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Optimal usage of coal, gas and oil in a power plant

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Abstract: This study addresses the problem of an optimal actuator selection when economic value is considered. The objective is to minimise the economical cost of operating a given plant. The problem has been formulated using mathematic notions from economics. Functionals describing the business objectives of operating a power plant has been established. The selection of actuator configuration has been limited to the fuel system which in the considered plant consists of three different fuels – coal, gas and oil. The changes over 24 h of operation is established and a strategy for using a plant utilising the three fuels is developed which will yield a greater profit than a coal fired plant.

1 Introduction

The requirements for a complex process control system are usually derived from a top level (business) requirement to the entire system which is to maximise the income or profit of the company. However, the requirements specification for the process control system rarely includes profit maximisation directly and instead the designer works with requirements to settling time, rise time, bandwidth, disturbance rejection and so on, as these are easy to evaluate through simulation and well defined with respect to transfer functions and the pole placement of the closed-loop system. All of these measures assume that a set of actuators and sensors is given. However, the choice of actuators and sensors influences the cost and performance of the system greatly – this will be addressed in this paper.

The selection of sensors and actuators has, to a great extent, depended on the designer's system knowledge and experience; however, in recent years more focus has been payed to developing tools to aid the designer during this phase as processes are becoming more complex and difficult to assess. One such tool is the relative gain array (RGA), which can be used to pair inputs and outputs in a multiple input multiple output system to enable a decentralised control (single input single output control) [1. p. 90]. Further advances using RGA have been examined in [2] where it is generalised to multiple output multiple input control structures.

The placement of sensors and actuators has been studied for different specific applications especially flexible structures in the aerospace industry for which the methods are usually based on search algorithms; however, these methods are difficult to generalise to other applications [3] as they consider the physical placement of actuator along a vibrating beam.

More general purpose methods for selecting and placing sensors and actuators have been evaluated in [4, 5], which include for example methods relying on controllability measures such as state reachability and more sophisticated methods using robust performance measures. It is also concluded in [4] that the choice of sensors and actuators dictates the expenses for hardware, implementation, operation and maintenance.

The methods mentioned above do not directly consider the cost/profit associated with the selection of actuators and sensors. The economical cost of sensors and actuators has, on the other hand, been considered in the selection method presented in [6], where the precision of a sensor or an actuator is assumed to be proportional to its cost. By introducing a bound on the economical cost of the instrumentation it is possible to formulate the design problem as a convex optimisation. This helps the designer to select the right instrumentation. However, this method only considers the implementation cost and not the operational cost which in many cases is the main concern for minimisation [7].

As the requirements for a process control system usually are derived from business objectives it would be natural to include these business objectives when configuring the sensor/actuator layout of a plant. An attempt of this has been presented in [8] where functionals describing the business objectives are maximised. In [8] heuristics was used to solve the problem and the functionals encapsulated both the economical value and business objective measures.

The work in [9] was extended to utilise notions from production economics. When viewing a market from the production perspective one usually defines a number of companies and the goods they are capable of producing. The firms are viewed as a black box able to transform inputs to outputs [10]. In [9] this approach was used by formulating functionals which describe DONG Energy's (DONG Energy is a Danish energy supplier) objectives for a power plant, which is a complex process control system, as outputs and the amount of fuel used as input. The price of producing the output and price of using the fuel/input was described by approximating data from a power trading market.

This paper will use two of the three business functionals from [9] which the third objective, availability, is discarded as it does not depend on the actuator selection. The results are in this paper, furthermore, extended to real price and demand data and a scenario with only partial production capabilities in the coal and gas system will be considered, which is interesting as most coal plants are started using gas or oil. This paper shows that a power plant capable of using coal as well as gas and oil will be able to generate a larger profit during normal operating conditions than a purely coal fired plant – in particular June 29, 2008 is considered, however, the result would be similar for any given day. During this day a profit increase of 12% is possible.

The work in this paper should be seen in relation to the plug and play process control (P3C) project [11]. The P3C project is investigating how to develop control algorithms and infrastructure to make plug-and-play, as known from the personal computer industry, possible in process control system. However, when should new hardware be plugged in and what are the benefits? These kinds of questions are investigated in this paper using a power plant as an example, for example two questions are addressed; when should 'new' hardware (fuel systems) be used and what is the benefit (economical profit).

1.1 Outline

The plant considered in this work is presented in Section 2 and then the problem is formulated. Two of DONG Energy's business objectives – efficiency and controllability – are described in Section 3 as static models for three different actuator systems; coal, gas and oil. In Section 4 the problem of profit maximisation is solved using the static models and the results are presented. The static models are expanded in Section 5 to include the dynamic nature of electricity prices and production reference during 24 h. The dynamic formulation is solved in Section 6 and it is shown that a power plant with multiple fuels can provide a greater profit than a traditional coal fired power plan. Finally a discussion about the results is brought in Section 9.

2 Problem formulation

The problem in this work has been formulated in collaboration with DONG Energy – a Danish power company. The goal of any company is to maximise its profit and for DONG Energy the profit maximisation has been divided into four individual business objectives which can be described by efficiency, controllability, availability, and lifetime (to simplify the model only the first two objectives are considered in this work) which will be defined in Section 3. The problem formulated is based on a model of a coal fired boiler – a vital component of a power plant – which is augmented with two additional fuels system; gas and oil.

2.1 Plant description

The power plant considered in this paper consists of the following components:

- *Fuel system:* The fuel system prepares the different fuels for burning, for example the coal mills grind the coal to small dust particles which burn quickly and efficiently.
- *Burners:* The burners deliver the fuel to specific places in the boiler such that the heat transfer is maximised.
- *Boiler:* The boiler is a module where the fuels are burned thereby heat is delivered to the evaporator.
- *Evaporator:* The evaporator is fed with water, which is evaporated under high pressure by the heat from the burners.
- *Superheater:* The superheater (super) heats the steam from the evaporator.
- *Economiser:* The economiser uses some of the remaining heat in the flue gas to preheat the feed water before it enters the evaporator.

The individual parts of the model are illustrated in Fig. 1.

The power plant has the possibility to use three different fuels which have certain advantages and disadvantages, for example gas is easy to control but an expensive fuel. Some of the characteristics of the different fuels are:

- *Coal* is advantageous when considering the price per stored energy; however, it is difficult to control as unmeasurable

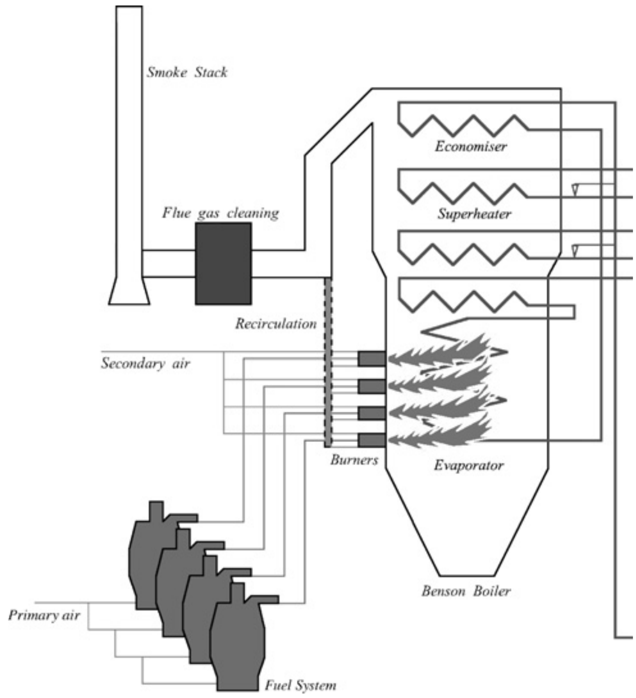

Figure 1 *Power plant model including the different modules from fuel processing to steam delivery*

fluctuations in the coal flow are introduces by the coal mill when the coal is ground to coal dust. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind and dry the coal which needs to be considered.

- *Gas* is more expensive than coal and energy is not converted to steam as efficient with gas as with coal because of the layout of the chosen boiler. However, gas arrives at the power plant under high pressure which is lowered using a turbine generating electrical energy. Furthermore, gas is much easier to control as it is possible to measure the flow.

- *Oil* is, with the current market prices, the most expensive of the three fuels and has to be heated before entering the boiler. This process demands energy itself. Nevertheless, oil is considered in this work as it is possible to measure the oil flow into the boiler making it easy to control. Furthermore, oil is present in most existing coal fired plants as oil is used in the period of starting the plant.

2.2 Problem

The focus of this work is to derive a mixture of the three fuels, described above, which will yield the greatest profit under consideration of the two business objectives; efficiency and controllability. The idea is to develop simple models of the business objectives to evaluate if there is an economical gain of mixing fuels. If it is advantageous to mix fuels a strategy for using the fuels will be developed. The idea in this work is not to develop controller for the plant as it is assumed this is done or will be done by other known methods.

3 Static plant model

In the sequel, models of the efficiency and controllability objectives will be derived for the input of coal, gas and oil. Furthermore, the input and output spaces are described. The input space is a polytope (more precisely a simplex) in a Euclidean space. Its coordinates are flows of coal, gas and oil. The power plant production is characterised by a map taking the fuel flow into a pair of production objectives: efficiency – actual power production in MW and controllability – ability to adjust the production to instantaneous needs of the market. The production objectives have associated price which is related to markets demands. The profit can now be calculated as the revenue from efficiency and controllability minus the expenses of using fuel. The article applies static optimisation to devise a fuel utilisation plan for coal, gas and oil such that the profit is maximal and the demand for production is satisfied.

Let $\mathbb{R}^3_+$ denotes the positive quadrant in $\mathbb{R}^3$, that is, $\mathbb{R}^3_+ = \{\mathbf{v} \in \mathbb{R}^3 | \mathbf{v} \geq \mathbf{0}\}$ where the inequality is to be understood coordinate wise (this notation will be used throughout this work).

The input space X is now given by

$$X = \{\mathbf{v} \in \mathbb{R}^3_+ | 0 \leq (\mathbf{v}|\mathbf{u}) \leq c\} \tag{1}$$

where $(\cdot|\cdot)$ is the Euclidean inner product, and the vector $\mathbf{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ with $\mathbf{u} > \mathbf{0}$ and scalar $c \in \mathbb{R}$ are to be determined later. Note that X is the three-simplex (in $\mathbb{R}^3_+$) with vertices $\mathbf{0}$, $(c/u_1, 0, 0)$, $(0, c/u_2, 0)$ and $(0, 0, c/u_3)$. Each input

$$\mathbf{x} = (x_c, x_g, x_o) \in X \quad (\text{kg/s},\ \text{kg/s},\ \text{kg/s})$$

to the system describes the flow of coal, gas and oil, respectively, measured in kg/s. In the sequel we let $\mathcal{I} = \{\text{c},\ \text{g},\ \text{o}\}$ where the elements of the index set $\mathcal{I}$ refers to the three different fuels. Occasionally, the identification $(\text{c}, \text{g}, \text{o}) = (1, 2, 3)$ will be used.

The output space $Y = Y_1 \times Y_2$ is a subset of $\mathbb{R}^2$ where each output (MW is an abbreviation for mega watt)

$$\mathbf{y} = (y_e, y_c) \in Y \quad (\text{MW},\ \text{MW/s})$$

of the system describes one of the two objectives; efficiency and controllability, respectively, that is, y_e is a measure of the efficiency and y_c is a measure of the controllability. Both of these quantities contain contributions from coal, gas and oil as will be explained next, where simple functions describing these two business objectives at steady state are derived.

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3.1 Efficiency

The efficiency objective, y_e, expresses how much electricity is produced from a certain amount of fuel. Three affine functions describing the contribution of the individual fuels to the efficiency objective have been established using measurement data from two Danish power plants. These functions are given by

$$y_{ec}(x_c) = e_c x_c + e_c'$$
$$y_{eg}(x_g) = e_g x_g + e_g'$$
$$y_{eo}(x_o) = e_o x_o + e_o'$$

where

$$(e_c, e_g, e_o) = (10.77, 18.87, 15.77)$$

are measures of how much energy is stored in the individual fuels [in MJ/kg, where MJ is an abbreviation for mega joule] and

$$(e_c', e_g', e_o') = (-1.76, 1.85, -0.37)$$

are the own consumptions of the different fuels (in MW) as explained in Section 2.1. The values above have been established using measurement data provided by DONG Energy.

The total amount of efficiency (at steady state) is described by the function

$$X \to Y_1;\ \mathbf{x} \mapsto y_e(\mathbf{x}) = \sum_{i\in\mathcal{I}} y_{ei}(x_i) = (\mathbf{x}|\mathbf{u}) + c' \qquad (2)$$

where $c' = \sum e_i'$ and $\mathbf{u} = (e_c, e_g, e_o)$ which also should be used in (1). The constant c in (1) can now be determined by $c = 400 - c'$, where 400 refers to the maximum efficiency (in MW) produced by the plant and c' is an expression of the own consumption of the complete plant which is lost in the electricity production. Finally Y_1 can be determined by $Y_1 = (0, 400]$.

3.2 Controllability

The controllability objective, y_c, gives a measure of how fast the production of electricity can be changed. Allowed change in the production is limited to a certain gradient depending on the current efficiency, y_e. The reason for this limit is a compliance to maximum temperature gradients in the boiler (the temperature gradients have not been explicitly modelled and are therefore indirectly considered this way). When running the plant in ranges 0–200 MW and 360–400 MW it is allowed to change production by 0.133 MW/s independent of fuel. However, in the range 200–360 MW the allowed changes are dependent of which fuel is used. If coal is used it is allowed to change production by 0.267 MW/s and when using oil and gas the allowed change is 0.534 MW/s. The changes allowed is modelled as piece-wise constant functions

$$h_i : Y_1 \to \mathbb{R} \quad (\mathrm{MW} \mapsto \mathrm{MW/s}), \quad i \in \{1, 2, 3\}$$

given by

$$h_i(y_1) = \begin{cases} 0.133, & y_1 \in (0, 200) \cup (360, 400], \\ 0.267i, & y_1 \in [200, 360], \end{cases} \quad i = 1, 2 \qquad (3)$$

$$h_2 = h_3 \qquad (4)$$

If a mixture of the three fuels is used it is assumed that the allowed change is a certain convex combination of the allowed change of the individual fuels. More precisely, the total amount of controllability is expressed by the function

$$X \to Y_2; \quad y_c(x) = \sum_{i\in\mathcal{I}} y_{ci}(x) \qquad (5)$$

where

$$y_{cc}(x) = \frac{y_{ec}(x_c)}{y_e(x)} h_1(y_e(x))$$
$$y_{cg}(x) = \frac{y_{eg}(x_g)}{y_e(x)} h_2(y_e(x))$$
$$y_{co}(x) = \frac{y_{eo}(x_o)}{y_e(x)} h_3(y_e(x))$$

The values in this model have been established in collaboration with DONG Energy.

3.3 Prices

At steady state the cost of using input $\mathbf{x}$, revenue from production of output $\mathbf{y}$ and the profit of operating the power plant can now be determined. The above constructions yield a product (or output) function, $\mathbf{y}_P$, of the system given by

$$\mathbf{y}_P : X \to Y; \quad \mathbf{x} \mapsto (y_e(\mathbf{x}), y_c(\mathbf{x}))$$

For the system, the growth of cost and growth of revenue are defined by the following functions (DKK is an abbreviation for the Danish currency)

$$g_C : X \to \mathbb{R}; \quad \mathbf{x} \mapsto (\mathbf{x}|\mathbf{p}_C)\ \mathrm{DKK/s}$$
$$g_R : Y \to \mathbb{R}; \quad \mathbf{y} \mapsto (\mathbf{y}|\mathbf{p}_R)\ \mathrm{DKK/s}$$

with price vectors

$$\mathbf{p}_C = (p_{C1}, p_{C2}, p_{C3}) = (1.20, 3.74, 6.00)$$
$$\mathbf{p}_R = (p_{R1}, p_{R2}) = (0.16, 247)$$

fixed and in units DKK/kg for p_{Ci}, DKK/MWs for p_{R1} and DKK/MW for p_{R2}. The prices correspond to the maximum market prices June 29, 2008 (see Section 5).

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The growth of profit is defined by the function

$$X \times Y \to \mathbb{R}; \quad (\mathbf{x}, \mathbf{y}) \mapsto g_{\mathrm{R}}(\mathbf{y}) - g_{\mathrm{C}}(\mathbf{x})$$

which for the system yields

$$g_{\mathrm{P}} : X \to \mathbb{R}; \quad \mathbf{x} \mapsto g_{\mathrm{R}}(\mathbf{y}_{\mathrm{P}}(\mathbf{x})) - g_{\mathrm{C}}(\mathbf{x})$$

Hence the profit is given by

$$P : \mathbb{R}_+ \to \mathbb{R}; \quad t \mapsto \int_0^t g_{\mathrm{P}}(\mathbf{x})\,\mathrm{d}\tau$$

4 Static optimisation

In the following we wish to find the optimal static fuel configuration, $\mathbf{x}^*$, such that the growth of profit, and thus the profit, is maximised. For a given efficiency $y_{\mathrm{r}} \in Y_1$, we consider the maximum growth of profit

$$\max_{\mathbf{x}\in y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})} g_{\mathrm{P}}(\mathbf{x}) \tag{6}$$

where we note that $y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})$ is the two-simplex (in $X \subset \mathbb{R}^3_+$) with vertices

$$\begin{aligned} \mathbf{v}_1^* &= ((y_{\mathrm{r}} - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= (0, (y_{\mathrm{r}} - c')/u_2, 0) \\ \mathbf{v}_3^* &= (0, 0, (y_{\mathrm{r}} - c')/u_3) \end{aligned} \tag{7}$$

Since g_{P} restricted to the set $\{\mathbf{x} \in X | \mathbf{x} \in y_{\mathrm{e}}^{-1}(y_1)\}$ is affine, the optimal configuration is given by

$$\mathbf{x}^* = \arg \max_{\mathbf{x}\in y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})} g_{\mathrm{P}}(\mathbf{x}) \in \{\mathbf{v}_i^*\} \tag{8}$$

for each y_{r}, that is

$$\max_{\mathbf{x}\in y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})} g_{\mathrm{P}}(\mathbf{x}) \in \{g_{\mathrm{P}}(\mathbf{v}_i^*)\}$$

and that we may describe the maximum growth of profit and the optimal configuration as functions of the efficiency by

$$Y_1 \to \mathbb{R}; \quad y_{\mathrm{r}} \mapsto \max_{\mathbf{x}\in y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})} g_{\mathrm{P}}(\mathbf{x}) \tag{9}$$

$$Y_1 \to X; \quad y_{\mathrm{r}} \mapsto \arg \max_{\mathbf{x}\in y_{\mathrm{e}}^{-1}(y_{\mathrm{r}})} g_{\mathrm{P}}(\mathbf{x}) \tag{10}$$

Fig. 2 (top) depicts the graph of (9), that is, the maximum growth of profit against the efficiency. The bottom figure depicts the graph of (10), that is, the optimal configuration against the efficiency where the values on the second axis should be read with the identification $(1, 2, 3) = (\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*)$. As seen in the figure the optimal configuration is changed from using only coal to using only gas when the efficiency is in the range [200, 360]. The gradient of the growth of profit is negative when using gas which is caused by the higher gas price. However, the growth of profit caused by the controllability, y_{c}, still makes gas advantageous.

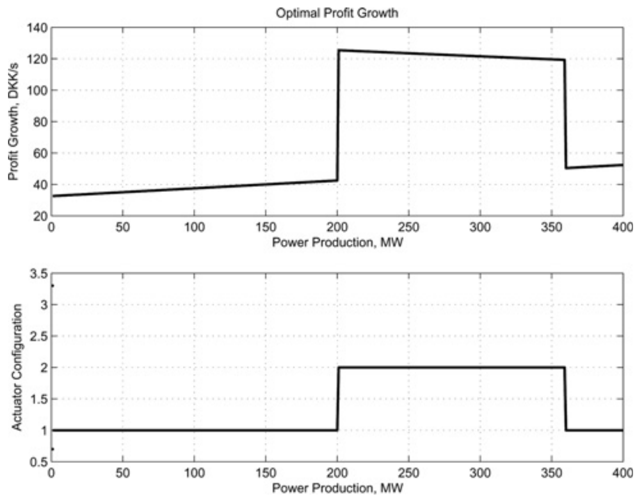

Figure 2 *Top: optimal profit growth; bottom: fuel configuration [second axis should be read with the identification (1, 2, 3) =* $(\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*)$*]*

The results above suggest that gas should be used whenever the efficiency is in the range [200, 360] and coal otherwise. However, things are not as obvious as it seems because the prices of the objective, p_{R} change during the day. These changes of the prices will be considered in the following section.

5 Dynamic plant model

The electricity production of a power plant is not constant during the year or even during 24 h. However, prediction of the demand of power 24 h into the future makes it possible to plan production ahead of time. During this planning for the entire electrical grid (consisting of multiple power plants throughout Denmark) a production plan is fitted to the capabilities of the individual plants, that is a production plan (y_{e} reference) is delivered to each power plant. The prices of efficiency and controllability are also established during this planning. In the following, these changes will be described and models of the effects will be derived.

5.1 Production plan

The total power production in West Denmark over 24 h during 30 days is depicted in Fig. 3. The data used to generate this plot have been obtained from Nord Pool [Nord Pool is a marketplace for trading power contracts (www.nordpool.dk)] and the graphs for the individual days have been normalised by the maximum production during that day.

In West Denmark there are multiple power plants and the total power production is obviously a sum of the production of these individual plants. It is expected that the production

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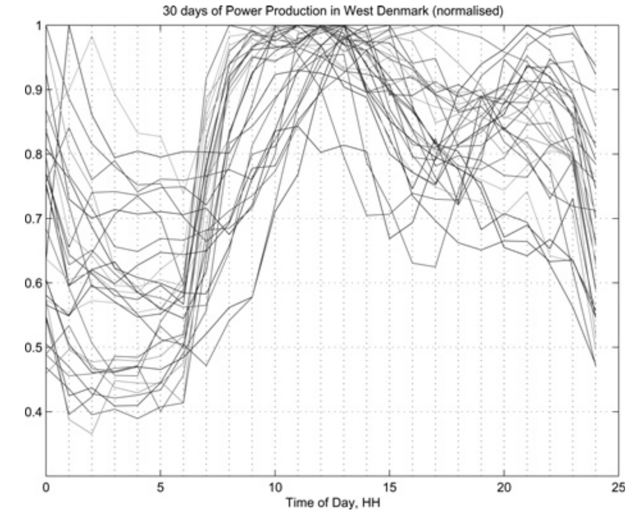

Figure 3 *Total power production over 24 h during 30 days*

The data used to generate this plot have been found on www.nordpool.dk

plan for the individual plants follows the trends in Fig. 3. Hence, the production is low at night and in the morning around 6:00 there is a large increase in production and finally, in the afternoon the production fluctuates a bit. In this work we will consider a particular day, where the relevant data have been provided by DONG Energy and Nord Pool. However, the methods presented can be used for any given day of the year. The production plan for the day considered in this work is depicted in Fig. 4. The graph depicts the production from midnight June 29, 2008 and 24 h ahead. As seen in the figure the production is rather low during the night but at 6:00–7:00 in the morning there is a steep gradient caused by the increase in consumption when people and companies start to use electricity. During the afternoon and evening some fluctuations are seen. The production plan is modelled as an approximation of the graph depicted in Fig. 4 and is denoted

$$t \mapsto y_r(t) \tag{11}$$

5.2 *Efficiency price*

The price of electricity, p_{R1}, changes during the day as the demand changes, that is, during the middle of the day when the demand is greatest the price is also higher than during the early morning. The trading prices for electricity over 24 h during 30 days is depicted in Fig. 5 where the average is depicted as well.

The electricity price from the day considered in this work (June 29, 2008) is depicted in Fig. 6 where the data have been found at the archive at Nord Pool. The price is modelled as an approximation of this graph and is denoted

$$t \mapsto p_{R1}(t) \tag{12}$$

5.3 *Controllability price*

Large gradients in the production plan, as seen in Fig. 4 around 6:00–7:00, yield a high price on controllability as it is likely that some plants are not capable of generating the gradients needed.

According to DONG Energy, the controllability price would, in general, be related to the derivative of the production plan. Hence, the price is higher during the periods in the morning and afternoon/evening where there exists steep gradients as seen in Fig. 4. The approximation

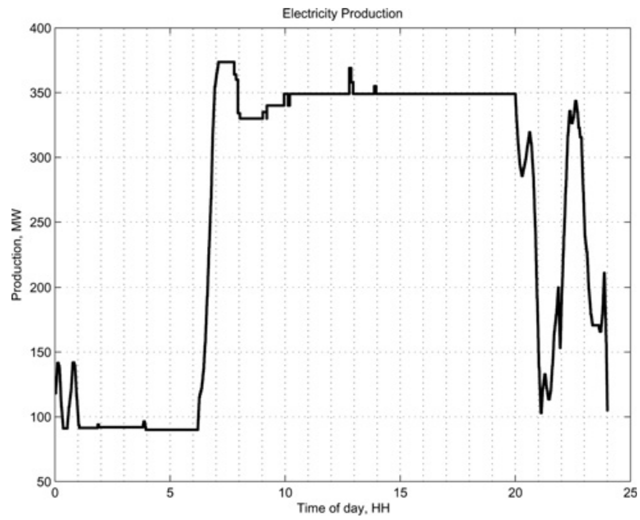

Figure 4 *A production plan over 24 h June 29, 2008*

The data used to generate this plot have been provided by DONG Energy

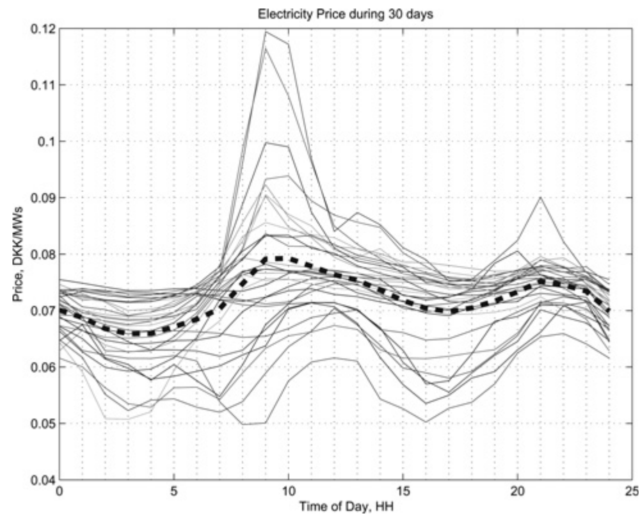

Figure 5 *Efficiency price over 24 h during 30 days and average price (thick dashed)*

The data used to generate this plot have been found on www.nordpool.dk

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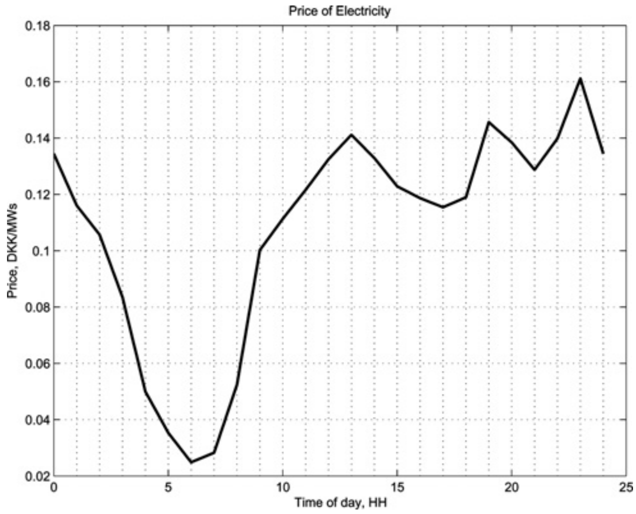

Figure 6 *Efficiency price during the June 29, 2008*

The data used to generate this plot have been found on www.nordpool.dk

of the controllability price is defined as

$$t \mapsto p_{R2}(t) = \beta \left| \frac{\mathrm{d}}{\mathrm{d}t} y_e(t) \right| \tag{13}$$

where $\beta = 1000$ is a factor which has been determined in collaboration with DONG Energy. We remark that established model is simplifying a complicated price model but is considered sufficient for this work. The modelled controllability price, p_{R2}, is depicted in Fig. 7.

5.4 Fuel price

Obviously the fuel prices change over time, however, these changes are slow compared to the changes described in the previous sections. The time span is a matter of weeks and is therefore, compared to the above, roughly constant and therefore the fuel prices given in Section 3 are used.

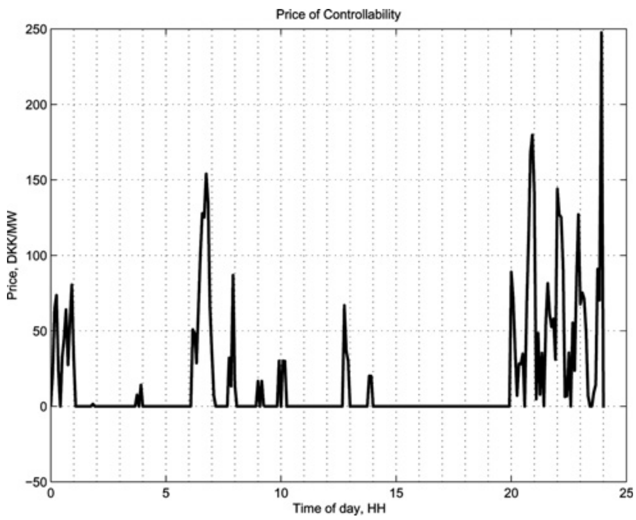

Figure 7 *Modelled controllability price during June 29, 2008*

The data in this plot have been established in collaboration with DONG Energy

5.5 Discussion of prices

The average price for efficiency is 0.11 DKK/MWs and the average price for controllability is 17.2 DKK/MW which might seem as a large difference or an unrealistic high price on controllability. However, the values of the efficiency measure and controllability measure are also different as the efficiency output is in the range (0,400] and controllability output is in the range [0.133,0.534]. At a load of 300 MW the instantaneous income (here the term instantaneous income is used instead of growth of profit as only the revenue of efficiency and controllability is considered) from efficiency is 32 DKK/s and from controllability the instantaneous income is between 4.6 and 9.2 DKK/s (using the average prices). At 6:30 the instantaneous incomes are 3.9 and 11 DKK/s for efficiency and controllability, respectively. On average, that is, the determining factor for revenue is the efficiency measure but at certain periods during the day the controllability measure becomes significant.

6 Fuel selection in dynamic case

In the following the static optimisation problem given in Section 4 is expanded to include the time dependence described in Section 5. The growth of profit and the profit is maximised during 24 h of operation.

Since the prices on the outputs are time dependent the growth of revenue for the system will now be defined by

$$g_R : Y \times \mathbb{R}_+ \to \mathbb{R}; \quad (\mathbf{y}, t) \mapsto (\mathbf{y}|\mathbf{p}_R(t))$$

where $\mathbf{p}_R(t) = (p_{R1}(t), p_{R2}(t))$ with the coordinate functions as defined in(12) and (13).

Hence, the growth of profit will be time dependent and given by

$$X \times Y \times \mathbb{R}_+ \to \mathbb{R}; \quad (\mathbf{x}, \mathbf{y}, t) \mapsto g_R(\mathbf{y}, t) - g_C(\mathbf{x})$$

which for the system yields

$$g_P : X \times \mathbb{R}_+ \to \mathbb{R}; \quad (\mathbf{x}, t) \mapsto g_R(\mathbf{y}_P(\mathbf{x}), t) - g_C(\mathbf{x}) \tag{14}$$

The objective is now to let the efficiency, y_e follow some predefined time-dependent reference signal (see Section 5.1), that is, $y_e = y_r(t)$.

For given t^* we consider the maximum growth of profit

$$\max_{\mathbf{x} \in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*)$$

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Hence, as in Section 4 we obtain

$$\mathbf{x}^*(t^*) = \arg \max_{\mathbf{x}\in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*) \in \{\mathbf{v}_i^*(t^*)\}$$

and for each t^*

$$\max_{\mathbf{x}\in y_e^{-1}(y_r(t^*))} g_P(\mathbf{x}, t^*) \in \{g_p(\mathbf{v}_i^*(t^*), t^*)\}$$

where the $\mathbf{v}_i^*$'s are as in (7) with y_r replaced by $y_r(t^*)$. The optimal fuel configuration is now described by the curve

$$\mathbb{R}_+ \to \{\mathbf{v}_i^*\}; \quad t \mapsto \mathbf{x}^*(t) \tag{15}$$

so the maximum growth of profit and maximum profit as functions of time are given by

$$G_P : \mathbb{R}_+ \to \mathbb{R}; \quad t \mapsto g_P(\mathbf{x}^*(t), t) \tag{16}$$

$$P : \mathbb{R}_+ \to \mathbb{R}; \quad t \mapsto \int_0^t G_P(\tau)\,\mathrm{d}\tau \tag{17}$$

In the following results the real data sets have been used for $y_r(t)$, $p_{R1}(t)$, and $p_{R2}(t)$. Fig. 8 (top) shows the graph of G_P, that is, the maximum growth of profit against time and the bottom figure depicts the graph of (15), that is, the optimal fuel configuration against time, where the identification $(1, 2, 3) = (\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*)$ is used.

The growth of profit is, as seen in the figure, negative during the early morning hours where the price of efficiency is low (see Fig. 6). Furthermore, some spikes are present around 6:00–7:00 and between 20:00 and 24:00 which are caused by shifting fuel from coal to gas and vice versa. As depicted in the figure, coal is used during most of the day. The use of coal at night is partially expected from the static optimisation as the efficiency reference is low, however, because of a low price on controllability during the middle of the day coal is used instead for gas as expected from the static optimisation. In the evening gas is used to cope with the changes in the demand of electric power.

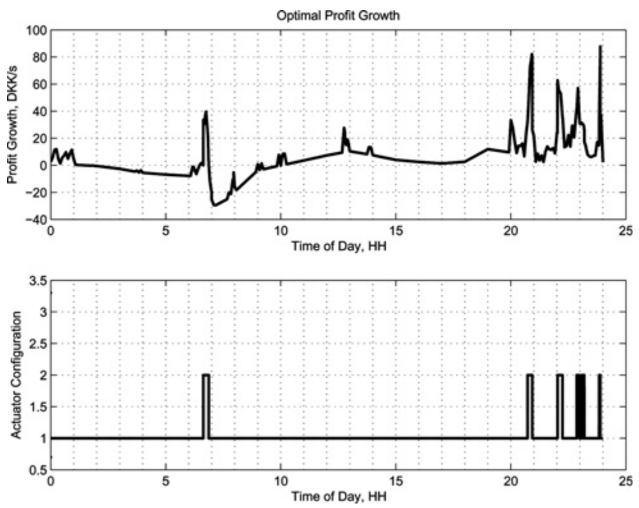

Figure 8 *Top: growth of profit; bottom: optimal fuel configuration*

Both plotted over 24 h of operation June 29, 2008

In Fig. 9 the graph of P, defined in (17), is depicted, that is, the maximum profit against time. The profit is low during the morning and actually negative most of the day until around 19:00, however, during the evening when the efficiency price is high the profit grows.

In Fig. 10 the profit is compared to a plant using only coal. Plants using only gas or oil will at the end of the day have a deficit of, respectively, 1.4 and 5.5 million DKK and these are, therefore, not depicted. As seen in the figure the profit

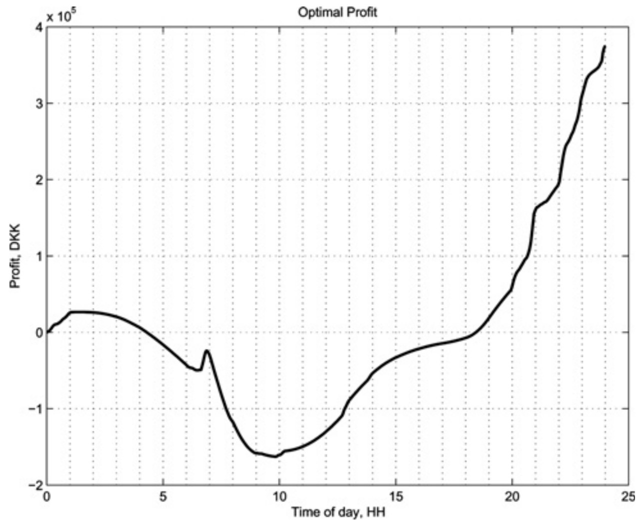

Figure 9 *Optimal profit over 24 h June 29, 2008*

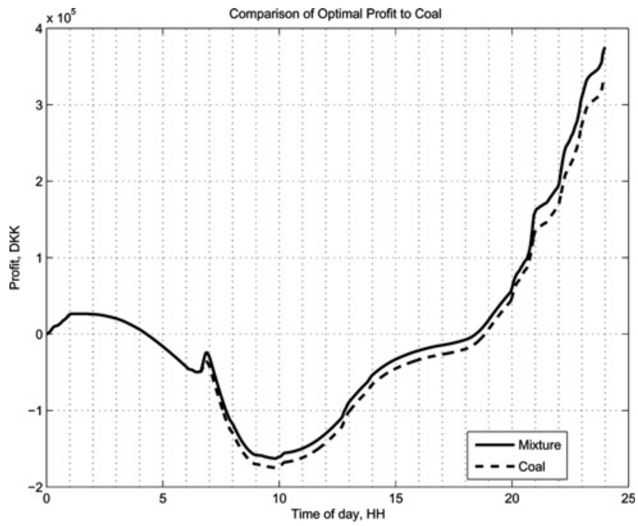

Figure 10 *Profit for a plant using a mixture of fuels is compared to a plant using only coal over 24 h of operation June 29, 2008*

from the two plants is equal until around 7:00 where gas is used in the mixed fuel plant. The difference in profit is during the day enlarged and at the end of the day the gain by using a mixed fuel is around 40 000 DKK or 12% more compared to the plant using only coal.

7 Change of parameters

In this section a discussion is made about how the results change when two of the parameters in the model of the plant are changed. The parameters considered are the controllability price and the production capabilities of oil and gas.

7.1 Controllability price

This section discusses how the results are influenced by changing β in the controllability price [see (13)]. If the fuel configuration in Fig. 8 is compared to the controllability price in Fig. 7 it can be observed that gas is chosen when the controllability price is above 100 DKK/MW and thus changing β will influence how often and how long time gas is used. If β is enlarged it is expected that gas will be used more often and thus it will be more valuable to be able to use both gas and coal. The optimal actuator configuration is depicted in Fig. 11 where $\beta = 10\ 000$ and 100 are used. As seen gas is not selected when $\beta = 100$ is used but as expected gas is selected more during the day when $\beta = 10\ 000$.

7.2 Partial production capabilities

The three different fuel systems considered in this work are comprised of multiple actuators, for example the coal system consists of four coal mills and the gas and oil system consists of 16 burners each. Furthermore, it can be argued that three systems capable of delivering fuel to full production might not be feasible as the cost of implementing this is large when 2/3 of the actuation power is not in use. Therefore in this section it will be investigated how the result changes when the gas and oil systems only consist of four burners each, that is, 25% of what is considered in Section 6. This configuration is interesting because the burners are usually implemented in sets of four and at least one set is present in existing coal fired plants as it is necessary in order to start up the plant.

The solution to this problem follows the procedure from the previous sections where $y_e^{-1}(y_r)$ in (7) changes from a simplex to a polytope of dimension 2 depending on the value of y_r. More precisely, the vertices of $y_e^{-1}(y_r)$ becomes

$$\left.\begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= (0, (y_r - c')/u_2, 0) \\ \mathbf{v}_3^* &= (0, 0, (y_r - c')/u_3) \end{aligned}\right\}, \quad y_r \in (0, 100]$$

$$\left.\begin{aligned} \mathbf{v}_1^* &= ((y_r - c')/u_1, 0, 0) \\ \mathbf{v}_2^* &= ((y_r - 100 - c')/u_1, (100 - c')/u_2, 0) \\ \mathbf{v}_3^* &= ((y_r - 100 - c')/u_1, 0, (100 - c')/u_3) \\ \mathbf{v}_4^* &= (0, (100 - c')/u_2, (y_r - 100 - c')/u_3) \\ \mathbf{v}_5^* &= (0, (y_r - 100 - c')/u_2, (100 - c')/u_3) \end{aligned}\right\}, \quad y_r \in (100, 200]$$

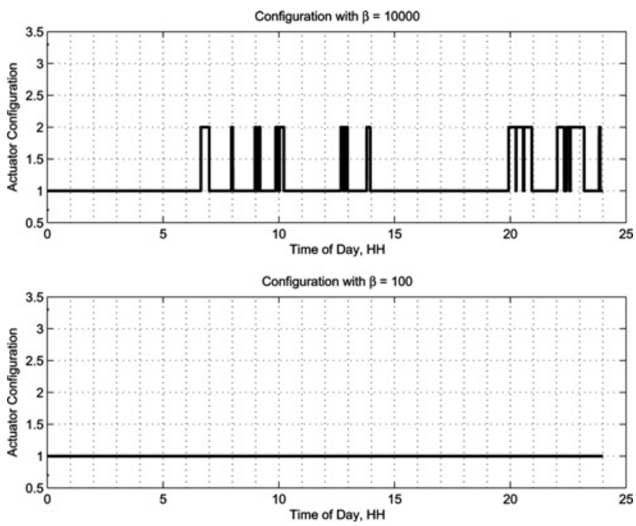

Figure 11 *Optimal actuator configuration with $\beta = 10\ 000$ and 100 over 24 h of operation during the June 29, 2008*

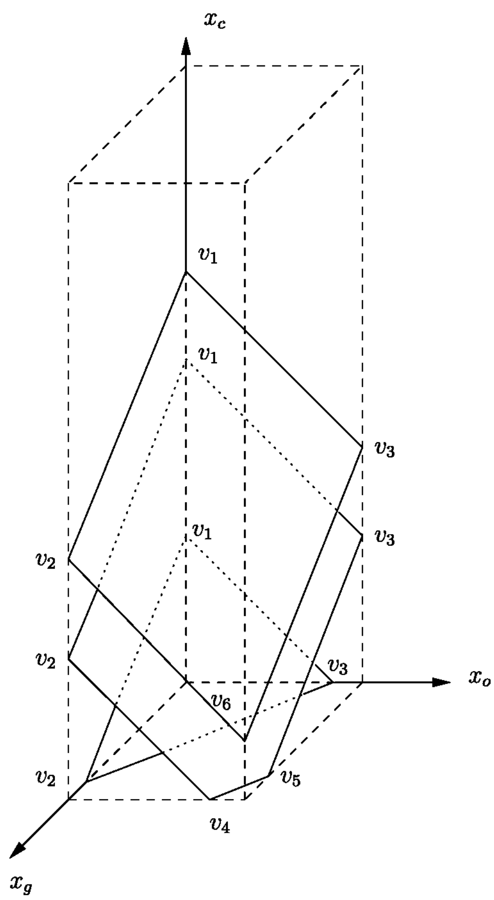

Figure 12 *Illustration of the input space where the optimal configuration is located on one of the vertices*

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$$\left.\begin{array}{l}\mathbf{v}_1^* = ((y_r - c')/u_1, 0, 0)\\ \mathbf{v}_2^* = ((y_r - 100 - c')/u_1, (100 - c')/u_2, 0)\\ \mathbf{v}_3^* = ((y_r - 100 - c')/u_1, 0, (100 - c')/u_3)\\ \mathbf{v}_6^* = ((y_r - 200 - c')/u_1, (100 - c')/u_2, (100 - c')/u_3)\end{array}\right\},$$

$$y_r \in (200, 400]$$

The vertices and thus the potential optimal configurations are illustrated in Fig. 12; it arises as the intersection between the efficiency plane and the constraint set.

The results from the static optimisation are depicted in Fig. 13 where the top graph is the growth of profit as a function of the efficiency. The bottom graph depicts the fuel configuration with the identification $(1, 2, 3, 4, 5, 6) = (\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*, \mathbf{v}_4^*, \mathbf{v}_5^*, \mathbf{v}_6^*)$, with $\mathbf{v}_i^*$ defined as above. As the figure shows the oil system is now used in the interval [200, 240].

The results of introducing the limit in the gas and oil system in the dynamic case are depicted in Fig. 14, where the top graph is the profit during 24 h of operation and the bottom graph is the fuel configuration with the identification as above. This is very similar to the results without the limit and it can be concluded that oil is not used at all. A limit of 25% of full production in gas and oil results in a gain of 16 000 DKK or 5% compared to the case of only using coal, that is, a reduction of 75% in production capabilities of the two fuels results in a reduction of 60% of the net income.

8 Including plant dynamics

In this section a brief discussion will be made of the optimisation problem when plant dynamics is considered.

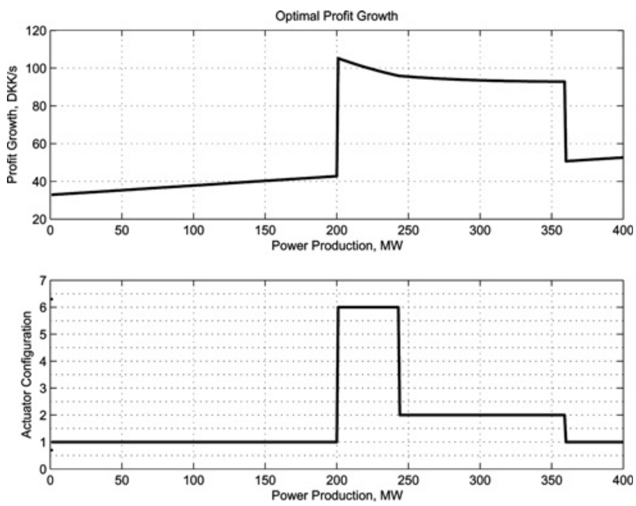

Figure 13 *Top: optimal profit growth with 25% production capabilities of gas and oil; bottom: fuel configuration [second axis should be read with the identification (1, 2, 3, 4, 5, 6) = ($\mathbf{v}_1^*, \mathbf{v}_2^*, \mathbf{v}_3^*, \mathbf{v}_4^*, \mathbf{v}_5^*, \mathbf{v}_6^*$)]*

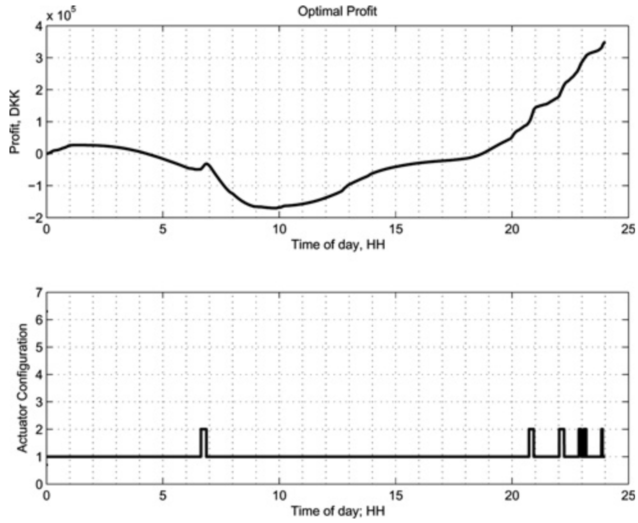

Figure 14 *Profit and optimal fuel configuration over 24 h of operation during June 29, 2008 with 25% production capabilities of gas and oil*

First, let

$$Z = \{\mathbf{z} = (z_1, z_2, \ldots, z_9) \in \mathbb{R}^9 | (z_1, z_4, z_7) \in X\}$$

be an auxiliary state space, which is used when describing the dynamics of the fuel flows. The fuel flow, $\mathbf{x}(t)$, into the power plant is governed by third-order differential equations (these equations also include a simple model for the power plant dynamics). The control signal to the valves controlling these flows is denoted $\mathbf{u} = (u_c, u_g, u_o) \in U$ and the system equations are given by

$$\begin{aligned}\dot{\mathbf{z}}(t) &= \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{x}(t) &= \mathbf{C}\mathbf{z}(t)\end{aligned} \tag{18}$$

where

$$\mathbf{A} = \begin{bmatrix}\mathbf{A}_c & \mathbf{0}_{3x3} & \mathbf{0}_{3x3}\\ \mathbf{0}_{3x3} & \mathbf{A}_g & \mathbf{0}_{3x3}\\ \mathbf{0}_{3x3} & \mathbf{0}_{3x3} & \mathbf{A}_o\end{bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ k_{i_1} & k_{i_2} & k_{i_3}\end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix}\mathbf{B}_c & \mathbf{0}_{3x1} & \mathbf{0}_{3x1}\\ \mathbf{0}_{3x1} & \mathbf{B}_g & \mathbf{0}_{3x1}\\ \mathbf{0}_{3x1} & \mathbf{0}_{3x1} & \mathbf{B}_o\end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix}0\\ 0\\ k_{i_0}\end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix}\mathbf{C}_1 & \mathbf{0}_{1x3} & \mathbf{0}_{1x3}\\ \mathbf{0}_{1x3} & \mathbf{C}_1 & \mathbf{0}_{1x3}\\ \mathbf{0}_{1x3} & \mathbf{0}_{1x3} & \mathbf{C}_1\end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

and k_{i_j}, $i \in \mathcal{I}$, are constants describing the dynamics of the three fuel systems which are obtained from transfer functions of the form $H_i(s) = (\tau_i s + 1)^{-3}$ where τ_i, $i \in \mathcal{I}$, is 90, 60 and 70, respectively. In the sequel the control set U is assumed compact and convex.

Moreover, the function

$$h(\mathbf{z}, t) = \mathbf{Y}\mathbf{z} + \psi(t) \tag{19}$$

is introduced with

$$\mathbf{Y} = \begin{bmatrix} \gamma^{\mathrm{T}}\mathbf{Q} \\ -\gamma^{\mathrm{T}}\mathrm{Q} \end{bmatrix}$$

$$\psi(t) = \begin{bmatrix} \gamma^{\mathrm{T}}\mathbf{b} - y_{\mathrm{r}}(t) + \alpha \\ -\gamma^{\mathrm{T}}\mathbf{b} + y_{\mathrm{r}}(t) + \alpha \end{bmatrix}$$

Hence h is constructed such that the set $Z' = \{(\mathbf{z}, t) | h(\mathbf{z}, t) \geq 0\}$ determines a 'reference band' around the reference, $y_{\mathrm{r}}(t)$. Here α should be thought of as a parameter dictating the size of the reference band.

In the sequel the map g_{P}, defined by (14), needs to be continuous. To obtain this it is assumed that the non-continuous contributions, that is the maps h_i defined by (3), are replaced by continuous approximations. The obtained map will, by abuse of notation, also be denoted by g_{P}.

Combining the above the optimisation problem is formulated as

$$\max_{(z(t),u(t))\in\Omega} \int_0^T g_{\mathrm{P}}(\mathbf{C}\mathbf{z}(t), t)\,\mathrm{d}t \tag{20}$$

subject to

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}u(t), \quad 0 \leq t \leq T \tag{21}$$

$$u(t) \in U, \quad 0 \leq t \leq T \tag{22}$$

$$h(\mathbf{z}(t), t) \geq 0, \quad 0 \leq t \leq T \tag{23}$$

where Ω is the set of admissible [that is $z(t)$ is absolutely continuous, $u(t)$ is (Lebesgue) measurable and $z(t)$, $u(t)$ satisfying (21)–(23)] pairs $(z(t), u(t))$. Note that by choosing the control set U and parameter α in (19) appropriate Ω becomes non-empty.

Now since the set

$$Q(\mathbf{z}, t) = \{(s, \mathbf{q}) | s \geq g_{\mathrm{P}}(\mathbf{z}, t), \ \mathbf{q} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u}, \ \mathbf{u} \in U\}$$

is convex for every $(\mathbf{z}, t) \in Z'$, the Filippov Existence Theorem (see [12, p. 199]) may be used to conclude that the above optimisation problem has an absolute maximum in Ω.

The approach described above will be studied in detail in future papers. In particular, we remark that some results have been obtained in the paper [13] where linear programming is used to solve the problem. This is obtained by approximating g_{P} by a piece-wise affine function and converting the dynamics, profit function and constraint function into discrete time.

9 Discussion

In this work models of two of DONG Energy's business objectives (efficiency and controllability) have been formulated such that a selection between three different fuels can be performed in an optimal manner. Profit maximisation is considered as a optimality measure as this is an important measure for companies today.

A static modelling and optimisation is performed such that the optimal configuration can be found for a given production setpoint. The developed optimisation method is then expanded to handle changes in prices and production reference. The result from this expansion is compared to a case where only coal is present and the use multiple fuels does increase the profit by 12% over 24 h of operation.

How the result is affected by a reduction of 75% in the gas and oil system is, furthermore, examined. The gain of mixing the fuels is reduced, however, during 24 h of operation the difference in profit compared to only using coal is 8%.

The result from this work can be used in two way; online to determine which fuels to use during the day and offline to determine if a plant could be instrumented with additional fuels such that the profit is increased.

An extension to fault detection could be relevant as this works could be used online in combination with fault detection methods [14]. Two possible scenarios are relevant depending on the seriousness of the detected fault; rerun the planning to optimise the profit given the new conditions or schedule maintenance during periods the failed actuator system is not in use.

Furthermore, with the changes in the demand for environmental friendly energy the current electric market is going to change dramatically during the next couple of years where more renewable energy will come into play. As many of the renewable energy systems are dependent of the forces of nature, the use of decentral short-time storage of energy will increase (e.g. electric cars [15, 16]). These short-time storage sources could be seen as an additional actuator in the methods presented in this work and thus planing for the entire electrical grid (in some region) is a possible extension of this work. Similar, work in this direction has been seen in [17] for Norwegian hydro-power plants.

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