Abstract—The grid voltage modulated direct power control (GVM-DPC) is a novel power regulation method for grid-connected voltage source inverters (VSI). However, interactions between real and reactive powers still exist in GVM-DPC under unbalanced grid voltage condition, which may deteriorate its transient performance. This paper proposes a power decoupling method for GVM-DPC-based VSI in unbalanced systems based on the dynamic feedforward power compensation (DFPC) strategy, which can reduce the coupling magnitudes of real and reactive powers in the transient stages. Firstly, the power coupling mechanism of positive and negative sequence real and reactive powers of VSI under the unbalanced voltage condition is analyzed. Then, the power coupling magnitudes (PCM) are derived according to relationships among the positive sequence components of real power, reactive power and PCC voltages. Next, the PCM are compensated into the GVM-DPC for the better decoupling performance. Furthermore, a stability analysis of the proposed control system is studied based on the impedance model, and a comparison with the traditional power decoupling method based on virtual impedance is conducted to show the superiority of the proposed method. Finally, simulations, hardware-in-loop test and experiment are conducted to validate the effectiveness of the proposed method.

Index Terms—grid-voltage-modulated direct power control, power decoupling, voltage source inverter, unbalanced system.

I. INTRODUCTION

W ith the installation of renewable generations, voltage source inverter (VSI) is usually utilized to integrate them into the power grids [1]. Different control strategies have been proposed to improve the VSI reliability, efficiency and safety [2]-[7]. Among them, the direct power control (DPC) is proposed for VSI to regulate output real and reactive powers directly without using phase-locked loop (PLL), which can achieve better dynamic performance and robustness comparing with conventional vector-oriented controls (VOC) [8]-[10].

Inspired by the direct torque control, a look-up-table DPC (LUT-DPC) is investigated which dynamic performance is proved to be faster than VOC [9]. However, the switching frequency is a time-varying value, which will result in the undesirable broadband harmonic fluctuations in output powers. To tackle this issue, different DPC methods are explored [11]-[19]. For example, a sliding-mode-control-based DPC is proposed to realize the constant switching frequency and robustness [11], [12]. Nevertheless, large ripples in output powers still exists and the convergence of its operation point cannot be guaranteed. Besides, a model predictive control DPC is proposed in [13], [14], which selects the optimal voltage vector to reduce the mutual interactions between the real and reactive powers based on the minimization of cost function. However, the evaluation of optimal voltage vector is time consuming. Recently, a novel grid voltage modulated-DPC (GVM-DPC) is proposed in [15]-[19], which makes the system a linear time-invariant system and solves the problems of large power harmonics induced by the nonlinear control characteristics of conventional DPC methods [15]. However, the coupling mechanism of real and reactive powers in the transient stages has not been given a full consideration under the unbalanced grid voltage condition, which may significantly degrade the dynamic performance and stability of low-inertia power systems [20], [21]. Thus, it is essential to develop power decoupling strategies to address the coupling problem suffered by the existing GVM-DPC in unbalanced system.

There are 4 types of power decoupling methods for VSI in distribution grids: 1) virtual power method (VPM) [22], 2) virtual frequency and voltage-based method (VFVM) [23], 3) virtual impedance method (VIM) [24]-[26], and 4) coupling compensation method (CCM) [31],[32]. The core ideas of VPM and VFVM are to rotate the vectors of real and reactive powers or vectors of voltage and frequency by an impedance angle. The VIM is to reshape the connecting line to be inductive by embedding virtual impedance into virtual synchronous generator control loop. The CCM adds coupling components of real and reactive powers into power control loops for the decoupling. However, all these power decoupling methods are designed for conventional virtual synchronous generator controls or droop controls, without considering the dynamic performance of GVM-DPC. Moreover, above methods only propose the power decoupling methods for VSI in 3-phase balanced system, without considering the power coupling mechanism under unbalanced voltage conditions.

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A parallel compensator (PC) is proposed for GVM-DPC under unbalanced grid voltage condition to improve the steady state performance [33]. In this case, output currents are balanced and real and reactive power ripples induced by negative current components are reduced. However, the GVM-DPC is based on an ideal grid condition, assuming that the voltage is not distorted by output powers of VSI, however, this assumption is not suitable for VSI connected to weak grids that the variation of voltage angle difference from PCC to grid side buses cannot be neglected, which will result in the severe power coupling issues. Therefore, in Section III, the traditional analysis considering the power decoupling effect, transient stages. DPC, this paper focuses on the decoupling strategy under the proposed extended-power-theory-based method. Noted that since the power coupling components in unbalanced network condition. This assumption is not suitable for VSI under balanced and unbalanced grid voltage conditions. It points out that traditional VSI control methods need to add power compensation values by extracting positive and negative sequence components to eliminate negative sequence components of output currents under unbalanced network condition. On the contrary, this sequence extraction is not required in the extended-power-theory-based DPC, which further simplifies the structure of the system. However, the real and reactive power coupling mechanism in transient stages is not considered in this method, since it also assumes that both positive and negative sequence voltages rotate with a fundamental angular frequency without considering the variation of angle difference between PCC voltage and grid voltage, which is not reasonable when VSI output powers change in the weak grid system. This assumption can lead to severer power coupling issues, which is the focus of this paper, and the conventional GVM-DPC method is selected as an example to clarify the contribution, revealing that the assumed ideal voltage condition in GVM-DPC will lead to power coupling phenomenon, which is also illustrated by phasor diagram of voltage and current. Then, the VSI powers can be decoupled by the proposed compensating power coupling magnitudes into the DPC loop, which can improve the power tracking performance.

Since there is no research focusing on the power coupling issues of GVM-DPC-based VSI under unbalanced voltage condition system, this paper proposes a power decoupling strategy for GVM-DPC under the unbalanced system based on the dynamic feedforward power compensation (DFPC) algorithm. The main contributions are summarized as follows:

1) The physical essences of power coupling in positive and negative sequence are visualized by phasor diagrams.

2) The limited power decoupling capability of existing GVM-DPC under unbalanced system is pointed out by mathematical derivation.

3) The power coupling components in the transient stages can be eliminated effectively based on the proposed DFPC method for VSI in the unbalanced system. Simulations and experiment are established to verify the effectiveness of the proposed method. Noted that since the power coupling components in steady state are effectively eliminated by conventional GVM-DPC, this paper focuses on the decoupling strategy under transient stages.

4) Stability of the VSI system is studied by impedance model analysis considering the power decoupling effect.

The rest of this paper is organized as follows. In Section II, the power coupling characteristics of GVM-DPC under unbalanced system are analyzed. In Section III, the traditional GVM-DPC is introduced, and then the power decoupling strategy is proposed. In Section IV, simulations are conducted to validate the effectiveness of the proposed method. A hardware-in-loop (HIL) test performed on RTDS is presented in Section V. Finally, the conclusions are drawn in Section VI.

II. POWER COUPLING CHARACTERISTICS OF GVM-DPC CONNECTED TO AN UNBALANCED SYSTEM

Fig. 1 shows the conventional two-level GVM-DPC-based VSI connected to an unbalanced grid voltage system where an asymmetric fault occurs [33]. A stiff DC power source is utilized in this paper, which ignores the dynamic influences from DC side. An inductor in series of resistor constitute the filter of VSI.

In Fig. 1, \( i_{abc} \) and \( v_{pcc,abc} \) are the 3-phase currents and PCC voltages measured for the calculation of instantaneous real and reactive powers in positive and negative sequence (\( P_{f}, Q_{f} \), \( P_{f}, Q_{f} \)), \( P_{f}, Q_{f} \) are control inputs of traditional GVM-DPC to regulate output powers according to power references, and \( P_{f}, Q_{f} \) are the control inputs of PC to balance the 3-phase currents. Delay signal cancellation method in [39] is used to extract positive and negative sequence components. Please note that there is a strong assumption in conventional GVM-DPC, i.e., the PCC voltages in positive sequence after the band pass filter (BPF) (denoted as \( v_{pcc,\alphaeta}^{\alpha\beta} \)) are ideal voltages without considering the angle difference from PCC to \( V_{g} \) as shown in Fig. 1 and (1), which will cause serious real and reactive power coupling issues of VSI in the transient stages. In (1), \( V_{pcc} \) is the magnitude of fundamental voltage, and \( \omega_{f} \) is the rated angular frequency of the fundamental voltage at PCC.

\[
\begin{align*}
\begin{align*}
\begin{align*}
V_{pcc} &= V_{pcc}\cos(\omega_{f}t) \\
V_{pcc} &= V_{pcc}\sin(\omega_{f}t)
\end{align*}
\end{align*}
\tag{1}
\end{align*}
\]

To illustrate the power coupling mechanism under the unbalanced grid voltage system, different sequence components of PCC voltage and current are shown in Fig. 2.

In the unbalanced system, GVM-DPC usually excites the negative and 3rd order positive sequence currents, i.e., \( i_{1}, i^{3}\), shown in Fig. 2(a), which will be eliminated by the PC control block in our control system shown in Fig. 1 to balance the output currents of VSI [33], as shown in Fig. 2. As a result, the PCC voltage \( v_{pcc} \) contains the positive and negative sequence components \( v_{pcc}^{\alpha\beta} \) and the output current of VSI \( i_{1} \) only comprises positive sequence quantity \( i_{1} \), as shown in Fig. 2(b). Then, different sequence real and reactive powers can be
derived in (2) as follows, where \( v'_\text{pecf} \) and \( i'_\text{pecf} \) are the positive sequence voltages and currents at PCC and \( v'_\text{pccf} \), \( i'_\text{pccf} \) are the negative sequence components.

\[
P_{11} = \frac{3}{2} (v_{\text{pecf}}^+ i_{\text{pecf}}^+ + v_{\text{pecf}}^- i_{\text{pecf}}^-), \quad Q_{11} = \frac{3}{2} (v_{\text{pecf}}^+ i_{\text{pecf}}^+ - v_{\text{pecf}}^- i_{\text{pecf}}^-) \tag{2}
\]

It can be observed that the output powers only contain DC components \((P_{11}, Q_{11})\) and twice-order harmonic components \((P_{21}, Q_{21})\). Since the magnitude of negative sequence voltage at PCC \((v^-_{\text{pccf}})\) is usually smaller than that of positive sequence voltage \((v^+_{\text{pccf}})\) for most asymmetric faults [33], i.e., \(P_{21}, Q_{21}\) are far less than \(P_{11}, Q_{11}\), thus this paper only considers the effect of power coupling between \(P_{11}\) and \(Q_{11}\) shown in Fig. 3 below, whereas the coupling between \(P_{21}\) and \(Q_{21}\) is not taken into account.

As a result, the projection of \(\Delta i_{f}\) on the \(q_{\text{new}}\) axis is not null (denoted as \(\Delta i_{q} \) in Fig. 3(a)), which will increase the reactive power. Thus, coupling reactive power component is produced after considering effect of the \(\Delta k_{p}\).

Similarly, in Fig. 3(b), the reactive current \(i_{q0}\) will increase by \(\Delta i_{q}\) when the \(Q_{ref}\) increases, as marked by red solid arrow. Then, the PCC voltage in positive sequence will changed from \(v^+_{\text{pccf}} \) to \(v^+_{\text{pccm}} \) with an angle variation \(\Delta \phi_{0}\) and magnitude deviation \(\Delta v^+_p\), as labeled in Fig. 3(b).

As a result, the projection of \(\Delta \phi_{0}\) on the vector of \(v'_\text{pccm}\) is not null (denoted as \(\Delta \phi_{p}\) in Fig. 3(b)), which is the coupling real power component after considering the effect of voltage angle variation \(\Delta \phi_{0}\). Please note that \(\Delta k_{p}, \Delta \phi_{0}\) can be analytically quantified and used for the calculation of power coupling magnitudes, which can be utilized as the power compensation for power decoupling of GVM-DPC. This mechanism is explained in detail in Section III-B below.

In conclusion, the neglect of voltage angle variation of PCC in conventional GVM-DPC will certainly lead to power coupling issues. In the following section, the detailed GVM-DPC model is introduced and the power-coupling components are verified according to derivations. Then a power decoupling strategy is proposed to improve the transient performance of GVM-DPC method.

III. THE POWER DECOUPLING STRATEGY FOR GVM-DPC

A. Limited Capability of Power Decoupling of GVM-DPC

In Fig. 1, one can obtain the state space equation of positive sequence output powers \(P^+_f, Q^+_f \) (denoted as \(P_{11} \) and \(Q_{11}\) in Eq. (2)):

\[
\begin{align*}
\frac{dP^+_f}{dt} &= \frac{3}{2} (v^+_{\text{pecf}} i^-_{\text{pecf}} + v^-_{\text{pecf}} i^+_{\text{pecf}} + v^+_{\text{pccf}} i^-_{\text{pccf}} + v^-_{\text{pccf}} i^+_{\text{pccf}}) \\
\frac{dQ^+_f}{dt} &= \frac{3}{2} (v^+_{\text{pecf}} i^-_{\text{pecf}} - v^-_{\text{pecf}} i^+_{\text{pecf}} - v^+_{\text{pccf}} i^-_{\text{pccf}} + v^-_{\text{pccf}} i^+_{\text{pccf}})
\end{align*}
\]

where \(v^+_{\text{pccf}}, v^-_{\text{pccf}} \) are assumed to be ideal voltages without considering voltage angle variation \(\Delta \phi_{0}\) in conventional GVM-DPC, shown in (1). This assumption lacks rationality when VSI connects to weak grids with higher grid side impedance, as discussed in Section II. Thus, the grid voltages can be modified as follows:

\[
\begin{align*}
\frac{dP^+_f}{dt} &= \frac{3}{2} (v^+_{\text{pecf}} i^-_{\text{pecf}} + v^-_{\text{pecf}} i^+_{\text{pecf}} + v^+_{\text{pccf}} i^-_{\text{pccf}} + v^-_{\text{pccf}} i^+_{\text{pccf}} + v^+_{\text{d}} i^-_{\text{d}} + v^-_{\text{d}} i^+_{\text{d}}) \\
\frac{dQ^+_f}{dt} &= \frac{3}{2} (v^+_{\text{pecf}} i^-_{\text{pecf}} - v^-_{\text{pecf}} i^+_{\text{pecf}} - v^+_{\text{pccf}} i^-_{\text{pccf}} + v^-_{\text{pccf}} i^+_{\text{pccf}} + v^+_{\text{d}} i^-_{\text{d}} - v^-_{\text{d}} i^+_{\text{d}})
\end{align*}
\]

where the angle difference \(\phi(t)\) can be represented as,

\[
\phi(t) = \text{arctan} \frac{P^+_f X_q - Q^+_f R_q}{P^+_f R_q + Q^+_f X_q + (V'_{fe})^2}
\]

One can differentiate (4) w. r. t. the time \(t\) and substitute it into (3):

\[
\begin{align*}
\frac{dP^+_f}{dt} &= \frac{R}{L} P^+_f (\omega_{0} + \phi(t)) Q^+_f + \frac{3}{2L} u_p \\
\frac{dQ^+_f}{dt} &= (\omega_{0} + \phi(t)) P^+_f - \frac{R}{L} Q^+_f - \frac{3}{2L} u_q
\end{align*}
\]

where \(u_p\) and \(u_q\) are defined variables used for feedback linearization method [33]. It can be founded that the coefficient \(\omega_{0} + \phi(t)\) in (6) is a time-varying value corresponding to output

---

Fig. 2. The phasor diagrams of the relationship between positive and negative sequence PCC voltage and current. (a) is the original vector before the balance of current, and (b) is the vector after the current is balanced.

Fig. 3. The phasor diagrams of the power coupling mechanism for positive sequence components including (a) reactive power coupled with real power, and (b) real power coupled with reactive power.
powers of VSI \((P', Q')\), thus it cannot be decoupled by conventional feedback linearization method in GVM-DPC, i.e., set \(u_f\) and \(u_o\) as follows:

\[
\begin{align*}
u_f &= \frac{2L\omega_p}{3}Q' + K_R e_f + K_P \int e_f(t)dt \\
u_o &= \frac{2L\omega_p}{3}P' + K_Q e_o + K_P \int e_o(t)dt
\end{align*}
\tag{7}
\]

where \(e_f\) and \(e_o\) are the errors between the reference \((P_{ref}, Q_{ref})\) and \(K_{P_f}, K_{Q_f}, K_{Q_{ref}},\) and \(K_{Q_o}\) are the coefficients of PI controllers for real and reactive powers. In the conventional GVM-DPC, the modulated voltages for PWM \((V_{inv, \alpha}, V_{inv, \beta}\) shown in Fig. 4) can be obtained by substituting (7) into (6):

\[
\begin{align*}
v_f &= v_{ref, \alpha} + v_{ref, \beta} + u_o + V_{pcc}^2 v'_{ref, \alpha} \\
v_o &= v_{ref, \alpha} - v_{ref, \beta} + u_p + V_{pcc}^2 v'_{ref, \beta}
\end{align*}
\tag{8}
\]

where \(V_{pcc}\) is the amplitude of PCC voltage. Please note that substituting (7) into (6) yields:

\[
\begin{align*}
\frac{dP'}{dt} &= \frac{R}{L} P' - \phi(\tau)Q' + 3K_{P_f} e_f + 3K_{Q_f} e_o \int e_f(t)dt \\
\frac{dQ'}{dt} &= \phi(\tau)P' - \frac{R}{L} Q' + 3K_{P_f} e_f + 3K_{Q_f} e_o \int e_o(t)dt
\end{align*}
\tag{9}
\]

Eq. (9) shows that the positive sequence real and reactive powers are not fully decoupled after using feedback linearization. Thus, it is essential to propose a power decoupling method for GVM-DPC-based VSI under unbalanced voltage system, which will be introduced in Section III-B as follows. Please note that in the steady state, the phase angle difference of PCC voltage and grid side voltage is nearly a constant value, so its deferential signal \(\phi'\) equals 0. In this case, the real and reactive powers are effectively decoupled in conventional GVM-DPC as shown in (9). Thus, this paper only considers the dynamic process of power coupling issues.

\section{Proposed Power Decoupling Strategy for VSI Connected to the Unbalanced System}

To decouple the coupling components of positive sequence real and reactive powers, two DFPCs are added into the power references in GVM-DPC, i.e., \(P_{com}\) and \(Q_{com}\), shown in Fig. 4. The \(P_{com}\) and \(Q_{com}\) are coupling magnitudes of positive sequence real and reactive powers in accordance with \(\Delta P'\) and \(\Delta Q'\) in Fig. 3, which is derived as follows.

According to Fig. 3, the output real and reactive powers w.r.t. the active and reactive current deviations \(\Delta P'\) and \(\Delta Q'\) are:

\[
\begin{align*}
P' &= v'_{pcc} \left[ |P_p| + \Delta P' \sin \Delta \delta_p + |P_o| \cos \Delta \delta_o \right] \\
Q' &= v'_{pcc} \left[ |Q_p| + \Delta Q' \sin \Delta \delta_p + |Q_o| \cos \Delta \delta_o \right]
\end{align*}
\tag{10}
\]

where \(P_p, Q_p\) are the real and reactive powers of VSI in accordance with currents change by \(\Delta P'\) and \(\Delta Q'\), and the \(i_{P_p}^0\) and \(i_{Q_p}^0\) represent the corresponding positive sequence currents at equilibrium point. Eq. (10) can be derived based on the analysis of Fig. 3 as follows.

When real power reference \((P_{ref}\) in Fig. 1) increases, active current \(i_{P_p}^0\) of \(i_{\alpha}\) will increase by \(\Delta P'\) based on the GVM-DPC, as marked by red solid arrow in Fig. 3(a). Then the positive sequence voltage \(v'_{pcc}\) will change to \(v'_{pcc+}\) with angle variation \(\Delta \delta_p\) by considering the angle difference on grid side impedance. In this case, the new reactive power \(Q_{ref}^0\) can be derived by multiplication of voltage magnitude \(|v'_{pcc+}|\) and active current \(|i_{P_p}^0|\sin \Delta \delta_p + |i_{Q_p}^0|\cos \Delta \delta_p\).

Similarly, when reactive power reference \((Q_{ref} \) in Fig. 1) increases, reactive current of \(i_{Q_p}^0\) will accordingly increase by \(\Delta Q'\), as marked by red solid arrow in Fig. 3(b). Thus, \(v'_{pcc}\) will increase to \(v'_{pcc+}\) with angle variation \(\Delta \delta_p\). Then, the new real power \(P_{ref}^0\) can be calculated by multiplication of voltage magnitude \(|v'_{pcc+}|\) and active current, i.e., \(|i_{P_p}^0|\sin \Delta \delta_p + |i_{Q_p}^0|\cos \Delta \delta_p\).

In (10), the voltage angle variation of angle difference between PCC voltage and grid voltage, i.e., \(\Delta \delta_p\) and \(\Delta \delta_o\), are derived as follows:

\[
\begin{align*}
\Delta \delta_p &= K_{AP} \Delta P \\
\Delta \delta_o &= K_{AQ} \Delta Q
\end{align*}
\tag{11}
\]

where \(\Delta P\) = \(P_{ref} - P'_f\), \(\Delta Q\) = \(Q_{ref} - Q'_f\), and both of them are denoted by \(e_f\) and \(e_o\) setting as control inputs of the DFPC blocks, as shown in Fig. 4.

The coefficients \(K_{AP}, K_{AQ}\) in (11) can be calculated by the inverse of the Jacobian matrix in (12), as shown in (13).

\[
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial U} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta U \end{bmatrix}
\tag{12}
\]
where $v_{pccf}$ can be measured by using conventional delay signal cancellation (DSC) [39], which can extract positive and negative sequence signals by introducing a 1/4 period time delay shown in Fig. 5. $x^{(a)\theta}(t)$ is a time-series signal in $\alpha$-$\beta$ reference frame. $x_{p}^{(a)\theta}(t)$ and $x_{N}^{(a)\theta}(t)$ are the corresponding positive and negative sequence signals.

$$x^{(a)\theta}(t) = \begin{cases} \frac{T_y}{4} + t & \text{for } x_{p}^{(a)\theta}(t) \\ \frac{T_y}{4} - t & \text{for } x_{N}^{(a)\theta}(t) \end{cases}$$

Fig. 5. Delay signal cancellation

In (14), $\delta_i$ represents the steady-state angle difference between positive sequence components of PCC voltage ($v^{p}_{pccf}$) and grid side voltage ($V_\phi \angle \delta_0$), which can be pre-calculated by PMU measuring. Also, the grid side impedance ($R_g, X_g$) can be measured by online impedance detection techniques [42]-[44].

Finally, the DFPC for real power channel, i.e., $P_{com} = P_{r0} - P_{r0}^{\prime}$ can be obtained as (15), where $P_{r0}$ is the positive sequence real power at equilibrium point. $|v^{p}_{pccf}|$ is the amplitude of positive sequence voltage, and $\Delta \delta$ denotes its amplitude deviation derived as (16), where $K_{AUP}$ and $K_{AQU}$ can be obtained by (13).

$$P_{com} = |v^{p}_{pccf}| \left[ |P_{r0} + \Delta P| \sin \Delta \delta_0 + |P_{r0} | \cos \Delta \delta_0 \right] - P_{r0}^{\prime}$$

$$= Q_r \sin \Delta \delta_0 + \left( 1 + \frac{\Delta \delta}{\Delta \delta_{ref}} \right) P_{r0} \cos \Delta \delta_0 - P_{r0}^{\prime}$$

$$\Delta \delta_{ref} = K_{AUP} \times \Delta P$$

$$\Delta \delta = K_{AQU} \times \Delta Q$$

(15)

(16)

For the simplification of real power compensation, $\sin \Delta \delta$ and $\cos \Delta \delta$ in (15) are linearized as $\Delta \delta_0$ and 1, respectively. Then, (15) is simplified to (17) based on (11) and (16), where $Q_r$ is the positive sequence reactive power of VSI.

$$P_{com} \approx K_{AUP} Q_r e^{\ast} + \frac{K_{AQU} e^{\ast}}{|v^{p}_{pccf}|} P_{r0}^{\prime}$$

(17)

Similarly, the reactive power compensation block can be simplified to (18), where $P_{r0}^{\prime}$ is the measured positive sequence real power of VSI.

$$Q_{com} = K_{AUP} P_{r0}^{\prime} e^{\ast} + \frac{K_{AQU} e^{\ast}}{|v^{p}_{pccf}|} Q_{r0}$$

(18)

Please note that the proposed method will not affect the steady-state performance and ripple cancelation of GVM-DPC, since it only extract positive sequence PCC voltage and current to generate coupling real and reactive powers as two compensation components for GVM-DPC, and the power errors (denoted as $P_{ref}-P_{r0}^{\prime}$, $Q_{ref}-Q_{r0}$ in Fig. 4) are zero in steady-state, so the outputs of the proposed DFPC block are nearly zero, which will not affect the steady-state performance.

C. Discussion of Proposed Power Decoupling Strategy

Most of the power decoupling researches focus on power coupling mechanism in the high resistance/inductance ratio (R/X) connecting line systems [24]-[29], indicating that when the resistance equals inductance, the power coupling of VSI will be severer. On the contrary, when the connection line from VSI to power grid is inductive, the powers will be decoupled. Thus, the virtual inductance methods are proposed to reshape the connecting line to be inductive by adding a virtual inductance into the VSG control loop. However, the decoupling performance can be deteriorated since the method only considers the effect of impedance angle, ignoring the influences from PCC voltage angle variation and its magnitude perturbation when the output powers of VSI change, which is illustrated by phasor diagram in Fig. 3 of our paper. Please note that the power coupling in positive sequence is severer even the connecting line is inductive, and its proof is shown in Appendix, where some conclusions can be drawn from (44):

i) If the VSI is connected to a strong grid where grid side inductance is small enough (i.e., $X_g = 0$), the output powers of VSI can be fully decoupled since the real power only depends on active current and reactive power only depends on reactive current.

ii) On the contrary, if the VSI is connected to a weak grid where grid side inductance $X_g$ increases, the coupling coefficients between real power and reactive current, reactive power and active current will increase accordingly, indicating the severer occurrence of power coupling.

D. Stability Analysis of GVM-DPC-based VSI in Unbalanced System based on Impedance Model

The stability of the control system under unbalanced grid voltage condition is analyzed in this section based on the impedance model. Firstly, the impedance model of VSI system is established in part (1) where the closing loop relationship between PCC voltage and current is described, then the Nyquist stability criteria is introduced in part (2) based on the impedance model. Please note that the VSI terminal voltage is separately controlled in positive and negative sequence under unbalanced grid voltage system, where positive sequence terminal voltage is controlled as (8) shown and negative sequence terminal voltage is controlled in traditional PC block to eliminate negative sequence output currents as shown in [19]. The VSI stability can be guaranteed when both positive and negative sequence impedance models satisfy Nyquist stability criteria. In this paper, only positive sequence impedance is introduced.
since this paper only considers the power decoupling effect in positive sequence system, which is proved that it will not affect negative sequence impedance in this section. The negative sequence impedance can be derived in a similar way.

(1) Impedance Model of VSI Considering Power Decoupling Effect

Firstly, the dynamics of VSI terminal voltage \( v_{\text{inv}} \) in terms of \( v_{\text{pcc,h}} \) and \( i_{\text{f},\text{h}} \) under stationary reference frame should be deduced. By substituting (7) into (8), the terminal voltage can be derived as:

\[
\begin{align*}
    \dot{v}_{\text{inv}} &= L(K_p(A_\alpha + B_\alpha) + K_vD_\alpha + C_v) + v_{\text{pcc,h}} \\
    \dot{v}_{\text{pcc,h}} &= L(K_v(A_\beta + B_\beta) + K_vD_\beta + C_v) + v_{\text{pcc,h}}
\end{align*}
\]

where \( A_\alpha, A_\beta, B_\alpha, B_\beta, C_\alpha, C_\beta, D_\alpha \), and \( D_\beta \) can be represented as (20), (21). \( K_p \) and \( K_v \) are parameters of PI controller, which are denoted with \( K_{P,\alpha}, K_{P,\beta}, K_{Q,\alpha}, \text{ and } K_{Q,\beta} \) in (7).

\[
\begin{align*}
    A_\alpha &= -2(P_{\text{ref}}v_{\text{pcc,h}}^* + Q_{\text{ref}}v_{\text{pcc,h}}^*)/3|v_{\text{pcc,h}}|^2 \\
    B_\alpha &= (2(P_{\text{ref}}v_{\text{pcc,h}}^* - Q_{\text{ref}}v_{\text{pcc,h}}^*)/3|v_{\text{pcc,h}}|^2 \\
    C_\alpha &= -2\alpha_\alpha(P_{\text{ref}}v_{\text{pcc,h}}^* - Q_{\text{ref}}v_{\text{pcc,h}}^*)/3|v_{\text{pcc,h}}|^2 \\
    D_\alpha &= 2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2 \int (P_{\text{ref}} - P_f) + \frac{2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2}{\int (Q_{\text{ref}} - Q_f)} \\
    B_\beta &= 2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2 \int (P_{\text{ref}} - P_f) - \frac{2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2}{\int (Q_{\text{ref}} - Q_f)}
\end{align*}
\]

(21)

Substituting instantaneous powers \( P_{\text{ref}}, Q_{\text{ref}} \) in (2) into (19) to simplify \( A_\alpha, A_\beta, C_\alpha, \) and \( C_\beta \) as follows,

\[
A_\alpha = i_{\text{f},\alpha}, A_\beta = i_{\text{f},\beta}, C_\alpha = \omega_\alpha i_{\text{f},\alpha}, C_\beta = -\omega_\beta i_{\text{f},\beta}
\]

(22)

For the linearization of \( B_\alpha \) and \( B_\beta \) the transfer function of Band-Pass Filter (BPF) in Fig. 1 is defined as:

\[
F_i = \frac{2\omega_\alpha s}{s^2 + 2\zeta_\alpha \omega_\alpha s + \omega_\alpha^2}
\]

(23)

where \( \omega_\alpha \) and \( \zeta_\alpha \) are the natural frequency and damping ratio of BPF. The coefficients \( B_\alpha, B_\beta \) can be linearized as (24), assuming that \( |v_{\text{pcc,h}}|^2 \) is a constant value after the BPF [30].

\[
\begin{align*}
    B_\alpha &\approx \frac{2P_{\text{ref}}}{3|v_{\text{pcc,h}}|^2} v_{\text{pcc,h}}^* + \frac{2Q_{\text{ref}}}{3|v_{\text{pcc,h}}|^2} v_{\text{pcc,h}}^* \\
    B_\beta &\approx \frac{2P_{\text{ref}}}{3|v_{\text{pcc,h}}|^2} v_{\text{pcc,h}}^* - \frac{2Q_{\text{ref}}}{3|v_{\text{pcc,h}}|^2} v_{\text{pcc,h}}^*
\end{align*}
\]

(24)

For the linearization of \( D_\alpha \) and \( D_\beta \), \( P_f \) and \( Q_f \) are presented as (25) where \( \bigcirc \) and \( \bigotimes \) denote the dot product and cross product of two complex vectors. \( P_{\text{ref}} \) and \( Q_{\text{ref}} \) are DC components where subscript \( \text{h} \) in \( F_{\text{L, h}}, v_{\text{pcc,b}} \) denotes fundamental frequency component. \( P_{\text{ac}} \) and \( Q_{\text{ac}} \) are harmonics with oscillation frequency \( \omega_f = \omega_0 \) where \( \omega_0 \) is fundamental frequency and \( \omega_f \) is the harmonic frequency. \( v_{\text{pcc,h}} \) and \( i_{\text{f},\text{h}} \) can be written as (26), (27), where subscripts \( \text{h} \) denotes the harmonic order.

\[
\begin{align*}
    P_f &\approx -\frac{3}{2}i_{1,\text{h}} \bigotimes v_{\text{pcc, h}}^* + -\frac{3}{2}i_{3,\text{h}} \bigotimes v_{\text{pcc, h}}^* \\
    Q_f &\approx \frac{3}{2}i_{1,\text{h}} \bigotimes v_{\text{pcc, h}}^* + \frac{3}{2}i_{3,\text{h}} \bigotimes v_{\text{pcc, h}}^*
\end{align*}
\]

(25)

\[
\begin{align*}
    v_{\text{pcc, h}} &\approx v_{\text{pcc, h}}^* + v_{\text{pcc, h}}^* \\
    i_{\text{f},\text{h}} &\approx \sum_{k=1,3} F_k(j\omega_k) v_{\text{pcc, h}}^* + \frac{1}{j\omega_k} + \sum_{k=1,3} F_k(j\omega_k) v_{\text{pcc, h}}^* + \frac{1}{j\omega_k}
\end{align*}
\]

(26)

From (21), (25), it can be assumed that the integrals of \( P_{\text{ref}} \) and \( Q_{\text{ref}} \) are approximately zero in stationary states, i.e., \( \int P_{\text{ref}} \approx 0, \int Q_{\text{ref}} \approx 0 \), which will not affect the harmonic stability of the control system. However, this approximation lacks rationality when real or reactive power changes, since the power coupling components still exist and shown in (17), (18) above. When \( P_{\text{ref}} \) changes, one can obtain \( D_\alpha \) and \( D_\beta \) by substituting coupling reactive power component in (18) into (21):

\[
\begin{align*}
    D_\alpha &= \frac{2v_{\text{pcc,h}}^*}{3|v_{\text{pcc,h}}|^2} \int (P_{\text{ref}} - P_f) + \frac{2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2}{\int (Q_{\text{ref}} - Q_f)} \\
    D_\beta &= \frac{2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2}{\int (P_{\text{ref}} - P_f)} - \frac{2v_{\text{pcc,h}}^*/3|v_{\text{pcc,h}}|^2}{\int (Q_{\text{ref}} - Q_f)}
\end{align*}
\]

(27)

(28)

(29)

(30)
Finally, the transfer matrix of \( \mathbf{v}'_{\text{pcc,inv}} \) and \( \mathbf{v}'_{\text{pcc}} \) to \( \mathbf{v}'_{\text{inv}} \) can be derived by substituting (22), (24), (30) into (19), shown as (37). Considering the harmonic components relation of \( \mathbf{v}'_{\text{inv}} \) and \( \mathbf{v}'_{\text{pcc,inv}} \) shown in (31), (37) can be reorganized as (32), where \( \mathbf{Z}_{\text{VSI}} \) represents the impedance model of the proposed DFPC-based VSI system. Similarly, when \( Q_{\text{grid}} \) changes, power coupling components in (17) in \( D_c \) and \( D_p \) can be decoupled and the impedance is the same as shown in Eq. (32).

\[
\begin{bmatrix}
\mathbf{v}'_{\text{pcc,inv}} \\
\mathbf{v}'_{\text{pcc}}
\end{bmatrix} = \begin{bmatrix} R + sL & 0 \\ 0 & R + sL \end{bmatrix} \begin{bmatrix} \mathbf{i}_{\text{inv}} \\
\mathbf{i}_{\text{pcc}}
\end{bmatrix} + \begin{bmatrix} \mathbf{v}'_{\text{pcc,inv}} \\
\mathbf{v}'_{\text{pcc}}
\end{bmatrix} (31)
\]

\[
\begin{bmatrix}
\mathbf{v}'_{\text{pcc,inv}} \\
\mathbf{v}'_{\text{pcc}}
\end{bmatrix} = \left( I - \mathbf{Z}_{\text{VSI}}^{-1} \right)^{-1} \mathbf{Z}_{\text{VSI}} \begin{bmatrix} \mathbf{i}_{\text{inv}} \\
\mathbf{i}_{\text{pcc}}
\end{bmatrix} (32)
\]

(2) Stability Criteria

The impedance model in (32) is transformed into a sequence-domain diagonal matrix by using the method proposed in [30], then the stability can be estimated by SISO Nyquist stability criterion.

The equivalent sequence domain impedance can be derived as,

\[
\begin{align*}
\mathbf{Z}_{\text{eq.p}}(s) = & \frac{\mathbf{Z}_{\text{eq.p}}(s) + \mathbf{Z}_{\text{eq.n}}(s)}{2} \\
\mathbf{Z}_{\text{eq.n}}(s) = & \frac{\mathbf{Z}_{\text{eq.n}}(s) + \mathbf{Z}_{\text{eq.p}}(s)}{2}
\end{align*}
\]

where subscripts \( p \) and \( n \) denote positive and negative sequence components. The negative sequence impedance is zero due to the symmetrical characteristic of the impedance matrix, i.e., \( \mathbf{Z}_{\text{eq.p}}(s) = \mathbf{Z}_{\text{eq.p}}(s), \mathbf{Z}_{\text{eq.n}}(s) = \mathbf{Z}_{\text{eq.n}}(s) \), only \( \mathbf{Z}_{\text{eq.p}} \) is selected for stability analysis. Please note that the zero \( \mathbf{Z}_{\text{eq.p}} \) indicates that the positive sequence voltage will not induce negative sequence current, and negative sequence impedance will not be affected by the proposed DFPC values in positive sequence.

The weak grid impedance can be derived as follows:

\[
\mathbf{Z}_{\text{eq.p}}(s) = \mathbf{L}_s + \mathbf{R}_s
\]

Then, the closed-loop transfer function \( H_c \) can be derived as (36):

\[
H_c = \frac{1}{1 + G_{\text{grid}}(s)}
\]

where \( G_{\text{grid}}(s) = \frac{\mathbf{Z}_{\text{grid}}(s)}{\mathbf{Z}_{\text{VSI}}(s)} \) is the feedback characteristic equation, which can be treated as the open-loop transfer function of \( H_c \). According to linear-control theory, the stability of the close-loop transfer function \( H_c \) can be guaranteed if the open-loop transfer function \( G_{\text{grid}}(s) \) satisfies the Nyquist stability criterion.

IV. CASE STUDY

In this section, case study is carried out in MATLAB/Simulink, SImscape Power Systems to verify the proposed power decoupling method for GVM-DPC under unbalanced voltage system shown in Fig. 4. The parameters of controller and grid system are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SYSTEM PARAMETERS IN THE SIMULATION SYSTEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Parameter Value Unit</td>
</tr>
<tr>
<td>( V_d )</td>
<td>Nominal grid voltage</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>DC-link voltage</td>
</tr>
<tr>
<td>( f )</td>
<td>Nominal grid frequency</td>
</tr>
<tr>
<td>( R )</td>
<td>Filter resistor</td>
</tr>
<tr>
<td>( L )</td>
<td>Filter inductance</td>
</tr>
<tr>
<td>( L_g )</td>
<td>Grid side resistor</td>
</tr>
<tr>
<td>( f_{sc} )</td>
<td>Switching frequency</td>
</tr>
<tr>
<td>( K_c )</td>
<td>Proportional control parameter</td>
</tr>
<tr>
<td>( K_i )</td>
<td>Integral control parameter of PI controller</td>
</tr>
</tbody>
</table>

First, in Subsection A, the coupling situation of GVM-DPC is tested under the unbalanced grid voltage condition with phase-A dropping to 0.9 p. u.. In this case, coupling components of positive sequence real and reactive powers are presented, which are the control inputs of the proposed DFPC power decoupling method. Also, the power decoupling performance of the DFPC is presented. In subsection B, C, different voltage drop magnitudes and power references are tested to verify the effectiveness of decoupling performance, and in Subsection D, the robustness of DFPC-based VSI is validated by setting grid side impedance parameter mismatches and distorted grid voltage conditions.

A. Phenomenon of Power Coupling under Unbalanced Grid Voltage Condition

The performance of power coupling of VSI under unbalanced grid voltage condition (phase-A dropping to 0.9 p. u.) using the traditional GVM-DPC is presented in Fig. 6, where power ripples \( P_{12}, Q_{12} \) in (2) cannot be eliminated when output currents of VSI are controlled to be balanced as shown in real and reactive power figures \( (P, Q) \) in Fig. 6, thus, the real and reactive powers of VSI cannot be directly utilized as the control inputs of DFPC-based control block, which will
introduce more harmonic components into the output powers of VSI. Alternatively, positive sequence real and reactive powers \( (P_{\text{ref}}, Q_{\text{ref}}) \) can be calculated via DSC as the control inputs of DFPC, which avoids the influences of power ripples on decoupling operation as shown in Fig. 6. In Fig. 6(c), (d), the coupling components \( (\Delta P \text{ and } \Delta Q) \) of real and reactive powers are reduced from 500W and 200Var to zero respectively when DFPC is adopted, with 100% effective reduction. It is noted that when the coupling powers are reduced to the power ranges: \( P_{\text{ref}} \pm 100W \) or \( Q_{\text{ref}} \pm 100Var \), the coupling magnitudes \( (\Delta P \text{ and } \Delta Q) \) in figures are assumed to be zero.

Also, the steady-state performance of output powers after utilizing the proposed DFPC method is discussed as follows. For the reactive power, as shown in Fig. 6(a), (c), the magnitude of power ripple after using DFPC is 100Var when real power increases from 500W to 2000W at 0.1s, which is almost the same with that of conventional GVM-DPC. For the steady-state operation of real power as shown in \( P \) of Fig. 6(b), (d), the power ripple magnitude is 100W after using DFPC when reactive power increases from 0Var to 2000Var at 0.25s, which is nearly the same with that of GVM-DPC. Moreover, it can be seen that small amounts of harmonics are included in the output powers in steady state after using the proposed method, but it is within an acceptable range which could also be eliminated by output filter design of VSI.

**B. Phenomenon of Power Coupling under Different Voltage Drop Magnitudes**

To test the decoupling performance of the proposed method under grid conditions with different voltage drop magnitudes, phase-A voltage magnitude is set to be 0.9 p. u., 0.8 p. u., 0.7 p. u., and 0.6 p. u., respectively, shown in Fig. 7 below. From Fig. 7(a), (b), it can be observed that when the real power increases from 500W to 2000W at 0.1s, the reactive power coupling magnitudes \( \Delta Q \) are nearly the same, both of which reach 150Var under 0.9 p. u. and 0.8 p. u. phase-A voltage conditions. After utilizing the proposed DFPC method, the reactive power coupling magnitudes \( \Delta Q \) are reduced to zero, with 100% reduction performance, as shown in \( Q_{\text{decoup}} \) in Fig. 7(a), (b).

![Fig. 6. Dynamic comparison of VSI using traditional GVM-DPC and the proposed DFPC method under the unbalanced voltage system with phase-A drop to 0.9 p. u. (a) reactive power coupling when real power changes, (b) real power coupling when reactive power changes, (c) reactive power decoupling result by the proposed DFPC, and (d) real power decoupling result by the DFPC.](image)

![Fig. 7. Performance of VSI power coupling by traditional GVM-DPC and the decoupling results by the proposed control method under different voltage drop magnitudes. (a) and (b) are reactive power coupling results of traditional GVM-DPC and decoupling results by DFPC under grid voltage conditions with 0.9 p. u. and 0.8 p. u. phase-A voltages. (c) and (d) are real power coupling result of GVM-DPC and decoupling results by DFPC under 0.7 p. u. and 0.6 p. u. phase-A voltages.](image)
From Fig. 7(c), (d), when reactive powers increase from 0Var to 2000Var at 0.1s, the real power coupling magnitudes \( \Delta P \) are 450 W and 400W respectively under 0.7 p. u. and 0.6 p. u. phase-A voltage conditions, and they are reduced to zero with 100% reduction after adopting the proposed method. Please note that larger power ripples exist in steady state when the voltage drops from 0.9 p. u. to 0.6 p. u. as seen from real and reactive power figures in Fig. 7(a)-(d), due to the occurrence of the larger negative sequence voltages in output powers \( P_{12}, Q_{12} \) in (2). Moreover, from Fig. 7 it can be seen that there is no impact on steady-state performance and power ripple cancelation performance after using the proposed power decoupling method, as analyzed in Fig. 6 above. Nevertheless, this paper only considers the power coupling issues of positive sequence real and reactive powers in transient stages, the steady state power ripples are not the focus.

C. Performance of Power Decoupling under Different Power References of GVM-DPC

In this section, power decoupling performance with different power references in GVM-DPC under grid condition with two-phase-to-ground fault (Phase A&B voltage drop to 0.9 p. u.) is tested and shown in Fig. 8, where in Fig. 8(a) the real power \( P \) increases from 500W to 1400W, 1800W, 2200W at 0.1s, respectively. It can be founded that, the coupling magnitudes of reactive power increases in accordance with the increases of real power references, reaching the maximum value of \( \Delta Q = 210 \text{Var} \), as shown in the zoom in figure of \( Q \) (denoted as \( Q_{\text{zoom}} \) in Fig. 8(a)). In Fig. 8(b), the coupling magnitudes of real power increases as well when reactive powers \( Q \) increase from 0Var to 2200Var at 0.1s, reaching the maximum value of \( \Delta P = 480\text{W} \), as shown in \( P_{\text{zoom}} \) in Fig. 8(b). Fig. 8(c) and (d) show the decouling results of real and reactive powers after using the proposed method. It can be seen that the reactive power coupling magnitude \( \Delta Q \) is reduced from 210Var to 0Var with ideally 100% reduction capability, and the real power coupling magnitude \( \Delta P \) is reduced from 480W to 30W, with 93% reduction performance.

Fig. 8. Performance of power decoupling under different power references in GVM-DPC when two-phases-to-ground fault occurs. (a) and (b) are power coupling results in GVM-DPC when \( P \) changes from 500W to 1.4kW, 1.8kW, 2.2kW and \( Q \) changes from 0 Var to 1.4kVar, 1.8kVar, 2.2kVar. (c) and (d) are power decoupling results in accordance with (a) and (b).

D. Robustness of the Proposed DFPC under Parameter Mismatch and Harmonic Distorted Grid Voltage Condition

As described in (14), the grid side impedance \( R_g, L_g \) is required for the DFPC-based power decoupling method. Hence, it is necessary to test the robustness of the proposed method under different percentage of errors of parameter mismatches, as shown in Fig. 9(a). It can be seen that the coupling magnitude of reactive power (\( \Delta Q \)) reaches 150Var when real power increases from 500W to 2000W at 0.1s. After using the proposed DFPC method with accurate parameter detection, \( \Delta Q \) drops to 0Var with 100% reduction performance. However, when the parameter mismatch error is set to be 20% and 30%,
decoupling results with parameter mismatches reaching 30% errors under unbalanced system. (c) is the coupling and decoupling results under unbalanced system when a 3rd harmonic component is emulated for phase-A voltage. (d) is the power decoupling results of the proposed method under voltage swell system when single-phase-to-ground fault occurs.

the coupling magnitudes will be increased from 0Var to 50Var, with 67% reduction performance of decoupling, which is still within acceptable range. Besides, different grid voltage conditions with 3-phase balanced voltages and harmonic distorted voltage are also explored, as shown in Fig. 9(b), (c). From (b), it can be seen that the proposed method is also suitable for the balanced voltage condition, which can effectively reduce the coupling magnitude of reactive power from $\Delta Q=150\text{Var}$ to $\Delta Q=25\text{Var}$ when real power increase from 500W to 2000W at 0.1s, with 83% reduction performance. From Fig. 9(c), a 3rd order harmonic component is set for phase-A voltage with maximum magnitude of 0.9 p. u. It can be observed that the coupling magnitude of reactive power can be reduced from $\Delta Q=60\text{Var}$ to $\Delta Q=5\text{Var}$ with 91% reduction. Consequently, the robustness of the proposed DFPC method against different voltage conditions and parameter mismatch is verified. Finally, voltage swell condition is also tested in this section, which often occurs as a result of single-phase-to-ground (SPTG) fault in an ungrounded system. From Fig. 9(d), the SPTG fault happens at 0.1s, with phase-A, B voltages swelling to 1.1 p. u., and phase-C voltage dropping to 0.9 p. u.. It can be seen that when real power increases to 2000W at 0.1s, the coupling reactive power magnitude is effectively reduced from 270Var to 50Var, with 81% reduction performance.

Noted that the large power ripple in Fig. 7, 8, 9 cannot be further eliminated when output 3-phase currents are controlled to be balanced and sinusoidal in this paper, thus the output powers are mostly 2nd order harmonic components denoted by $P_{12}, Q_{12}$ as shown in (2). It has already been proved that the ripples in real and reactive powers cannot be eliminated simultaneously [12]. Normally, there are 3 control methods to constrain power ripples, e.g., maintaining the sinusoidal currents, keeping constant real power, or removing the ripples of reactive power [12]. This paper selects the first control scheme to obtain the 3-phase balanced-sinusoidal currents, so the power ripples in real and reactive powers cannot be fully eliminated.

E. Comparison with traditional virtual-inductance-based power decoupling method

Conventional virtual impedance method (VIM) for virtual synchronous generator control (VSG) is selected as a comparison to clarify the contribution of the proposed DFPC method for GVM-DPC [24]. The VIM is widely adopted in VSG to reshape the predominately inductive connecting line so as to reduce the coupling magnitudes.

As discussed in Section III-C, the “predominant inductive line” indicator cannot reduce the coupling magnitudes when grid side impedance increases in weak grids, since it only considers the effect of impedance angle without considering voltage angle variation and its amplitude perturbation when injected powers change. To verify this, Fig. 10 shows the power decoupling results of VSG-based VSI using VIM and GVM-DPC-based VSI with the proposed DFPC in the unbalanced system (phase-A voltage amplitude dropping to 0.9 p. u.). The control parameters of GVM-DPC can be seen in Table I, and VSG refers to [24], [45]. The grid side impedance is set to be 12mH, which is large enough to test the drawback of VIM. From Fig.10(a), (b), when real power of VSG-based VSI increases to 1 p. u. at 1s with virtual impedance $x_v=0$ p. u., the reactive power decreases from 0 to -0.24 p. u. accordingly. When $x_v$ increases from 0 to 0.6 p. u., coupling magnitudes of reactive powers increase as well, indicating that more reactive powers will be absorbed by VSI from power grids which will eventually leads to system destabilization. However, the coupling reactive power can be effectively decoupled by the proposed DFPC method, with magnitude dropping from 0.22 p. u. to 0.03 p. u., as shown in Fig. 10(d).

F. Stability case study

The stability of the proposed control system is tested in this case, which is studied by the Nyquist stability analysis on $G_{df}$ (s).

For a weak grid system with phase-A voltage dropping to 0.9 p. u., the grid side impedance is set to be 10mH. All the parameters are set according to Table I above. When the VSI is controlled to output real powers from 0W to 1000W, 1300W, 1400W and 1500W respectively, the Nyquist diagram near the critical point (-1, 0) is drawn in Fig. 11 below. It can be seen that, when the $P_{ref}$ increases from 1000W to 1400W, characteristic loci of $G_{df}(s)$ move close to the critical point without encircling it, indicating the systems are stable. However, the locus begins to encircle the critical point when $P_{ref}=1500W$, indicating that the system begins to destabilize.
Fig. 11. Nyquist diagram of $G_{\alpha\beta}(s)$ under different power references after power decoupling. (a) is the Nyquist diagram and (b) is its zoom in plot near critical point.

A time-domain simulation is carried out to prove the correctness of stability analysis. Fig. 12 shows the stability performance of the GVM-DPC-based VSI system when $P_{\text{ref}}$ changes from 0W to 1000, 1300, and 1500W at 0.1s, respectively, where Fig. 12(a) shows the conventional GVM-DPC performance and Fig. 12(b) shows the simulation results of the proposed DFPC-based VSI system. From Fig. 12(b), it can be seen that when $P_{\text{ref}}$ changes from 0W to 1000, 1300, and 1400W at 0.1s, the systems are still stable while it is destabilized when $P_{\text{ref}}$ reaches 1500W. Simulation results of DFPC-based VSI show consistency with the impedance analysis.

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Reactive Power (Var) Real Power (W)
(a)
Fig. 12. Real and reactive powers when rated real powers are 1000W, 1300W, and 1500W. (a), (b) are powers before power decoupling and (c), (d) are powers after power decoupling.

V. HARDWARE IN LOOP TEST

The proposed DFPC power decoupling strategy is also validated through an HIL test system shown in Fig. 13. The control parameters and grid side impedance can be changed via communications with a control computer. The grid-connected VSI using GVM-DPC and DFPC scheme is represented by detailed EMT models in the RTDS. The proposed DFPC method is implemented in VSI control system established in RSCAD. The sampling time is 1 $\mu$s to capture the switching dynamics of converter. Also, the PC control block is included in the system to control negative sequence real and reactive powers to be zero, so as to obtain 3-phase balanced output currents of VSI. To eliminate the positive sequence quantity, the delay signal cancellation (DSC) method [39] is adopted in PC control block for the sequence separation. The DSC extracts the positive and negative sequence with a time delay of $1/4$ of the signal period, which is widely used to improve PLL or VSI control under unbalanced grid conditions [39]-[41].

Fig. 13. HIL test setup with RTDS

Two different cases are tested, i.e., balanced grid voltage condition and single-phase-to-ground fault condition. For the balanced system, the proposed DFPC method can effectively eliminate coupling powers in the transient stage, as shown in Fig. 14(a) below. It can be observed that when real power increases from 0W to 2kW at 0.4s, the coupling magnitude of reactive power reaches 1200Var at 0.45s. When the proposed method is utilized, the coupling magnitude of reactive power is reduced from 1200Var to 0Var at 0.45s, with 100% reduction of coupling component. Besides, the dynamic of real power is improved by using the proposed method.

Fig. 14. Power decoupling performance under different grid voltage conditions. (a) is 3-phase voltages balanced system and (b) is the unbalanced system. (c) is the angle variation result ($\delta_P$) when $P_{\text{ref}}$ change from 0W to 2000W.

For the unbalanced grid voltage system with phase-A drops to 0.9 p. u., the decoupling performance is verified and shown in Fig. 14(b). From Fig. 14(b), it can be seen that large power ripples are produced due to the unbalanced grid voltage system. When real power increases from 0W to 2kW at 0.4s, the coupling magnitude of reactive power reaches 1300Var at 0.45s. When the proposed DFPC is utilized for GVM-DPC, the coupling magnitude is reduced to nearly zero at 0.45s with 100% reduction performance. Fig. 14(c) shows the trajectories of the angle difference of PCC voltage and grid side voltage in positive sequence denoted by $\Delta\theta_P$ in Fig. 3 above. It can be seen that the $\Delta\theta_P$ increases from $0$ rad to $0.2$ rad when real power rises.
from 0W to 2000W at 0.4s, which indicates the occurrence of power coupling in that short period. Then it decreases to zero when real power remains at a steady state, and the power coupling component does not exist in this situation.

VI. EXPERIMENTAL RESULTS

An experimental setup is established to validate the effectiveness of the proposed power decoupling method. A 3-leg 3-phase 1kVA Danfoss inverter with an L filter is connected to a chroma grid simulator via a grid side inductor as Fig. 15 shown. The control strategy is implemented in dSPACE1006 system to generate PWM signals for VSI, where the switching pulses are generated via DS5101 digital waveform output board. The unbalanced grid voltage condition is set in grid simulator with phase-B dropping to 0.9 p. u.. The signals are measured by LEM boards as shown in Fig. 15(a) with sampling frequency of 10kHz. All the parameters are set according to Table II.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_g$</td>
<td>Nominal line-to-line grid voltage</td>
<td>133</td>
<td>V</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>DC-link voltage</td>
<td>250</td>
<td>V</td>
</tr>
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<td>$f$</td>
<td>Nominal grid frequency</td>
<td>50</td>
<td>Hz</td>
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<td>$R$</td>
<td>Filter resistor</td>
<td>0</td>
<td>Ω</td>
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<tr>
<td>$L$</td>
<td>Filter inductance</td>
<td>7.2 mH</td>
<td></td>
</tr>
<tr>
<td>$R_g$</td>
<td>Grid side resistor</td>
<td>0</td>
<td>Ω</td>
</tr>
<tr>
<td>$L_g$</td>
<td>Grid side inductance</td>
<td>1.8 mH</td>
<td></td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
<td>10 kHz</td>
<td></td>
</tr>
<tr>
<td>$f_a$</td>
<td>Sampling frequency</td>
<td>10 kHz</td>
<td></td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional control parameter</td>
<td>12000</td>
<td></td>
</tr>
<tr>
<td>$K_i$</td>
<td>Integral control parameter of PI controller</td>
<td>3000</td>
<td></td>
</tr>
</tbody>
</table>

As seen in Fig. 16 (a)(b), the output 3-phase currents are balanced under the unbalanced voltage system based on the PC control block in Fig. 1. From Fig. 16(c), it can be observed that coupling reactive power is produced when real power increases from 0W to 550W, with coupling magnitude of 160Var. Then, the coupling reactive power decreases to zero when the VSI system operates in the steady states. After utilizing the proposed power decoupling method, the coupling magnitude of reactive power ($\Delta Q$) reduces to zero with 100% reduction performance, as shown in Fig. 16(d). Moreover, it can be seen that the steady-state performances of real and reactive powers are not distorted by the proposed method, which mainly include twice order harmonic ripple with period 0.01s, as shown in Fig. 16(c)(d). Also, small amounts of higher order harmonics can be seen in real power in steady states before and after decoupling as shown in Fig. 16 (c)(d), due to the harmonic distortion in voltage condition.

For the discussion of robustness of the proposed method, different parameter mismatches are set in the proposed DFPC control system, and the VSI operates in a harmonic distorted unbalanced voltage condition. From Fig. 17(a), it can be observed that the reactive power compensation magnitude is larger than realistic coupling magnitude when the estimated grid side inductance is larger than real value with +20% error (estimated value: 2.16mH, realistic value: 1.8mH), the $\Delta Q$ equals 20Var, with 75% reduction performance. To sum up, the proposed method still could eliminate over 70% coupling magnitude under 25% parameter mismatch.
Fig. 16. Test results of the proposed power decoupling method. (a), (b) are 3-phase voltages and currents at PCC detected by LEM board under unbalanced grid voltage condition with phase-B dropping to 0.9 p. u.; (c) and (d) are the coupling component of reactive power and the decoupling performance of the proposed method.

VII. CONCLUSIONS AND FUTURE WORK

This paper proposes a power decoupling method for GVM-DPC of the three-phase VSI connected to the weak grids under unbalanced grid voltage system, where the DFPC is implemented into the power control loop in GVM-DPC to compensate the coupling powers. Compared with the traditional GVM-DPC, the proposed method can effectively eliminate the power coupling issues in weak grids with consideration of positive sequence component of PCC voltage variations. Case studies show that the proposed method can reduce over 80% coupling magnitudes of both real and reactive powers for the normal power regulation of VSI under unbalanced voltage condition with different voltage drop from 0.9 p. u. to 0.6 p. u.. Besides, the robustness of the proposed DFPC is verified in terms of harmonic distorted unbalanced grid voltage and parameter mismatches, with performance of over 67% reduction of coupling powers considering 30% error of parameter mismatch and over 90% reduction of coupling powers under harmonic distorted unbalanced grid voltages. Also, experimental work validates the proposed power decoupling method under normal and harmonic distorted system, indicating that the reduction of coupling components performance can be over 70% under 25% parameter mismatch errors. Moreover, impedance analysis of the VSI system is conducted after the powers are decoupled in the transient stage, and case study result using different control parameters verifies the stability analysis. The power decoupling of multiple VSIs based on the GVM-DPC constitutes the future work.

APPENDIX

To prove the power coupling phenomenon of VSI connected to the unbalanced grid voltage system via inductive connecting line, the $R_g$ is set to zero in Fig. 1. The relationship of positive sequence components of PCC voltage, active and reactive...
currents is given as (38)
\[ v_{\text{ref}}^p \angle \delta = j X_i (i_p^r - j i_q^r) + V^* \angle 0 \] (38)
where the PCC voltage amplitude is given by (39) and it can be linearized according to active and reactive currents as shown in (40).
\[ v_p = v_p^* + j X_i i_p^r + j X_i i_q^r = \left( V^p + X_i i_p^r \right)^2 + \left( X_i i_q^r \right)^2 \] (39)
\[ \Delta V_{\text{pcc}} = \left( \frac{V^p + X_i i_p^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right) \Delta i_p^r + \left( \frac{V^p + X_i i_q^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right) \Delta i_q^r \] (40)

Meanwhile, the output real power \( P \) can be linearized in accordance with PCC voltage amplitude \( \Delta V_{\text{pcc}} \) and active current \( i_p^r \):
\[ P^0 = i_p^r \Delta V_{\text{pcc}}^r + v_{\text{pcc}0} \Delta i_p^r \] (41)
Substituting (40) into (41) yields,
\[ P^0 = \left[ \frac{V^p + X_i i_p^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right] \Delta i_p^r + \left( \frac{V^p + X_i i_q^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right) \Delta i_q^r \] (42)
Similarly, the linearization of reactive power can be derived,
\[ Q^0 = \left( \frac{V^p + X_i i_q^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right) \Delta i_p^r + \left( \frac{V^p + X_i i_p^r}{|V^p + X_i i_p^r|^2 + |X_i i_q^r|^2} \right) \Delta i_q^r \] (43)
Finally, positive sequence power coupling of VSI can be expressed by (44), where \( c \) is shown in (45).
\[ \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} X_i i_p^r + v_{\text{pcc}0} \\ X_i i_q^r \end{bmatrix} \begin{bmatrix} V^p + X_i i_p^r \\ V^p + X_i i_q^r \end{bmatrix} \begin{bmatrix} \Delta i_p^r \\ \Delta i_q^r \end{bmatrix} \] (44)
\[ c = \frac{X_i i_p^r + v_{\text{pcc}0}}{X_i i_q^r} \] (45)

REFERENCES


