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Dynamical orders of decentralized H_∞ controllers

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The problem of decentralized control is addressed, i.e. the problem of designing a controller where each control input is allowed to use only some of the measurements. It is shown that, for such problems, there does not always exist a sequence of controllers of bounded order which obtains near-optimal control. Neither does there exist an infinite-dimensional optimal controller. Using the insight of the line of proof of these results, a heuristic design algorithm is proposed for designing near-optimal controllers of increasing orders.

1. Introduction

In several industrial environments, implementing a full multivariable controller which combines all measurements and all control signals is not possible, practical, or desirable. For a distributed plant, installing a full multivariable controller could mean that a complex communication network had to be installed. Moreover, in terms of reliability, a full multivariable controller could have the effect that a breakdown in a single unit, no matter how peripheral to the system, could have plantwide consequences. Examples of application areas where full multivariable controllers are unacceptable are: distributed power systems (where the controllers for each station should be independent), steel milling (where the controllers for each stand should not interfere), and large-scale space systems (where the modules should be autonomous).

To formalize such requirements, known as decentralized control specifications, we consider a state-space plant model of the form

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \\ z &= C_1 x + D_{11} w + D_{12} u, \\ y &= C_2 x + D_{21} w + D_{22} u, \end{aligned} \tag{1}$$

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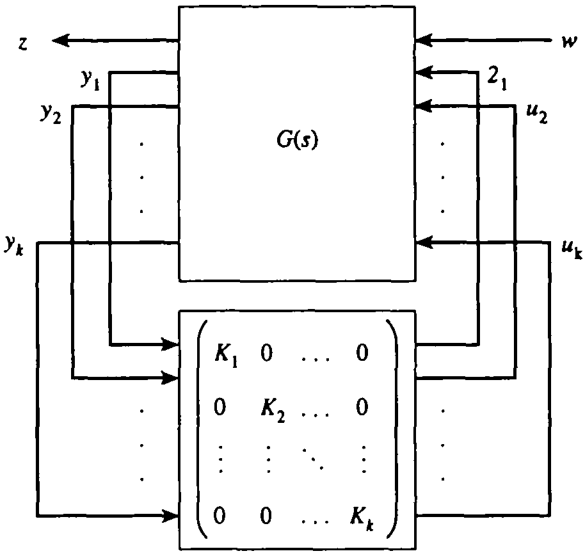

FIG. 1. Decentralized control

where u and y are partitioned as

$$u = \begin{bmatrix} u_1 \\ \vdots \\ u_k \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix}. \tag{2}$$

Now, the problem is to design k controllers

$$u_i = K_i y_i \quad (i = 1,\dots, k) \tag{3}$$

such that the resulting transfer function from w to z meets the specifications. In this paper we shall assume that the specifications are posed in terms of an H_∞ norm constraint of the transfer function from w to z. However, this choice is not crucial, and the argument found below would hold for many other types of performance specification.

Rewriting (3) using (2), we get

$$u = Ky, \qquad K = \begin{bmatrix} K_1 & 0 & \dots & 0 \\ 0 & K_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & K_k \end{bmatrix}.$$

The decentralized control problem is depicted in Fig. 1.

The theory of decentralized control has been widely studied in the literature. The classical theory which especially addresses the issue of decentralized stabilization is surveyed by Davison (1984). Two excellent textbooks dealing with decentralized control are those of Vidyasagar (1985) and Özgüler (1994). More recently, H_∞ decentralized control has been introduced by Paz (1993), and robust and reliable decentralized control has been studied, (e.g. Veilette *et al.* 1992).

Most published results on decentralized control are based on sufficient conditions only. In contrast, Sourlas & Manousiouthakis (1995) suggest an optimization-based approach. Their method uses a parametrization which enables an infinite-dimensional optimization problem to be approximated by a finite-dimensional one. In the example studies, controller orders grow rapidly as the optimization approaches the optimum. Sourlas & Manousiouthakis blame their method rather than the decentralized control problem itself. Indeed, since the problem formulation is finite-dimensional, it is tempting to believe that a decentralized control problem always can be solved by fixed-order controllers. In this paper, we shall prove to the contrary that, for decentralized control problems, all controllers can have dynamic orders that tend to infinity as the optimum is approached.

2. Main results

The main result of this paper is that near-optimal decentralized H_∞ control problems can require controllers of arbitrarily large orders as the optimum as approached. To state this in more precise terms, we introduce the following two sets of controllers:

$$\mathcal{K}_\gamma(G) =$$
$$\left\{ K = \begin{bmatrix} K_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & K_k \end{bmatrix} : K \text{ is internally stabilizing, and } \|\mathcal{F}_l(G, K)\|_\infty < \gamma \right\},$$

$$\mathcal{K}_\gamma^N(G) =$$
$$\left\{ K = \mathrm{diag}(K_i : i = 1, \dots, k) \in \mathcal{K}_\gamma(G) \;:\; K_i \; (i = 1, \dots, k) \text{ is of dynamical order } \leqslant N \right\}.$$

THEOREM 1 There exists a nonempty class $\mathcal{G}^\infty$ of systems such that, for each $G \in \mathcal{G}^\infty$, the inequality

$$\inf\left\{\gamma \;:\; \mathcal{K}_\gamma(G) \neq \emptyset\right\} < \inf\left\{\gamma \;:\; \mathcal{K}_\gamma^N(G) \neq \emptyset\right\}$$

holds for any N.

The interpretation of Theorem 1 is that there exists systems for which no sequence of *fixed-order* decentralized controllers approach the optimal value.

REMARK 1 It is tempting, yet incorrect, to conclude from Theorem 1 that this implies the existence of an optimal infinite-dimensional decentralized controller. We shall prove that, in general, there is no optimal decentralized controller yielding a closed-loop system analytical in the open right half plane.

Before embarking the proof of Theorem 1, we shall need the following result from functional analysis

COROLLARY 2 Let $\mathcal{D}$ denote a closed subset of the complex plane. Consider $f(\cdot) \in \mathrm{H}^p$, with f analytic in $\mathcal{D}$, and assume $f(z) = 0$ on a set of positive measure on the boundary of $\mathcal{D}$. Then $f \equiv 0$.

This observation is evident from the following result, which can be found in Jensen's (1899) paper. (This paper illustrates how use of the word 'new' in the title can be misleading.)

THEOREM 3 Let $\mathcal{D}$ denote the unit disc or $\mathbb{C}^+$. If $f(\cdot) \in \mathrm{H}^p(\mathcal{D})$, with $f \not\equiv 0$, then

$$\frac{1}{2\pi}\int_{\partial D} \log|f(\mathrm{e}^{\mathrm{i}\theta})|\,\mathrm{d}\theta > -\infty.$$

Finally, we shall use the following technical result.

LEMMA 4 Let

$$G(s) = \frac{B(s)}{A(s)} = \frac{\beta_0 s^N + \beta_1 s^{N-1} + \cdots + \beta_N}{s^N + \alpha_1 s^{N-1} + \cdots + \alpha_N}$$

be an irreducible Nth-order proper rational function, and let $\{\omega_1, \ldots, \omega_{2N+1}\}$ be a set of distinct real values for which $A(\mathrm{i}\omega_i) \neq 0$ $(i = 1, \ldots, 2N+1)$. Define the numbers

$$\gamma_i = \frac{B(\mathrm{i}\omega_i)}{A(\mathrm{i}\omega_i)} = \frac{\beta_0(\mathrm{i}\omega_i)^N + \beta_1(\mathrm{i}\omega_i)^{N-1} + \cdots + \beta_N}{(\mathrm{i}\omega_i)^N + \alpha_1(\mathrm{i}\omega_i)^{N-1} + \cdots + \alpha_N}. \tag{4}$$

Then there exists a neighbourhood of $(\gamma_1, \ldots, \gamma_{2N+1})$ such that the map $F : \mathbb{C}^{2N+1} \to \mathbb{C}^{2N+1}$, which maps the complex $(2N+1)$-vector $(\gamma_1, \ldots, \gamma_{2N+1})$ to the $(2N+1)$-dimensional (possibly complex) parameter vector $(\alpha_1, \ldots, \alpha_N, \beta_0, \ldots, \beta_N)$ of a rational function in the form (4), is a continuous bijection.

Proof. Let us first establish uniqueness of F at $(\gamma_1, \ldots, \gamma_{2N+1})$. To that end, assume that the parameters $\tilde{\alpha}_1, \ldots, \tilde{\alpha}_N, \tilde{\beta}_0, \ldots, \tilde{\beta}_N$ satisfy (4), i.e.

$$\gamma_i = \frac{\tilde{B}(\mathrm{i}\omega_i)}{\tilde{A}(\mathrm{i}\omega_i)} = \frac{\tilde{\beta}_0(\mathrm{i}\omega_i)^N + \tilde{\beta}_1(\mathrm{i}\omega_i)^{N-1} + \cdots + \tilde{\beta}_N}{(\mathrm{i}\omega_i)^N + \tilde{\alpha}_1(\mathrm{i}\omega_i)^{N-1} + \cdots + \tilde{\alpha}_N}. \tag{5}$$

However, from (4) and (5) we infer

$$A(\mathrm{i}\omega_i)\tilde{B}(\mathrm{i}\omega_i) - \tilde{A}(\mathrm{i}\omega_i)B(\mathrm{i}\omega_i) = 0 \quad (i = 1, \ldots, 2N+1).$$

The only polynomial of degree less than or equal to $2N$ having $2N+1$ zeros is the zero polynomial; hence

$$A(s)\tilde{B}(s) - \tilde{A}(s)B(s) \equiv 0. \tag{6}$$

Since $A(s)$ and $B(s)$ were assumed to be coprime, the only solutions to (6) of order less than or equal to N are

$$\tilde{A} = k \cdot A, \qquad \tilde{B} = k \cdot B,$$

where k is a unit in the ring of polynomials, i.e. a constant. Finally, since the coefficients of highest order in A and $\tilde{A}$ are fixed at 1, we conclude $k = 1$.

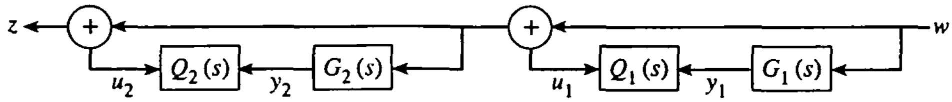

FIG. 2. Decentralized control for series connected model matching problems

From this argument, it follows that the map

$$F : \mathbb{C}^{2N+1} \to \mathbb{C}^{2N+1} : (\gamma_1, ..., \gamma_{2N+1}) \mapsto (\alpha_1, ..., \alpha_N, \beta_0, ..., \beta_N)$$

is well defined in any neighbourhood of $(\gamma_1, ..., \gamma_{2N+1})$ where the corresponding transfer function remains irreducible. Such a neighbourhood exists due to the continuity of the roots of a polynomial as functions of the coefficients, and due to the fact that the coefficients are computable by solving linear equations that depend continuously on the γ_i's. This also establishes continuity. Obviously, the inverse map is injective, due to the definition of the γ_i's. □

We are now able to prove our main result.

Proof of Theorem 1. To establish nonemptyness of $\mathcal{G}^\infty$, we shall study the decentralized control problem in Fig. 2. The system is a series connection of two 'model-matching problems', which can be thought of as a prototype of decentralized production-line control. In this interpretation, w is the amount of an impurity of the product eliminated in part by the controller Q_1 which is then transferred downstream, where the product is further refined by Q_2 before it is fully processed as z. The notation Q_i rather than K_i is introduced because we think of the Q_i's as YJBK parameters (Youla *et al.* 1976; Kučera 1975) rather than controllers. Specifically, we shall choose:

$$G_1(s) = \frac{s - z_1}{s + z_1}, \qquad G_2(s) = \frac{s - z_2}{s + z_2}, \qquad z_2 > z_1 > 0.$$

Note that internal stability is equivalent to stability of Q_1 and Q_2 since the G_i's are stable (though non-minimum phase.)

For this particular system, we shall prove that any sequence of fixed-order controllers stays boundedly away from the optimal value of γ, which for this example is 0 (see below). To that end, let N be fixed and assume to the contrary that we have a sequence of controllers $Q^\gamma = \begin{bmatrix} Q_1^\gamma & 0 \\ 0 & Q_2^\gamma \end{bmatrix}$, with Q_i^γ being Nth-order transfer functions, which satisfies $\|T_{zw}^\gamma(\cdot)\|_\infty < \gamma$ for all $\gamma > 0$, where $T_{zw}^\gamma(\cdot)$ is the closed-loop transfer function from w to z:

$$T_{zw}^\gamma(\cdot) = T_2^\gamma(\cdot)T_1^\gamma(\cdot) = \left[1 + Q_2^\gamma(\cdot)G_2(\cdot)\right]\left[1 + Q_1^\gamma(\cdot)G_1(\cdot)\right].$$

For any $\delta > 0$ we can perturb $G_1(\cdot)$ and $G_2(\cdot)$ by two irreducible $(N-1)$th-order stable proper rational functions $\hat{G}_1(\cdot)$ and $\hat{G}_2(\cdot)$:

$$\tilde{G}_1 = G_1 + \hat{G}_1, \qquad \tilde{G}_2 = G_2 + \hat{G}_2, \tag{7}$$

such that $\tilde{G}_1$ and $\tilde{G}_2$ are Nth-order stable non-strictly proper rational functions which have zeros in the right half plane and satisfy

$$\left\|\tilde{T}^{\gamma}_{zw}(\cdot)\right\|_{\infty} = \left\|\tilde{T}^{\gamma}_2(\cdot)\tilde{T}^{\gamma}_1(\cdot)\right\|_{\infty} = \left\|\left[1 + Q^{\gamma}_2(\cdot)\tilde{G}_2(\cdot)\right]\left[1 + Q^{\gamma}_1(\cdot)\tilde{G}_1(\cdot)\right]\right\|_{\infty} < \gamma + \delta.$$

Obviously, $\left\|\tilde{T}^{\gamma}_{zw}\right\|_{\infty} < \gamma + \delta$ implies that, for each frequency ω,

$$\text{either} \quad \left\|\tilde{T}^{\gamma}_1(\mathrm{i}\omega)\right\| < \sqrt{\gamma + \delta} \quad \text{or} \quad \left\|\tilde{T}^{\gamma}_2(\mathrm{i}\omega)\right\| < \sqrt{\gamma + \delta}.$$

Now, choose $4N + 2$ arbitrary but different frequencies. Then, for each γ, either $\left\|\tilde{T}^{\gamma}_1(\mathrm{i}\omega_i)\right\| < \sqrt{\gamma + \delta}$ or $\left\|\tilde{T}^{\gamma}_2(\mathrm{i}\omega_i)\right\| < \sqrt{\gamma + \delta}$ will be satisfied for at least $2N + 1$ of the chosen frequencies. Since there are only finitely many ways to choose $2N + 1$ frequencies among $4N + 2$ frequencies, there exists a subsequence $\{Q^{\hat{\gamma}}\}$ of $\{Q^{\gamma}\}$ for which one of the $Q^{\hat{\gamma}}_i(\cdot)$'s, which can be taken to be $Q^{\hat{\gamma}}_1(\cdot)$ without loss of generality, satisfies

$$\left\|\tilde{T}^{\hat{\gamma}}_1(\mathrm{i}\omega_i)\right\| < \sqrt{\hat{\gamma} + \delta}$$

for $2N + 1$ fixed frequencies $\omega_1, \ldots, \omega_{2N+1}$. Hence, for these $2N + 1$ frequencies,

$$\lim_{\hat{\gamma}\to 0} Q^{\hat{\gamma}}_1(\mathrm{i}\omega_i) \in \mathrm{B}(-\tilde{G}^{-1}_1(\mathrm{i}\omega_i), \delta), \tag{8}$$

where $\mathrm{B}(c, r)$ denotes the complex ball of radius r centred at c.

Let us consider a transfer-function† representation of $Q^{\hat{\gamma}}_1$:

$$Q^{\hat{\gamma}}_1(s) = \frac{\beta_0 s^N + \beta_1 s^{N-1} + \cdots + \beta_N}{s^N + \alpha_1 s^{N-1} + \cdots + \alpha_N}.$$

Now, since $\tilde{G}^{-1}_1$ is irreducible, we can apply Lemma 4. Indeed, by selecting δ sufficiently small, $\mathrm{B}(-\tilde{G}^{-1}_1(\mathrm{i}\omega_i), \delta)$ will be contained in some neighbourhood of $-\tilde{G}^{-1}_1(\mathrm{i}\omega_i)$ in which the operator F mentioned in Lemma 4 is continuous. Finally, by continuity of F and by the continuity of the roots of a polynomial in the coefficients, the denominator of $Q^{\hat{\gamma}}_1(s)$ will have roots in the open right half plane for $\hat{\gamma}$ and δ sufficiently small, since the denominator of $-\tilde{G}^{-1}_1(s)$ has. That is a contradiction, since $Q^{\hat{\gamma}}_1(s)$ was assumed to be stable. Hence, no fixed-order sequence of controllers achieve the infimal value of γ.

To establish the nonexistence of an infinite-dimensional optimal decentralized controller as mentioned in Remark 1, we assume to the contrary the existence of an optimal analytical function $Q^* = \begin{bmatrix} Q^*_1 & 0 \\ 0 & Q^*_2 \end{bmatrix}$, i.e. a function which is analytical in the open right half plane, and which makes the closed-loop transfer function from w to z equal to 0:

$$T_{zw}(\cdot) = \left[1 + Q^*_2(\cdot)G_2(\cdot)\right]\left[1 + Q^*_1(\cdot)G_1(\cdot)\right] = 0.$$

† The controller is allowed to be a *complex* transfer function in this argument. Thereby we prove a slightly stronger result.

From continuity of the transfer function $1 + Q_2^*(\cdot)G_2(\cdot)$, the transfer function $1 + Q_1^*(\cdot)G_1(\cdot)$ has to be identically equal to zero in a neighbourhood of $s = z_2$. Applying Corollary 2 for a set $\mathcal{D}$ contained in the (nonempty) intersection between this neighbourhood and the open right half plane, it follows from Corollary 2 that $1 + Q_1^*(\cdot)G_1(\cdot) \equiv 0$. This implies that $Q_1^*(\cdot) = -G_1(\cdot)^{-1}$, which is a contradiction since $G_1(\cdot)^{-1}$ is not analytic in the right half plane.

On the other hand, there does exist a sequence of controllers of increasing orders that makes T_{zw} tend to zero in H_∞ norm topology. Such a sequence is relatively easy to design. The main idea is to design $1 + Q_1^\gamma(\cdot)G_1(\cdot)$ to have low-pass characteristics and $1 + Q_2^\gamma(\cdot)G_2(\cdot)$ to have high-pass characteristics. Then the overall H_∞ norm is determined only at frequencies between z_1 and z_2 by the roll-off rates of these two transfer functions.

To achieve this, we introduce $P_B^N(s, \omega_B)$ to denote the Nth-order Butterworth polynomial with characteristic frequency ω_B. In terms of these polynomials, we can give explicit expressions for a possible controller sequence:

$$Q_1^N(s) = \frac{(s + z_1)[P_B^N(z_1, z_1) - P_B^N(s, z_1)]}{(s - z_1)P_B^N(s, z_1)},$$

$$Q_2^N(s) = \frac{(s + z_2)[s^N P_B^N(z_2, z_2^{-1}) - z_2{}^N P_B^N(s, z_2^{-1})]}{(s - z_2)z_2{}^N P_B^N(s, z_2^{-1})}.$$

Note that Q_1^N and Q_2^N are stable, considered as rational functions, since the two unstable denominator factors are cancelled by the numerators.

Now it can be verified, using a symbolic manipulation package, that the maximal value of $|T_{zw}(i\omega)|$ appears for $\omega = \sqrt{z_1 z_2}$, and that this maximal value tends to zero as N tends to infinity. The resulting design for $N = 5$ can be seen in Fig. 3. The dotted lines are the magnitudes of the two transfer functions $1 + Q_1^N(i\omega)G_1(i\omega)$ and $1 + Q_2^N(i\omega)G_2(i\omega)$, and the solid line is magnitude of their product, $T_{zw}(i\omega)$. The vertical lines indicate the two nonminimum-phase zeros z_1 and z_2. □

REMARK 2 It is not easy to determine the exact contents of the class $\mathcal{G}^\infty$. Theorem 1 shows that the class is nonempty. Indeed, from the line of proof, it could be anticipated that a majority of nonminimum-phase systems would be in the class. On the other hand, if G_1 or G_2 would be minimum-phase in the configuration in Fig. 2, there would exist a fixed-dimensional sequence of controllers, so the class does not comprise *all* decentralized control problems.

3. Near-optimal design of decentralized controllers

In the literature, few algorithms can be found for near-optimal decentralized control for arbitrary plants. The reason for this is likely to be found in Theorem 1, which eliminates the possibility of Riccati-type necessary and sufficient conditions for near-optimal problems. One result that facilitaties design for near-optimal control is that of Sourlas & Manousiouthakis (1995). This method, however, is based on a complex optimization procedure, and might be numerically infeasible for large-scale systems. Based on the line of proof above, however, a heuristic algorithm can be devised, which works for

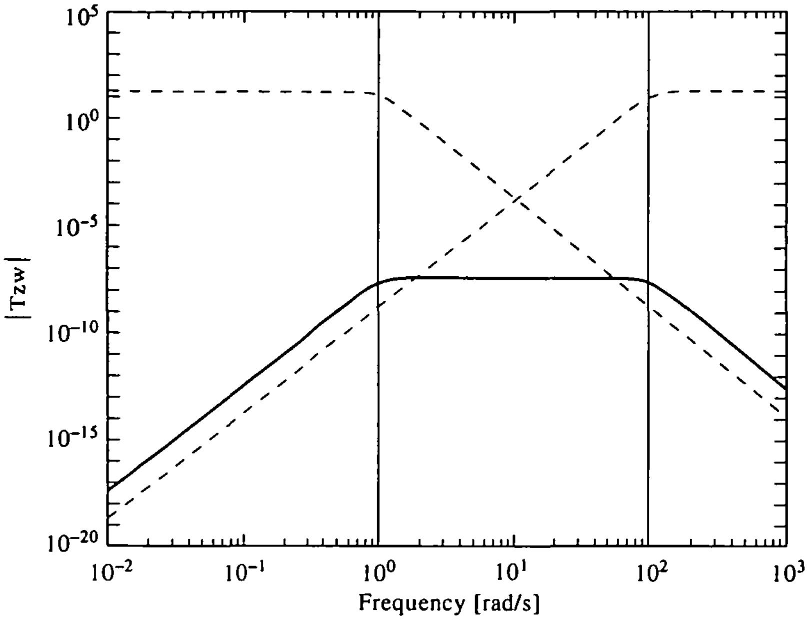

FIG. 3 5th-order Butterworth design

systems where individual subsystems have only a limited number of nonminimum-phase zeros, and where subsystems are only lightly coupled.

First, without loss of generality, we will rewrite (1) as k subsystems of the form

$$\Sigma_i \ : \ \begin{cases} \dot{x}_i = A_i x_i + B_{1,i} w + \sum_{j \neq i} B_{2,ij} z_j + B_{3,i} u_i, \\ z_i = C_{1,i} x_i + D_{11,i} w + \sum_{j \neq i} D_{12,ij} z_j + D_{13,i} u_i, \\ y_i = C_{2,i} x_i + D_{21,i} w + \sum_{j \neq i} D_{22,ij} z_j + D_{23,i} u_i. \end{cases} \tag{9}$$

The intuition of this form is that each controller 'looks into' a subsystem with two kinds of disturbance: the original exogenous signals w and the artificial disturbance vector

$$\tilde{w}_i = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_k)$$

which determine how the subsystems influence one another. Expanding the idea of the proof of Theorem 1, we obtain the following *conceptual* algorithm:

ALGORITHM 1

Step 1. Determine the nonminimum-phase zeros for each subsystem Σ_i with respect to each component of the input $\tilde{w}_i$

Step 2. Sort the zeros of all subsystems by magnitude, and assign either a low-pass (LP), a band-pass (BP), or a high-pass (HP) attribute to each input of each subsystem based on this. The assignment should consider the signals w_i also; i.e. by computing the zeros related to these inputs, the local interpolation constraints should be taken into account

Step 3. Design weightings for each subsystem such that the outputs for a subsystem with a HP attribute is input only to LP loops and vice versa

Step 4. Compute a controller for each subsystem with these weightings using H_∞ optimization

Step 5. Iterate from Step 3 by increasing the roll-off rate of the weightings until the specifications are met

At each iteration of the algorithm, the controller order will increase due to the increased order of the weightings.

REMARK 3 Obviously, if any of the involved systems are minimum-phase with respect to all input–output pairs, these subsystems can be made *uniformly* small (at the possible cost of robustness).

It is interesting to observe that, e.g. for systems comprising three subsystems with each having just one nonminimum-phase zero, it might be the case that (LP, LP, HP) and (LP, HP, HP) are both admissible assignments of attributes, leading to the same optimum. In fact, for a series connection, if it is possible to design two loops to have disjoint LP and HP characteristics, the remaining loops are completely free. Needless to say, the corresponding controllers will then be rather different. This type of non-uniqueness does not exist always in a full multivariable near-optimal design.

4. Conclusions

We have shown that, for a class of systems, the controller order of a decentralized H_∞ controller will not remain bounded as the H_∞ optimization tends to the optimum. In such cases, no sequence of controllers will converge, not even to an infinite-dimensional controller. The 'optimal' controller will be non-causal.

We believe that the proof of the main result in this paper provides insight which can guide the design of decentralized controllers. In particular, a heuristic design algorithm has been devised, which works for systems that are not too strongly coupled, or have not too many nonminimum-phase zeros.

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