An Occurrence Description Logic

Badie, Farshad; Götzsche, Hans

Published in:
Logical Investigations

DOI (link to publication from Publisher):
10.21146/2074-1472-2022-28-1-142-156

Publication date:
2022

Document Version
Publisher's PDF, also known as Version of record

Логические исследования
2022. Т. 28. № 1. С. 142–156
УДК 16

Logical Investigations
2022, Vol. 28, No. 1, pp. 142–156
DOI: 10.21146/2074-1472-2022-28-1-142-156

Логика и язык
Logic and language

Farshad Badie, Hans Götzsche

An occurrence description logic

Farshad Badie
Faculty of Computer Science & Informatics, Berlin School of Business & Innovation,
Potsdamer Straße 180-182, 10783 Berlin, Germany.
Aalborg University,
Rendsburggade 14, 9000 Aalborg, Denmark.
E-mail: badie@id.aau.dk

Hans Götzsche
Aalborg University,
Rendsburggade 14, 9000 Aalborg, Denmark.
E-mail: goetzsche@id.aau.dk

Abstract: Description Logics (DLs) are a family of well-known terminological knowledge representation formalisms in modern semantics-based systems. This research focuses on analysing how our developed *Occurrence Logic (OccL)* can conceptually and logically support the development of a description logic. OccL is integrated into the alternative theory of natural language syntax in *Deviational Syntactic Structures* under the label 'EFA(X)3' (or the third version of Epi-Formal Analysis in Syntax, EFA(X), which is a radical linguistic theory). From the logical point of view, OccL is a formal logic that mainly deals with the occurrences of symbols as well as with their priorities within linguistic descriptions, i.e. natural language syntax, semantics and phonology. In this article — based on our OccL-based definitions of the concepts of *strong implication* and *occurrence value* as well as of the logical concept *Identical Occurrence Constructor (IDOC)* that is the most fundamental logical concept in our formalism — we will model *Occurrence Description Logic* ($\mathcal{ODL}$). Accordingly, we will formally-logically analyse 'occurrence(s) of symbol(s)' within descriptions of the world in $\mathcal{ODL}$. In addition, we will analyse and assess the logical concepts of *occurrence* and *occurrence priority* in $\mathcal{ODL}$. This research can make a strong logical background for our future research in the development of a Modal Occurrence Description Logic.

Keywords: description logic, logic & language, occurrence, occurrence logic, occurrence priority, occurrence value

For citation: Badie F., Götzsche H. "An occurrence description logic", *Logicheskie Issledovaniya / Logical Investigations*, 2022, Vol. 28, No. 1, pp. 142–156. DOI: 10.21146/2074-1472-2022-28-1-142-156

1. Introduction

This research focuses on sketching out how our developed *Occurrence Logic (OccL)*, originally based on an alternative theory of natural language syntax, can conceptually and logically support the development of a description logic (as a formal description logic). OccL is integrated into the alternative theory of natural language syntax in 'Deviational Syntactic Structures' under the label 'EFA(X)3' (or the third version of Epi-Formal Analysis in Syntax, EFA(X), which is a radical linguistic theory), see [Götzsche, 2013] (Chapter 3). Beyond linguistics, OccL has been designed as an alternative kind of logic considering how some specific phenomena, including time, could be logically conceptualised and realised by means of a combinatorial system. [Götzsche, 2013] presents some conceptions about how the ensuing formal system can be transformed into a model of the cognitive mechanism that handles natural language syntax based on defined philosophical and theoretical foundations.

As mentioned above, this research focuses on the conceptual-logical analysis of a description logic based on OccL. Description Logics (DLs) are among the most widely-used knowledge representation formalisms. They are also known as terminological logics. DLs are designed based on semantic networks [Quillian, 1968] and frame-based systems [Minsky, 1974]. Most DLs are decidable fragments of predicate logic (PL). More specifically, DLs are PL-based terminological systems that attempt to represent knowledge, by means of a formal semantics, in order to establish a common ground for human and machine interactions, see [Baader et al., 2017a; Sikos, 2017].

This research, based on [Badie, 2018], provides a logical background for the development of an *Occurrence Description Logic* ($\mathcal{ODL}$). Accordingly, it focuses on formal-logical, as well as semantic, analysis of 'the occurrence(s) of symbols' within descriptions of the world. Correspondingly, the concepts of 'occurrence' and 'occurrence priority' are analysed in $\mathcal{ODL}$. We finally offer several fundamental axioms in order to deal with the possibility and the necessity of the $\mathcal{ODL}$-based descriptions of 'the occurrences of symbols and of their priorities' and to make a backbone for our future research in the development of a Modal Occurrence Description Logic.

2. Occurrence Logic

Occurrence Logic (OccL) is a formal logic the capability of which is to describe the occurrence of things/symbols and their interrelations as co-occurrences. In more adequate words, OccL mainly deals with the occurrences of individual symbols as well as with their precedence(s).

Definition (Strong Implication). OccL describes the term 'the symbol y occurs in case and only in case the symbol z occurs' by the description '$z \circ > y$'. This logical description is called *strong implication*. It should be noticed that z and y are not propositions, and therefore not associated with truth values, but only 'individual' symbols that either are present (do occur) or are absent (do not occur) in our descriptions.

Definition (Occurrence Value). The strong implication '$z \circ > y$' can logically be valid (and meaningful) or invalid (and meaningless). According to OccL, depending on the meaning of the operator '$\circ >$', the symbols z and y in '$z \circ > y$' either have the occurrence value of *occurrence* (that can be semantically represented by 1) or have the occurrence value of *non-occurrence* (that can be semantically represented by 0).

We shall draw your attention to the following valid logical argument:

(1) $z \circ > y$
(2) y occurs.
Conclusion: z occurs.

Regarding this argument — knowing the facts that (1) the symbol y occurs in case and only in case the symbol z occurs, and (2) the symbol y occurs, we can conclude, for certain, that the symbol z occurs. In fact, considering this argument, it follows by necessity from 'the occurrence of y' that 'z occurs'. Hence, (1) and (2) collectively give us a reason to accept that an occurrence can take place (i.e. has the occurrence value of 'occurrence').

3. Description Logics

Description Logics (DLs) are a family of well-known terminological knowledge representation formalisms, see [Baader et al., 2017a; Baader et al., 2017b; Sikos, 2017]. In DLs the formalism is structured based on 'individuals', 'concepts', and 'roles'. An *individual* is equivalent to a constant symbol in predicate logic. Individuals are the instances of concepts. A *concept* (or class) corresponds to one, or more, distinct entities. Concepts are interpreted to be equivalent to unary predicates in predicate logic. Also, a *role* expresses a relationship between individuals or it assigns a property to an individual. Any role is equivalent to a n-ary (for $n \geq 2$) predicate in predicate logic.

In DLs concepts and their interrelationships create terminologies. In fact, terminologies are expressible in the form of hierarchical structures. There are three kinds of atomic symbols (that are elementary descriptions from which we inductively build more-specified descriptions based on logical constructors): (i) individuals, e.g., `bob`, `pink`, (ii) atomic concepts, e.g., *Human*, *Colour*, and

(iii) atomic roles, e.g., *isEating*, *hasColour*. Taking into consideration the relations of various valences in DL-based descriptions, any individual symbol (e.g., `mary`, `google`, `y`) is related to itself by means of the relation of valence 0. The descriptions 'John is a philosopher' (formally speaking: $\mathit{Philosopher}(\texttt{john})$) and 'blue is a colour' (formally: $\mathit{Colour}(\texttt{blue})$) are structured based on the relations of valence 1. Also, the descriptions 'John is married to Mary' (formally: $\mathit{marriedTo}(\texttt{john}, \texttt{mary})$) and 'Bob is the friend of Alice' (formally: $\mathit{hasFriend}(\texttt{alice}, \texttt{bob})$) are structured based on the relations of valence 2.

The set of main logical symbols in $\mathcal{ALC}$ (the Attributive Concept Language with Complements that is the prototypical description logic) is conjunction ($\sqcap$), disjunction ($\sqcup$), negation ($\neg$), implication ($\rightarrow$), equivalence ($\equiv$), existential quantification ($\exists$), universal quantification ($\forall$), the concept of tautology/truth ($\top$), the concept of contradiction/falsity ($\bot$).

In DLs, the underlying collections of facts and assumptions (in any formal system) usually consists of terminological and assertional axioms. Terminological axioms are, in fact, terminological structures of formal descriptions. They are the fundamental terminological building blocks of a formal description. Considering C and D as two concepts, and R and S as two roles, terminological axioms are in the forms: (1) $C \sqsubseteq D$ that represents a *concept subsumption*, (2) $R \sqsubseteq S$ that represents a *role subsumption*, (3) $C \equiv D$ that represents a *concept equality*, and (4) $R \equiv S$ that represents a *role equality*. In addition to terminological axioms, assertional axioms are defined. Assertional axioms are the most fundamental descriptions of the world. Considering A as an atomic concept, r as an atomic role, and $\texttt{a}$ and $\texttt{b}$ as individual symbols, assertional axioms are either *concept assertions* (in the form of $A(\texttt{a})$) or *role assertions* (in the form of $r(\texttt{a}, \texttt{b})$).

In order to define a formal semantics, we need to utilise *terminological interpretations*. A terminological interpretation consists of (1) the interpretation domain, like Δ, that is a non-empty set, and (2) the interpretation function, like $\mathcal{I}$, that assigns every individual symbol $\texttt{a}$ to an element $\texttt{a}^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. Table 1 represents the syntax and semantics of concept constructors in $\mathcal{ALC}$. Also, Table 2 reports DLs terminological and assertional axioms.

4. An Occurrence Description Logic ($\mathcal{ODL}$)

In this research our most central assumption is that we need a formalism that utilises individual symbols (e.g., `y`, `z`, `john`, `red`) under the concept of *Occurrence*. Accept that we have defined the concept *StudentOfbrian* as 'those student(s) who are taught by Brian'. Formally speaking, we can offer the following DL-based concept description:

$$\mathit{StudentOfbrian} \equiv (\mathit{Student} \sqcap \exists \mathit{teaches}^{-}.\texttt{brian}) \quad (*)$$

Table 1. $\mathcal{ALC}$ Syntax and Semantics

Syntax	**Semantics**
A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
$\top$	$\Delta^{\mathcal{I}}$
$\perp$	$\emptyset$
$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \wedge D^{\mathcal{I}}$
$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \vee D^{\mathcal{I}}$
$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
$\exists r.C$	$\{\mathtt{a} \mid \exists \mathtt{b}.(\mathtt{a}, \mathtt{b}) \in r^{\mathcal{I}} \wedge \mathtt{b} \in C^{\mathcal{I}}\}$
$\forall r.C$	$\{\mathtt{a} \mid \forall \mathtt{b}.(\mathtt{a}, \mathtt{b}) \in r^{\mathcal{I}} \rightarrow \mathtt{b} \in C^{\mathcal{I}}\}$

Table 2. Terminological and Assertional Axioms

Name	**Syntax**	**Semantics**
concept subsumption axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role subsumption axiom	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
concept equality axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role equality axiom	$R \equiv S$	$R^{\mathcal{I}} = S^{\mathcal{I}}$
concept assertion	$C(\mathtt{a})$	$\mathtt{a}^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(\mathtt{a}, \mathtt{b})$	$(\mathtt{a}^{\mathcal{I}}, \mathtt{b}^{\mathcal{I}}) \in R^{\mathcal{I}}$

In this formal description, $\exists teaches^{-}$ is an inverse role that is formalised in order to relate us to the role *havingStudent* (of the individual `brian`) and in fact, to the specified concept *StudentOfbrian*. However, the salient problem is that this concept description would not work for the following two reasons: (1) `brian` cannot be both 'an individual' and 'a concept name of the predicate', and (2) if we were to allow Brian to be in the place of a concept (say *Brian*), we would need to say what this means for *Brian*'s interpretation. In fact, based on every interpretation $\mathcal{I}$, $Brian^{\mathcal{I}}$ would be an element of the interpretation domain, but concepts are interpreted as sets of elements.

According to [Baader et al., 2017b], in order to enable the use of individual names in concepts and avoid the problems mentioned, nominals can be utilised. In fact, we can accept that we need a description logic that has nominals as 'additional concepts'. Considering the individual symbol `a`, for the interpretation $\mathcal{I}$, the mapping $.^{\mathcal{I}}$ can be extended as $\{\mathtt{a}\}^{\mathcal{I}} = \{\mathtt{a}^{\mathcal{I}}\}$. Consequently, by utilising the interpretation $\mathcal{I}$, it is possible to redefine (*) by:

$$StudentOfbrian \equiv (Student \sqcap \exists teaches^{-}.\{\mathtt{brian}\})$$

In fact, by placing curly brackets around the individual name `brian`, we have transformed the individual `brian` into a concept. Now suppose that Mary is a student of Brian. According to the analysed formal model, we can offer the following logical description:

$$\{\mathtt{mary}\} \equiv (Student \sqcap \exists teaches^{-}.\{\mathtt{brian}\}) \quad (**)$$

Here the main question is that how we can reflect such a logical model in our OccL-based formal model in order to define an *Occurrence Description Logic ($\mathcal{ODL}$)*. It should be observed that $\mathcal{ODL}$ is analysable based on the logical assumption that the concept *Occurrence* is the most fundamental logical concept in our formalism. We are thus going to offer an $\mathcal{ODL}$-based description of the occurrence(s) of symbols in our logical descriptions.

Now we need to offer two important definitions:

Definition (Identical Occurrence Constructor). An *identical occurrence constructor (IDOC)*, or formally representing, '$^\circ(\quad)$', is defined in order to logically turn an individual symbol into an 'occurred'/'occurring' concept. It can be interpreted that any IDOC is a kind of role that expresses 'to be (and becoming) Occurred'. In more accurate words, an IDOC makes a logical interrelationship between an individual symbol and the concept of 'Occurrence'. Hence an IDOC makes an *identification* for an individual's occurrence.

Definition (Occurrence Interpretation). An *occurrence interpretation* (formally representing, $\mathcal{IO}$) is defined in order to interpret the occurrence values of individual symbols (that have actually been turned into occurring concepts) within logical descriptions. An $\mathcal{IO}$ is structured based on the following elements: (i) the *occurrence interpretation domain* ($^\circ\Delta$) that consists of any occurring concept that may occur within our descriptions, and (ii) the *occurrence interpretation function* (formally, $.^{\mathcal{IO}}$) that assigns every occurring concept, like $^\circ(\mathtt{a})$, to the element $^\circ(\mathtt{a})^{\mathcal{IO}}$ in order to express `a`'s occurrence (where `a` stands for an individual symbol). Accepting A as any possible atomic concept, we can define 'the occurrence of an atomic concept' by $^\circ A$. Obviously, as analysed above, any atomic concept is in the form of '$^\circ(\mathtt{a})$'. Therefore, semantically we have: $^\circ A^{\mathcal{IO}} = {}^\circ(\mathtt{a})^{\mathcal{IO}} \subseteq {}^\circ\Delta^{\mathcal{IO}}$.

At this point we shall stress the fact that this research relies on the assumption that the relationship(s) between occurrence interpretations and Kripke models (or, more specifically, the states of a Kripke frame that can be regarded as various states of the world ([Kripke, 1963]) is established as follows:

1. The states of the world are seen as the elements of the domain of an occurrence interpretation (that are occurring concepts that may occur within any possible description).

2. The propositions, which can be either valid or invalid at any state of the world, can be regarded as the products of possible interpretations of concepts over the interpretation domain. Assessed by $\mathcal{ODL}$, this means that a proposition — that is either expressing 'an occurrence in the form of being' or 'an occurrence in the form of not-being' at any state of the world — is produced based on the occurrence interpretation of the occurrence(s) of concept(s) over the occurrence interpretation domain in a DL world description.

According to (**), we can conclude that $|^\circ(\texttt{mary})|^{\mathcal{IO}} = 1$ and $|^\circ(\texttt{brian})|^{\mathcal{IO}} = 1$. In this example, the individual $\texttt{mary}$ is the instance of the concept *Student* (i.e. $\texttt{mary} \in \mathit{Student}$). Also, the occurring concept $^\circ(\texttt{mary})^{\mathcal{IO}}$ is interpreted as the sub-concept of the concept $^\circ\mathit{Student}$. Formally speaking, $^\circ(\texttt{mary}) \sqsubseteq\ ^\circ\mathit{Student}$. Therefore, we can interpret that $^\circ(\texttt{mary})^{\mathcal{IO}} \subseteq\ ^\circ\mathit{Student}^{\mathcal{IO}}$. In addition, the individual $\texttt{brian}$ is the instance of the concept *Teacher* (i.e. $\texttt{brian} \in \mathit{Teacher}$). Also, the occurring concept $^\circ(\texttt{brian})^{\mathcal{IO}}$ is interpreted as the sub-concept of the concept $^\circ\mathit{Teacher}$. Formally speaking, $^\circ(\texttt{brian}) \sqsubseteq\ ^\circ\mathit{Teacher}$. Then it can be interpreted that $^\circ(\texttt{brian})^{\mathcal{IO}} \subseteq\ ^\circ\mathit{Teacher}^{\mathcal{IO}}$.

Definition (Occurrence Negation). The *occurrence negation* (formally representing: '$^\circ\neg$') of 'the occurrence of an individual symbol' (that has, utilising an IDOC, been represented in the form of an occurring concept) expresses the negation of the occurrence value of an occurring concept. The logical expression '$^\circ\neg\ ^\circ(\texttt{a})$' represents the occurrence negation of the occurring concept $^\circ(\texttt{a})$.

- Suppose that the concept $^\circ(\texttt{a})$ has the occurrence value of 'occurrence'. Therefore, its negation, or $|^\circ\neg\ ^\circ(\texttt{a})|$, has the occurrence value of 'non-occurrence'.

- Suppose that the concept $^\circ(\texttt{a})$ has the occurrence value of 'non-occurrence'. Therefore, its negation, or $|^\circ\neg\ ^\circ(\texttt{a})|$, has the occurrence value of 'occurrence'.

Definition (Occurrence Conjunction). The *occurrence conjunction* (formally, $^\circ\sqcap$) of 'the occurrences of two, or more, individual symbols (that are represented in the form of two, or more, occurring concepts)' expresses the conjunction of the occurrence values of those occurring concepts. Accepting a and b as two individual symbols, $^\circ(\mathtt{a})\ {}^\circ\sqcap\ {}^\circ(\mathtt{b})$ expresses the occurrence conjunction of the occurrences of a and b in a description. Semantically: $|^\circ(\mathtt{a})\ {}^\circ\sqcap\ {}^\circ(\mathtt{b})|^{\mathcal{IO}} \to\ {}^\circ(\mathtt{a})^{\mathcal{IO}}\ {}^\circ\wedge\ {}^\circ(\mathtt{b})^{\mathcal{IO}}$.

Definition (Occurrence Disjunction). The *occurrence disjunction* (formally, $^\circ\sqcup$) of 'the occurrences of two, or more, individual symbols (that are represented in the form of two, or more, occurring concepts)' expresses the disjunction of the occurrence values of those occurring concepts. Taking into consideration the individual symbols a and b, $^\circ(\mathtt{a})\ {}^\circ\sqcup\ {}^\circ(\mathtt{b})$ expresses the occurrence disjunction of the occurrences of a and b in a description. Semantically: $|^\circ(\mathtt{a})\ {}^\circ\sqcup\ {}^\circ(\mathtt{b})|^{\mathcal{IO}} \to\ {}^\circ(\mathtt{a})^{\mathcal{IO}}\ {}^\circ\vee\ {}^\circ(\mathtt{b})^{\mathcal{IO}}$.

Definition (Occurrence Universal Quantification). The *occurrence universal quantification* of 'the occurrence of the individual symbol a (that has been expressed in the form of an occurring concept)' is represented by $^\circ\forall\ {}^\circ(\mathtt{a})$. The validity of $^\circ\forall\ {}^\circ(\mathtt{a})$ expresses the existence of the occurring concept $^\circ(\mathtt{a})$ in all possible descriptions.

Accept that $^\circ\forall\ {}^\circ(\mathtt{a})$ has the occurrence value of 'occurrence'. Therefore, we can conclude that the symbol a in our all possible descriptions occurs.

Definition (Occurrence Existential Quantification). The *occurrence existential quantification* of 'the occurrence of the individual symbol a (that has been expressed in the form of an occurring concept)' is represented by $^\circ\exists\ {}^\circ(\mathtt{a})$. The validity of $^\circ\exists\ {}^\circ(\mathtt{a})$ expresses the existence of the occurring concept $^\circ(\mathtt{a})$ in, at least, one possible description.

Accept that $^\circ\exists\ {}^\circ(\mathtt{a})$ has the occurrence value of 'occurrence'. Hence we can conclude that the symbol a in — at least — one possible description occurs.

Definition (Concurrency). The concept of *concurrency* is expressible based on the logical concept '*occurrence equality*' (formally, $^\circ =$). Any 'occurrence equality' is defined between two occurring concepts (e.g., $^\circ(\mathtt{a})\ {}^\circ = {}^\circ(\mathtt{b})$) in order to express that they occur concurrently. The logical concurrency $^\circ(\mathtt{a})\ {}^\circ = {}^\circ(\mathtt{b})$ is translatable into $(\mathtt{b}\ {}^\circ > \mathtt{a})\ {}^\circ\sqcap\ (\mathtt{a}\ {}^\circ > \mathtt{b})$. Equivalently we have: $\mathtt{b}\ {}^\circ > \mathtt{a}\ {}^\circ > \mathtt{b}$. This means that 'b occurs in case and only in case a occurs, and a occurs in case and only in case b occurs'. Hence, we have the concurrency (co-occurrence) of a and b.

5. Occurrence Priority in $\mathcal{ODL}$

Regarding the strong implication $\mathtt{z}\ {}^\circ > \mathtt{y}$, the individual symbol y occurs in case and only in case the individual symbol z occurs. According to such a logical interrelationship between the occurrences of two individual symbols, it can be interpreted that the individuals z and y have been related to each other by means of 'their occurrences'. Considering the strong implication $\mathtt{z}\ ^{\circ}>\mathtt{y}$, there is a logical relationship (let us name it '*priority relation*' to make it more understandable) between y and z. It shall be concluded that the 'priority' relation is the most significant occurring role in $\mathcal{ODL}$. We shall draw your attention to the following definition.

Definition (Identical Occurrence Priority Constructor). An *Identical Occurrence Priority Constructor (IDOPC)* of the occurring concepts $^{\circ}(\mathtt{a})$ and $^{\circ}(\mathtt{b})$ is represented by $^{\circ}(^{\circ}(\mathtt{a}),^{\circ}(\mathtt{b}))$. Any IDOPC is defined in order to turn the 'priority' relationship between two individual symbols into an occurring role.

According to '$^{\circ}(^{\circ}(\mathtt{a}),^{\circ}(\mathtt{b}))$', an IDOPC has made an *identifier* for the priority of a's occurrence. Consequently, we can regard any IDOPC as a kind of role that expresses the concept of 'to be (and becoming) preferentially occurred'.

6. Semantic Interpretation of Occurrence Priority

As mentioned above, an occurrence interpretation ($\mathcal{IO}$) is utilised in order to interpret the occurrence values of individual symbols (that have actually been turned into occurring concepts) within our logical descriptions. We have already analysed any occurrence interpretation based on (1) the occurrence interpretation domain and (2) the occurrence interpretation function. Here, based on the concept of 'occurrence priority', we refocus on the definition of occurrence interpretation and deploy it.

We need to take into account that the occurrence interpretation function must be able to assign to every priority role between the symbols a and b (that, in fact, links two occurring concepts $^{\circ}(\mathtt{a})$ and $^{\circ}(\mathtt{b})$ together), the relationship $^{\circ}(^{\circ}(\mathtt{a}),\ ^{\circ}(\mathtt{b}))^{\mathcal{IO}}$ in order to interpret the occurrence of the priority of $^{\circ}(\mathtt{a})$ in addition to the interpretation of the occurring concepts $^{\circ}(\mathtt{a})$ and $^{\circ}(\mathtt{b})$. Actually considering r as any possible atomic role in DL, we can define 'the occurrence of an atomic role' by $^{\circ}r$. Accordingly, any $^{\circ}r$ is in the form of $^{\circ}(^{\circ}(\mathtt{a}),\ ^{\circ}(\mathtt{b}))$ in $\mathcal{ODL}$. Therefore, semantically we have: $^{\circ}r^{\mathcal{IO}} = \ ^{\circ}(^{\circ}(\mathtt{a}),\ ^{\circ}(\mathtt{b}))^{\mathcal{IO}} \subseteq \ ^{\circ}\Delta^{\mathcal{IO}} \times \ ^{\circ}\Delta^{\mathcal{IO}}$.

Subsequently, taking into consideration the relationship(s) between occurrence interpretations and Kripke models ([Kripke, 1963]), we can state that the accessibility relations between various states of the world (within $\mathcal{ODL}$ descriptions), can be seen as the products of occurrence interpretations of occurrence priorities over the occurrence interpretation domain.

Accordingly, utilising an IDOPC, we can transform the strong implication $\mathtt{z}\ ^{\circ}>\mathtt{y}$ into the $\mathcal{ODL}$-based description $^{\circ}(^{\circ}(\mathtt{z}),\ ^{\circ}(\mathtt{y}))$ that is a priority relation between the individuals z and y (as well as between the occurring concepts

$^\circ(\mathtt{z})$ and $^\circ(\mathtt{y})$). Note that the validity of $^\circ(^\circ(\mathtt{z}),\ ^\circ(\mathtt{y}))$ expresses that there is, necessarily, a 'priority relation' between the occurring concepts $^\circ(\mathtt{z})$ and $^\circ(\mathtt{y})$. Focusing on $^\circ(^\circ(\mathtt{z}),\ ^\circ(\mathtt{y}))$, semantically we have: $|^\circ(^\circ(\mathtt{z}),\ ^\circ(\mathtt{y}))|^{\mathcal{IO}} = 1$. This means that the existence of the strong implication $\mathtt{z}\ ^\circ\!> \mathtt{y}$ concludes that $|^\circ(\mathtt{y})|^{\mathcal{IO}} = 1$ in case and only in case $|^\circ(\mathtt{z})|^{\mathcal{IO}} = 1$.

Let us offer an example. Brian is mentioning the names of his highest-ranked students. Brian: "The first award goes to James. Congratulations, James! Also, the second award goes to Mary". According to Brian's description, we can conclude that the concept $^\circ(\mathtt{mary})$ occurs in case and only in case the concept $^\circ(\mathtt{james})$ occurs. Formally speaking, $|^\circ(^\circ(\mathtt{james}),\ ^\circ(\mathtt{mary}))^{\mathcal{IO}}| = 1$ and $|^\circ(^\circ(\mathtt{mary}),\ ^\circ(\mathtt{james}))^{\mathcal{IO}}| = 0$. It is also, based on $|^\circ(^\circ(\mathtt{james}),\ ^\circ(\mathtt{mary}))^{\mathcal{IO}}| = 1$, interpretable that the individual `mary` occurs in case and only in case the individual `james` occurs (in Brian's description).

7. $\mathcal{ODL}$ Syntax and Semantics

Based on the analysed logical structure of $\mathcal{ODL}$, this section summarises the syntax and semantics of $\mathcal{ODL}$.

1. An atomic concept $^\circ A$ is syntactically in the form of $^\circ(\mathtt{a})$. Semantically: $^\circ A^{\mathcal{IO}} = \ ^\circ(\mathtt{a})^{\mathcal{IO}}$. Note that $^\circ(\mathtt{a})^{\mathcal{IO}} \subseteq \ ^\circ\Delta^{\mathcal{IO}}$. In fact, the occurring concept $^\circ(\mathtt{a})$ is interpreted to be a sub-concept of 'the occurrence interpretation domain'.

2. An atomic role $^\circ r$ is syntactically in the form of $^\circ(^\circ(\mathtt{a}),\ ^\circ(\mathtt{b}))$. Semantically: $^\circ r^{\mathcal{IO}} = \ ^\circ(^\circ(\mathtt{a}),\ ^\circ(\mathtt{b}))^{\mathcal{IO}}$. Also, $^\circ(^\circ(\mathtt{a}),\ ^\circ(\mathtt{b}))^{\mathcal{IO}} \subseteq \ ^\circ\Delta^{\mathcal{IO}} \times \ ^\circ\Delta^{\mathcal{IO}}$. Informally speaking, an atomic role is interpreted to be a sub-role of the Cartesian product of the interpretation domain with itself.

3. The OccL-based strong implication $\mathtt{a}\ ^\circ\!> \mathtt{b}$ expresses a relationship between two symbols. By utilising IDOC, the $\mathcal{ODL}$ produces $^\circ(\mathtt{a})\ ^\circ\!> \ ^\circ(\mathtt{b})$ that is an equivalent logical description. Semantically: $^\circ(\mathtt{a})^{\mathcal{IO}}\ ^\circ\!\rightarrow \ ^\circ(\mathtt{b})^{\mathcal{IO}}$. This means that the concept $^\circ(\mathtt{b})$ occurs in case and only in case the concept $^\circ(\mathtt{a})$ occurs. In fact, the individual symbol `b` occurs in case and only in case the individual symbol `a` occurs.

4. The logical symbol '$^\circ\top$', or *Top Occurrence Concept*, represents the concept of *Occurrence*. This logical concept semantically equals 'the occurrence interpretation of the whole occurrence interpretation domain ($^\circ\Delta^{\mathcal{IO}}$)'.

5. The logical symbol '$^\circ\bot$', or *Bottom Occurrence Concept*, represents the concept of *Non-Occurrence*. This logical concept semantically equals 'the occurrence interpretation of the non-occurrence (or formally, $^\circ\phi^{\mathcal{IO}}$)'.

6. $^\circ(\mathtt{a})\ ^\circ\sqcap\ ^\circ(\mathtt{b})$ is the occurrence conjunction of two occurring concepts. Semantically we have: $^\circ(\mathtt{a})^{\mathcal{IO}}\ ^\circ\wedge\ ^\circ(\mathtt{b})^{\mathcal{IO}}$.

7. $^\circ(\mathtt{a})\ ^\circ\sqcup\ ^\circ(\mathtt{b})$ is the occurrence disjunction of two occurring concepts. Semantically we have: $^\circ(\mathtt{a})^{\mathcal{IO}}\ ^\circ\vee\ ^\circ(\mathtt{b})^{\mathcal{IO}}$.

8. $^\circ\neg\ ^\circ(\mathtt{a})$ is the occurrence negation of an occurring concept. Semantically we have: $(^\circ\neg\ ^\circ(\mathtt{a}))^{\mathcal{IO}}$ that equals to $^\circ\Delta^{\mathcal{IO}} \setminus\ ^\circ(\mathtt{a})^{\mathcal{IO}}$. It expresses all possible occurring concepts but not $^\circ(\mathtt{a})$.

9. $^\circ(\mathtt{a})\ ^\circ = {}^\circ(\mathtt{b})$ represents the occurrence equality of two occurring concepts. Semantically: $|^\circ(^\circ(\mathtt{b}),\ ^\circ(\mathtt{a}))|^{\mathcal{IO}} = |^\circ(^\circ(\mathtt{a}),\ ^\circ(\mathtt{b}))|^{\mathcal{IO}}$. This means that $\mathtt{a}$ and $\mathtt{b}$ occur concurrently.

8. Axiomatisation

This section focuses on $\mathcal{ODL}$-based representations of the possibility, impossibility and necessity of 'the occurrences of symbols and of their priorities' within descriptions in order to offer the most fundamental axioms.

1. The logical description $|^\circ\forall\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the necessity of occurrence of the individual $\mathtt{a}$.

   Specific Analysis. According to $|^\circ\forall\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, there is, always, an occurrence value of 'occurrence' for the individual $\mathtt{a}$. Therefore, $\mathtt{a}$ necessarily occurs. Note that $^\circ\forall\ ^\circ(\mathtt{a})$ is equivalent to $^\circ\neg\ ^\circ\exists\ ^\circ\neg\ ^\circ(\mathtt{a})$. Considering $|^\circ\neg\ ^\circ\exists\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, there is no occurrence value of 'non-occurrence' for the symbol $\mathtt{a}$. In fact, it is impossible that $\mathtt{a}$ does not occur.

2. The logical description $|^\circ\forall\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the necessity of non-occurrence of the individual $\mathtt{a}$.

   Specific Analysis. According to $|^\circ\forall\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, there is, always, an occurrence value of 'non-occurrence' for the individual $\mathtt{a}$. In fact, there is, necessarily, an occurrence value of 'non-occurrence' for $\mathtt{a}$. More specifically, it is necessary that $\mathtt{a}$ does not occur. $^\circ\forall\ ^\circ(\mathtt{a})$ is equivalent to $^\circ\neg\ ^\circ\exists\ ^\circ\neg\ ^\circ(\mathtt{a})$. Considering $|^\circ\neg\ ^\circ\exists\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, there is, necessarily, an occurrence value of 'non-occurrence' for the symbol $\mathtt{a}$. In fact, it is impossible that $\mathtt{a}$ occurs.

3. The logical description $|^\circ\forall\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the necessity of the occurrence of the strong implication $\mathtt{b}\ ^\circ> \mathtt{a}$.

Specific Analysis. Suppose that $|^\circ\forall\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$. So there is, necessarily, an occurrence value of 'occurrence' for the occurrence priority $^\circ(\mathtt{b},\mathtt{a})$. Actually it is necessary that the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$ occurs.

4. The logical description $|^\circ\forall\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the necessity of the non-occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$.

   Specific Analysis. Suppose that $|^\circ\forall\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$. Then there is, necessarily, an occurrence value of 'non-occurrence' for $^\circ(\mathtt{b},\mathtt{a})$. Actually it is necessary that the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$ does not occur.

5. The logical description $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the impossibility of the occurrence of the individual $\mathtt{a}$.

   Specific Analysis. According to $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, there exists no occurrence value of 'occurrence' for the individual $\mathtt{a}$. This means that the occurring concept $^\circ(\mathtt{a})$ is necessarily invalid. In fact, it is impossible to have the occurrence of the individual $\mathtt{a}$ (and in fact, it is impossible that $\mathtt{a}$ occurs). Note that $^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})$ is equivalent to $^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})$. Thereby, regarding $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, we can conclude that: $|^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$.

6. The logical description $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the impossibility of the non-occurrence of the individual $\mathtt{a}$.

   Specific Analysis. According to $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, there exists no occurrence value of 'non-occurrence' for $\mathtt{a}$. In fact, it is impossible to have the non-occurrence of the individual $\mathtt{a}$. In other words, it is impossible that $\mathtt{a}$ does not occur. Regarding $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, we can conclude that: $|^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$.

7. The logical description $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the impossibility of the occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$.

   Specific Analysis. Regarding $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$, there exists no occurrence value of 'occurrence' for the occurrence priority $^\circ(\mathtt{b},\mathtt{a})$. This means that $^\circ(\mathtt{b},\mathtt{a})$ is invalid. In fact, it is impossible to have the occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$. In other words, it is impossible that $\mathtt{b}\ ^\circ > \mathtt{a}$ occurs. Considering $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$, we can conclude that: $|^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$.

8. The logical description $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the impossibility of the non-occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$.

Specific Analysis. According to $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$, there exists no occurrence value of 'non-occurrence' for the occurrence priority $^\circ(\mathtt{b},\mathtt{a})$. In fact, it is impossible to have the non-occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$. In more adequate words, $\mathtt{b}\ ^\circ > \mathtt{a}$ necessarily occurs. According to $|^\circ\neg\ ^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$, we can conclude that: $|^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$.

9. The logical description $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the possibility of the occurrence of $\mathtt{a}$.

   Specific Analysis. According to $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, there exists an occurrence value of 'occurrence' for the symbol $\mathtt{a}$. In fact, the occurring concept $^\circ(\mathtt{a})$ can be valid and meaningful. In other words, 'the symbol $\mathtt{a}$ is sometimes an occurred symbol' (i.e., $\mathtt{a}$ sometimes occurs). In fact, it is possible that $\mathtt{a}$ occurs. Note that $^\circ\exists\ ^\circ(\mathtt{a})$ is equivalent to $^\circ\neg\ ^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})$. Therefore, considering $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$, it can be concluded that: $|^\circ\neg\ ^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 1$.

10. The logical description $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the possibility of the non-occurrence of $\mathtt{a}$.

    Specific Analysis. According to $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, there exists an occurrence value of 'non-occurrence' for $\mathtt{a}$. In fact, 'the symbol $\mathtt{a}$ is sometimes a non-occurred symbol' (i.e., $\mathtt{a}$ sometimes does not occur). This means that it is possible that $\mathtt{a}$ does not occur. Regarding $|^\circ\exists\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$, it can be concluded that: $|^\circ\neg\ ^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{a})|^{\mathcal{IO}} = 0$.

11. The logical description $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$ expresses the possibility of the occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$.

    Specific Analysis. According to $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$, there exists an occurrence value of 'occurrence' for the occurrence priority $^\circ(\mathtt{b},\mathtt{a})$. Subsequently, it is possible to have the occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$. Informally, it is possible that '$\mathtt{a}$ occurs in case and only in case $\mathtt{b}$ occurs'. According to $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$, we can conclude that: $|^\circ\neg\ ^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 1$.

12. The logical description $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$ expresses the possibility of the non-occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$.

    Specific Analysis. According to $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$, there exists an occurrence value of 'non-occurrence' for the priority $^\circ(\mathtt{b},\mathtt{a})$. Subsequently, it is possible to have the non-occurrence of the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$. In other words, it is possible that the strong implication $\mathtt{b}\ ^\circ > \mathtt{a}$ does not occur. Regarding $|^\circ\exists\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$, we can conclude that: $|^\circ\neg\ ^\circ\forall\ ^\circ\neg\ ^\circ(\mathtt{b},\mathtt{a})|^{\mathcal{IO}} = 0$.

9. Concluding Remarks and Future Work

Occurrence Logic (OccL) is a formal logic that mainly deals with the occurrences of symbols as well as with their priorities within linguistic descriptions, i.e. natural language syntax, semantics and phonology. This research has focused on sketching out how OccL can, conceptually and logically, support the development of an Occurrence Description Logic ($\mathcal{ODL}$). The research is initially structured based on the concepts of 'strong implication' (in order to express that a symbol occurs in case and only in case another symbol occurs) and 'occurrence values' (of 'occurrence' and 'non-occurrence'). Subsequently, based on the logical concept 'identical occurrence constructor (IDOC)' that is the most fundamental logical concept in our formalism, $\mathcal{ODL}$ is modelled. More specifically, IDOC is defined in order to logically turn an individual symbol into an 'occurred'/'occurring' concept. In more proper words, any IDOC makes a logical interrelationship between an individual symbol and the concept of 'occurrence'. Later on, 'the occurrence interpretation' based on 'the occurrence interpretation domain' and 'the occurrence interpretation function' is designed. Subsequently, other fundamental logical symbols are, syntactically and semantically, analysed. In order to deal with the concept of 'occurrence priority', the research has defined the identical occurrence priority constructors (IDOPC). In fact, an IDOPC expresses the occurrence priority of a symbol (in connection with other occurring/occurred symbol). Consequently, relying on the concepts of 'IDOC' and 'IDOPC', a semantic analysis of occurrence priority is offered. The paper has finally presented a syntax and semantic as well as several axioms for $\mathcal{ODL}$. Note that the offered axioms have — based on 'the occurrence existential quantification' and 'the occurrence universal quantification' — offered a connection between 'the $\mathcal{ODL}$-based descriptions of the occurrences of symbols and of their priorities' and the 'concepts of "necessity" and "possibility"'. Subsequently, our next research will, based on the outcomes of this research, be focused on the development of a Modal Occurrence Description Logic.

References

Baader et al., 2017a – Baader, F., Horrocks, I., Lutz, C., and Sattler, U. *An Introduction to Description Logic*. Cambridge: Cambridge University Press, 2017.

Baader et al., 2017b – Baader, F., Horrocks, I., Lutz, C., and Sattler, U. "A Basic Description Logic", in: *An Introduction to Description Logic*. Cambridge: Cambridge University Press, 2017, pp. 10–49.

Badie, 2018 – Badie, F. "Towards a Formal Symbolic Occurrence Logic", in: *The Meaning of Language*, ed. by Götzsche, H. Newcastle: Cambridge Scholars Publishing, 2018, pp. 290–300.

Götzsche, 2013 – Götzsche, H. *Deviational Syntactic Structures.* London / New Delhi / New York / Sydney: Bloomsbury Academic, 2013.

Kripke, 1963 – Kripke, S. "Semantical Considerations on Modal Logic", *Acta Phil. Fennica*, 1963, Vol. 16, pp. 83–94.

Minsky, 1974 – Minsky, M. *A Framework for Representing Knowledge.* Cambridge, MA, USA: Massachusetts Institute of Technology, 1974.

Quillian, 1968 – Quillian, R. "Semantic Memory", in: *Semantic Information Processing.* MIT Press, 1968, pp. 216–270.

Sikos, 2017 – Sikos, L.F. *Description Logics in Multimedia Reasoning.* Springer International Publishing AG, 2017.