A New Perspective for Relating Virtual Inertia with Wideband Oscillation of Voltage in Low-Inertia DC Microgrid

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Abstract—Virtual synchronous generator (VSG) has been a grid-friendly integration control technique for the integration of grid-connected inverters. However, the emulated inertia and damping of VSG control technique can also be used in the field of DC systems. In this paper, a virtual synchronous control is proposed to dampen the wideband oscillation of DC voltage in a DC microgrid. The proposed control strategy contributes to maintaining synchronous operation of DC converter with the network. Besides, the relationships among damping, inertia, wideband oscillation, rate of change of voltage (RoCoV) as well as DC voltage nadir (DCVN) are studied. It is concluded that the RoCoV and DCVN are similarly as the oscillation frequency and fluctuation ranges of poorly-damped oscillation, respectively. A unified concept is proposed by connecting the oscillation-related stability with inertial transient response originated from the imbalanced powers/mismatched currents. Besides of this, the inertia plays the same role as damping because the inertia contributes to maintaining the original state and damping to impeding further change. A new feedback analytical method is proposed to clarify the important role of RoCoV and DCVN on the motion of DC voltage. Finally, the theoretical results are compared with simulations and experiments.

Index Terms—rate of change of voltage (RoCoV), DC voltage nadir (DCVN), inertia and damping, virtual synchronous generator (VSG), poorly-damped oscillations, feedback analytical method

I. INTRODUCTION

ENVIRONMENTAL pollution and energy crisis have been a troublesome issue, which needs to be tackled urgently. Aroused by this stimulation, large-scale renewable energy sources, energy-storage system and electric vehicles have been interfaced with the grid via power electronic converters to alleviate the burden of conventional fossil fuels [1]. Unfortunately undesirable interactive behaviors between power converter and grid may induce wideband oscillations due to the poor damping [2]. At the same time fast response from the converter can result in low-inertia and weak-damping, which can jeopardize the stable operation of the system [3].

Power converters with low-inertia and weak-damping can interact with the DC network, resulting in undesirable interactions or even instability of the systems [4]. The leading cause has been found in the form of a resonance loop at high frequency bands between two subsystems impedance. Similarly, the mechanism of the oscillation phenomenon has been illustrated by eigenvalue analysis, which found poorly-damped dominant oscillatory modes in DC microgrid [6]. In addition, a feedback analytical method was proposed to identify whether divergent oscillation or convergent oscillation of the voltage in a DC system [15]. In summary, the leading cause of voltage oscillation in DC microgrid can be summarized as three aspects: 1) undesirable impedance interaction between converter and the DC network, 2) poorly-damped critical modes, and 3) positive feedback leading to divergent oscillation.

To overcome those instability issues, virtual impedance (VI) [4]-[6] and virtual synchronous machine (VSM) [7]-[13] are the two main categories used to provide support for inertia and damping for the power system. Compared to VI control method, VSM control has certain advantages, e.g. provision of inertial support, relatively easier implementation, etc. VSM can be divided into two categories including AC VSM and DC VSM. AC VSM is generally used to improve the frequency stability and transient response during the disturbance [10]-[13]. DC VSM is usually used for inertia and damping support of DC voltage [14]-[17]. In addition, several papers have also investigated the VSM and virtual inertia, including parameter constraints [18], small signal modeling [19], [20] stability assessment [21], and the inertia design [22].

In some references, a virtual inertia control is used for improving the transient response of the system, i.e., alleviating the RoCoF and FN [23]-[25] or rate of change of voltage (RoCoV) or DC voltage nadir (DCVN) [16], [17]. No matter of RoCoF, FN, RoCoV, or DCVN, it is attributed to the imbalanced active powers which compels the frequency or DC voltage deviate from the nominal value, and thus a RoCoF (RoCoV) and FN (VN) has been emerging.

In fact, the main focus of inertia analysis is generally concentrated on the transient nadir and the rate of change of the state variables. However, for the topics of oscillation-related stability (ORS), the voltage oscillation amplitude (VOA) and oscillation frequency (OF) are usually the main focus. It can be seen in Fig. 1 that the transient nadir has the highly similarity with the oscillation amplitude, and the rate of change of voltage can be analogous to oscillation frequency of the voltage.

As shown in Fig. 1(a), a power disturbance occurs and thus...
the transient deviation of DC voltage appears. Besides, an attenuated oscillation occurs due to the weak damping of the system, as shown in Fig. 1(b). In fact, the expression of attenuated oscillation can be derived as a standard second-order system, as shown in Fig. 1(b).

\[ f(t) = e^{-\delta \omega t} \sin(\omega d t) \]  

(1)

\( f' \) is the slope of the transient deviation of DC voltage with the imbalanced power, the grey damping.

Fig. 1. Analogy between inertia analysis and oscillation-based stability analysis, (a) transient response during imbalanced power, (b) attenuated oscillation with weak damping.

Perturbing the above (1) at \( t=0 \), the slope of the curve at \( t=0 \) is maximum RoCoV, i.e.,

\[ f'(t) = -\delta \omega e^{-\delta \omega t} \sin(\omega d t) + \omega d e^{-\delta \omega t} \cos(\omega d t) \]

\[ \Rightarrow f'(t=0) = \omega d \]

(2)

From (2), it can be inferred that the maximum of RoCoV from the perspective of inertia analysis is equal to the VOF from the view of ORS analysis. Furthermore, the voltage nadir arises at RoCoV=0, i.e., slope of curve=0, within the first cycle.

The step response is a good way to identify the IR with consideration of RoCoV and DCVN when system is subjected to a disturbance of imbalanced powers. Thus, the second-order model of the attenuated oscillation can be derived as:

\[ H_{osc}(s) = \frac{\omega d}{s + \delta \omega} \]

where \( H_{osc}(s) \) can be used to identify the performance of transient deviations, i.e., RoCoV, and DCVN.

It can be seen from Fig. 2 that the RoCoV is justly equal to VOF, and the DCVN is justly as the same as VOA. In addition, the inertial response (IR), which is originated from mismatched powers, is as the same form as the oscillation-related stability due to weak damping and undesirable interactions, which can be shown in Fig. 2. It can be inferred that the IR of the DC voltage cause by the imbalanced powers has the similar characteristics with that of DC voltage oscillation due to the undesirable interactions instability.

To make clearer concepts for the classifications among RoCoF, RoCoV, FN, DCVN, VOA, and VOF, the difference between IR (subjected to imbalanced powers) and ORS (during small-perturbations) are identified by Table I.

In Table I, MFD represents maximum frequency deviations, MVD stands for maximum voltage deviations, IP denotes imbalanced powers, and RSP means random small perturbation.

<table>
<thead>
<tr>
<th>Items</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index classification</td>
<td>IR</td>
</tr>
<tr>
<td>RoCoF</td>
<td>RoCoV</td>
</tr>
<tr>
<td>Literature</td>
<td>[14]-[17]</td>
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<tr>
<td>Dynamic characteristic</td>
<td>IP</td>
</tr>
</tbody>
</table>

From the view of physical significance, inertia contributes to compelling the state variables to maintain the original state and thus not being changed. However, the effect of damping aims to impede the state variables further deviating from the nominal value and compel it convergent to the given value.

Based on this idea, it has been pointed out that transient voltage nadir has the similar characteristics with that of oscillation of DC voltage. Thus, the proposed virtual synchronous control can implement the “synchronized operation” of DC voltage. The main contributed works are organized as follows:

1) The principle of synchronization of DC converter with the grid is clarified. And the impact of inertia and damping on the synchronization of DC voltage is explored. A new perspective of voltage oscillation mechanism is clarified that voltage oscillation is originated from the periodic fluctuation of DC current and periodically charging or discharging current through DC capacitors. Perturbation
2) This paper put forward the unified concept for relating the RoCoV and DCVN with the interaction-related oscillation for the first time. And it builds the connection between RoCoV and oscillation frequency, and it also builds up the relationship between DCVN and oscillation magnitude. Besides, the impact of virtual inertia and virtual damping on RoCoV, DCVN, VOF, and VOA are discussed comprehensively.
3) It is found that larger size of virtual inertia can bring better damping performance. Furthermore, the larger inertia and stronger damping can make the better synchronization ability of the DC voltage with the DC microgrid.
4) The dynamic interaction between RoCoV and DCVN is clarified by the proposed feedback effect concept. Both of RoCoV and DCVN play important roles in the motion of the voltage.
5) The feedback analytical method and principle are initially proposed to illustrate the motion of voltage in a DC microgrid, with relating RoCoV and DCVN with the ORS. A unified concept is initially proposed by relating RoCoV with VOF, and relating DCVN with VOA.

The rest of this paper is organized as follows: Section II briefly introduces the mechanism of periodical fluctuation of DC voltage. Section III discusses the dynamic interaction
between RoCoV and DCVN, besides, the unified concept of relating RoCoV and DCVN with ORS is proposed. In Section IV, theoretical analysis is verified by simulations and experiments. Finally, Section V gives the conclusions and discussions.

\[
\frac{1}{J_{vir}} \omega_N \frac{d\omega_{vir}}{dt} = P_m - P_e - k_{\text{dam}} \omega_N (\omega_{vir} - \omega_N) \tag{4}
\]

where \(k_{\text{dam}} = k + D_{\text{dam}}\), which incorporate both damping effect and droop effect. \(\omega_N\) is an adjustable parameter which can be flexibly set. However, here it is set to 50 Hz (314 rad/s) to emulate the characteristics of synchronous generator. To provide inertia support for DC voltage, the virtual angular frequency \(\omega_{vir}\) is coupled with virtual DC voltage through a proportional gain \(f_{sr}\), i.e.,

\[
V_{\text{vir}} = \omega_{vir} f_{sr}
\]

where \(V_{\text{vir}}\) is the obtained virtual DC voltage, the virtual DC current is further derived by the following equation:

\[
I_{\text{vir}} = \frac{1}{k_{\text{vir}}} (V_{\text{vir}} + V_e) \tag{6}
\]

where the feedforward term \(V_{\text{vir}}\) can provide inertia and damping support for DC system by the emulated virtual inertia and virtual damping. Besides, \(k_{\text{vir}}\) is a supplementary damping term to provide damping support. It should be noted that (4)-(6) depict the dynamic behavior of virtual synchronous control, and the virtual synchronous control loop is cascaded between outer voltage control loop and inner current loop, which can be shown in Fig. 5(a).

Perturbing (4)-(6), the small signal model can be derived as:

\[
\frac{1}{J_{vir}} \omega_N \frac{d\Delta \omega_{vir}}{dt} = \Delta P_e - \Delta P_m - k_{\text{dam}} \omega_N (\Delta \omega_{vir} - \Delta \omega_N) \tag{7}
\]

\[
\Delta V_{\text{vir}} = f_{sr} \Delta \omega_{vir}
\]

\[
\Delta I_{\text{vir}} = \frac{1}{k_{\text{vir}}} (\Delta V_{\text{vir}} + \Delta V_e)
\]

where it can be seen that both variation of DC voltage and virtual DC voltage can produce the virtual DC current to provide inertial support to assist the synchronized operation of DC voltage, besides, it should be noted that the energy source to provide inertia and damping support incorporate two components, i.e., \(\Delta P_m\) and \(\Delta P_{\text{dam}}\).

Specifically, \(\Delta P_m\) is generated by the output of regulation of outer voltage control loop, and the output is reliable on the variation of DC voltage. That is to say, the much faster variation rate and larger fluctuation range can generate more virtual input power for support of inertia and damping. Moreover, the damping power contributes to impeding the further change of DC voltage, which can make DC voltage attenuated much faster. Meanwhile, the larger \(k_{\text{dam}}\) leads to less dynamic deviation of DC voltage and voltage nadir, and larger \(k_{\text{dam}}\) produces more damping powers to support inertia.

Indeed, the swing equation of the virtual synchronous control behaves as a low-pass filter, i.e.,

\[
H_{uv} = \frac{1}{k_{\text{dam}}} \frac{1}{s J_{sr} k_{\text{dam}}} + 1
\]

where \(H_{uv}\) behaves like a first-order inertial link, and \(J_{sr}\) and \(k_{\text{dam}}\) have the same effect to contribute to the inertia support,
which also verifies that the virtual inertia and virtual damping have the same effect on improving the transient response.

From physical view, larger inertia means that the DC voltage can better maintain the original state; meanwhile, the larger damping suggests that the DC voltage is impeded further changed. Thus, the inertia has the same effect as the damping in this control structure.

Moreover, larger $J_{\text{in}}$ and $k_{\text{dam}}$ means the faster response of the active power to provide more inertia and damping support for DC voltage during the transient responses.

Fig. 6 displays the effect of inertia and damping on the dynamic response of DC voltage by means of test of step response. It can be seen in Fig. 6(a) that the larger size of virtual inertia has several advantages, e.g., smaller fluctuation ranges, less DC voltage nadir, less settling time, and more robust transient response. Furthermore, it can be inferred that transient response caused by imbalanced powers is essentially has the same dynamics as the oscillation-related dynamics because it has been clarified that the periodical fluctuation of the current through DC capacitors is the main cause of periodical charging and discharging of DC capacitors.

It illustrates that larger virtual inertia can reduce the dynamic deviation of DC voltage. In addition, it can compel DC voltage to arrive at the steady state much faster with less oscillation. Fig. 6(b) shows that larger virtual damping factor owns several advantages, e.g., less settling time, less fluctuation ranges, less voltage nadir, and more stable DC voltage. When $k_{\text{dam}}$ arrives at 5, there exists no any oscillation and DC voltage can quickly restore to steady state.

It can be inferred that both virtual inertia and virtual damping contribute to compelling DC voltage maintain the original state and impeding it further changed. Thus, DC voltage can be divergent and stable with less time and less fluctuation.

III. STABILITY AND DYNAMICS ANALYSIS

In this Section, the dynamic interaction between RoCoV and DCVN is discussed by the proposed feedback analysis. Besides, a unified concept of relating virtual inertia with ORS is illustrated with feedback effect.

A. Nyquist criterion based on minor-loop gain

The Nyquist criterion based on minor-loop gain proposed by Middlebrook is an effective way to identify the interaction between two subsystems in a cascaded network [26]. Based on the laws of Kirchhoff and energy conservation, perturbing the circuit equation around the equilibrium point, one can obtain:

$$
\Delta V_{\alpha} - \Delta E_{\gamma} = L_{\text{dc}} \frac{dM_{\text{dc}}}{dt}$$
$$\Delta V_{\alpha}I_{\text{dc}} + V_{\text{dc}} \Delta I_{\text{dc}} = V_{\text{dc}}I_{\text{dc}} + V_{\text{dc}} \Delta I_{\text{dc}} + CV_{\text{dc}} \frac{d\Delta V_{\text{dc}}}{dt}$$

where $V_{\text{dc}}$ and $E_{\gamma}$ represent source voltage and inner potential of DC converter, $V_{\text{dc}}$ is the DC voltage across the capacitor, and $I_{\text{dc}}$ denotes the output DC current of the converter.

The control dynamic equations of virtual synchronous control are deduced as:

$$\begin{aligned}
\Delta V_{\alpha} &= \Delta V_{\text{ref}} + \frac{k_p}{s} V_{\text{dc}} + \frac{k_i}{s} \Delta I_{\text{dc}} \\
\Delta E &= \frac{k_p}{s} V_{\text{dc}} + \frac{k_i}{s} \Delta I_{\text{dc}}
\end{aligned}$$

Combining (7), (9), with (10), the output impedance of the converter with virtual synchronous control can be obtained as:

$$Z_{\alpha} = -\frac{\Delta V_{\text{dc}}}{\Delta I_{\text{dc}}}$$

where $Z_{\alpha}$ is the output impedance of the converter with the virtual synchronous control, besides, the input impedance includes the DC grid impedance as well as constant power load (CPL), i.e.,

$$Z_{\alpha} = R_{\text{g}} + sL_{\text{g}} + R_{\text{i}}$$

where it can be seen $Z_{\alpha}$ is contained in the closed loop control of DC voltage. With the minor-loop gain [26], the Nyquist criterion is developed to evaluate the interactive behavior in a cascaded system, i.e., $T_{\text{loop}} = Z_{\alpha}/Z_{\alpha}$.

Fig. 7 illustrates the results of Nyquist stability analysis, where the studied control system is the minor-loop gain, i.e., $Z_{\alpha}$ divided by $Z_{\alpha}$. Moreover, Fig. 7(a) shows the Nyquist curve without virtual synchronous control. It should be noted that the signs mean the length of DC lines. It can be seen that all of the curves encircled the point (-1, 0) with various length of DC line, indicating the system is unstable. Thus, conventional dual-loop control has a high risk of instability due to the unexpected impedance ratio. It should be noted that the direction of arrows means the direction of larger size of the studied parameters.

Fig. 7(b) shows the impact of virtual inertia factor on the
stability of the system. In fact, stability margin is better as \( J_{vir} \) gets larger. The similar effect can be seen in the variation of \( k_{diam} \) and \( k_{vir} \). It shows a superior performance of the control algorithm. Besides of this, the control has a wide control parameters ranges that can guarantee the stability of the system.

![Fig. 6. Unit step responses, (a) various \( J_{vir} \), (b) various \( k_{diam} \).](image)

![Fig. 7. Nyquist stability analysis, (a) without virtual synchronous control, (b) various \( J_{vir} \), (c) various \( k_{diam} \), (d) various \( k_{vir} \).](image)

**B. Eigenvalue analysis**

Fig. 8(a) depicts the impact of \( J_{vir} \) on the dominant poles of the system, and it shows that the oscillation frequency firstly increases and then declines as \( J_{vir} \) gets larger. Besides, the damping of the system is successively increased as \( J_{vir} \) gets larger. The phenomenon agrees well with the unit step response in Fig. 6(a). The unit step response indicates that the oscillated frequency gets higher and then becomes lower (finally equal to 0), and the damping is continually enlarged due to the oscillation range gets smaller.

Fig. 8(b) and (c) shows the impact of \( k_{diam} \) and \( k_{vir} \) on the stability of the system. The damping of the system gets larger as the parameter becomes larger, which indicates that the larger \( k_{vir} \) and \( k_{diam} \) contributes to stronger damping for the system. Thus, larger sizes of \( J_{vir} \), \( k_{vir} \) and \( k_{diam} \) are all good to the stability of the microgrid. It also suggests that the proposed control has a good robustness performance as well as wide operation range.

**C. Dynamic interactions between RoCoV and DCVN with feedback effect**

In this part, the dynamic interaction between RoCoV and DCVN is discussed, and those two indices in imbalanced transient response are related with the two indices, e.g., VOF, VOA, of ORS to clarify the motion of DC voltage.

**Remark 1:** dynamic deviation of DC voltage is defined as \( (V_{dc}-V_{ref}) \), and its maximum deviation is defined as DC voltage nadir (DCVN), which has been mentioned in the introduction. The RoCoV can be used for describing how fast the voltage changes, i.e., defined as \( dV_{dc}/dt \).

![Fig. 8. Eigenvalue analysis with virtual synchronous controller, (a) \( 0.2 \leq J_{vir} \leq 100 \) with a step of 1, (b) \( 2 \leq k_{diam} \leq 10 \) with a step of 0.05, (c) \( 6 \leq k_{vir} \leq 30 \) with a step of 0.5.](image)
dynamic deviation \(V_{dc} - V_{ref}\) is still always positive. It can be inferred that acceleration is inverse to the velocity. It forms a negative feedback effect that can make the DC voltage approach the equilibrium point. From the phenomenon, the motion of DC voltage is approaching the equilibrium point, which validates the analysis.

During from the instant \(t_2\) to instant \(t_3\), RoCoV is negative and dynamic deviation is negative as well, which suggests it forms a positive feedback, that the RoCoV can compel the dynamic deviations continuously changed. In this case, DC voltage cannot be stabilized at an equilibrium point because of positive feedback effect.

From instant \(t_3\) to instant \(t_4\), it forms a negative feedback effect, and the DC voltage is dynamically approaching the equilibrium point, and the dynamic deviation is gradually decreased to zero.

Discussion 2: The dynamic interaction between the RoCoV and DCVN is developed during the whole transient response from the instant of disturbance to the steady state (or to the divergent unstable state if instability). From the aforementioned analysis, the voltage waveform of transient response due to a mismatched/imbalance power can coincide with the voltage waveform of oscillation due to current oscillation. Besides, two indices, i.e., RoCoV and DCVN are similar with the oscillation frequency and oscillation magnitude, respectively.

Hence, it is significant to identify the impact of RoCoV as well as DCVN on the dynamics of oscillation. Besides, the dynamic interaction between RoCoV and DCVN is also discussed here to clarify the motion of DC voltage with feedback effect. It can be inferred from Fig. 9(a) that the maximum deviation, i.e., DCVN and RoCoV are declined simultaneously over time, which indicates the system forms a negative feedback effect. From another view, both RoCoV and DCVN are gets zero when system reaches to an equilibrium point. RoCoV not only can reflect the variation speed of DC voltage, but also can reflect the extent of imbalanced current. Motion of the voltage tends to be stable when and only when both DCVN and RoCoV are equal to zero. This phenomenon is as the same as that of oscillation. Vice versa, both RoCoV and DCVN get larger and larger if the system forms a positive feedback effect.

For example, with the imbalanced fluctuation of current, voltage is naturally charged and climb to the peak value until DC capacitor begins to discharge. However, the owned inertia of system can compel the voltage maintain the original state and not to climb. Thus, DC voltage will climb slower with less deviation to the nominal. Similarly, inertia makes the voltage drop slower with less deviation in the descending process. Hence, the larger inertia can make the DC voltage slower and deviate less, as shown in Fig. 10(a). It can be seen that although size of inertia can impose effect on both RoCoV and DCVN, compared to DCVN, the impact of inertia on RoCoV is greater. Greater inertia leads to much slower RoCoV as well as less DCVN, which can make the DC voltage smoother.

Furthermore, damping contributes to impeding the further changes of DC voltage during the disturbance as shown in Fig. 10(b). With the motivation of periodical fluctuation of DC current, voltage across DC capacitors will be oscillated around the nominal because of periodical charging and discharging of the capacitors. However, the damping effect can hinder it to be further changed, and the attenuated energy makes it deviate less in the next cycle. The stronger damping can impede motion of DC voltage and thus deviates less.
For robustness performance analysis, it is required to define the peak value of $S$ and $T$ by means of $H_\infty$ norm, i.e.,

$$
N_S = \max_{\omega} |S(j\omega)| = \|S\|_\infty,
$$

$$
N_T = \max_{\omega} |T(j\omega)| = \|T\|_\infty.
$$

(14)

Thus, to acquire the good enough robustness performance and robust stability margins, the infinite norm of both $S$ and $T$ are described as:

$$
\begin{align*}
RS & \iff \|T\|_\infty \leq 1 \\
RP & \iff \|S\|_\infty + \|T\|_\infty \leq 1
\end{align*}
$$

(15)

As shown in Fig. 11, both $S$ and $T$ are acquired to identify the robustness performance and robust stability. Fig. 11(a) and (b) depicts the frequency response of $S$ and $T$ with conventional dual loop control method. And the resonance peaks appear in both $S$ and $T$, which indicates poor robust stability and robustness performance.

As shown in Fig. 11(c) and (d), the resonance peaks are weakened by the proposed virtual synchronous control method, and the infinite norm of both $S$ and $T$ are below 1 (absolute value 1 means 0 dB). Hence, the proposed virtual synchronous control can weaken the resonance peaks and enhance robustness performance and improve the robust stability margins.

E. The energy source of inertia

As can be seen in Fig. 5(b), the gain of current-loop $G_{i_D}$ can be seen as unit since the response in time scale of inductor-current is much faster than other loops, which leads to $\Delta i_{i_D}=\Delta i_D$. That is, the virtual DC current is equal to output current of the converter, i.e., the controlled current. Indeed, the virtual synchronous controller will take actions to release the required energy to provide inertial support, when a disturbance appears which leads to the variation of DC voltage. According to the swing equation of virtual synchronous in (4), it can be known that the damping power contributes to enhancing damping performance meanwhile provide inertial support, i.e.,

$$
J_{\omega} = \frac{d\omega}{dt}
$$

(16)

where the virtual inertia $J_{\omega}$ is equivalent to the obtained inertia by damping power, it can be also expressed as:

$$
J_{\omega} = \int k_{\text{dam}} \omega (\omega_{\text{vir}} - \omega) dt
$$

(17)

where it is derived based on energy conservation, and $T_c$ means the time that DC voltage reaches the new steady equilibrium point. Indeed, the supplementary of inertia and damping is supported by the built virtual DC voltage $V_{\text{vir}}$ and the obtained virtual current $I_{\text{vir}}$, which are the output of the virtual synchronous controller.

However, the energy of built virtual voltage is originated from the energy storage, which supplies the required energy to support inertia and damping. The dynamic response during support of inertia as well as damping can be described by the energy conservation equation, i.e.,

$$
\int_0^T \frac{V_{\text{vir}}^2 + V_{\text{dc}}^2}{k_{\text{vir}}} dt = \int_0^T f_c V_{\text{vir}} dt + \int_0^T CV_{\text{dc}} \frac{dV_{\text{dc}}}{dt} dt
$$

(18)

Besides, the virtual current $I_{\text{vir}}$ is desired to be equal to $I_{\text{dc}}$ in the steady equilibrium point. Thus, the larger $k_{\text{vir}}$ can make the built virtual voltage $V_{\text{vir}}$ higher, which means that it can provide larger inertia support and more required energy. It can be inferred that $k_{\text{vir}}$ adds a control freedom degree for the system to regulate the damping and inertia for the DC microgrid.

Fig. 12. The flow chart of tuning control parameters of the proposed virtual synchronous control method.

F. Parameters selection

The selection of system specifications is essential for the design of DC-DC converter which contains both the hardware part and control part.

Once the hardware parameters have been determined, it can be hardly changed. Thus, a good way is to select the circuit hardware parameters firstly as the control parameters are flexible to be controlled to satisfy the dynamic performance of the system. Hence, the parameters selection can be developed step by step as followings:

1) Control parameters selection of $J_{\omega}$, $k_{\text{dam}}$ and $k_{\text{vir}}$

The final step of the controller parameters tuning is the selection of the presented control algorithm, i.e., selection of $J_{\omega}$, $k_{\text{dam}}$, and $k_{\text{vir}}$. It should be noted that the premise is that the proportional integral parameters are fixed prior to it. Those can be determined from various perspectives, i.e.,

1) Perspective of assurance of good stability

As the hardware parameters and control parameters of dual-loop have been determined in superior step, tuning of $J_{\omega}$, $k_{\text{dam}}$, and $k_{\text{vir}}$ can be developed by the eigenvalues guidelines from perspective of assurance of good stability.

Fig. 12 displays the tuning procedure of the control parameters of virtual synchronous control strategy, e.g., tuning of $J_{\omega}$, $k_{\text{dam}}$, and $k_{\text{vir}}$. To obtain the expected dynamic performance of the system, appropriate control parameters can be flexibly tuned by procedure in Fig. 12.
The first step is to develop eigenvalue analysis by varying $I_{dc}$ but with fixed $k_{dam}$ and $k_{virt}$. Afterwards, an optimum $I_{dc}$ can be found out to satisfy the good stability and desirable dynamic performance, signed with the selected $I_{dc_{opt}}$. Then, the second step aims to develop eigenvalue analysis by varying $k_{dam}$ but with fixed $k_{virt}$ and the selected $I_{dc_{opt}}$. Afterwards, the optimum value of $k_{dam}$ signed with $k_{dam_{opt}}$ can be found out by identifying the desirable damping performance region. Finally, the third step is to carry out the eigenvalue analysis by varying $k_{virt}$, to find out the optimum $k_{virt_{opt}}$. And the detailed procedure can be seen in Fig. 12. For this operation, the optimum tuning can be realized for the three control parameters.

**2) Perspective of transient inertial support**

As shown in Fig. 5, the virtual synchronous (VS) DC current $I_{dc}$ is the controlled current of DC converter, which is justly equal to the output current, when system reaches to an equilibrium point. It can be seen in Fig. 5(b), the small perturbation increment of the virtual DC voltage $\Delta V_{dc}$ can be used for the inertial support and thus generate the required current $\Delta I_{dc}$ for inertial support.

It can be seen in Fig. 8(c) that $k_{virt}$ cannot be selected too small to guarantee the positive damping of the dominant poles. In addition, the greater $I_{dc}$ can lead to less dynamic voltage deviations and less oscillation magnitude of the voltage. Besides, the greater $I_{dc}$ can also decrease the settling time of the voltage which returns back to the steady state point much faster.

Fig. 6(b) indicates the greater $k_{dam}$ results in less restoration time and less fluctuation range of the voltage, and the better damping and inertial support can be provided.

Actually, the tuning should take the remaining capacity of the power converter which can realize the inertial support into account. Hence, the control parameters tuning should be considered comprehensively with the discussed indices aforementioned to satisfy the superior performance of the system.

**IV. SIMULATIONS AND EXPERIMENTS**

**A. Simulations**

In this Section, simulations and experiments are developed to demonstrate the proposed ideas based on a setup of a DC microgrid with three DC boost converters, which is connected to DC bus via DC line with feeding DC loads. Table II shows the specifications of the systems in the simulations.

Fig. 13 displays the waveforms of DC voltage across DC capacitor and DC current through DC lines, respectively. It can be seen the voltage and current oscillation is induced, but is dampened when virtual synchronous control is put into operation.

Fig. 14 depicts the transient IRs when the DC microgrid is disturbed in a form of load increase. The units of measurement of the control parameters, e.g., $k_{virt}$, $k_{dam}$, $k_{virt}$ are The impact of various size of virtual damping $k_{dam}$ on the transient response of DC voltage is discussed. It can be seen in Fig. 14(b) that larger virtual damping leads to smaller dynamic deviation of DC voltage, i.e., smaller DCVN.

It can be seen in Fig. 14(c) that the voltage oscillation occurs when $k_{virt}=10$, which illustrates that small enough $k_{virt}$ can provide insufficient damping for DC microgrid and causes the voltage oscillation. As $k_{virt}$ increases, the voltage oscillation disappears, which indicates that the larger size of $k_{virt}$ can bring stronger damping for the system.

**B. Experiments**

Figs. 15 and 16 show the Star-Sim HIL results of DC voltages and currents. It should be noted that DCVO and DCCO means the oscillation of DC voltage and DC current. DCVS means virtual synchronous control put into operation.

![Fig. 13. Simulation results of DC voltage oscillation and oscillation damping by VSG control.](image)

**Table II. System Specifications of Simulations**

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<th>Parameters</th>
<th>Description</th>
<th>Value</th>
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<td>$V_{dc}$</td>
<td>Given DC voltage</td>
<td>320 V</td>
</tr>
<tr>
<td>$P_{l}$</td>
<td>Load power (single)</td>
<td>9.6 kW</td>
</tr>
<tr>
<td>$L_{f}$</td>
<td>Filter inductance</td>
<td>0.3 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Filter capacitance</td>
<td>700 μF</td>
</tr>
<tr>
<td>$R_{g}$</td>
<td>Line resistance</td>
<td>0.3 Ω</td>
</tr>
<tr>
<td>$L_{g}$</td>
<td>Line inductance</td>
<td>0.3 mH</td>
</tr>
<tr>
<td>$k_{d}$</td>
<td>Outer proportional constant</td>
<td>0.0873 (0.3) A/V</td>
</tr>
<tr>
<td>$k_{i}$</td>
<td>Outer integral constant</td>
<td>63.73 (50) A/(V.s)</td>
</tr>
<tr>
<td>$k_{p}$</td>
<td>Inner proportional constant</td>
<td>1.4 (5) V/A</td>
</tr>
<tr>
<td>$k_{q}$</td>
<td>Inner integral constant</td>
<td>1051 (0) V/(A.s)</td>
</tr>
<tr>
<td>$J_{in}$</td>
<td>Moment of virtual inertia</td>
<td>10 rad/(V^2)</td>
</tr>
<tr>
<td>$k_{virt}$</td>
<td>Virtual damping</td>
<td>3 Nm/rad</td>
</tr>
<tr>
<td>$k_{dam}$</td>
<td>Virtual resistance</td>
<td>20 Ω</td>
</tr>
<tr>
<td>$k_{ref}$</td>
<td>Switching frequency</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

**Fig. 14. The impact of virtual synchronous control parameters on the DCVN and RoCoV: (a) changing $k_{virt}$, (b) changing $k_{dam}$, (c) varying $k_{ref}$.**

![Fig. 15. Results of DC current](image)
maintain the synchronization operation. Moreover, the current quality is improved as well.

Fig. 16 The impact of virtual inertia on voltage oscillation damping and RoCoV, (a) $J_1 = 0.5$, (b) $J_1 = 1$, (c) $J_1 = 2$, (d) $J_1 = 5$.

Table III Comparison Performance Indicators Between Virtual Synchronous Control and Conventional Dual-loop Control

<table>
<thead>
<tr>
<th>Control</th>
<th>PI</th>
<th>RoCoV (max)</th>
<th>DCVN (max)</th>
<th>VOF</th>
<th>VOA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional dual loop control</td>
<td></td>
<td>226 V/s</td>
<td>5 V</td>
<td>226 Hz</td>
<td>5 V</td>
</tr>
<tr>
<td>Proposed virtual synchronous control ($J_1 = 0.5$)</td>
<td></td>
<td>0.5 V/s (attenuated to 0 V/s)</td>
<td>5 V (attenuated to 0 V)</td>
<td>0.5 Hz (gradually to be stable)</td>
<td>5 V (finally to 0 V)</td>
</tr>
</tbody>
</table>

Table IV System Specifications of Experiments

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{dc}$</td>
<td>DC operation voltage</td>
<td>160 V</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Load resistance</td>
<td>57.5 ohm (adjustable)</td>
</tr>
<tr>
<td>$R_T$</td>
<td>Total load resistance</td>
<td>57.5/3</td>
</tr>
<tr>
<td>$L_I$</td>
<td>Filter inducance</td>
<td>0.8 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>Filter capacitance</td>
<td>1100 μF (2200/2)</td>
</tr>
<tr>
<td>$N_T$</td>
<td>Amount of paralleled $R_L$</td>
<td>3</td>
</tr>
<tr>
<td>$N_I$</td>
<td>Total amount of L-filter</td>
<td>3</td>
</tr>
<tr>
<td>$N_{DC}$</td>
<td>Total amount of DC capacitors</td>
<td>6</td>
</tr>
<tr>
<td>$N_{DC_{boost}}$</td>
<td>Amount of DC boost converters</td>
<td>3</td>
</tr>
<tr>
<td>$J_{vir}$</td>
<td>Moment of virtual inertia</td>
<td>adjustable (rad/($N_c^2$))</td>
</tr>
<tr>
<td>$k_{dam}$</td>
<td>Virtual damping</td>
<td>adjustable (N/m/rad)</td>
</tr>
<tr>
<td>$k_{res}$</td>
<td>Virtual resistance</td>
<td>adjustable (Ω)</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>Switching frequency</td>
<td>5 kHz</td>
</tr>
</tbody>
</table>

Fig. 18 shows the d-Space experimental results. This case of experiment is to validate the synchronization performance of the control algorithm. Thus, no switches of the control modes are considered in the cases. It can be seen that a small voltage dip occurs after the disturbance, and recovers to the nominal value with a short transients without any oscillation. It shows a good synchronization performance of the system with good damping performance.

V. Conclusions and Discussions

This paper proposes a virtual synchronous control strategy to maintain the synchronization operation of DC converter with grid. Besides, it can improve the damping performance, inertia, RoCoV, DCVN, and robustness as the control parameters is flexible to be settled.

This paper relates the RoCoV and DCVN, which usually appears in the IR, with the ORS for the first time. A unified concept is put forward for relating the RoCoV and DCVN with the ORS. Especially, a comparison of performance indicators among RoCoV, DCVN, voltage oscillation frequency, and voltage oscillation magnitude is developed. Through theoretical analysis, it is found that transient response of DC voltage due to imbalanced currents or mismatched powers is
essentially the voltage oscillation which is originated from periodical fluctuation of DC current.

The dynamic interaction between RoCoV and DCVN is discussed with the feedback effect. It can be concluded that both RoCoV and DCVN gets smaller and smaller thereby finally tends to be zero when the system forms a negative feedback effect. While both RoCoV and DCVN gets greater and greater thereby finally tends to be infinite when the system forms a positive feedback effect. The RoCoV and DCVN is related with ORS through feedback effect, and those can be recognized as indicators of motion of voltage.

The inertia can be recognized as another type of damping to impede the further change of the state variables and maintain close to the original state for synchronization operation. Thus, the inertia plays a similar role as the damping.

Finally, quantification analysis between the two most important indices, RoCoV and DCVN, as well as the impact on the motion of DC voltage will be studied in our future works in a DC microgrid.

REFERENCES


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