Robustness of Structures

theoretical framework

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Robustness of structures – theoretical framework

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Abstract

An important aspect of the COST Action TU0601 "Robustness of structures" concerns the development of a theoretically sound basis for the assessment of robustness and acceptance criteria for structural robustness which can form the basis for development of practical relevant methods for ensuring robust design as well as strategies for maintaining the robustness of existing structures throughout their service life. This paper describes an overall theoretical framework for assessing robustness of structures developed within WG1 "Robustness of structures".

Robustness can be defined in different ways and on different levels of complexity / applicability. On the most general level robustness is assessed taking basis in decision analysis theory by estimating both direct risk, which is associated with the direct consequences of potential damages to the structure, and indirect risk, which corresponds to the increased risk of a damaged structure. Indirect risk can be interpreted as risk from consequences disproportionate to the cause of the damage. Robustness of a structure can therefore be measured by the contribution of the indirect risks to the total risk.

The risk-based approach for implementation for robustness is described and different measures of robustness are described and discussed – a risk-based, a reliability-based and a deterministic measure. These measures require probabilistic models to be formulated for the important failure modes and the uncertain parameters related to loads, strengths and models. Further, for quantification of the risk-based measure of robustness, modeling of the consequences of failures is needed. These probabilistic and consequence models are in general difficult to establish and not directly applicable for recommendations for practical applications. But the risk and reliability based robustness measures can be used as a rational basis for formulating recommendations for practice.

Keywords

Robustness; Risk analysis; Reliability

1. Background / Introduction

An important aspect of the COST Action TU0601 "Robustness of structures" concerns the development of a theoretically sound basis for the assessment of robustness and acceptance criteria for structural robustness which can form the basis for development of practical relevant methods for ensuring robust design as well as strategies for maintaining the robustness of existing structures throughout their service life. This paper describes an overall theoretical framework for assessing robustness of structures developed within WG1 "Robustness of structures".

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This paper is based on the fact sheet "J.D. Sørensen, E. Rizzuto and M. H. Faber: Robustness – theoretical framework", see (Köhler et al. 2010), (Sørensen et al. 2010), (Vrouwenvelder and Sørensen 2009) and the COST TU 601 guideline document on the "Theoretical framework on structural robustness" (Sørensen 2011). Details can be found in these references where also a discussion on acceptance criteria and system effects can be found.

2. Implementation of structural robustness

In Eurocode EN 1990:2002 (CEN 2002), the basic requirement to robustness is given in clause 2.1 4(P): "A structure should be designed and executed in such a way that it will not be damaged by events such as explosion, impact, and the consequences of human errors, to an extent disproportionate to the original cause."
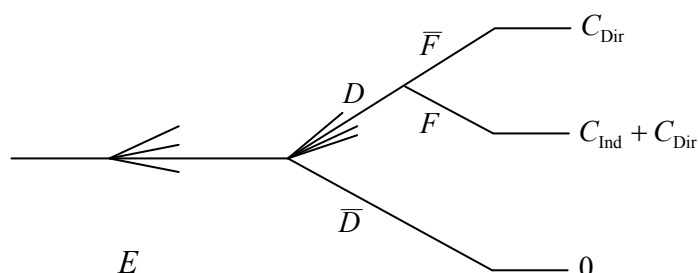

Figure 1. An event tree for robustness quantification (Baker et al. 2008).

Figure 1 presents the idea behind assessment of robustness, see (Baker et al. 2008). The assessment starts with the consideration and modeling of exposures (E) that can cause damage to the components of the structural system. The term "exposures" refers to extreme values of design loads, accidental loads and deterioration processes but also includes human errors in the design, execution and use of the structure. The term "damage" refers to reduced performance or failure of individual components of the structural system. After the exposure event occurs, the components of the structural system either remain in an undamaged state ($\overline{D}$) as before or change to a damage state (D). Each damage state can then either lead to the failure of the structure (F) or no failure ($\overline{F}$).

Consequences are associated with each of the possible damage and failure scenarios, and are classified as either direct (C_{dir}) or indirect (C_{ind}). Direct consequences are considered to result from damage states of individual component(s). Indirect consequences are incurred due to loss of system functionality or failure and can be attributed to lack of robustness (Baker et al. 2008) and (JCSS 2008).

The basic framework for risk analysis is based on the following equation in which risk contributions from local damages (direct consequences) and comprehensive damages (follow-up/indirect consequences), are added, see (Baker et al. 2008) and (JCSS 2008):

$$R = \sum_i \sum_j C_{dir,ij} P\left(D_j|E_i\right) P(E_i) + \sum_k \sum_i \sum_j C_{ind,ijk} P\left(S_k|D_j \cap E_i\right) P\left(D_j|E_i\right) P(E_i) \tag{1}$$

where

$C_{dir,ij}$ consequence (cost) of damage (local failure) D_j due to exposure E_i

$C_{ind,ij}$ consequence (cost) of comprehensive damages (follow-up/indirect) S_k given local damage D_j due to exposure E_i

$P(E_i)$ probability of exposure E_i

$P(D_j|E_i)$ probability of damage D_j given exposure E_i

$P(S_k|...)$ probability of comprehensive damages S_k given local damage D_j due to exposure E_i

The optimal design (decision) is the one minimizing the sum of costs of mitigating measures and the total risk R. A detailed description of the theoretical basis for risk analysis can be found in (JCSS 2008).

It is noted that an important step in the risk analysis is to define the system and the system boundaries. This includes the definition/modeling of the structure itself, but also the effect of a possible collapse of the structure on the environment/surrounding society. It is noted that in some cases the failure of a structure can cause extensive indirect consequences for the society. These are important to include when defining the system to be considered in the risk analysis.

The total probability of comprehensive damages/collapse associated to (1) is:

$$P(\text{collapse}) = \sum_i \sum_j P(\text{collapse}|D_j \cap E_i) P(D_j|E_i) P(E_i) \tag{2}$$

where $P(\text{collapse}|D_j \cap E_i)$ is the probability of collapse (comprehensive damage) given local damage D_j due to exposure E_i. Note that compared to (1) only one comprehensive damage (collapse) is included in (2).

The terms $P(\text{collapse}|D)$ and $P(D|E)$ are related to the concepts damage tolerance and vulnerability, respectively. The product $P(\text{collapse}|D)P(D|E)$ can be considered as a structure dependent measure of the robustness.

From equation (2), it is obvious that the probability of collapse can be reduced by:
- Reducing one or more of the probabilities of exposures $P(E_i)$ – i.e. prevention of exposure or event control.
- Reducing one or more of the probabilities of damages $P(D_j|E_i)$ – i.e. related to element/component behavior.
- Reducing one or more of the probabilities $P(\text{collapse}|D_j \cap E_i)$.

If the consequences are included in a risk analysis, then reduction of direct (local) consequences, $C_{\text{dir,ij}}$ and comprehensive (indirect) consequences, $C_{\text{ind,ij}}$ are also important.

3. Measures of structural robustness

(Baker et al. 2008) proposed a definition of a robustness index based on risk measures. The approach divides consequences into direct consequences associated with local component damage (that might be considered proportional to the initiating damage) and indirect consequences associated with subsequent system failure (that might be considered disproportional to the initiating damage). An index is formulated by comparing the risk associated with direct and indirect consequences. The index of robustness (I_{rob}) is defined as:

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3}$$

where R_{Dir} and R_{Ind} are the direct and indirect risks associated with the first and the second term in equation (1). The index takes values between zero and one, with larger values indicating larger robustness.

As mentioned above, the optimal decision is the one which minimizes the total risk obtained by equation (1). This could equally well be by reducing the first or the second term in equation (1). This implies that the definition of a robustness index by

equation (3) is not always completely consistent with a full risk analysis, but can be considered as a helpful indicator based on risk analysis principles. It is noted that since the direct risks typically are related to code based limit states, they can generally be estimated with higher accuracy than the indirect risks.

A difficult step in the risk assessment is to model and quantify the probability of the exposures. Therefore, it can be very convenient and helpful to use a conditional index of robustness obtained using risks $R_{Dir|\text{exposure}}$ and $R_{Ind\|\text{exposure}}$ conditioned of a given exposure:

$$I_{rob|\text{exposure}} = \frac{R_{Dir|\text{exposure}}}{R_{Dir|\text{exposure}} + R_{Ind|\text{exposure}}} \tag{4}$$

(Frangopol and Curley 1987) and (Fu and Frangopol 1990) proposed some probabilistic measures related to structural redundancy – which also indicates the level of robustness. A redundancy index (*RI*) is defined by:

$$RI = \frac{P_{f(\text{damaged})} - P_{f(\text{intact})}}{P_{f(\text{intact})}} \tag{5}$$

where $P_{f(\text{damaged})}$ is the probability of failure for a damaged structural system and $P_{f(\text{intact})}$ is the probability of failure of an intact structural system. The redundancy index provides a measure on the robustness / redundancy of the structural system. The index takes values between zero and infinity, with smaller values indicating larger robustness.

A simple and practical measure of structural redundancy (and robustness) used in the offshore industry is based on the so-called *RIF*-value (Residual Influence Factor), (ISO 2008).

Other simple measures of robustness have been proposed based on e.g. the determinant of the stiffness matrix of structure with and without removal of elements.

4. Conclusions

A risk-based approach for implementation for robustness is described and different measures of robustness are described and discussed – a risk-based, a reliability-based and a deterministic measure. These measures require probabilistic models to be formulated for the important failure modes and the uncertain parameters related to loads, strengths and models. Further, for quantification of the risk-based measure of robustness, modeling of the consequences of failures is needed. These probabilistic and consequence models are in general difficult to establish and not

directly applicable for recommendations for practical applications. But the risk and reliability based robustness measures can be used as a rational basis for formulating recommendations for practice.

Estimation of the probability of extensive failure and collapse requires system models of the failure modes to be formulated. Especially the importance of ductility is investigated and shows that the level of ductility should be at least 1.5 - 2.0 before a significant increase in system reliability is observed for redundant structural systems.

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