of language, and also of formal languages. Symbolic structuring provides a way of nominating something as primary and other things as secondary. Such a grammar-based primary-secondary ranking makes up one layer of a language-instilled metaphysics.

In order to clarify further the primary-secondary ranking, I will consider two distinctions that can be associated with mathematics. One distinction is related to the formulation of the mechanical world view, while the other is related to the formulation of what I refer to as the formal world view.[6] By paying attention to these two distinctions, I try to point out two features of symbolic power that can be associated to mathematics.

3.1. Appearance and reality. Through the scientific revolution an intimate relationship between mathematics and the natural sciences was

[5]This applies to all different forms of mathematics, and also to the different forms of ethnomathematics. In the following, however, I will concentrate on what can be referred to as academic mathematics, in particular as it is realised through its applications.

[6]For a discussion of these two distinctions see Skovsmose (2009). See also Skovsmose (2005) for a discussion of mathematics and power. My work with these distinctions is inspired by my cooperation with Ole Ravn, see Skovsmose and Ravn (draft).

established. This relationship, however, was only made possible through the distinction between appearance and reality. The establishment of the heliocentric world view illustrates clearly what this distinction is about. Looking at the sun in the morning, one sees how it rises in the sky. During the course of the day one can follow its movements. In the evening we can see the beautiful colours of the western sky, when it sinks below the horizon. Literature is awash with variations of sunrise-sunset descriptions. Let us imagine that we were to collect all these descriptions and from them try to extract an insight into the sunrise-sunset phenomenon. According to the scientific revolution, we would never attain any insight at all, as we would remain trapped by the appearance of the phenomenon. At the heart of the formulation of the heliocentric world view is the assumption that one needs to get around the appearances of phenomena in order to grasp the structures of reality.

The distinction between appearance and reality has been emphasised by many. However, I will refer to a particular text by Galileo Galilei as paradigmatic for formulating the appearance-reality distinction. In *The Assayer*, first published in 1623, Galilei discusses the notion of heat[7] We all have experiences of this phenomenon. One may be burned by the rays of the sun, touch a warm kettle, come too close to a fire, gulp a spoonful of too hot soup, etc. One could try to register a broad variety of experiences with heat and on this basis try to formulate an insight as to what heat really is. Galilei's point is that such an approach brings us nowhere: our experiences of heat do not reveal anything about the real nature of heat. The "mechanics" of heat, according to Galilei, is very different from whatever might be gleaned from of our sense experiences, just as the mechanism of sunrise and sunset is different from our experiences of these phenomena. According to Galilei, to provoke an experience of heat "nothing is required in external bodies except shapes, numbers, and slow or rapid movements" (Galilei 1957: 276–277).

The mechanical world view presents reality as a tremendous mechanism composed of material units, characterised by their shape, number and movements, governed by certain laws. Such a mechanism is behind experienced phenomena like heat as well as sunsets. It is behind any of our experiences. The appearance-reality distinction facilitates the formulation of the mechanical world-view and brings mathematics into a prominent position: it becomes the principal tool for describing reality. While natural language is useful for expressing experiences, mathematics is capable of depicting the underlying reality. It does so in terms of shapes, numbers and movements in other words: in terms of a mechanism.

If we assume that reality is *in fact* a mechanical structure, then mathematics can be assigned a tremendous descriptive power: it turns out to be not only necessary but also sufficient for grasping reality. If we think of this descriptive

[7]The Assayer is reprinted in Galilei (1957: 229-280). See also Skovsmose (2009) for a discussion of The Assayer.

power as a form of symbolic power, the whole outlook of the scientific revolution would bring us to celebrate the symbolic power of mathematics. This celebration has brought about the claim that mathematics is the language of sciences; it is the universal symbolism of knowledge. However, if we do not consider the mechanical world as a given to be discovered, but rather as invented, then we reach quite a different interpretation of this symbolic power. The mechanical reality is not described by means of mathematics but rather established through mathematics as a projection of the grammar of mathematics, which seems designated to talk about entities like shape, number, movements, etc. The mechanical world view is due to the way mathematics nominates certain phenomena as primary and ignores others as secondary. The mechanical world view can be seen as a frightening metaphysics rooted in the grammar of mathematics. Thus the mechanical world view becomes a demonstration of a symbolic power associated with mathematics. Through applications of mathematics the mechanical world view becomes imposed not only on nature, but on any domain that is mathematised: business, management, forms of production, marketing, etc.

3.2. Sense and reference. While in *The Assayer*, Galilei formulated a distinction between appearance and reality, Gottlob Frege, in *Über Sinn und Bedeutung*, first published in 1892, formulated a distinction between sense (*Sinn*) and reference (*Bedetung*).[8] The distinction between appearance and reality is linked to the scientific revolution, while the distinction between sense and reference can be linked to a formal revolution. Both distinctions specify what is to be considered primary and what secondary. While the distinction between appearance and reality concerns our perception of nature and physical environment, the distinction between sense and reference concerns our perception of logic and rationality. Frege sought to grasp the nature of logical reasoning.

To illustrate the distinction between sense and reference we can look at the notion of a triangle. In order to indicate the sense of the word triangle, one could try to explain that we are dealing with a geometric figure composed of three straight lines. If, however, one were to indicate the reference of the concept "triangle", one would look to the set of all triangles. As Frege was a Platonist, he would see the reference as the collection of ideal objects. More generally, the reference of a concept is the set of objects that "fall under" that concept.

Frege also applies the distinction between sense and reference to statements. If we state that "the sum of the angles in a triangle is 180°", then one could try to clarify the sense of that sentence, maybe by showing some of the steps in the proof of the statement. The sense of the statement has to do with the content of what is stated. However, according to Frege, the reference of the statement

[8] *Über Sinn und Bedeutung* is reprinted in Frege (1969: 40-65). See also Skovsmose (2009) for a discussion of *Über Sinn und Bedeutung*.

is something quite different. He suggests that the reference of a statement is its truth value. Furthermore, he assumes that there are only two such values: "true" (or T) and "false" (or F). This means that the reference of the statement "the sum of the angles in a triangle is 180°" is "true". If we were to consider all possible statements, they would have lots of different senses, but their references would be either "true" or "false". The domain of references of sentences would be a very small universe, namely consisting of only two objects, the two possible truth values, "true" and "false".

Such a claim may appear absurd. However, it makes it possible for Frege to formulate his main point: in order to clarify the reality of logical reasoning, one needs to concentrate on the domain of references of concepts and statements. References can be considered primary, while senses are secondary and can be ignored. In fact, when it comes to logical investigation, the dimension of sense only confuses analysis.[9]

The distinction between sense and reference has also been expressed in terms of intension and extension, corresponding to sense and reference. Thus, the extension of a concept is the set of objects that fall under the concept, while the intension can be understood as its sense. The extension of a statement is its truth value, while its intention refers to the content of what is stated. With this terminology Frege's claim is that the logical aspects of language are located in the domain of extensions, while the intentional aspects are carriers of psychological aspects. If one wants to grasp the reality of logical reasoning, one has to focus on the extensional aspects of language.[10] If one pays attention to the intentional aspects, one might get bewildered by the appearance of rationality. This appearance may reveal just as little about logic as the experience of heat reveals about movements of molecules, or as the beauty of sunrise and sunset reveals about the rotation of the Earth.[11]

Frege's apparently absurd idea paved the way for a tremendous development of formal systems, formalisations of deduction, automation of reasoning and for the proliferation of formal languages, including all variations of computer languages. Frege's ranking of primary and secondary with respect to logical reasoning is crucial for the development of artificial intelligence. It is crucial for establishing any automatic manipulation of formal systems.

[9]According to Frege, many have suffered such confusion. Mill, for instance, who found that in order to understand both the nature of logical reasoning and the foundation of mathematics, one had to grasp their inductive origin. See Mill's presentation in A System of Logic and Frege's harsh critique of Mill in The Foundation of Arithmetic.

[10]Frege's idea was nicely condensed by Wittgenstein in the Tractatus, where he presented a truth-table logic.

[11]An important step towards giving logic an extensional format was presented by Frege in his Begriffschrift, which was published in 1879. Later Frege provided a new careful elaboration of formal logic in Grundgesetze der Arithmetik, which appeared in two volumes in 1893 and 1903. Many studies have followed, and Whitehead's and Russell's Principia Mathematica, published in three volumes in 1910-1913, reworked many of Frege's ideas and established a more powerful symbolism than the one originally suggested by Frege.

However, one need not assume that the distinction between sense and reference reveals a basic reality of logical rationality. One may instead consider the possibility that the sense-reference distinction is imposed on the domain of investigation. It might be a proposal for implementing a primary-secondary ranking within the domain of logic. The ranking may represent a profound metaphysics with respect to rationality. Maybe a new logic is not discovered through the sense-reference distinction, so much as a new logic is created and brought into action. We might be dealing with an imposition that represents symbolic power. And this symbolic power is exercised with respect to all the different domains within business, management, forms of production, marketing, etc.— taken into custody by automatic manipulations for formal systems.

4. Mathematics in Action

Symbolic power connected to mathematics reaches beyond any primary-secondary imposition. It is manifested in mathematics-based actions. In this section I will illustrate the range of mathematics-based actions within technology. I use "technology" as an almost all-embracing concept referring to any form of design and construction (of machines, artefacts, tools, robots, automatic processes, networks, etc.) decision-making (concerning management, promotion, economy, etc.), and organisation (with respect to production, surveillance, communication, money-processing, etc.).

Like any action, so also a mathematics-based action can be described in general terms, and I will point out some of its dimensions: (1) Any action includes visions about what could be done, and by *technological imagination* I refer to the tentative formulation of technological possibilities. (2) As part of investigating a possible action, *hypothetical reasoning* is important. Through such reasoning one addresses consequences of not-yet-realised technological constructions and initiatives. Through an if-then reasoning one tries to estimate how feasible it might be to carry out an action. (3) An action may require *justification*. Some such justification may take place before one carries out the action, although one can also try to justify actions after their completion. In many ways, justification might take the form of a questionable *legitimisation*. (4) When completed, an action comes to make part of reality, and *realisation* of mathematics refers to the fact that mathematics itself may come to make part of reality. (5) One can think of an acting person as being responsible for the action. However, in many examples of mathematics-based actions, it is not easy to identify an acting subject, and a *dissolution of responsibility* might occur.[12]

[12]For presentations and discussions of mathematics in action see Skovsmose (2005, 2009); Skovsmose and Yasukawa (2009); Christensen and Skovsmose (2007); Christensen, Skovsmose and Yasukawa (2007); Skovsmose, Yasukawa and Ravn (draft); and Skovsmose and Ravn (draft). The following presentation of mathematics in action draws on this material.

4.1. Technological imagination. Often technological imagination is mathematics - based. As a paradigmatic example, one can think of the conceptualisation of the computer. The mathematical construct, in terms of the Turing machine was investigated in every detail.[13] Even the computational limits of the computer were worked out before the construction of the first computer had taken place. If we consider the computational approach in all its dimensions, we can talk about the formal revolution, and this revolution is directly related to the sense-reference distinction. Algorithmic procedures which could be handled mechanically were related to the extensional aspect of language.[14]

All features of modern information and communication technology are deeply rooted in mathematics-based imagination. To illustrate: great potential for cryptography was identified through mathematical clarifications of number-theoretical properties. Of particular importance was the identification of what could be referred as a one-way function This is a function, f, where it is easy to calculate $y = f(x)$, when x is given, but impossible in any feasible way to calculate $f^1(y)$, when only f and y are given.[15] The straightforward calculation of y from the value of x can be associated with encryption, and breaking the code, i.e. calculating x from the value of y, remains impossible.[16] In this way a mathematical construct, a one-way function, provided new technological possibilities. There is no commonsense-based imagination equivalent to mathematics-based imagination. Furthermore, it must be noted that mathematics-based imagination operates beyond any scheme of prediction; instead it brings about contingencies as a characteristic feature of technological development.

Mathematics-based technological imagination plays a crucial role in economy and business, for instance in establishing schemes for prices and payment of goods. We can take air-fares as an example: airlines deliberately overbook as one element of such schemes.[17] The overbooking is carefully planned; in particular, the degree to which a flight can be overbooked needs to be estimated from the statistics of the numbers of no-shows for the departures in question.

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[13]See, Turing (1965) as well as Skovsmose (2009) for a discussion for this example.

[14]It is worth noting that intensional logic has developed tremendously, for instance through the work of Montague (1974), who was keen to develop a Frege semantics, acknowledging Frege's contribution to logic and the analysis of language. Montague demonstrated how apparently intentional features of language could be incorporated in a Frege semantics and, in this way, provided with an extensional foundation. This insight is crucial for developing computational linguistic features, and, for instance, for establishing automatic forms of translation.

[15]That it is possible to construct one-way functions is based on number theoretical insight, and in particular on the observation of the extreme complexity of factorising a product of two very large (say at least 50 digits) unknown prime numbers.

[16]See Skovsmose and Yasukawa (2009), as well as more general presentations in Schroeder (1997) and Stallings (1999). See also Diffie and Hellman (1976) for the presentation of the original idea.

[17]See Clements (1990). See also Skovsmose (2005) for a discussion of this example. There is a great amount of papers and comments about the phenomenon of overbooking at the internet. See for instance "Why do Airlines Overbook Flights" (http://weakonomics.com/2009/12/29/why-do-airlines-overbook-flights/).

(A "no-show" refers to a passenger with a valid ticket who does not show up for the departure.) The costs of bumping a passenger need to be estimated as well. ("Bumping" a passenger means not allowing a passenger with a valid ticket to board the plane.) The predictability of a passenger for a particular departure being a no-show is naturally an important parameter in designing the overbooking policy. The whole overbooking policy can be mathematically experimented with until a price-setting is reached that maximises profits, this in turn becoming an ongoing algorithmic-based process. Mathematics-based technological imagination is crucial, not only for the construction of new technological artefacts, but also for the identification of new schemes for, say, production, management, decision-making, etc. It is an imagination, however, that exists within a certain space. It is an imagination that assumes the mechanical world view, and it is an imagination that assumes rationality to be of a certain format.

4.2. Hypothetical reasoning. Hypothetical reasoning is counterfactual, as it is of the form: "if p then q, although p is not the case". This form of if-then reasoning is essential to any kind of technological enterprise.

If we do p, what would be the consequence? It is important to address this question before in fact doing p. In order to carry out any more specific hypothetical reasoning within the domain of technology, mathematics is brought in action. A mathematical model comes to represent an imagined situation, and the model becomes the basis for identifying what could be the implications of doing what was imagined. However, the model-determined implications are just *calculated* implications. It is far from obvious what might be the relationship between such *calculated* implications and real-life consequences of completing the technological enterprise. The identification of implications, based on formal calculations, assumes that the mathematical model adequately represents what is to be implemented. But this assumption rests upon the mechanical world view claiming that the primary-secondary distinction imposed by the mathematical format of the model is adequate for identifying implications. In other words the assumption is that what the model downgrades as secondary is in fact secondary for identifying implications. However, this is a deeply metaphysical assumption. It is a questionable assumption that relevant implications are of a mechanical nature, and can be indentified through formal calculations. Yet this assumption accompanies any mathematics-based hypothetical reasoning.[18]

4.3. Legitimation or justification. According to a classic perspective in philosophy, justification refers to a proper and genuine logical support of a statement, of a decision, or of an action, while the notion of legitimation does not include such an assumption. The point of providing a legitimation of an action might be to make it appear, *as if* it is justified. When a mathematical

[18]Risks emerge from the fact that mathematical modelling is, in this way, a technique for overlooking. The emergence of the risk society is partly due to of the development of mathematics-based hypothetical reasoning that to mathematics-based actions in general.

model is brought into effect, it can serve as both a legitimation and a justification. It can help to provide priorities, although the basis for doing so might be obscure.

Let me try to illustrate this with a quotation from an article "The Predator War" by Jane Mayer in *The New Yorker*, which addresses US use of unmanned aircraft which can be used for identifying targets and for launching missiles. The Pentagon has created formulas to help the military develop a taxonomy of targets: "A top military expert, who declined to be named, spoke of the military's system, saying, 'There's a whole taxonomy of targets.' Some people are approved for killing on sight. For others, additional permission is needed. A target's location enters the equation, too. If a school, hospital, or mosque is within the likely blast radius of a missile, that too is weighed by a computer algorithm before a lethal strike is authorized."[19]

Although the particular details of such "elaborate formulas" for helping the military most likely will remain a military secrete, we can speculate about the kind of rationality that is reflected in the taxonomy of targets. In principle, one could assume that an automatic connection between the processes of calculation and the military action has been established. However, according to the article one should assume that the decision—firing or not firing—is a human decision, although guided by the taxonomy.

We could imagine that the development of the taxonomy is of a cost-benefit format. On the benefit side must be counted the importance of the target, and the likelihood that the target will in fact be eliminated by the strike. But, most certainly, many other military gains could be considered. The costs of the action also have to be estimated, which implies a range of parameters to be considered. First one could think of the death of American soldiers, but as in this case we are dealing with unmanned aircraft this parameter might not enter into the cost-calculations. However, the value of the airplane must be included, although reduced by the rather small likelihood that the plane will get lost in the operation. The value of the missile fired will clearly represent a cost. But there are more parameters to consider: non-targeted people might be killed, and, as pointed out, the target could be located close to schools, hospitals or mosques. How does a school become "weighted" by a computer algorithm? Through the number of school children expected killed? Or through the economic value of such a child? Or perhaps it is not the school children as such that are valued, but the negative PR the bombing of school might cause?

The crucial point of cost-benefit analysis is that costs and benefits are measured by the same units. But which? What is the shared unit for cost and benefits, encompassing the value of fired missiles, American soldiers, school

[19]Brian Greer drew my attention to this quotation. See the whole article at: http://www.newyorker.com/reporting/2009/10/26/091026fa_fact_mayer?currentPage=all See also Greer (in print).

children, hospitals, mosques, etc.? One might label the stipulation of shared units of measurement for cynical equations. Such equations are necessary for any cost-benefit analysis and for turning a process of decision-making into a process of calculation. Cynical equations are made possible when a mechanical world-view is forced on the domain in question. All human matters are nominated "appearance", while reality is constituted by what might be captured by mathematics. Originally, the appearance-reality distinction nominates the mechanical world view with respect to nature. However, when mathematics is applied to human enterprises, the appearance-reality distinction makes human matters secondary with respect to the enterprise in question. The "primary" takes a mechanical format captured by predesigned scales of measurement—and cynical equations might come to appear both natural and neutral. Cynical equations stem from the imposed mechanical world view, and they enter smoothly into the automated procedures for formal manipulations. The formulation of cynical equations blurs the distinction between legitimisations and justifications. This not only applies to military action, but to any action—in engineering, economy, business, administration—where a mathematics-based taxonomy might provide a suspicious legitimation with a glimmer of justification.

4.4. Realisation. A mathematical model can become part of our environment. Our life-world is formed through techniques and practices as well as through categories and discourses emerging from mathematics in action. Technology is not something "additional" which we can put aside, as if it were a simple tool, like a hammer. We live in a technologically structured environment, a techno-nature. Our life-world is situated in this techno-nature, and we cannot even imagine what it would mean to eliminate technology from our environment. Just try to do the subtraction piece by piece. We remove the computer, the credit card, the TV set, the phone. And we continue by removing medicine, newspapers, cars, bridges, streets, shoes. We have no idea about what kind of life-world such a continued subtraction would bring us. In this sense our life-world is submerged in techno-nature.[20]

Mathematics is an integral part of both techno-nature and life-world. Thus computers, credit cards, TV sets, phones, medicine, newspapers, cars, bridges, streets, and shoes are today produced by means of processes packed with mathematics. But not only the objects which make part of our techno-nature are formatted through mathematics; so are many practices. Mathematics establishes routines: in production, in business, in all economic affairs, in daily life.

The whole domain of relevant knowledge for decision making at the stock market—buying or selling—is mathematised and made available through figures and diagrams. In this way mathematics can provide a highly relevant descriptive tool. One could also imagine that algorithms make proposals as to which decisions to make. However, there is a step more that can be taken. One might

[20]For a discussion of the notion of life-word, see Skovsmose (2009).

imagine that the very decision about selling and buying is in fact made by a mathematical algorithm. The Danish newspaper, *Politiken*, in its edition of the 24$^{\text{th}}$ of February 2010 contains an article, "Maskinen overtager den globale brshandel" ("The machine takes over the global stock market") by Jeremy Grant and Michael Mackenzie, whose point is exactly that the very decision-making is placed in the hands of algorithms. Furthermore the newspaper contains an article by Per Thiemann stating that 20% of the selling and buying at the Copenhagen Stock Market is conducted by the computer. This is an example of mathematics coming to be a direct part of the economic reality.

The overall implication of this is that the nature into which we are submerged is of a mechanical format. Techno-nature is a complex mathematics-based construction. Through mathematics in action, we are in fact bringing our social, political, and economic environment deeper into a mechanical format.

5. Dissolution of Responsibility

An action may be associated with an acting subject, this being a person or institution that conducts the action. Generally, the acting subject is held responsible for the action. This responsibility, however, can be questioned if the acting subject might have been forced to perform the action, or if they had been unaware of the full range of implications of the action.

However, mathematics-based actions often appear to be missing an acting subject. As a consequence, mathematics-based actions easily appear to be conducted in an ethical vacuum. As an illustration, one could think of automatic selling-buying decisions made at the stock market, as referred to previously. Such decisions are merged into automated clusters of decisions, and large quantities of such clusters have implications far beyond what is normally expected. It is in fact possible to relate features of the world-wide economic crisis to such mathematics-based avalanches of decisions. But who could be held responsible? Somehow responsibility seems to dissolve.

An example of such a possible dissolution of responsibility is presented by Mario Snchez (2009, 2010) in his discussion of a "marginalisation index". This index has been applied in a Mexican context in trying to invent measures for the degree of marginalisation which certain communities might suffer. Naturally, there can be many different ways of measuring marginalisation, but whatever modelling is applied, some parameters have to be introduced and related, and some standards have to be introduced so that the entire social, political and economic processes of marginalisation emerge in a modelled format. Here may occur an extreme form of primary-secondary ranking, where the experienced characteristics of marginalisation are "abstracted away", in favour of only concentrating on quantitative and "mechanical" features of marginalisation. On this basis political action might be taken, or not taken. Such an approach has many implications, one of which might be that new criteria are formulated and claimed to be "objective".

It might be claimed that mathematics helps to establish objectivity in calculations and that mathematics-based actions are well-considered and represent the optimal course to be taken. However, mathematics might also introduce a certain amount of arbitrariness into the decision-making process, as can be illustrated by the "cynical equations". Arbitrariness might be covered by an overwhelming mass of formal calculations and formalities that may endow the result with a perceived necessity, although a subjective and impart necessity. This impartation draws on the whole metaphysics that accompanies mathematics. It does so by imposing a mechanical world view. This also applies to the mathematical marginalisation model. Through the impartation of necessity, elimination of responsibility becomes part of mathematics in action.

6. Symbolic Power, Beyond Good and Evil?

The duality between good and evil is deep-rooted in many philosophical discourses. But when we consider the symbolic power associated with mathematics, it might be relevant to try to step outside the good-evil duality. Symbolic power opens a space for technological enterprises that can be problematic, unfair, blind, helpful, ruthless, benevolent, productive, risky, innovative, etc. Such qualities cannot be described along a good-evil axis.

It could well be that mathematics imposes much on the domain it is assumed to describe. Mathematics can impose priorities concerning what is primary and what to relegate as secondary. Mathematics can become part of action by forming conceptions about what can be constructed, designed and accomplished. It can structure the as-if reasoning through which the viability of an action is addressed. It can provide patterns for justification and legitimation. It can come to make part of reality as an integral part of what has been implemented. Finally, mathematics in action might miss an "acting subject" and let responsibility dissolve. However, we cannot assume that we are in a position to provide any straightforward evaluation of such features of symbolic power.

My conclusion is not to try to eliminate or to obstruct the symbolic power that might be rooted in mathematics. Thus there is no point in claiming that the distinctions between appearance and reality and between sense and reference are "bad" distinctions, nor can they be claimed to be "good". They are distinctions that may facilitate powerful symbolic actions. My point is to address this power explicitly, and to try to identify its possible dimensions.

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the second will serve as a basis for my participation in the conference *Mathematics Education and Contemporary Theory*, to be held in Manchester in July 2011.

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