Aalborg Universitet

Application of Leakage Localization Framework for Water Networks with Multiple Inlets in Smart Water Infrastructures Laboratory at AAU

Rathore, Saruch Satishkumar; Kallesøe, Carsten; Wisniewski, Rafal

Published in:
IFAC-PapersOnLine

DOI (link to publication from Publisher):
10.1016/j.ifacol.2022.07.170

Creative Commons License
CC BY-NC-ND 4.0

Publication date:
2022

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Rathore, S. S., Kallesøe, C., & Wisniewski, R. (2022). Application of Leakage Localization Framework for Water Networks with Multiple Inlets in Smart Water Infrastructures Laboratory at AAU. *IFAC-PapersOnLine*, *55*(6), 451-457. https://doi.org/10.1016/j.ifacol.2022.07.170

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

Take down policy
If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from vbn.aau.dk on: December 06, 2025

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 55-6 (2022) 451–457

Application of Leakage Localization Framework for Water Networks with Multiple Inlets in Smart Water Infrastructures Laboratory at AAU

Saruch Satishkumar Rathore * **Carsten Skovmose Kallesøe** *,** **Rafal Wisniewski** *

* *Aalborg University, Fredrik Bajers vej 7, DK-9220 Aalborg, Denmark (e-mail: {ssr,csk,raf}@ es.aau.dk).*
** *Grundfos Holding A/S, Poul Due Jensens vej 7, DK-8850 Bjerringbro, Denmark (e-mail: ckallesoe@grundfos.com)*

Abstract:
In this paper we extend the work presented in Rathore et al. (2021) to leakage localization in water distribution networks with multiple inlets. A self-adaptive reduced order model is used to estimate network pressures under nominal condition. These estimated pressures are compared to the measured network pressure to generate pressure residuals. Further, the pressure residuals are compared to expected residual signatures for leakage localization. For the reduced order model to be valid for water distribution network with multiple inlets, a control requirement is to be placed on the network. In this paper we also present the test results from a laboratory setup, which is present at Aalborg University, Denmark, to demonstrate the localization framework.

Copyright © 2022 The Authors. This is an open access article under the CC BY-NC-ND license (https://creativecommons.org/licenses/by-nc-nd/4.0/)

Keywords: Fault detection and isolation, water resources, graph theory, data-driven model, sensitivity analysis

1. INTRODUCTION

According to 2030 Water Resources Group (2020), 700 million people could be displaced by water scarcity by 2030. Liemberger and Wyatt (2019) estimate the volume of non-revenue water at 126 billion cubic meter with an associated financial loss of 39 billion USD annually. A significant part of this non-revenue water is water loss due to leakages and therefore water utilities put substantial efforts on leakage detection and isolation. A comprehensive review and comparison of the existing technologies in the field of leakage detection and isolation in water distribution networks is presented in Chan et al. (2018); Adedeji et al. (2017). Several of the commercially available techniques rely on Minimum Night Flow, (Mazzolani et al., 2017), acoustic noise loggers, (Sánchez et al., 2005), or Ground Penetrating Radar (GPR) Demirci et al. (2012) for leakage detection and identification. These methods may not require a mathematical model of the network, however, they need expensive equipment, manual labour and are time-consuming.

A leakage in the network typically causes a sudden and unexpected deviation in pressure at the critical points in the network. Leakage detection and isolation using this effect has been previous employed in several previous works, such as Perez et al. (2014) and Jensen et al. (2018). Perez et al. (2014) utilizes a standard hydraulic simulation model for making the comparison between the measured pressure and the nominal expected pressure, whereas Jensen et al. (2018) utilizes a data driven reduced order model to do the same. Both works present the use of sensitivity matrix, which represents the relation between leakages and the network pressure, for leakage localization. Rathore et al. (2021) extends this work with derivation of sensitivity matrix using reduced order model and considering leakages as a change in the distribution among the consumers rather than a change in consumption. Rathore et al. (2021) also presents test results for leakage localization on an EPANET model of a hydraulic network which is part of a city's water distribution network.

In this work we use similar approach as presented in Rathore et al. (2021) for residual generation and derivation of sensitivity matrix. The residuals are generated using pressure and flow measurements along with the self-adaptive reduced order network model, and the sensitivity matrix is derived using small signal model based on the reduced order network model. The scheme presented in Rathore et al. (2021) is only limited to networks with single inlet, however in this work we extend it to networks with multiple inlets. In this work we will show that with a simple flow control constraints at inlets the reduced order model for single inlet network can be extended to network with multiple inlets. Further, the pressure variation model based on this extended reduced order model is presented, which are the main contribution of this work. The generated residuals are then compared to this pressure variation model for different leakage scenarios to localize the leakage. Moreover, the localization framework is tested on Smart

⋆ This work was supported by Poul Due Jensen Foundation.
AAU stands for Aalborg University, Denmark

2405-8963 Copyright © 2022 The Authors. This is an open access article under the CC BY-NC-ND license.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2022.07.170

Water Infrastructures Laboratory (SWIL) (Ledesma et al., 2021) at Aalborg University, Denmark. With the test in the laboratory setup we demonstrate the performance of the framework in case where the residuals are subjected to noise.

The rest of the paper is organized as follows. The framework for the water distribution networks model under consideration is presented in section 2. In section 3, the pressure variation model for network with multiple inlet is derived based on the reduced order model. Further, the leakage localization framework is presented in section 4. The laboratory setup and the test results of the localization framework are presented in section 5. Finally, section 6 concludes the work.

2. REDUCED ORDER NETWORK MODEL FOR NETWORK WITH MULTIPLE INLETS

In this section we extend the reduced order model for single inlet network, first presented in Kallesøe et al. (2015), to multiple inlet network. This model will be the base for the leakage detection and localization framework. Firstly, we would simply recall the graph theory based hydraulic model presented in Kallesøe et al. (2015) in the following equations; (1) and (2). A detailed derivation of the model can be found in Kallesøe et al. (2015).

$$\bar{p}(t) = \bar{H}_{\mathcal{T}}^{-T}\lambda_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}q_{\mathcal{C}}(t)+\bar{H}_{\mathcal{T}}^{-1}\bar{d}(t))-(\bar{z}-\mathbb{1}z_n)+\mathbb{1}p_n(t) \tag{1}$$

$$\lambda_{\mathcal{C}}(q_{\mathcal{C}}(t)) - \bar{H}_{\mathcal{C}}^{T}\bar{H}_{\mathcal{T}}^{-T}\lambda_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}q_{\mathcal{C}}(t) + \bar{H}_{\mathcal{T}}^{-1}\bar{d}(t)) = 0 \tag{2}$$

Here, the pipes are modelled as the edges and the connection points as the nodes of the graph. The supply flow and the consumer demands are modelled by assigning independent nodal demands to a subset of the nodes. Moreover, the n^{th} node is set as the reference node. The underlying network graph had been divided into arbitrary spanning tree $\mathcal{T}$ and its corresponding chords $\mathcal{C}$. $\bar{H}_{\mathcal{C}}$ is the reduced incidence matrix corresponding to the chords and $\bar{H}_{\mathcal{T}}$ to the spanning tree. Similarly, the vector function $\lambda_{\mathcal{T}}(\cdot)$ and $\lambda_{\mathcal{C}}(\cdot)$ models the flow dependent pressure drops in the spanning tree $\mathcal{T}$ and the chords $\mathcal{C}$ respectively. $q_{\mathcal{C}}(t)$ is the vector of flows through the chords. $\bar{d}(t) \in \mathbb{R}^{(n-1)}$ the vector of independent nodal demands at the non-reference nodes. $\bar{p}(t) \in \mathbb{R}^{(n-1)}$ is the vector of pressure at the non-reference nodes and $p_n(t) \in \mathbb{R}^n$ is the reference node pressure. Similarly, $\bar{z} \in \mathbb{R}^{(n-1)}$ is the vector of pressures due to geodesic levels at the non-reference nodes and $z_n \in \mathbb{R}^n$ is the pressure due to geodesic level at the reference node. Also, $\mathbb{1}$ represents a vectors of 1s.

The water network considered in this project are District Metering Areas (DMAs) with multiple supply points but without an elevated reservoir. With that the independent nodal demands can be partitioned as $d(t) = [d_c(t)\ d_p(t)]^T$, where $d_p(t) \in \mathbb{R}^p$ is the vector of nodal demands at the inlet nodes and $d_c(t) \in \mathbb{R}^{(n-p)}$ is the vector of nodal demands at the non-inlet nodes. For the non-inlet nodes, $i = 1, \cdots, (n-p)$, where the flow is out of the network, $d_i(t) \leqslant 0$ and for the inlet nodes $i = (n-p+1), \cdots, n$, where the flow is into the network, $d_i(t) \geqslant 0$. Further, we set one of the inlet (or supply) node as the reference node and also denote it as the n^{th} node. Now, due to mass conservation in the network, the flow at the reference node, n^{th} node, can be written as the negative sum of all the other independent nodal flows in equation (3); and further they would collectively be refereed as non-reference nodal demands, $\bar{d}(t)$.

$$d_n(t) = -\left(\sum_{i=1}^{n-p} d_i(t) + \sum_{i=n-p+1}^{n-1} d_i(t)\right) = -\mathbb{1}^T\bar{d}(t) \tag{3}$$

The network is considered to be sectionalized into DMAs such that the consumers in the DMA are of the same type, i.e. either residential or industrial. This consideration implies that the consumer demand pattern for all the consumers in the DMA are same in average, though scaled for each consumer, i.e. the consumption flow for each consumer has the same time function but multiplied by different scaling factors. Further, we consider that the reference node is pressure controlled and all the other inlet nodes are flow controlled. Similar control structure is present in the water distribution network in Randers, Denmark operated by the company Verdo and is common in operation of water networks. The flow controllers are designed to ensure that the distribution of flow from the inlets is always constant. These conditions are further stated in the following assumption.

Assumption 1. The distribution between the $n-p$ non-inlet demands, $d_c(t)$, is fixed in time, i.e. $\exists\ w_c \in \mathbb{R}_+^{n-p}$ with the property $\sum_{i=1}^{n-p} w_i = 1$. Also, the distribution between the $p-1$ inlet demands of $\bar{d}_p(t)$ is fixed in time, i.e. $\exists\ w_p \in \mathbb{R}_-^{p-1}$ with the property $\sum_{i=n-p+1}^{n-1} w_i = -1 + \kappa$; where κ is the constant ratio of the reference inlet flow, $d_n(t)$, to the total inlet flow (or the total non-inlet demand), $\gamma(t)$, $\kappa = \frac{d_n(t)}{\gamma(t)}$ such that,

$$\bar{d}(t) = -w\gamma(t) = -\begin{bmatrix} w_c \\ w_p \end{bmatrix}\gamma(t) \quad ; \quad \sum_{i=1}^{n-1} w_i = \kappa\ . \tag{4}$$

With that, the set-point reference for the non-reference inlet flows would be $\bar{d}_p^*(t) = -w_p\gamma(t)$. Furthermore, following assumption is made about $\lambda(\cdot)$ from (2) and (1).

Assumption 2. The hydraulic resistance $\lambda_i : \mathbb{R} \to \mathbb{R}$ takes the form (Swamee and Sharma, 2008),

$$\lambda_i(x_i) = f_i|x_i|x_i\ , \tag{5}$$

with $0 < f_i$.

Here x represents the argument of the function. With Assumption 2, λ_i is a homogeneous function with degree 2, i.e. $\lambda_i(a_i x) = \lambda_i(a_i)x^2$ for $x \geqslant 0$.

Under Assumption 1, with a simple flow control on the non-reference inlet nodes the distribution of nodal demands among the non-reference nodes is constant; and with that Lemma 1, proven in Kallesøe et al. (2015), holds for networks with multiple inlets.

Lemma 1. Kallesøe et al. (2015) Under Assumptions 1 and 2, for a given distribution of flow at the non-reference nodes, w, there exist a unique vector a, such that $q(t) =$

$a\gamma(t)$, where $q(t)$ is is the vector of flows through the network.

Lemma 1 implies that for a constant distribution of nodal demands among the non-reference nodes, w, there exists a constant distribution of flow among all the edges of the network, a. This distribution is independent of the magnitude of the total demand $\gamma(t)$.

Now, with Assumption 1 and 2, and Lemma 1, the nodal pressure equation, (1), can be written as,

$$\bar{p}(t) = \gamma(t)^2 g(a_{\mathcal{C}}, w) + b p_n(t) + c \ , \tag{6}$$

and the vector function and vectors g, b, and c are,

$$g(a_{\mathcal{C}}, w) = \bar{H}_{\mathcal{T}}^{-T}\lambda_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}w - \bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}a_{\mathcal{C}})$$
$$b = \mathbb{1} \quad , \quad c = \mathbb{1}z_n - \bar{z} \ .$$

Here, the vector $a_{\mathcal{C}}$ is the part of a from Lemma 1 that relates to the chord flows of the spanning tree. Note that the vectors w and a are time invariant by Assumption 1 and Lemma 1 respectively.

Similarly, the chord flows equation, (2), can be written as,

$$h(a_{\mathcal{C}}, w)\gamma(t)^2 = 0 \ , \tag{7}$$

where,

$$h(a_{\mathcal{C}}, w) = \lambda_{\mathcal{C}}(a_{\mathcal{C}}) - \bar{H}_{\mathcal{C}}^T\bar{H}_{\mathcal{T}}^{-T}\lambda_{\mathcal{T}}(-\bar{H}_{\mathcal{T}}^{-1}w - \bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}a_{\mathcal{C}}) \ .$$

And with that the reduced order model for networks with single inlet, from Kallesøe et al. (2015), is extended to network with multiple inlets with the control structure proposed in this paper.

3. PRESSURE VARIATION MODEL

In this section a small signal variation model is presented which based on the reduced order model given by (6) and (7). Subsequently, the variation model is used to define the sensitivity matrix for leakage localization. As the structure of (6) and (7) is same as the model structure for networks with single inlet as presented in Rathore et al. (2021), we employ the same development step for the pressure variation model here. Therefore, the derivation of the pressure variation model is not presented and simply the final equations are presented.

Before presenting the pressure variation model we first introduce some terms. Let $\bar{D}(t)$ denote the nominal nodal demands at the non-reference nodes and $Q(t)$ denote the nominal flow in the edges; and we define nominal nodal demand distribution, W, as $\bar{D}(t) = W\Gamma(t)$ and nominal edge flow distribution, A, as $Q(t) = A\Gamma(t)$, where $\Gamma(t)$ is the nominal total inlet flow. With a leakage in the network the actual non-reference nodal demand would differ from the nominal demands and is given by $\bar{d}(t) = \bar{D}(t) + \delta\bar{d}(t)$, where $\delta\bar{d}(t)$ represents the leakage flow. With leakages the actual nodal demand distribution and the actual edge flow distribution also differs from their nominal values and is given by,

$$a(t) = A + \delta a(t) \quad , \quad w(t) = W + \delta w(t) \ , \tag{8}$$

where, $\delta w(t)$ is the variation of the non-reference nodal demand distribution and $\delta a(t)$ the variation of the edge flows distribution due to leakage $\delta\bar{d}(t)$. Further, let $\delta p(t)$ denote the variation of the nodal pressures around the nominal nodal pressures, $\bar{P}(t)$.

From Rathore et al. (2021), $\delta p(t)$ is given as,

$$\delta\bar{p}(t) = -\gamma(t)^2 S\delta w(t) \ , \tag{9}$$

where S is termed as the resistance matrix and is given by,

$$S = \partial_{a_{\mathcal{C}}} g|_{A_{\mathcal{C}},W}(\partial_{a_{\mathcal{C}}} h|_{A_{\mathcal{C}},W})^{-1}\partial_w h|_{A_{\mathcal{C}},W} - \partial_w g|_{A_{\mathcal{C}},W} \ .$$

The notation $\partial_x f|_{X,Y}$ is used to denote partial derivative of f with respect to x evaluated at X, Y. Further, the S matrix can be expressed in terms of graph matrices, by calculating the partial derivative and using Woodbury Matrix Identity (Hager, 1989) as,

$$S = (\bar{H}_{\mathcal{T}}\partial\lambda_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{T}}^T + \bar{H}_{\mathcal{C}}\partial\lambda_{\mathcal{C}}^{-1}\bar{H}_{\mathcal{C}}^T)^{-1} \ . \tag{10}$$

The S matrix only depends on the nominal distribution of the demands, W. Moreover, from Assumption 2, $\partial\lambda_{\mathcal{T}}$ and $\partial\lambda_{\mathcal{C}}$ are diagonal matrices implying that S is symmetric. Again, from Assumption 2, $\partial\lambda_i > 0$ for $q_i \neq 0$. In a real-life water network $\partial\lambda_{\mathcal{C}}$ and $\partial\lambda_{\mathcal{T}}$ are always non-singular, as all edges are pipes with an inherent flow resistance.

Now, a relation between $\delta w(t)$ and $\delta\bar{d}(t)$ is derived to obtain relation between $\delta\bar{p}(t)$ and $\delta w(t)$, for network with multiple inlets, from (9).

As mentioned before, the actual non-reference nodal demand in the network with a leakage, $\bar{d}(t)$, can be represented as deviation $\delta\bar{d}(t)$ from nominal $\bar{D}(t)$.

$$\bar{d}(t) = \bar{D}(t) + \delta\bar{d}(t) \tag{11}$$

The nominal non-reference nodal demand $\bar{D}$ and the non-reference nodal demand $\bar{d}$ with a leakage $\delta\bar{d}$ is described by,

$$\bar{D}(t) = -W\Gamma(t) \qquad , \qquad \bar{d}(t) = -(W + \delta w(t))\gamma(t) \ . \tag{12}$$

With mass conservation in the network, (3), the negative sum of all non-reference nodal demands $\bar{d}(t)$ (and $\bar{D}(t)$) must equal the flow at the reference node $d_n(t)$ (and $D_n(t)$), respectively. Further, from Assumption 1 with flow control at all the non-reference inlet nodes, the flow at the reference inlet node can be represented as a fixed ratio, κ, of the total inlet flow, $\gamma(t)$ (and $\Gamma(t)$).Therefore, $\kappa\gamma(t) = -\mathbb{1}^T\bar{d}(t)$ (and $\kappa\Gamma(t) = -\mathbb{1}^T\bar{D}(t)$). With that rewriting (11) gives,

$$\Gamma(t) - \gamma(t) = \frac{1}{\kappa}\mathbb{1}^T\delta\bar{d}(t) \ . \tag{13}$$

Substituting (12) in (11) and solving for δw using (13) leads to the following relation between $\delta w(t)$ and $\delta d(t)$,

$$\delta w(t) = -\frac{1}{\gamma(t)}\left(I - \frac{1}{\kappa}W\mathbb{1}^T\right)\delta\bar{d}(t) \ . \tag{14}$$

The following can be stated about the matrix $(I - \frac{1}{\kappa}W\mathbb{1}^T)$ and is proven in Appendix A.

Lemma 2. Let $M = I - \frac{1}{\kappa}W\mathbb{1}^T$, where $\sum_i W_i = \kappa$, then M has a non-trivial kernel,

$$\ker(M) = \text{span}\{W\} \ .$$

Now, the relation between leakages and the pressure variation can be given by substituting expression for δw from (14) in (9).

$$\delta\bar{p}(t) = \gamma(t)S\left(I - \frac{1}{\kappa}W\mathbb{1}^T\right)\delta\bar{d}(t) \tag{15}$$

In real-life water network for all i, $q_i \neq 0$, making S full rank, therefore with Lemma 2, $\delta\bar{p}(t)$ is zero only when $\delta\bar{d}(t) = 0$ or $\delta\bar{d}(t) \in \text{span}\{W\}$. Here, $\delta\bar{d}(t)$ is used to model leakages in the network, hence $\delta\bar{d}(t) = 0$ implies no leakage in the network. Further, Lemma 2 implies that as long as the non-reference nodal demand distribution between the nodes, in case of a set of leaks, is different from the nominal distribution W, the pressure residuals $\delta\bar{p}(t)$ would be non-zero and hence the leakages would be visible in the pressure residuals. Therefore, in practice a leakage at a single node would always be detectable. In the following section, leakage localization using (15) is presented, assuming that a leakage appears only one node at the time.

4. LEAKAGE LOCALIZATION

Now that we have the pressure variation model for networks with multiple inlets, we can use it for leakage localization. Leakage localization in networks with single inlet using pressure variation model is presented in Rathore et al. (2021) and here, we follow a similar scheme to extend it to networks with multiple inlets. The leakage localization approach is based on pressure residuals, which are calculated as a difference between measured nodal pressure and estimated nominal nodal pressure. The nominal pressures are estimated using (6). The obtained residuals are then compared to expected pressure change due to leakages, given by the variation model (15), for leakage localization.

4.1 Residual generation

Given the reference inlet pressure $p_n(t)$, total inlet flow $\gamma(t)$, the non-reference nodal pressures in the network under nominal conditions can be estimated using (6). The estimated nominal nodal pressure at the i^{th} node can be given as,

$$\hat{P}_i(t) = \alpha_i d_n(t)^2 + \beta_i + p_n(t), \tag{16}$$

where, from (6), α_i is the i^{th} element of $\bar{H}_{\mathcal{T}}^{-T} f_{\mathcal{T}}(\bar{H}_{\mathcal{T}}^{-1}W - \bar{H}_{\mathcal{T}}^{-1}\bar{H}_{\mathcal{C}}A_{\mathcal{C}})$ and β_i is the i^{th} element $-(\bar{z} - \mathbb{1}z_n)$. The parameters α_i and β_i can be estimated given time series data of measured pressure at the i^{th} node, $P_i(t)$, $\gamma(t)$ and $p_n(t)$, for the network operating under nominal conditions, these parameters can be identified using linear regression (Madsen, 2007). Note that the parameters can only be identified for the nodes where the pressure is measured and subsequently the nominal pressure can only be estimated for those nodes.

With the estimated pressure, given by (16), the pressure residual can be calculated as,

$$r_i(t) = p_i(t) - \hat{P}_i(t)\ , \tag{17}$$

where p_i is the measured pressure at the i^{th} node and $\hat{P}_i$ is its estimated value in the non-leaking case. In the following section we use the residual vector r for leakage localization.

4.2 Leakage localization

A leakage is seen as an unexpected change in the end-user demands. We assume leaks to occur only at non-inlet nodes and a leak at only one node at a time. Therefore, a leakage at node $l \in \mathbb{Z} : l \in [1, (n-p)]$ can be represented as a change in demand such that $\delta\bar{d} = \begin{bmatrix} e_l{}^T & w_p^T \end{bmatrix}^T \zeta(t)$, where $e_l \in \{0,1\}^{(n-p)}$ is a unit vector with 1 at l^{th} position and w_p is the distribution between the $p-1$ non-reference inlet demands; and $\zeta(t) < 0$ is the magnitude of the leakage. To localize a leakage we compare the measure residual and the expected residual, therefore the expected residual, $\hat{r}_l(t)$, should only include the nodes where the pressure is measured. These nodes are nothing but a subset of the non-reference nodes, hence we use a binary matrix F to extract those nodes from (15) which gives an expression for $\hat{r}_l(t)$ as,

$$\hat{r}_l(t) = -\gamma(t)G\begin{bmatrix} e_l \\ w_p \end{bmatrix}\zeta(t) = \gamma(t)|\zeta(t)|G\begin{bmatrix} e_l \\ w_p \end{bmatrix}\ . \tag{18}$$

Here, G is the sensitivity matrix obtained from (15) and is given by,

$$G = -FS\left(I - \frac{1}{\kappa}W\mathbb{1}^T\right)\ , \tag{19}$$

G gives the directional relation between the residual and any leak in the network nodes. From (18) it can be seen, that the direction of $r_l(t)$ is neither impacted by total inlet flow, $\gamma(t)$, nor by the magnitude of the leakage $\zeta(t)$. Moreover, G only depends on the distribution of the nominal non-reference nodal demands, W, which can be deduced from billing data and inlet flow control. Note that, theoretically the leakage size does not impact the leakage localization however, in laboratory tests or real-life scenarios there is higher possibility of better localization with larger leakage size, as in these cases the data is impacted by the stochastic nature of the consumers and measurement noises.

The leaking node is identified by comparing the direction of the actual residual, $r(t)$ from (17), with the set of possible reference directions described by $\hat{r}_i$ $i = 1, \cdots, n-p$ from (18), leading to following decision signal,

$$\psi_i(r(t)) = \frac{\left\langle r(t), G\begin{bmatrix} e_i \\ w_p \end{bmatrix}\right\rangle}{|r(t)|\left|G\begin{bmatrix} e_i \\ w_p \end{bmatrix}\right|}\ . \tag{20}$$

The notation $\langle\cdot,\cdot\rangle$ denotes vector inner product. This approach for comparing measured residual to leakage signature has been previously presented in Perez et al. (2014) and Rathore et al. (2021).

From (20) we know $-1 \leqslant \psi_i \leqslant 1$; and for a node a value of ψ closet to 1 would indicate the best directional fit and a leakage at that node. A leakage indicator can also be obtained by truncating the decision signal $\psi_i(r)$ to be zero or larger than zero,

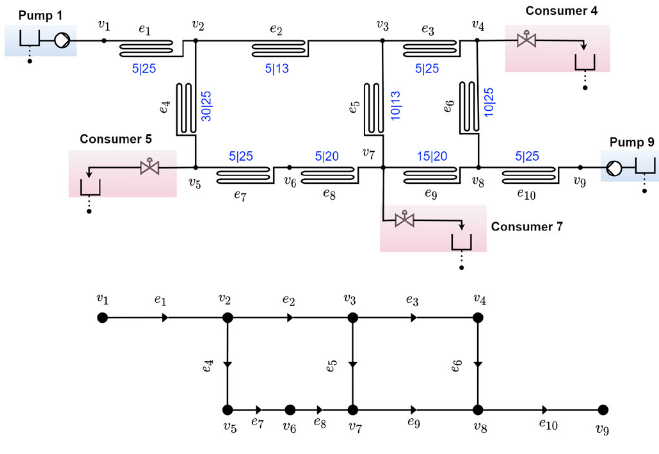

Fig. 1. The SWIL setup and it corresponding network graph used for laboratory test.

$$\mu_i(t) = \max\{\psi_i(r(t)), 0\}_{i=1,\cdots,n-1} \ , \tag{21}$$

where $\mu_i \in [0, 1]$ is the leakage indicator for the i^{th} node and again a value closet to 1 indicates a leakage at that node. However, a real-life water network is susceptible to uncertainties, mainly from immediate consumer demands which would bring uncertainty to nominal distribution, W and thereby to the sensitivity matrix, G. Thus creating a dubiety towards (21) to point to the correct node. Therefore, one should expect the leakage indicator to point towards a set of nodes, having higher likelihood of a leakage with μ value close to 1; meaning that the leakage indicator signal leads the utility to an area of the network that should be inspected for the leakage.

5. LABORATORY TEST RESULTS

5.1 Laboratory setup

To evaluate the performance of the leakage localization algorithm, a test is carried out on a re-configurable laboratory test-bed, viz. Smart Water Infrastructures Laboratory (SWIL), present at Aalborg University, Denmark. Detail of SWIL with it's application is presented in Ledesma et al. (2021).

The SWIL setup and it's corresponding network graph is presented in figure 1. Both the hydraulic network and the graph is labelled with nodes, $v_1 \cdots v_9$, the pipes in the network, which are considered as the edges of the network graph, are labelled with $e_1 \cdots e_{10}$; the direction considered for the edges are also presented in the network graph. Apart from that, the length and diameter for each pipe is mentioned in *blue* with the following notation $x|y$, where x is the length of the pipe in m and y is the diameter of the pipe in mm. The geodesic levels for the nodes is presented in table 1.

Table 1. Geodesic levels for the nodes

Node	v_1	v_2	v_3	v_4	v_5
Geodesic level $[m]$	0	0	0	1.5	1.5
Node	v_6	v_7	v_8	v_9	
Geodesic level $[m]$	0	1.5	0	0	

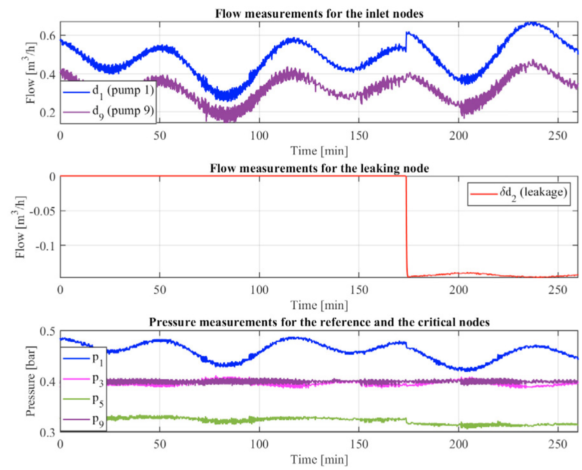

Fig. 2. Pressure and flow measurements from the laboratory test on the setup presented in figure 1.

In the laboratory setup, the pump 1 and 9 are connected at v_1 and v_9 respectively; whereas the consumer 4, 5 and 7 are connected at v_4, v_5 and v_7 respectively. The numbering for the pumps and consumer is for easy association to the nodes to which they are connected. The node v_9 has been considered as the reference node, and here the reference node pressure p_n is measured. The inlet flow at both the pump nodes are also measured and the sum of both these flows is the total inlet flow, γ. Apart from that, the pressure is also measured at nodes v_1, v_3 and v_5; these nodes are termed as the critical nodes and these are the nodes for which the pressure residuals are generated using (17).

5.2 Laboratory results for residual generation and leakage detection

In-line with Assumption 1, the pump 9, which is connected to the reference node, is pressure controlled and the pump 1 is flow controlled. In the laboratory, this is achieved by a simple Multiple Input Multiple Output State Space controlled; we use the same controller as developed and presented by Nielsen et al. (2021). The consumer flows is manipulated by controlling the valves for each consumers using individual PI controllers. The set-point reference for the consumer demands is a periodic signal with a time period of 120 min; this is to imitate 24 hrs of real-life operation in 120 min in the laboratory setup.

Figure 2 presents the pressure and flow measurement data from the laboratory test. The 1^{st} sub-plot presents inlet flows from both the pumping stations. The flow from the pump 1 is controlled at a set-point reference of 0.6 times the total inlet flow and with that $\kappa = 0.4$. A leakage is emulated at 173 min at node v_2 by simply opening a valve which is connected at node v_2 and is opened to atmospheric pressure; this results in a leakage flow of a around 0.14 m^3/h from 173 min till the end of test. The leakage flow measurement is presented in the 2^{nd} sub-plot. The pressure measurements from the laboratory test is presented in the 3^{rd} sub-plot. The pump 9 pressure is controlled at a set-point reference of 0.4 bar and this is labelled as p_9 in the figure. The other measured pressures

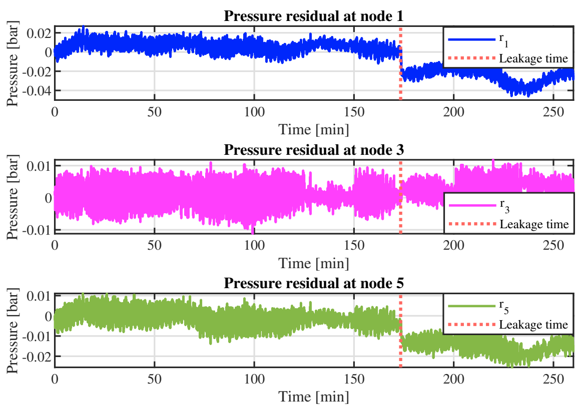

Fig. 3. Pressure residual signals for the critical nodes v_1, v_3 and v_5.

at node v_1, v_3 and v_5 are labelled as p_1, p_3 and p_5 in the figure.

The network parameters, α_i and β_i, are identified for the critical nodes from the normal operation data from the laboratory. Further by (17), the pressure residuals are generated for the critical nodes. These pressure residuals are presented in figure 3, where 1^{st}, 2^{nd} and 3^{rd} sub-plots each presents residual for node v_1, v_3 and v_5 respectively. In the figure it can be seen that after the leakage has occurred the mean value of the residuals diverge from zero, however the residuals are impacted by noise and therefore leakage detection using a simple decision signal of $r_i \neq 0$ won't be apt. The leakage detection in this work is done by using Generalised Likelihood Ratio (GLR) algorithm (Blanke et al., 2006) for detecting change in mean of the residuals and the mean value of the residuals under normal operation is assumed to be 0. The GLR decision function, $\phi_i(k)$, for the i^{th} residual at time instance k is given by,

$$\phi_i(k) = \frac{1}{2\sigma_i^2} \max_{k-M_w+1 \leqslant j \leqslant k} \frac{1}{k-j+1} \left(\sum_{s=j}^{k} r_i(s) \right)^2 , \quad (22)$$

where σ_i is the standard deviation of the residual signal, M_w is the sliding window and r_i is the i^{th} residual signal. A fault or a leakage is said to have happened when any one of the GLR decision function crosses the threshold value, thr; i.e. if $\phi_i(k) > thr_i$ for any i, a leakage is detected. The GLR decision functions for the pressure residuals of the critical nodes and their individual thresholds are presented in figure 4. The parameters of GLR, $M_w = 300$ and thr for individual residuals are selected based on experimental tests. In the figure it can be seen that before the leakage time the GLR decision functions for all the three residuals are below their individual thresholds and after sometime of the leakage the GLR decision functions crosses the threshold, implying a leakage has occurred. A leakage detection alarm is generated as soon as any one of the GLR decision functions crosses the threshold.

5.3 Laboratory results for leakage localization

The sensitivity matrix, G is obtained with the known parameters of the laboratory setup and the known distribution of non-reference flows. And once a leakage is detected, the generated residuals are compared to the expected residual signatures using (20) and (21). This comparison leads to leakage indicators for the all the nodes. The leakage indicator result for the laboratory test is presented in figure 5. For the laboratory data to reduce the impact of the noise from the residuals, the mean value of 300 samples of the residual after the leakage is detected is considered when comparing with the expected residual signatures. In the figure, a colour scale presents the likelihood of leakage at a node in the laboratory setup. Again from (21), the colour close to 1 represents maximum likelihood of leakage and the colour close to 0 represents least likelihood of leakage.

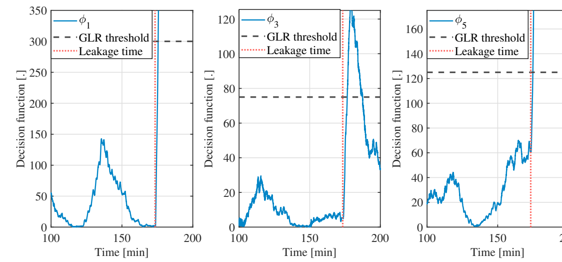

Fig. 4. The GLR decision functions to detect change in mean for pressure residuals presented in figure 3.

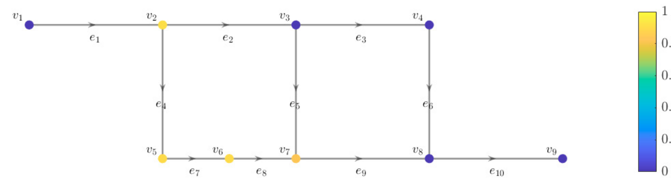

Fig. 5. The leakage localization indicators for each node for the laboratory test.

From figure 5, it can seen that the leakage indicators point towards a set of nodes on the left side of the network. And the leakage is marginally more likely to be at node v_2, which is the actual leaking node.

6. CONCLUSION

This paper presents leakage localization framework for water supply networks with multiple inlets. The framework utilizes the reduced order model for generation of pressure residuals. This reduced order model is only valid under a special operational control structure, which is pressure control for one of the inlet pumps and flow control for the rest of the inlet pumps. The generated pressure residuals are compared to expected residual signatures, which are derived from a small signal model. The expected residual signatures are represented by a sensitivity matrix. In this work, the sensitivity matrix is obtained by just the known parameters of the network and the distribution of demands between the end-users. Further, the leakage localization framework is tested on a laboratory setup which is impacted by noise. The experimental results shows that the localization framework is able to locate the leakage to a set of nodes which includes the actual leaking node. Considering the stochastic nature of the consumer demands in the model itself for leakage localization could be a possible future work.

ACKNOWLEDGEMENTS

The authors would like to thank Esben H. Nielsen, Mark Christensen and Simon N. Gnap, Control and Automation Master's students at AAU for development of the controller for the laboratory setup during their Master's thesis project from which the data for this work has been collected. The authors would also like to thank Peter Nordahn from the utility company Verdo for provinding us insight into the operation of water distribution networks.

REFERENCES

2030 Water Resources Group (2020). 2020 annual report: Valuing water, enabling change. URL https://www.2030wrg.org/wp-content/uploads/2020/12/WRG-Annual-Report_2020_Web.pdf.

Adedeji, K.B., Hamam, Y., Abe, B.T., and Abu-Mahfouz, A.M. (2017). Towards achieving a reliable leakage detection and localization algorithm for application in water piping networks: An overview. *IEEE Access*, 5, 20272–20285.

Blanke, M., Kinnaert, M., Lunze, J., Staroswiecki, M., and Schröder, J. (2006). *Diagnosis and fault-tolerant control*, volume 2. Springer.

Chan, T.K., Chin, C.S., and Zhong, X. (2018). Review of current technologies and proposed intelligent methodologies for water distributed network leakage detection. *IEEE Access*, 6, 78846–78867.

Demirci, S., Yigit, E., Eskidemir, I.H., and Ozdemir, C. (2012). Ground penetrating radar imaging of water leaks from buried pipes based on back-projection method. *Ndt & E International*, 47, 35–42.

Hager, W.W. (1989). Updating the inverse of a matrix. *SIAM review*, 31(2), 221–239.

Jensen, T.N., Puig, V., Romera, J., Kallesøe, C.S., Wisniewski, R., and Bendtsen, J.D. (2018). Leakage localization in water distribution using data-driven models and sensitivity analysis. *Ifac-Papersonline*, 51(24), 736–741.

Kallesøe, C.S., Jensen, T.N., and Wisniewski, R. (2015). Adaptive reference control for pressure management in water networks. In *2015 European Control Conference (ECC)*, 3268–3273. IEEE.

Ledesma, J.V., Wisniewski, R., and Kallesøe, C.S. (2021). Smart water infrastructures laboratory: Reconfigurable test-beds for research in water infrastructures management. *Water*, 13(13), 1875.

Liemberger, R. and Wyatt, A. (2019). Quantifying the global non-revenue water problem. *Water Supply*, 19(3), 831–837.

Madsen, H. (2007). *Time series analysis*. CRC Press.

Mazzolani, G., Berardi, L., Laucelli, D., Simone, A., Martino, R., and Giustolisi, O. (2017). Estimating leakages in water distribution networks based only on inlet flow data. *Journal of Water Resources Planning and Management*, 143(6), 04017014.

Nielsen, E.H., Christensen, M., and N, G.S. (2021). Leakage detection and localization in water distribution networks with multiple inlets. URL https://projekter.aau.dk/projekter/files/421098402/CA10_Leakage_localization.pdf.

Perez, R., Sanz, G., Puig, V., Quevedo, J., Escofet, M.A.C., Nejjari, F., Meseguer, J., Cembrano, G., Tur, J.M.M., and Sarrate, R. (2014). Leak localization in water networks: A model-based methodology using pressure sensors applied to a real network in barcelona [applications of control]. *IEEE control systems magazine*, 34(4), 24–36.

Rathore, S.S., Kallesøe, C.S., Wisniewski, R., and Jensen, T.N. (2021). Leakage localization in municipal water supply using self adaptive reduced network models and sensitivity analysis. In *2021 5th International Conference on Control and Fault-Tolerant Systems (SysTol)*, 199–204. IEEE.

Sánchez, E., Ibáñez, J., and Cubillo, F. (2005). Testing applicability and cost effectiveness of permanent acoustic leakage monitoring for loss management in madrid distribution network. *Leakage 2005*, 12–14.

Swamee, P.K. and Sharma, A.K. (2008). *Design of water supply pipe networks*. John Wiley & Sons.

Appendix A. PROOF OF LEMMA 2

Let $M = I - \frac{1}{\kappa} w \mathbb{1}^T$, then for $x \in \ker(M)$

$$0 = (I - \frac{1}{\kappa} w \mathbb{1}^T)x \Leftrightarrow x = \frac{1}{\kappa}\left(\sum_{i=1}^{n-1} x_i\right) w \ ,$$

which clearly has a trivial solution. For this equation to have a non-trivial solution the following must hold true

$$\kappa \frac{x}{\sum_{i=1}^{n-1} x_i} = w \tag{A.1}$$

This equation has a solution if and only if $\sum_i w_i = \kappa$ otherwise the kernel of M is trivial. Therefore, for $\sum_i w_i = \kappa$ then $\ker(M) = \mathrm{span}\{w\}$, which completes the proof.