A model for equilibrium swelling of the UCST-type thermo-responsive hydrogels

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Abstract
Thermo-responsive gels of the UCST (upper critical solution temperature)-type shrink below their critical temperature and swell above it. Changes in water uptake by these gels are driven by thermally induced dissociation of hydrogen or ionic bonds between chains. A simple model is developed for the description of equilibrium swelling of thermophilic gels with hydrogen bonds. To confirm its ability to describe observations, equilibrium swelling diagrams are fitted on poly(acrylamide-acrylic acid) and poly(acrylamide-acrylonitrile) macroscopic gels and microgels with various structures (copolymer gels, gels with interpenetrating network, nanocomposite gels), as well as on biocompatible poly(N-acryloyl-glycinamide) and poly(allylurea-co-allylamine) gels. Numerical simulation reveals that material parameters evolve consistently with molar fractions of cross-linker, hydrophobic and hydrophilic comonomers in the feed, degree of ionization of functional groups, and concentration of nanofiller. It is shown that the model can also be applied to describe observations on thermophilic zwitterionic gels.

Key-words: Thermo-responsive gels; UCST-type gels; Zwitterionic gels; Hydrogen bonds; Equilibrium swelling.

INTRODUCTION
Thermo-responsive (TR) gels form to a special group of stimuli-sensitive hydrogels whose equilibrium degree of swelling is strongly affected by temperature T. Three types of TR gels are conventionally distinguished. Gels of the LCST (low critical solution temperature) type (gels with negative temperature sensitivity or thermophobic gels [1]) swell pronouncedly at temperatures below their volume phase transition temperature \( T_c \) and collapse above \( T_c \) [2]. Gels of the UCST (upper critical solution temperature) type (gels with positive temperature sensitivity or thermophilic gels [3]) swell noticeably when temperature exceeds their critical temperature \( T_c \) and shrink below
Hydrogels of the mixed type (also known as doubly thermo-responsive or "schizophrenic" gels [4, 5]) are in the swollen state at low (below $T_{c1}$) and high (above $T_{c2}$) temperatures and collapse in an intermediate interval ($T_{c1}, T_{c2}$).

In the past three decades, substantial attention has been focused on the mechanical properties of thermophobic macroscopic gels and microgels and their equilibrium and transient swelling. This is explained by a wide spectrum of possible biomedical and bio-technological applications of LCST-type gels as smart materials for (i) controlled delivery and release of drugs and genes [6], (ii) adhesive dressings for wound healing [7], and (iii) scaffolds for tissue regeneration [8].

Coil-globule transformations in aqueous solutions of thermophilic polymers and swelling-deswelling transitions in the UCST-type gels have recently become a hot topic [9-12]. This interest is driven by the unique abilities of thermophilic gels (i) to capture and transfer proteins [13] and (ii) to form reversible multi-cellular spheroids [14, 15]. Discovery of novel biomedical applications of UCST-type gels has led to an active search for biocompatible thermophilic polymers exhibiting volume phase transition under physiological conditions [16, 17]. Advanced design strategies for synthesis of thermophilic gels were discussed in [18-20]. Modeling the thermo-mechanical response of these materials plays an important role in the development of novel routes for their preparation.

The objective of this study is three-fold: (i) to derive a simple model with a relatively small number of material constants able to describe equilibrium swelling diagrams on UCST-type gels, (ii) to find adjustable parameters by fitting experimental data on several thermophilic homo- and copolymer gels and gels with interpenetrating networks (IPN), and (iii) to examine how the critical temperature $T_c$ and the shape of the swelling curves are affected by micro-structure of the gels, molar fraction of cross-linker, concentration of nanofiller, degree of ionization of functional groups, and ionic strength of aqueous solutions.

According to the Flory-Rehner concept [21], equilibrium degree of swelling $Q$ of a non-ionic gel is determined by the balance between the energy stored in the polymer network (elastic forces inducing contraction of a gel) and the energy of interactions between water molecules and segments of chains (osmotic pressure that causes its volumetric expansion) [22]. Changes in temperature $T$ induce evolution of the micro-structure of a thermo-responsive gel (due to changes in strength of the secondary interactions between segments of chains), which, in turn, affects the mechanical energy of the network.

In TR gels of the LCST-type, thermally induced changes in the micro-structure are driven by hydrophobic interactions between segments. The following scenario is conventionally accepted for their description [23-25]. Chains in a polymer network contain hydrophilic and hydrophobic segments. At temperatures $T < T_c$, each hydrophobic segment is surrounded by a cage-like structure formed by water molecules bridged by hydrogen bonds. An increase in temperature leads to the growth of intensity of thermal fluctuations that destabilize the cage-like structures. Their breakage results in "release" of hydrophobic segments from the cages and their direct contact with water molecules. This is observed as an increase in the overall hydrophobicity of the network (characterized by the Flory-Huggins (FH) parameter $\chi$) with $T$. When molar fraction of "released" hydrophobic segments reaches its critical value (characterized by the ultimate value $\chi_{max}$ of the FH
parameter), these segments begin to aggregate into hydrophobic clusters from which water molecules are expelled. The temperature at which aggregation starts is identified as the volume phase transition temperature $T_c$. As hydrophobic aggregates are enveloped by hydrophilic segments (with a fixed degree of hydrophilicity), further growth of the effective FH parameter at $T > T_c$ does not occur (the effective FH parameter remains constant above $T_c$). The clusters formed by hydrophobic segments above $T_c$ serve as extra physical bonds between polymer chains. Their presence induces a pronounced increase in the elastic energy of the network. The latter is confirmed by experiments demonstrating a pronounced growth of the elastic modulus with temperature above $T_c$ [26, 27]. As a result of the aggregation process, the balance between elastic forces and osmotic pressure changes, water molecules are expelled from the gel, and degree of swelling $Q$ decreases strongly at $T > T_c$.

A similar picture is suggested to describe thermally induced evolution of the micro-structure in TR gels of the UCST-type. Two kinds of these gels are conventionally distinguished [1]: gels with (i) hydrogen bonds between neutral segments, and (ii) ionic bonds between charged functional groups. Poly(acrylamide)/poly(acrylic acid) (PAAm/PAAc) IPN gel [28] provides an example of TR gels of the first kind (hydrogen bonds are formed between amide groups of AAm and non-ionized carboxylic groups of AAc). Polyzwitterionic sulfobetaine gels [29, 30] serve as an example of gels of the other type (ionic bonds bridging chains arise due to formation of ion pairs between cationic and anionic functional groups).

When temperature $T$ is far below the critical temperature $T_c$, physical cross-links between chains formed by hydrogen and ionic bonds are relatively stable (their concentration remains practically unaffected by temperature). With an increase in temperature, the intensity of thermal fluctuations grows. This growth induces breakage of hydrogen and ionic bonds (a decay in the total concentration of physical cross-links between chains). The latter results in a decrease in the elastic energy stored in polymer chains. A pronounced decay in the elastic modulus with temperature was reported in [31] for gels with hydrogen bonds and [32] for gels with ionic bonds. This decrease leads to a shift of the balance between elastic forces in the polymer network and osmotic pressure of water molecules to higher values of the equilibrium degree of swelling $Q$. At temperatures $T$ strongly exceeding $T_c$, all physical bonds between chains are broken, and $Q$ reaches its ultimate (temperature-independent) value.

Although thermophobic and thermophilic gels demonstrate some mirror symmetry (the equilibrium degree of swelling of LCST-type gels decreases, while that of UCST-type gels increases with temperature; incorporation of hydrophobic monomers leads to a reduction in $T_c$ in LCST-type gels [33] and an increase in $T_c$ in UCST-type gels [34]), this symmetry is only partial for two reasons.

(I) In majority of thermophobic homopolymer gels (we refer to poly($N$-isopropylacrylamide) (PNIPAm) and poly($N$-vinylcaprolactam) (PVCL) as examples), monomers contain both hydrophilic and hydrophobic segments, which implies that the kinetics of aggregation of hydrophobic moieties is weakly affected by the structure of a polymer network. On the contrary, hydrogen bonds between chains in thermophilic homopolymer gels (poly(acrylic acid) (PAAc), poly(vinyl alcohol) (PVA) [35] and poly(acrylonitrile) (PAN) [36] serve as examples) are relatively weak. To prepare UCST-type gels
exhibiting noticeable changes in their equilibrium degree of swelling with temperature, incorporation of comonomers is required (for example, acrylamide (AAm)) that form stronger hydrogen bonds with the above monomers. As this incorporation can be performed in different ways (copolymerization, formation of interpenetrating networks, etc.), swelling diagrams on UCST-type gels are heavily influenced by the structure of polymer networks (see Fig. 2 below).

(II) The characteristic size of hydrophobic aggregates in LCST-type gels grows pronouncedly with temperature when $T > T_c$. In the close vicinity of $T_c$, this size is relatively small, and hydrophobic clusters rearrange under the influence of thermal fluctuations. The latter is observed in mechanical tests as the viscoelastic response of thermophobic gels [37,38]. Further growth of temperature induces a strong increase in the size of hydrophobic clusters, which makes them practically insensitive to thermal fluctuations (when $T$ exceeds $T_c$ by 10 K, the response of thermophobic gels becomes merely elastic). The situation with UCST-type gels is different as rearrangement of hydrogen and ionic bonds induces their viscoelastic response at all temperatures under consideration [36] (although concentration of these bonds changes with temperature, their strength remains temperature-independent). The equilibrium degree of swelling of a network with permanent and reversible bonds is determined by the balance of osmotic pressure of water molecules and elastic forces in chains bridged by (i) covalent cross-links and (ii) entanglements that have no sufficient time for disentanglement (due to reptative diffusion [39]) when temporary bonds between the ends of appropriate chains dissociate and re-associate. As concentration of these entanglements decreases with temperature (due to thermally induced acceleration of the reptative diffusion), the structure of the polymer network does not remain fixed, but changes with temperature. This implies that (i) the equilibrium swelling diagrams on thermophilic gels are strongly affected by the methods of their preparation (formation of the network), (ii) these curves demonstrate pronounced hysteresis [40] when water uptake tests are conducted under heating and cooling, and (iii) alternative heating-cooling processes induce formation of several phases coexisting in UCST-type gels [41].

Two empirical formulas were proposed for the equilibrium degree of swelling of thermophilic gels. The first was suggested in [42] to describe swelling curves with an abrupt increase in $Q$ near the critical temperature $T_c$,

$$Q(T) = Q_{\text{min}} + \frac{Q_{\text{max}} - Q_{\text{min}}}{1 + \exp(-\alpha(T - T_c))},$$

(1)

where $Q_{\text{min}}$, $Q_{\text{max}}$ stand for the equilibrium degrees of swelling far below and above $T_c$, and $\alpha$ is an adjustable parameter. The other relation was introduced in [43] to describe swelling diagrams with a slow increase in $Q$ with temperature,

$$Q(T) = Q_{\text{min}} \exp(\alpha T^2).$$

(2)

A shortcoming of Eqs. (1) and (2) is that they cannot predict how the equilibrium water uptake by UCST-type gels is affected by their composition, methods of preparation and experimental conditions.
Several theoretical models have been developed for the analysis of swelling of TR gels with hydrogen bonds. Their common feature consists in modification of the conventional expression for the free energy of interaction between water molecules and segments of chains $\Psi_{int}$.

Erman and Flory [44] proposed to treat the Flory-Hyggins (FH) parameter $\chi$ as a polynomial function of the volume fraction of polymer network with temperature-dependent coefficients and described quantitatively the UCST-type behavior of a cyclohexane-polystyrene network. A similar approach (where some coefficients in the polynomial expansion increase, while the other decrease with $T$) was used in [45] to predict equilibrium swelling of double thermo-responsive gels.

Another version of this approach was suggested by the Tanaka group [46]. The FH parameter $\chi$ (treated as a measure of hydrophobicity) was presumed to grow with concentration of hydrogen bonds (the overall hydrophobicity of a polymer network increased when these bonds concealed hydrophilic segments).

Explicit expressions for $\Psi_{int}$ based on the lattice theories accounting for hydrogen bonding between polymer segments were derived in [47-50]. However, the ability of these models to describe swelling diagrams on UCST-type gels was not examined.

Another way to calculate the effect of hydrogen bonds on the specific energy of interaction between water molecules and polymer chains was proposed in [51]. Based on this approach, governing equations for equilibrium swelling of gels with intra- and inter-chain hydrogen bonds were developed in [52]. A shortcoming of these relations is that they involve a large number of adjustable parameters, which complicates their use in the analysis of observations.

Unlike previous studies, we adopt the conventional expression for the specific free energy of interactions $\Psi_{int}$. To account for temperature induced breakage of hydrogen bonds, the strain energy density of the polymer network is modified. We presume the specific mechanical energy to consist of two parts: (i) the energy stored in polymer chains connected by covalent cross-links (this quantity remains independent of temperature), and (ii) the strain energy of chains bridged by entanglements (this quantity decreases with temperature due to partial disentanglement of the network driven by breakage of hydrogen bonds that connect these chains with the network).

Treatment of hydrogen bonds as physical cross-linking between chains was proposed in [53]. The assumption that stiffness (and the specific mechanical energy) of polymer chains is affected by concentration of intra-chain hydrogen bonds was introduced in [54] to describe multiple phase transitions in stimuli-responsive gels.

**MODEL**

A thermophilic gel is modeled as a two-phase medium composed of an equivalent polymer network and water molecules. The solid and fluid phases are treated as immiscible interpenetrating continua.

The initial state of the gel coincides with that of an undeformed dry specimen at some temperature $T_0$ (for definiteness, we presume $T_0$ to be close to 0 °C). Transformation of the initial state into the
actual state at temperature $T$ is described by the deformation gradient $F$. The deformation gradient for macro-deformation obeys the molecular incompressibility condition

$$\det F = 1 + Q,$$

where $\det$ stands for the determinant, $Q = C \nu$ is the degree of swelling, $C$ denotes concentration of water molecules (number of molecules per unit volume in the initial state), and $\nu$ stands for their characteristic volume.

The polymer network consists of two sub-networks. The first sub-network is formed by covalently cross-linked chains. The other sub-network consists of chains whose ends are merged with the network by hydrogen bonds. To simplify the analysis, we disregard clustering of hydrogen bonds. Each bond is modeled as a separate temporary bridge between chains that can break and reform independently of other bonds. When the hydrogen bonds break, chains of the second network are transformed into the dangling state (where stresses in these chains vanish) and do not contribute into the strain energy of the network (Fig. 1).

![Figure 1](image-url)

**Figure 1.**

Figure 1: A - At relatively low temperatures, an element of a network consists of chains of the first sub-network (green) connected by covalent cross-links (red) and chains of the second sub-network (orange) bridged by physical bonds (blue). B - At elevated temperatures, some physical bonds break, chains of the second sub-network (orange) are transformed into the dangling state, and the same element involves only two active chains (green) connected by the covalent cross-links.

Following [55], we describe the kinetics of rearrangement of temporary bonds by means of the Bell model [56]. Changes in the number of hydrogen bonds $n$ (per unit volume in the initial state) are governed by the equation

$$\frac{dn}{dt} = k_1(N - n) - k_2 n,$$

where $N$ denotes the total concentration of moieties able to form hydrogen bonds, and $k_1, k_2$ are the rates of formation (association) and breakage (dissociation) of bonds. Eq. (4) states that the number of bonds broken per unit time is proportional to the number of existing bonds, while the number of new bonds developed per unit time is proportional to the number of segments available for their formation.

Bearing in mind that the characteristic time for rearrangement of bonds is small compared with that for water uptake by a gel, we determine $n$ as a steady-state solution of Eq. (4),

$$n = \frac{N}{1 + r}, \quad r = \frac{k_2}{k_1}.$$
The rates of breakage, \( k_2 \), and reformation, \( k_1 \), of hydrogen bonds are affected by two factors: temperature \( T \) and degree of swelling \( Q \). To describe the influence of temperature, we presume the rate of breakage \( k_2 \) to grow with \( T \). Adopting the Arrhenius dependence, we write

\[
k_2 = k_2^0 \exp \left( \frac{E_a}{k_B} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right),
\]

where \( T \) is the absolute temperature, \( k_B \) is the Boltzmann constant, \( E_a \) denotes some activation energy, and \( k_2^0 \) is the rate of breakage of hydrogen bonds at temperature \( T_0 \). When the difference \( T - T_0 \) is relatively small and \( T_0 \) corresponds to 0 °C, Eq. (6) is simplified,

\[
k_2 = k_2^0 \exp(aT),
\]

where \( T \) is measured in Celsius, and \( a = \frac{E_a}{k_B T_0^2} \).

To account for the effect of swelling, we presume the rate of reformation of hydrogen bonds \( k_1 \) to increase linearly with volume fraction of the polymer network,

\[
k_1 = k_1^0 \phi_n,
\]

where \( k_1^0 \) is a pre-factor, and

\[
\phi_n = \frac{1}{1 + Q}, \quad \phi_w = \frac{Q}{1 + Q}
\]

are volume fractions of the polymer network and water, respectively. Eq. (8) means that broken hydrogen bonds reform when the distance between appropriate segments of chains remains sufficiently small, and the rate of restoration vanishes when chains are located far away from each other (due to water uptake by the polymer network). Inserting Eqs. (7)-(9) into Eq. (6), we calculate the intensity of hydrogen bonding

\[
S = \frac{n}{N}
\]

and arrive at the formula

\[
S = \left[ 1 + R \exp(aT) (1 + Q) \right]^{-1}
\]

with \( R = \frac{k_2^0}{k_1^0} \).

Adopting the affine hypothesis, we suppose that deformations of the sub-networks coincide with macro-deformation of the gel. The deformation gradient for macro-deformation \( F \) is connected with the deformation gradient for elastic deformation of the \( m \)th sub-network \( F_e^{(m)} \) by the multiplicative decomposition formula

\[
F = F_e^{(m)} \cdot f^{(m)},
\]

where \( f^{(m)} \) is the deformation gradient for transition from the initial to the reference (stress-free) state of the \( m \)th sub-network, and the dot stands for inner product (the index \( m \) equals 1 for the
sub-network with covalently cross-linked chains and 2 for the sub-network of chains connected by physical bonds).

Focusing on the analysis of equilibrium swelling tests with the conventional program (characterized by a monotonic increase in temperature), we suppose that transformation of the initial state into the reference state for both sub-networks is determined by the same deformation gradient $f^{(1)} = f^{(2)} = f$ with

$$f = (1 + Q)^{\frac{1}{3}} I,$$

where $I$ is the unit tensor, $Q_0 = C_0 \nu$ is the degree of swelling, and $C_0$ stands for the concentration of water molecules in the reference state. A shortcoming of Eq. (13) is that it cannot predict hysteresis of swelling diagrams (the difference between the swelling curves observed under heating and cooling). To model the latter phenomenon, Eq. (13) is to be replaced by a more sophisticated relation that accounts for the evolution of the deformation gradient $f^{(2)}$ induced by thermal history.

Following the conventional approach [21], we treat the Helmholtz free energy of the gel (per unit volume in the initial state) as the sum of three components,

$$\Psi = \Psi_1 + \Psi_2 + \Psi_{\text{int}},$$

where $\Psi_1$ is the specific energy of water molecules not interacting with segments of chains, $\Psi_2$ is the specific energy of polymer chains not interacting with water, and $\Psi_{\text{int}}$ is the specific energy of mixing of water molecules with segments of chains.

The specific energy of water molecules is given by the conventional formula

$$\Psi_1 = \mu^0 C,$$

where $\mu^0$ is the chemical potential of water molecules not interacting with the polymer network.

The specific energy of the polymer network (involving two sub-networks) reads

$$\Psi_2 = \sum_{m=1}^{2} W_m \left( t_{e_1}^{(m)}, t_{e_2}^{(m)}, t_{e_3}^{(m)} \right).$$

The specific mechanical energy $W_m$ stored in chains of the $m$th sub-network depends on the principal invariants $t_{e_1}^{(m)}, t_{e_2}^{(m)}, t_{e_3}^{(m)}$ of the corresponding Cauchy-Green tensor for elastic deformation

$$B_e^{(m)} = F_e^{(m)} \cdot (F_e^{(m)})^T,$$

where $T$ stands for transpose.

The neo-Hookean expressions are adopted for the functions $W_m$,

$$W_m = \frac{1}{2} G_m \left[ \left( t_{e_1}^{(m)} - 3 \right) - \ln t_{e_3}^{(m)} \right].$$
where $G_m$ stands for the shear modulus of the $m$th sub-network. The physical meaning of Eq. (17) was discussed in [57], where this formula was re-derived within the concept of entropic elasticity. More sophisticated expressions for the functions $W_m$ were developed and verified by comparison with observations in [58, 59].

For the sub-network with covalent bonds, the coefficient $G_1$ is independent of temperature. For the sub-network of chains connected by physical bonds, the coefficient $G_2$ is treated as a function of the intensity of hydrogen bonding $S$. For definiteness, we set

$$G_2 = G S^\kappa,$$  \hspace{1cm} (18)

where the pre-factor $G$ and the exponent $\kappa$ are material constants. Eqs. (17) and (18) describe adequately a decrease in the elastic moduli with temperature observed in small-amplitude oscillatory tests on poly(N-acryloyl glycinamide-co-acrylamide) P(NAGA-AAm) gels [60].

The specific energy of mixing of water molecules with segments of chains reads [21]

$$\Psi_{int} = k_B T_0 \left( C \ln \phi_w + \chi \frac{C}{C} \phi_m \right),$$  \hspace{1cm} (19)

where the FH parameter $\chi$ is presumed to be constant. The first term in Eq. (19) characterizes the entropy and the other term describes the enthalpy of mixing of water molecules and segments of chains. Presuming the difference $T - T_0$ to be relatively small, we replace $T$ with the initial temperature $T_0$ in the thermodynamic factor $k_B T$.

Under unconstrained swelling of a thermophilic gel with hydrogen bonds, its equilibrium degree of swelling $Q$ is governed by the nonlinear equation

$$\ln \frac{Q}{1 + Q} + \frac{1}{1 + Q} + \frac{\chi}{(1 + Q)^2} + \frac{g_1 + g_2}{1 + Q} \left[ \left( \frac{1 + Q}{1 + Q_0} \right)^3 - 1 \right] = 0,$$  \hspace{1cm} (20)

where

$$g_m = \frac{G_m v}{k_B T_0}$$  \hspace{1cm} (21)

are dimensionless elastic moduli. Eq. (20) is developed by means of the free energy imbalance inequality by using a method similar to that used for thermophobic gels in [61].

It follows from Eq. (21) that the coefficient $g_1$ is constant. The coefficient $g_2$ is found from Eqs. (11), (18) and (21),

$$g_2 = g \left[ 1 + R \exp(aT) (1 + Q) \right]^{-\kappa}$$  \hspace{1cm} (22)

with $g = \frac{G v}{k_B T_0}$.

Eqs. (20) and (22) provide a coupled set of governing equations for equilibrium swelling of UCST-type gels with hydrogen bonds. These relations involve seven adjustable parameters: (i) the coefficient $\chi$ characterizes hydrophobicity of chains in the polymer network, (ii) $Q_0$ stands for the degree of swelling in the reference (stress-free) state, (iii) dimensionless shear moduli $g_1$ and
\( g \) describe mechanical properties of the sub-networks with covalently and physically cross-linked chains, respectively, (iv) \( a \) and \( R \) account for the temperature-induced decrease in concentration of hydrogen bonds, (v) \( \kappa \) characterizes topology of the network and describes how dissociation of hydrogen bonds affects concentration of active chains in the second sub-network.

**FITTING OF EXPERIMENTAL DATA**

To demonstrate the ability of the model to describe observations on thermophilic homo- and copolymer gels with hydrogen bonds, we approximate experimental swelling diagrams on several macro- and microgels. In simulation, Eqs. (20) and (22) are solved by the Newton-Raphson algorithm. Adjustable parameters are calculated by the nonlinear regression method to minimize the expression

\[
\sum (Q_{exp} - Q_{sim})^2,
\]

where summation is performed over all temperatures \( T \) under consideration, \( Q_{exp} \) stands for the degree of swelling measured in a test, and \( Q_{sim} \) is determined by Eq. (20).

A common difficulty of matching observations in equilibrium water uptake tests is that the number of material constants for gels exceeds the number of experimental points. For example, three coefficients (the FH parameter \( \chi \), the elastic modulus \( g_1 \), and the degree of swelling in the reference state \( Q_0 \)) are required to characterize the only experimental point \( Q_{exp} \) for a temperature-insensitive gel. To find a unique set of material constants, observations in additional tests are needed (to determine \( \chi \) and \( g_1 \) independently) or introduction of special hypotheses (regarding the reference state of the polymer network) [62].

To reduce the number of material constants, we set \( g = 1 \) in the analysis of experimental data, and choose a physically reasonable value of \( \chi \) for each gel under consideration. The remaining five parameters are determined as follows. The coefficient \( g_1 \) is calculated by fitting observations far above the critical temperature \( T_c \). The parameter \( Q_0 \) is found by matching observations at the lowest temperature at which observations in swelling tests are provided. \( R \) is determined by approximation of experimental data below \( T_c \) (this parameter characterizes the slope of a swelling curve at \( T < T_c \)). The exponent \( \kappa \) is calculated by matching observations above \( T_c \) (it characterizes the slope of a swelling curve at \( T > T_c \)). The coefficient \( a \) is found by fitting the data in the close vicinity of \( T_c \) (the temperature at which a pronounced upturn of the graph \( Q(T) \) is observed).

**Poly(acrylamide-acrylic acid) gels**

We begin with study of equilibrium swelling of thermophilic macro- and microgels based on acrylamide (AAm) and acrylic acid (AAc) monomers. Hydrogen bonds are formed between amide groups of AAm and non-ionized carboxyl groups of AAc. Our aim is to examine how adjustable parameters in the model are affected by (i) molar fraction of a cross-linker, (ii) micro-structure of
the polymer network (gels with interpenetrating network (IPN) of PAAm and PAAc chains versus copolymer P(AAm-AAc) gels), (iii) molar fraction of hydrophobic comonomers, (iv) degree of ionization of carboxyl groups, and (v) mass fraction of filler in nanocomposite gels.

**PAAm/PAAc IPN microgels**

First, experimental data are fitted in equilibrium water uptake tests on microgels with PAAm/PAAc interpenetrating networks (Owens et al. [64]). Composition of the microgels and their preparation procedure are described in Supplementary Material. To ensure that most carboxyl groups remain neutral (and form hydrogen bonds), swelling tests were performed in an aqueous solution with pH=3. Hydrodynamic radius of microgel particles $R_h$ was measured by dynamic light scattering in the interval of temperatures between 25 and 55 °C. The equilibrium degree of swelling $Q(T)$ is determined by the equation [63]

$$1 + Q(T) \frac{1}{1 + Q(T^*)} = \left( \frac{R_h(T)}{R_h(T^*)} \right)^3$$

with $T^* = 25$ °C and $Q(T^*) = 3$.

Experimental swelling curves on microgels with molar fractions of cross-linker $N,N'$-methylenebis(acrylamide) (BIS) $\psi = 0.001, 0.0025$ and $0.005$ (the same for PAAm and PAAc networks) are depicted in Fig. 2 together with results of numerical simulation with the material parameters collected in Tab. S-1 and reported in Fig. S-1. Following [64], the relative swelling volume

$$R = \frac{1 + Q}{1 + Q(T^*)}$$

is plotted versus temperature $T$ in this figure.

**Figure 2.**

Figure 2: Relative swelling volume $R$ versus temperature $T$. Circles: experimental data [64] on PAAm/PAAc IPN microgels with various molar fractions of cross-linker $\psi$ (A - $\psi=0.0010$; B - $\psi=0.0025$, C - $\psi=0.0050$). Solid lines: results of simulation.

Figs. 2A to 2C show that an increase in concentration of cross-linker results in a pronounced reduction in $Q$ far above the critical temperature $T_c$ and strong decrease in the slope of the swelling diagrams in the vicinity of $T_c$, but affects the critical temperature rather weakly. In the fitting procedure, the parameters $\chi, Q_0, R$ are fixed. Each set of data is matched separately by means of 3 parameters: $g_1, a, \kappa$. Evolution of these quantities with $\psi$ is illustrated in Fig. S-1, where the data are approximated by the equations

$$\log g_1 = g_1^0 + g_1^1 \psi, \quad a = a^0 + a^1 \psi, \quad \kappa = \kappa^0 + \kappa^1 \psi,$$  

(24)
where \( \log = \log_{10} \), and the coefficients \( g_1^k, a_k, \kappa_k \) \( (k = 0,1) \) are calculated by the least-squares technique. Fig. S-1 reveals that \( g_1 \) increases strongly with \( \psi \) (in accord with the scaling laws for polymer networks [62]), \( a \) remains practically constant (which reflects the fact that \( T_c \) is independent of concentration of cross-linker), and \( \kappa \) decreases linearly with \( \psi \) (due to changes in topology of the sub-network formed by entangled chains).

**P(AAm-AAc) copolymer microgels**

To assess the difference between the thermo-mechanical responses of IPN microgels and copolymer microgels with the same composition, we approximate observations (Owens et al. [64]) in Fig. 3. This figure demonstrates good agreement between the experimental data and results of simulation with the adjustable parameters listed in Tab. S-2.

![Figure 3](image-url)

> **Figure 3.**
>
> Figure 3: Relative swelling volume \( R \) versus temperature \( T \). Symbols: experimental data on PAAm/PAAc IPN microgels (open circles) and P(AAm-AAc) copolymer microgels (filled circles). Solid lines: results of simulation.

This table shows that changes in the network topology (random copolymerization of monomers instead of formation of the IPN structure) do not affect the coefficients \( \chi, Q_0, R \). Copolymerization of AAm and AAc monomers results in (i) an insignificant increase in \( g_1 \), (ii) a growth of the coefficient \( a \) (reflecting the shift of \( T_c \) to smaller values), and (iii) in a pronounced decay in \( \kappa \) (responsible for the slope of the swelling diagram above \( T_c \)).

Bouillot and Vincent [65] explain the difference between the responses of IPN and copolymer gels with reference to the “zipper effect,” according to which polymer chains are connected by ladder-like structures of hydrogen bonds whose dissociation progresses sequentially along the chains. According to this picture, the coefficient \( \kappa \) can be treated as a characteristic length of the ladder-like structures formed by hydrogen bonds: it is small when these structures are short (copolymer gels) and becomes large when these complexes are long (IPN gels).

**P(AAm-BMA)/PAAc IPN gels**

To examine how adjustable parameters in the model are affected by incorporation of hydrophobic monomers into the polymer network, we analyze experimental swelling diagrams (Katono et al. [34]) on macroscopic gels with interpenetrating networks formed by poly(acrylamide-co-butyl methacrylate) (P(AAm-BMA)) and PAAc chains. Composition of the gels and their preparation are described in Supplementary Material.
Experimental data on gels with various mass fractions of BMA monomers $\psi$ (ranging from 0 to 0.2) are presented in Fig. 4 together with results of simulation with the material constants collected in Tab. S-1 and reported in Fig. S-2.

Figure 4: Degree of swelling $Q$ versus temperature $T$. Circles: experimental data [34] on P(AAm-BMA)/PAAc IPN gels with various mass fractions $\psi$ of BMA monomers in P(AAm-BMA) network (A - $\psi = 0$, B - $\psi = 0.05$, C - $\psi = 0.10$, D - $\psi = 0.20$). Solid lines: results of simulation.

Tab. S-1 shows that $\chi, Q_0, R$ adopt similar values for PAAm/PAAc and P(AAm-BMA)/PAAc IPN gels (a slightly higher value of $\chi$ is used to account for the presence of hydrophobic BMA monomers). The effect of molar fraction of BMA monomers on parameters $g_1, a, \kappa$ is illustrated in Fig. S-2, where the data are approximated by Eq. (24). This figure shows that $g_1$ increases noticeably with $\psi$ (hydrophobic clusters formed by BMA monomers serve as extra cross-links between polymer chains). The coefficient $a$ decreases slightly with $\psi$ (this decrease reflects a shift of the critical temperature $T_c$ to higher values), while $\kappa$ decays pronouncedly with $\psi$ (thermally induced breakage of hydrogen bonds between PAAm and PAAc chains is decelerated due to the presence of hydrophobic monomers).

**P(AAc-CD)/PAAm IPN gels**

To evaluate how (i) preparation of IPN networks (PAAm network as the first and PAAc network as the second or vice versa) and (ii) incorporation of cyclic oligosaccharides grafted to polymer chains affect water uptake by UCST-type gels with hydrogen bonds, we analyze observations in equilibrium swelling tests on PAAc/PAAm and poly(acrylic acid-co-β-cyclodextrin)(PAAc-CD)/PAAm IPN gels (Wang et al. [66]). The chemical composition of the gels and the preparation procedure are provided in Supplementary Material.

Thermo-mechanical properties of cyclodextrin-based gels have recently attracted noticeable attention because these materials are biocompatible and non-cytotoxic [67, 68]. Due to the ability of cyclodextrin to form supramolecular bonds based on the host-guest inclusion complexation, these gels are widely used for controlled delivery of proteins and nucleic acids [69].

Experimental swelling diagrams on (PAAc-CD)/PAAm IPN gels with two molar fractions $\psi = 0$ and 0.1 (with respect to AAc monomers) of β-CD functionalized with maleic anhydride (MAH) are reported in Fig. 5 together with results of simulation with the material constants listed in Tab. S-3.

Figure 5.
This table reveals that grafting of MAH-functionalized CD to PAAc chains causes a slight increase in the elastic modulus $g_1$ and a weak decrease in the parameter $a$ (reflecting a slight increase in the critical temperature $T_c$). The growth of the elastic modulus may be attributed to formation of extra hydrogen bonds between chains and hydroxyl groups on the outer surface of CD molecules (treated as truncated cones), while the shift of $T_c$ may be explained by an increase in hydrophobicity of the network due to the presence of hydrophobic cavities in the truncated cones [70].

**PAAm/PAAc IPN gels**

To examine how ionization of functional groups affects the equilibrium degree of swelling of thermophilic gels with hydrogen bonds, we match experimental water uptake diagrams on PAAm/PAAc IPN gels with controlled degrees of ionization of carboxyl groups (Illmain et al. [28]). The chemical composition of the gels and the preparation procedure are given in Supplementary Material.

Experimental data on PAAm/PAAc gels with degrees of ionization $\alpha = 0, 0.03$ and 0.06 are depicted in Fig. 6 together with results of numerical analysis with the material constants listed in Tab. S-1 and presented in Fig. S-3.

Figs. 6A to 6C show that an increase in degree of ionization of functional groups induces a strong increase in the equilibrium degree of swelling $Q$ above the critical temperature $T_c$ (due to electrostatic repulsion of bound charges), a noticeable decrease in $T_c$ (associated with an increase in hydrophilicity of the network driven by ionization), and a pronounced growth of slopes of the equilibrium swelling curves in the vicinity of the critical temperature. These changes are reflected by the model as evolution of the coefficients $g_1, a, \kappa$ with $\alpha$. The influence of degree of ionization on these quantities is demonstrated in Fig. S-3, where the data are approximated by the equations

$$\log g_1 = g_1^0 + g_1^1 \alpha, \quad a = a^0 + a^1 \alpha, \quad \kappa = \kappa^0 + \kappa^1 \alpha$$

(25)
with the coefficients determined by the least-squares method. Fig. S-3 shows that $g_1$ decreases noticeably with $\alpha$ (the presence of NaOH in the solution under preparation of the gels results in a reduction in concentration of covalent cross-links), the coefficient $a$ decays with $\alpha$, and the parameter $\kappa$ increases strongly with degree of ionization of carboxyl groups.

**P(AAm-AAc) copolymer nanocomposite microgels**

To evaluate the influence of nanofiller on equilibrium water uptake by gels with hydrogen bonds, we fit swelling diagrams on P(AAm-AAc) copolymer microgels reinforced with various mass fractions $\psi$ of magnetite nanoparticles (Echeverria and Mijangos [71]). The chemical composition, details of preparation and the experimental procedure are described in Supplementary Material.

Experimental swelling diagrams on P(AAm-AAc) microgels with mass fractions of nanoparticles $\psi = 0, 0.05, 0.10$ and $0.15$ are depicted in Fig. 7 together with results of simulation with the material constants collected in Tab. S-1 and reported in Fig. S-4. In accord with Fig. 2, the value $Q(T_*) = 3$ is used in numerical analysis.

**Figure 7.**

Figure 7: Relative swelling volume $R$ versus temperature $T$. Circles: experimental data [71] on P(AAm-AAc) copolymer microgels with various mass fractions of magnetite nanoparticles $\psi$ (A - $\psi = 0$, B - $\psi = 0.05$, C - $\psi = 0.10$, D - $\psi = 0.15$). Solid lines: results of simulation.

Fig. 7 shows that an increase in concentration of nanofiller leads to a pronounced decay in the equilibrium degree of swelling at temperatures $T > T_c$, weakly affects $T_c$, and results in a strong decrease in slopes of the swelling curves in the vicinity of the critical temperature. These changes in shapes of the swelling curves are described in the model as evolution of the coefficients $g_1, a, \kappa$ with $\psi$. The influence of $\psi$ on these coefficients is illustrated in Fig. S-4, where the data are approximated by Eq. (24).

Fig. S-4 reveals that the elastic modulus $g_1$ grows with $\psi$ (due to reinforcement of gels with nanoparticles), $\alpha$ remains practically independent of $\psi$, whereas $\kappa$ is reduced noticeably with mass fraction of nanofiller. The decay in $\kappa$ in Fig. S-4 is similar to that in Fig. S-1 (driven by an increase in concentration of cross-links) and Fig. S-2 (induced by the presence of hydrophobic moieties in the network).

**Discussion**

Figs. 2 to 7 demonstrate good agreement between the results of numerical analysis and the experimental data in equilibrium swelling tests on thermophilic gels based in the acrylamide-acrylic
The entire set of adjustable parameters is split into two groups. The coefficients $\chi$, $Q_0$ and $R$ are independent of preparation conditions and additives and adopt similar values for all gels under consideration (Tab. S-1). The parameters $g_1$, $a$ and $\kappa$ are affected by micro-structure of gels (IPN versus random copolymer), incorporation of hydrophobic moieties and nanofiller, and degree of ionization of functional groups. The dimensionless shear modulus $g_1$ increases with concentration of permanent cross-links (due to the presence of extra bonds caused by formation of hydrophobic clusters or incorporation of nanofiller). The coefficient $\kappa$ (this quantity characterizes topology of the sub-network of chains bridged by physical bonds in Eq. (18)) decreases when concentration of chemical and physical cross-links between chains grows (Figs. S-1, S-2 and S-4) and increases when this concentration is reduced (Fig. S-3). The coefficient $a$ in Eq. (7) characterizes thermally-induced acceleration of the dissociation process for hydrogen bonds. When ionization of functional groups does not occur, this parameter changes insignificantly (Figs. S-1, S-2 and S-4), but it decays noticeably when degree of ionization $\alpha$ grows (Fig. S-3).

**Biocompatible thermophilic gels**

Although UCST-type gels based on the acrylamide-acrylic acid system demonstrate sharp changes in their volume and a strong increase in the equilibrium degree of swelling above the critical temperature $T_c$, their biomedical and biotechnological applications are restricted for two reasons: (i) a strong dependence of the thermomechanical response on pH and ionic strength of aqueous solutions (induced by the presence of AAc monomers in the feed), and (ii) high neurotoxicity and cytotoxicity of AAm monomers [72, 73]. This leads to an active search for novel thermophilic polymers that are free from the above shortcomings. As the number of observations in swelling tests on these gels is rather limited, we confine ourselves to the analysis of three systems in order to demonstrate the ability of the model to describe observations.

**P(AAm-AN) copolymer gels**

We begin with fitting experimental data in equilibrium water uptake tests on poly(acrylamide-acrylonitrile) (P(AAm-AN)) copolymer gels (Auge and Zhao [36]). Their chemical composition and the preparation procedure are described in Supplementary Material.

Experimental swelling diagrams on P(AAm-AN) gels with various molar fractions $\psi$ of BIS as a cross-linker (ranging from 0.0015 to 0.0150) are reported in Fig. 8 together with results of simulation with the material parameters collected in Tab. S-4 and presented in Fig. S-5.
Figure 8: Degree of swelling $Q$ versus temperature $T$. Circles: experimental data [36] on P(AAm-AN) copolymer gels with various molar fractions of cross-linker $\psi$ (A - $\psi = 0.0015$, B - $\psi = 0.0046$, C - $\psi = 0.0076$, D - $\psi = 0.0150$). Solid lines: results of simulation.

Figs. 8A to 8D show that an increase in concentration of the cross-linker results in (i) a strong shift of the critical temperature $T_c$ to higher values, and (ii) a slight increase in the slope of swelling curves above $T_c$. Unlike the data on PAAm/PAAc IPN gels (Fig. 2 shows that the slopes of swelling diagrams below $T_c$ are practically unaffected by $\psi$), Fig. 8 demonstrates their noticeable growth with molar fraction of cross-linker. This feature is reflected in the model by a pronounced (by three orders of magnitude) increase in the coefficient $R$ (Tab. S-4). The difference between the thermo-mechanical responses of these systems may be explained by the fact that in PAAm/PAAc gels, an increase in temperature leads to dissociation of hydrogen bonds, whereas in P(AAm-AN) gels, it induces breakage of hydrogen and van der Waals (dipole-dipole) bonds simultaneously [74].

Fig. S-5 shows that the increase in the elastic modulus $g_1$ with molar fraction of cross-linker $\psi$ is described adequately by Eq. (24). The coefficient $a$ decreases, whereas $\kappa$ increases with $\psi$, which differs from the results reported in Fig. S-1. Changes in $\kappa$ are determined by Eq. (24), while the reduction in $a$ (that reflects a pronounced increase in $T_c$ with $\psi$ observed in Fig. 7) is governed by the equation

$$a = a^0 + \frac{a^1}{\psi}. \quad (26)$$

**PNAGA macro- and micro-gels**

We proceed with fitting experimental swelling diagrams on poly(N-acryloyl-glycinamide) (PNAGA) gels. The preparation procedure for these gels and their compositions are given in Supplementary Material.

Experimental data (Liu et al. [3]) on PNAGA gels with molar fractions $\psi$ of BIS as a cross-linker (varied in the interval between 0.01 and 0.46) are presented in Fig. 9A to 9C together with results of simulation with the material parameters collected in Tab. S-5 and reported in Fig. S-6.

Figure 9: Degree of swelling $Q$ versus temperature $T$. Circles: experimental data on PNAGA gels with various molar fractions of cross-linker $\psi$ (A - $\psi = 0.010$, B - $\psi = 0.015$, C - $\psi = 0.460$ [3]) and PNAGA microgel (D - $\psi = 0.019$ [75]). Solid lines: results of simulation.
Fig. 9 shows that an increase in molar fraction of the cross-linker induces (i) a strong reduction in the equilibrium degree of swelling $Q$ and (ii) a shift of the critical temperature $T_c$ to higher values. Fig. S-6 demonstrates that these changes in the shape of swelling diagrams are induced by the growth of the elastic modulus $g_1$ (described by Eq. (24)) and the decay in the coefficient $a$ (determined by Eq. (26)). Unlike P(AAm-AN) copolymer gels (that reveal a pronounced increase in $\kappa$ with molar fraction of cross-linker), this parameter remains independent of $\psi$ in PNAGA gels.

For comparison, observations in equilibrium water uptake test are presented in Fig. 9D on PNAGA microgel (Yang et al. [75]). Numerical analysis is performed with $Q(T_*) = 1.45$ and the material constants reported in Tab. S-6. The same values of $\chi$ and $R$ are used in simulation for the macroscopic gels and the microgel. Comparison of adjustable parameters for macro- and microgels shows that the coefficients $Q_0$ and $\kappa$ adopt similar values, whereas $g_1$ and $a$ differ slightly. The latter may be explained by the difference in preparation conditions and the presence of surfactant in microgels.

**PAU copolymer microgel**

Finally, we approximate experimental data in equilibrium swelling tests on poly(allylurea-co-allylamine) (PAU) copolymer microgel (Shimada et al. [76]). Biocompatibility and non-cytotoxicity of PAU were verified in [77, 78].

To incorporate hydrogen bonds between chains (which induce positive thermo-responsiveness of the polymer), 86 % of amino groups of poly(allylamine) were transformed into ureido groups by chemical modification. Physical cross-links in the microgel were formed between pendant ureido groups (containing carbonyl groups as hydrogen bond acceptors) and non-ionized amino groups as hydrogen bond donors [79].

To ensure that amino groups remain unionized, experiments were conducted in a HEPES buffer with pH=7.5 and 150 mM of NaCl. Although poly(allylamine) is a weak cationic polymer with pKa ranging from 8.6 [80] to 8.7 [81], its apparent pKa remained below pH under the experimental conditions (based on a rough estimate that 1 mM of a monovalent salt in an aqueous solution induces reduction of pKa by 0.1 [80]). Details of preparation of the microgel are provided in Supplementary Material.

The experimental swelling diagram is plotted in Fig. 10 together with the results of simulation with $Q(T_*) = 2$ and the material constants reported in Tab. S-7.

**Figure 10.**

Figure 10: Relative swelling volume $R$ versus temperature $T$. Circles: experimental data [76] on AP-PAU microgel in HEPES buffer with pH=7.5 and 150 mM of NaCl. Solid line: results of simulation.

**Discussion**
Figs. 8 to 10 reveal an acceptable agreement between the observations in swelling tests and results on simulation (however, some scatter of the data in Figs. 8B and 9B is to be mentioned). This confirms the ability of the model to describe experimental data on biocompatible thermophilic gels.

Swelling diagrams on these gels are characterized by two features: (i) a noticeable increase in degree of swelling with temperature far below the critical temperature $T_c$ (it is reflected in the model by relatively large values (compared with AAm-AAc gels) of the coefficient $R$), and (ii) a rather weak growth of $Q$ above $T_c$ (this property is described in the model with low values of $\kappa$).

Unlike AAm-AAc copolymer gels whose coefficient $a$ is practically independent of molar fraction of cross-linker $\psi$ (Fig. S-1), these gels demonstrate a strong decay in $a$ with $\psi$ (Figs. S-5 and S-6). Keeping in mind the physical meaning of $a$ (a dimensionless activation energy for thermally-induced dissociation of hydrogen bonds), we conclude that breakage on hydrogen bonds between chains in biocompatible UCST-type gels is strongly affected by their architecture.

**Zwitterionic thermophilic gels**

A substantial increase in the equilibrium degree of swelling $Q$ of thermophilic gels with inter- and intra-chain hydrogen bonds is conventionally explained by the growth of intensity of thermal fluctuations with temperature $T$. Above the critical temperature $T_c$, this intensity exceeds the strength of hydrogen bonds (which is presumed to be independent of $T$). As a result, hydrogen bonds (that serve as physical cross-links between chains) disassociate, and $Q$ increases (due to a decay in the elastic energy of the network).

It seems natural to describe the thermo-mechanical response of zwitterionic gels (where chains are bridged by covalent cross-links and ionic pairs between anionic and cationic functional groups) by a similar scenario. When temperature remains sufficiently low (far below $T_c$) and the strength of ionic complexes exceeds the intensity of thermal fluctuations, these complexes are stable, and the equilibrium degree of swelling $Q$ remains practically independent of $T$. An increase in temperature induces a growth of intensity of thermal fluctuations, which causes dissociation of ionic pairs. When $T$ exceeds the critical temperature $T_c$ and ionic complexes (physical bonds between chains) are broken, the mechanical energy of the network is reduced. The balance between osmotic pressure, repulsion of ionized functional groups, and elastic forces in the polymer network leads to an increase in the equilibrium degree of swelling.

The validity of this scenario was first confirmed by Georgiev et al. [30] in tests on poly($N,N$-dimethyl(methacroylethyl) ammonium propanesulfonate) (PDMMAPS) and poly($N,N$-dimethyl(acrylamidopropyl) ammonium propanesulfonate) (PDMAAPS) gels. An increase in the equilibrium degree of swelling with temperature was recently reported in [82] for zwitterionic macroscopic gels and in [83, 84] for zwitterionic microgels.

It should be noted that the absence of hydrophobic segments in polymer chains (able to form clusters from which water molecules are expelled with an increase in temperature) provides the necessary condition for the above scenario. When hydrophobic moieties are incorporated into the
polymer network, zwitterionic gels demonstrate the LCST-type response [85] or the behavior typical of doubly thermo-responsive gels [86].

As water uptake by zwitterionic gels is strongly affected by pH and ionic strength of aqueous solutions, advanced constitutive models are required for its description. Our aim is to demonstrate the ability of the model to describe the influence of temperature on equilibrium swelling diagrams phenomenologically.

We analyze observations in water uptake tests on PDMAAPS gels with various mass fractions \( \psi \) of ethylene glycol dimethacrylate (EGDMA) as a cross-linker (Georgiev et al. [30]). Chemical composition of the gels and their preparation conditions are given in Supplementary Material.

Experimental swelling diagrams on PDMAAPS gels with \( \psi \) ranging from 0.005 to 0.125 are depicted in Fig. 11 together with results of simulation with the material constants collected in Tab. S-8 and reported in Fig. S-7.

Figure 11.

Each set of data in Fig. 11 is determined by two parameters only (the elastic modulus \( g_1 \) and the volume fraction of polymer network in the reference state \( \phi_{m0} = (1 + Q_0)^{-1} \)). The influence of \( \psi \) on these quantities is illustrated in Fig. S-7. The data are approximated by the linear equations

\[
g_1 = g_1^0 \psi, \quad \phi_{m0} = \phi_{m0}^0 + \phi_{m0}^0 \psi
\]

(27)

with the coefficients calculated by the least-squares technique.

To examine how adjustable parameters are affected by ionic strength, we fit equilibrium swelling diagrams on PDMAAPS gel with the lowest mass fraction of cross-linker \( \psi = 0.005 \) in aqueous solutions of NaCl with various molar fractions \( \theta \) belonging to the physiological interval from 0 to 0.17 M. Experimental data [30] are presented in Fig. 12 together with results of simulation with the material parameters reported in Tab. S-8 and Fig. S-8.

Figure 12.
Figure 12: Degree of swelling $Q$ versus temperature $T$. Circles: experimental data [30] on PDMAAPS gel with mass fraction of cross-linker $\psi = 0.005$ in aqueous solutions of NaCl with various molar fractions $\theta$ (A - $\theta = 0$, B - $\theta = 0.021$, C - $\theta = 0.171$ M). Solid lines: results of simulation.

Observations in Fig. 12 shows that an increase in molar fraction of salt $\theta$ results in (i) a pronounced increase in degree of swelling $Q$ below the critical temperature $T_c$, (ii) a slight shift of the critical temperature $T_c$ to higher values, and (iii) a strong decay in $Q$ above $T_c$. These changes are accounted for by means of three parameters ($\psi_{n0}, a, g_1$). Evolution of these quantities with $\theta$ is illustrated in Fig. S-8. We approximate the data by the equations

$$
\begin{align*}
    g_1 &= g_1^0 + g_1^1 \theta, \\
    \phi_{n0} &= \phi_{n0}^0 + \phi_{n0}^1 \theta, \\
    a &= a^0 + a^1 \theta
\end{align*}
$$

with the coefficients determined by the least-squares method.

Figs. 11 and 12 reveal good agreement between the experimental swelling diagrams and results of numerical analysis. Figs. S-7 and S-8 show that the adjustable parameters in the model change consistently with chemical composition (molar fraction of cross-linker $\psi$) and external conditions (molar fraction of NaCl in aqueous solutions $\theta$). This confirms the ability of the model to describe the effect of temperature on swelling of thermophilic zwitterionic gels.

A characteristic feature of zwitterionic gels under consideration (that distinguishes their response from that of UCST-type gels with hydrogen bonds) consists in pronounced changes in their reference state (determined by the coefficient $\phi_{n0}$) with concentration of cross-linker and molar fraction of salt. Although these changes (induced by electrostatic repulsion of bound charges) can be described within a more sophisticated model that accounts for interactions between mobile ions and ionized functional groups [87], we do not dwell on such an extension of the model as it leads to a substantial increase in the number of adjustable parameters.

CONCLUDING REMARKS

Unlike thermophobic (LCST-type) gels that swell at temperatures below their volume phase transition temperature $T_c$ and collapse above $T_c$, UCST-type gels demonstrate the inverse response: their equilibrium degree of swelling remains rather low below the critical temperature $T_c$ and grows pronouncedly above $T_c$. This behavior is conventionally associated with thermally-induced breakage (dissociation) of hydrogen bonds that serve as physical cross-links between chains.

A simple model is developed for the effect of temperature on equilibrium water uptake by TR gels with hydrogen bonds. Unlike traditional models, where the specific energy of interactions between water molecules and polymer segments is modified to account for the presence of hydrogen bonds, we use the standard formula for this energy, but modify the specific mechanical energy of the polymer network. This network is presumed to consist of two sub-networks (formed by covalently cross-linked chains and chains bridged by hydrogen bonds as physical cross-links). The number of
chains in the former network remains fixed, while the number of chains in the latter network decreases with temperature due to breakage of hydrogen bonds.

An advantage of the model (20) and (22) is that it involves only six adjustable parameters. Three of them (the FH parameter $\chi$ as a measure of hydrophobicity of chains, the dimensionless elastic modulus of the covalently cross-linked network $g_1$, and the volume fraction of the polymer network in the stress-free state $\phi_{m0}$) are the same as in conventional models for swelling of gels. The remaining three coefficients ($R$, $a$, $\kappa$) have transparent physical meaning: $R$ is the ratio of rates for dissociation and association of hydrogen bonds, $a$ is an analog of the activation energy for their breakage, and $\kappa$ stands for the characteristic length of ladder-like complexes formed by hydrogen bonds. Given an equilibrium swelling diagram on a thermophilic gel with hydrogen bonds, $R$ and $\kappa$ describe slopes of this curve below and above the critical temperature $T_c$, and $a$ characterizes position of the critical temperature on the graph.

To confirm the ability of the model to describe observations in equilibrium swelling tests, we fit experimental data on conventional systems (acrylamide-acrylic acid and acrylamide-acrylonitrile), as well as biocompatible poly($N$-acryloylglycinamide) and poly(allylurea-co-allylamine) gels. Figs. 2 to 10 demonstrate good agreement between results of simulation and experimental data on macroscopic gels and microgels. Figs. S-1 to S-6 show that the material parameters change consistently with molar fractions of cross-linker, hydrophobic and hydrophilic comonomers, degree of ionization of functional groups, and mass fraction of nanofiller.

Although the model is derived to predict the thermo-mechanical response of UCST-type gels with hydrogen bonds, we demonstrate that it can also be applied to describe observations in swelling tests on zwitterionic gels whose thermophilic behavior is induced by breakage of ionic pairs between anionic and cationic functional groups (Figs. 11 and 12). We treat this approach as merely phenomenological because an adequate account for electrostatic interactions between bound charges requires more sophisticated models.

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APPENDIX. SUPPLEMENTARY MATERIAL

REFERENCES


\begin{figure}
\centering
\includegraphics[width=\textwidth]{pi_6304_fig-09.eps}
\caption{Graphs showing the relationship between temperature ($T$) and a parameter ($Q$).}
\end{figure}
A model is developed for equilibrium swelling of UCST-type gels with hydrogen and ionic bonds. Its ability to describe swelling diagrams is confirmed by fitting observations on macro- and microgels.