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RESEARCH ARTICLE

Highlights in the Development of Tense-Logic

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Abstract. In 1954, on the 27th August, Arthur Norman Prior presented his ideas of tense-logic[1] for the first time. He developed the field further in many publications until his death in 1969. His books *Time and Modality* (1957a), *Past, Present and Future* (1967), and *Papers on Time and Tense* (1968) were clearly vital milestones. Much of Prior's personal motivation had to do with his struggle with the logical tension between the theological doctrines of divine foreknowledge and human freedom. It turned out that tense-logic gave rise to a powerful tool for dealing with this and similar problems. Furthermore, important highlights in Prior's tense-logic were the development of branching time and the introduction of instant propositions (leading to what has later been called 'hybrid logic'). Since Prior's death, many further developments of formal tense-logic and its semantics have been presented and carefully investigated. In philosophical logic, many researchers have focused on discussions regarding 'the true future' and the notion of 'the thin red line'.

Keywords: Tense-logic; A.N. Prior; time; modality; determinism.

[1] The hyphenated term 'tense-logic' and the nonhyphenated 'tense logic' are both used in the literature. In this paper the hyphenated term will be chosen as this is what Prior did in his important books, *Time and Modality* (1957a) and *Past, Present and Future* (1967).

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1. The beginning of tense-logic

Tense-logic is one of the most important contributions to logic during the 20th century. It offers a new formalism in which tenses (past, future, etc.) are represented as propositional operators (P, F, etc.). Tense-logic also reintroduced into modern logic the ancient idea that the truth value of a proposition can change from time to time.

The founding father of tense-logic is Arthur Norman Prior (1914–69). He presented his basic ideas of tense-logic for the very first time at a conference in Wellington, New Zealand, in 1954 (see [Prior 1958]), and within a few years, this new development of logic became known by logicians all over the world. However, it should be mentioned that there were other scholars who had played important roles in the work that led to tense-logic. One of them was Henrik von Wright (1916–2003), who had been a great inspiration to Prior. In fact, Prior explicitly mentioned von Wright's important work on the logic of futurity in his famous lecture in Wellington in 1954. Even more important was probably Prior's philosophical and theological interests in fundamental and existential questions regarding determinism and human freedom. As we shall see, his struggle with such problems during the 1930s and 1940s apparently led him to look for a new logic of time.

2. Prior's motivation for the struggle toward a new logical framework for the study of time

Much of Prior's personal motivation for working with problems regarding time had to do with his study of the logical tension between the Christian doctrines of divine foreknowledge and human freedom.

Prior's interest in the problem of determinism and its philosophical and theological aspects can be traced back to his early years. Already, as a teenager, he rejected the Methodism of his parents and became a Calvinist. Actually, in 1931, when he was only 17 years old, he wrote some rather detailed essays on problems related to determinism and time (see [Jakobsen et al. 2021]). He found Bergson's arguments against Einstein's space-time

unconvincing and, having accepted Einstein's ideas, he found that everything that has happened and everything that is going to happen must be accessible from God's perspective. He concluded that this means that there cannot be any human freedom of choice.

During the 1930s and 1940s, Prior carried out numerous Calvinistic studies, and he became a very active member of the Presbyterian Church, although he also had periods of doubt, particularly after 1940. He emphasized that there is a long tradition in theology of rejecting the doctrine of free will:

> ... a whole line of Christian thinkers, running from Augustine (to trace it back no further) through Luther and Calvin and Pascal to Barth and Brunner in our own day, have attacked freewill in the name of religion.
> … Jonathan Edwards, the 18th–century New England divine who produced a novel defence of Calvinism … simply demonstrating the absurdity of freewill itself… (Prior 2022a, 1)

During the 1940s Prior gradually changed his view on human freedom and finally he became a defender of free choice. Around 1950, when Prior worked as a senior lecturer at Canterbury University College, Christchurch, he was apparently looking for a logical framework that would be useful for the further studies of the relations between the doctrines of human freedom and divine foreknowledge. Prior accepted that there are future contingents, i.e., propositions about the future that are neither necessary nor impossible. But what can be said about the truth values of such propositions? Clearly, a believer in complete divine foreknowledge will have to say that God knows the truth values of all future contingents. Prior found that we need a precise formalism to explore the logical possibilities if we want to hold on to the doctrine of human freedom, along with the belief in God's complete foreknowledge. After intensely researching modal logic in the following few years, Prior realised that to analyse such problems, tenses would have to be taken seriously in logic; that is, we must include the relevant tenses in the formalism we are using to carry out the logical analysis.

In 1951, Prior became an elder of the Presbyterian Church in Christchurch. The same year, he also attended the Philosophical Congress in Sydney, Australia, where he made important friendships with J.L. Mackie and

J.J.C. Smart and other philosophers who were interested in topics related to logic and time.

In 1952, Prior was appointed as professor of philosophy at Canterbury University College, Christchurch. In 1953, he organised the first national conference on philosophy in New Zealand, and he became president of the *New Zealand Section of the Australasian Association of Psychology and Philosophy.*

In 1953, Prior also published a paper dealing with the problem of future contingency in terms of a three-valued logic (1953). For a few years, Prior thought that the use of a third truth value 'was the only way to present an indeterminist tense-logic' (1967, 128-29). However, later he was able to show that there are interesting alternatives (1957a, 94 ff.). As he had realised that a three-valued logic could give rise to complications, he found that it would be better to stick to a traditional, bivalent logic.

3. Toward a tense-logical formalism

In the beginning of June 1954, Prior was preparing his presidential address, which he was supposed to give at *The Second Philosophical Congress organized by New Zealand Section of the Australasian Association of Psychology and Philosophy*, Wellington 27–30 August 1954. On 6 June 1954, he wrote a letter to his wife, Mary, who was then in hospital. In this letter, Prior explained that he intended to present 'a long thing on "The Syntax of Time Distinctions", which is going to be a classic'. (Prior 2022b) It is obvious that he had very high expectations of this presentation of his ideas on time and logic. His hope was apparently that this would mark the beginning of a new approach to the study of logic and time. He even indicated something about the next steps in this work. He wrote, 'Later on I may work on interaction between tense-logic and deontic logic, but that's way up in the air at present.' (Prior 1954).

Prior had got important inspiration from reading a footnote by John Findlay, and he told Mary about it when he visited her (probably at the hospital):

> … he came and sat on the bed in high excitement. He read the all important footnote. He felt he could formalise tense distinctions, drawing inspiration from this footnote of Findlay's. (Interview with Mary Prior, included in [Prior 2003, 297])

The footnote in question is as follows:

> And our conventions with regard to tenses are so well worked out that we have practically the materials in them for a formal calculus... The calculus of tenses should have been included in the modern development of modal logics. It includes such obvious propositions as that
>
> x present = (x present) present
>
> x future = (x future) present = (x present) future;
>
> also such comparatively recondite propositions as that
>
> (x).(x past)future; i.e. all events, past and future will be past. (Findlay 1941, 233)

Early in 1954 Prior had studied Benson Mates book, *Stoic Logic* (1953). In particular, he was interested in the Master Argument which Mates presented in his book in the following manner:

> Diodorus argued that the following three propositions could not all be true.
>
> (1) Every proposition true about the past is necessary.
>
> (2) An impossible proposition may not follow from a possible one.
>
> (3) There is a proposition which is possible, but which neither is true nor will be true.
>
> Since, according to Epictetus, the first two propositions seemed to Diodorus to be more plausible than the third, he dropped the third, and this accounts for his definition of the possible as 'that which either is true or will be true'. (Mates 1953, 38)

According to Mates (1953), the details of the Diodorean Master Argument are not known. However, Prior was eager construct an argument by which it can be demonstrated that the denial of (3) follows from (1) and (2) in the Diodorean argument. In fact, he managed to do so using a formal language like the one suggested by Findlay. His proof was written in early 1954 and published in (1955). It shows that given (1) and (2) the possible is 'that

which either is true or will be true'. Prior referred to this Diodorean concept of possibility in his presidential address in August 1954 (1958a, 110), and in the following years he often returned to the Master Argument and similar arguments. As we shall see in section 5, he even discussed the Diodorean argument in his most important book, *Past, Present, and Future* (1967). There can be no doubt that what Prior learned from Mates was one of the important highlights in the development of tense-logic. In a letter to Mates dated 6 August 1954, Prior wrote: 'It goes without saying that I've enjoyed & profited by your book immensely' (2022e).

Prior admitted that the formalisation of the use of tenses may be a very complicated project. In his presidential address, he quoted C.S. Peirce, who in 1903, had stated:

> Time has usually been considered by logicians to be what is called 'extra-logical' matter. I have never shared this opinion. But I have thought that logic had not yet reached the state of development at which the introduction of temporal modifications of its forms would not result in great confusion; and I am much of that way of thinking yet. (Peirce 1931, 4.523)

However, Prior maintained that it would now be possible to carry out what Peirce hesitated to do in 1903:

> What the time was not ripe for in 1903, it may well be ripe for now, for in the intervening period, we have acquired a vast fund of knowledge about the possible structures of modal systems, and (as the scholastic logicians knew) tense and mood are species of the same genus. (Prior 1958a, 106)

Prior apparently found that one of the things that had made the formulation of tense-logic possible was a deeper understanding of what 'the scholastic logicians knew'. In fact, there is a strong emphasis on the importance of wisdom formulated in scholastic logic, mainly in the logic developed by William of Ockham (c. 1287–1347), who wrote a book that significantly inspired Prior (William of Ockham 1945). According to Ockham (and medieval logic in general) logic should include the study of propositions 'in the ancient and medieval sense' (Prior 1958a, 105 & 113), i.e., propositions that may change their truth-values relative to the time of uttering. Furthermore,

Ockham (1945) had demonstrated the importance of dealing not only with propositions in the present tense, but also with propositions in the past and the future tenses. With his tense-logic, Prior wanted to formulate this medieval wisdom in terms of modern symbolic logic.

Prior's presidential address in Wellington in 1954 marked the beginning of the worldwide development of tense-logic. The ideas presented in this lecture quickly became known among important logicians in several countries, and Prior himself was very active in the further development of the new field. In particular, it was of great importance that Prior was invited to deliver the John Locke Lectures for 1955–56 at the University of Oxford. These lectures led to the publication of the first book on the topic of time and modality (1957a). The publication of the Wellington lecture, the presidential address from 27 August 1954, had to wait until 1958, when it was published in *Franciscan Studies* (1958a). Given Prior's emphasis on Ockham's logic in the lecture, it was natural to submit it to this journal. After all, William of Ockham was a Franciscan friar.

4. Prior's approach to tense-logic in his presidential address, 27 August, 1954

In his famous Wellington lecture held in 1954 (1958a), Prior referred to Henrik von Wright's modal system, which may be understood as an extension of propositional logic with an operator, M (resp. 'It is possible that'), that obeys the following axioms:

B1: $p \supset Mp$

B2: $M(p \vee q) \equiv (Mp \vee Mq)$

and the rules

RB1: $\alpha \equiv \beta \rightarrow L\alpha \equiv M\beta$

RB2: $\alpha \rightarrow L\alpha$, where $L = {\sim}M{\sim}$

From this modal logic, the logic of futurity is obtained by excluding B1 and interpreting Mp as

F: 'it will be the case that …'

and assuming

F3: $FFp \equiv Fp$

Similarly, Prior obtained a logic of pastness from von Wright's modal system by interpreting Mp as

P: 'it has been the case that …'

and by assuming

P3: $PPp \equiv Pp$

Furthermore, Prior introduced two additional tense-logical operators:

G: 'it will always be the case that …'

H: 'it has always been the case that …'

defined as $G \equiv \sim F\sim$ and $H \equiv \sim P\sim$, respectively.

Finally, Prior established what he called the 'PF-calculus' by adding the following two axioms:

PF1: $p \supset GPp$

PF2: $p \supset HFp$

In his lecture, Prior demonstrated that the 'PF-calculus' is a rather powerful tool. As a nice example, he showed that the following is a theorem in the system:

$(p \vee Pp \vee Fp) \supset FPp$

Clearly, several other theorems can be proved in the system. In much of his later work with tense-logic, Prior concentrated on the exploration of what can be proved within the 'PF-calculus' and within other similar systems. However, in his presidential address in 1954, Prior emphasized that we may also discuss the logic of time in terms of another important formalism, namely, the so-called 'l-calculus', i.e., 'later than calculus' (1958a, 113). This alternative approach is based on the idea that time is a set of instants ordered by a before-after-relation, $(TIME, <)$.

For a modern reader, it may be surprising that Prior does not mention McTaggart's A- and B-series, which obviously correspond closely to the 'PF-calculus' and the 'l-calculus', respectively. However, as it appears from the preface of *Past, Present and Future,* Prior thought of McTaggart 'as an enemy' (1967, vi) until Peter Geach convinced him to revise his view.

In his presidential address in 1954, Prior pointed out that tenses may be introduced and further explored in terms of the 'l-calculus' using the following definition along with classical quantification theory:

$T(x,Fq) = \exists y\colon x<y \ \&\ T(y,q)$

$T(x,Pq) = \exists y\colon y<x \ \&\ T(y,q)$

This appears to suggest that tense-logic is just a by-product of the 'l-calculus'. However, in his lecture, Prior maintained that the metaphysics of time should in fact be conceived in the opposite manner:

> For 'now' is not the name of a date (it has the same meaning whenever it is used, but does not refer to the same date when it is used). In fact, the whole movement of events from the future through the present into the past is inexpressible in the l-calculus. If there is to be any 'interpretation' of our calculi in the metaphysical sense, it will probably need to be the other way round; that is, the l-calculus should be exhibited as a logical construction out of the PF-calculus rather than vice versa. (1958a, 116)

This view probably surprised the audience at the congress in Wellington, but the idea of the primacy of tense remained a cornerstone in Prior's philosophy of time until his death in 1969. As we shall see in Section 7, Prior introduced the so-called instant propositions to develop and support this idea.

In his lecture, Prior made it very clear that his formalism was designed to facilitate the exploration of some basic and classical problems within the metaphysics of time. In particular, the 'PF-calculus' should make it possible to study the problem of future contingency in a formal manner. Actually, the lecture marked the beginning of an extensive list of papers and book chapters dealing with the problems of (in)determinism, divine foreknowledge and human freedom in terms of Prior's 'PF-calculus' (i.e. his tense-logic).

At the end of the first day of the congress when Prior had given his important lecture on tense-logic, he wrote a letter to his wife explaining how it had been to present his important ideas at the congress:

> Darling, It's 2.15 a.m., & I'm at last in bed at the end of the 1st day of Congress, wh. has gone very pleasantly…… I put up my formulae on blackboard & started organising last night-&-this-morning's party; & then when the hour was due, delivered my piece. I felt very laboured in giving it, but was assured that it didn't look that way… (Prior 2022b)

Prior's main contribution to the study of time is his development of the 'PF-calculus' as a formalism from which the 'l-calculus' can be constructed. He had earlier, as a teenager, welcomed Bergson's intuitive ideas on time as a relevant response to determinism and the view of time as space. However, as a mature logician, he emphasized that much more is needed if we want to establish a proper and precise approach to the study of time and tense. In an undated note, he wrote:

> And I think it important that people who care for rigorism and formalism should not leave the basic flux and flow of things in the hands of existentialists and Bergsonians and others who love darkness rather than light, but we should enter this realm of life and time, not to destroy it, but to master it with our techniques. (Prior 2022c)

It should be noted that according to Prior, the concept of time as presented in terms of the 'PF-calculus' is very much like the understanding of time assumed in medieval logic (e.g. by William of Ockham), whereas the understanding of time as presented in terms of the '1-calculus' is very close to the idea used in medieval theology (e.g. by Thomas Aquinas). In his own words,

> Time, one might say, figures in the 1-calculus not as it does in medieval logic (which, as we have pointed out earlier, took tenses far more seriously than our own common logic does, and which already had such laws as our PF1), but rather as it does in medieval theology, in which God is said to behold all events in an unchanging present. (1958a, 117)

It appears that Prior's discussion of time according to medieval logic versus time according to medieval theology had given rise to some debate at the congress. In the letter to his wife, Prior wrote:

> There was a very pugnacious priest at the back who said that he was 'a Thomist & a strict Thomist', that this was the first exhibition he had seen of 'logistics', & that (this very aggressively & totally irrelevantly) he wanted to know if I was a 'realist'. I had a great deal of pleasure in telling him that I was far more of a realist than he was, & that he would in fact classify me as an 'extreme' realist. (2022b)

It should be mentioned that it is also evident from Prior's presidential address in 1954 that he knew his view on time may be seen as controversial, particularly by physicists and philosophers working with the key notions in Einstein's theories of relativity. It appears that Prior found that the burden of bothering with this discussion would be a necessary price to pay if we wanted to insist on proper freedom and indeterminism. At least there are interpretations of Einstein's special theory of relativity, which he wanted to question. He wrote:

> At least in many of its presentations, relativity theory seems to be as closely bound up with the 'spread-out-eternally' view of time underlying the l-calculus as medieval theology was. (1958a)

Prior clearly wanted to establish a formalism based on some fundamental philosophical assumptions on time and tense. In his undated note, *Some Free Thinking about Time*, he presented his basic beliefs in the following manner:

> …. what we see as a progress of events is a progress of events, a coming to pass of one thing after another, and not just a timeless tapestry with everything stuck there for good and all...
>
> This belief of mine... is bound up with a belief in real freedom. One of the big differences between the past and the future is that once something has become past, it is, as it were, out of our reach—once a thing has happened, nothing we can do can make it not to have happened. But the future is to some extent, even though it is only to a very small extent, something we can make for ourselves.... (Prior 2022d)

5. Further analysis of the Master Argument and similar arguments

As mentioned above Prior's study of the Master Argument of Diodorus played a very important role in his early development of tense-logic. Clearly, he found that the formal analysis of the Master Argument and similar arguments makes it possible to handle the struggle with the problems of determinism in a very precise and helpful manner.

Already in 1954, he wrote a paper suggesting a possible formalisation of the argument. This paper was published in (1955b). In (1958b) he published a new paper on the argument correcting a minor error in (1955b). In his very important book, *Past, Present and Future* (1967), Prior continued his work with the argument. Here he used a slightly different translation or paraphrase of the argument than the one found in (Mates 1953) claiming that the following three propositions cannot all be true (1967, 32):

D1. Every true proposition concerning the past is necessary.

D2. The impossible does not follow from the possible.

D3. Something that neither is nor will be is possible.

Obviously, the Master Argument was originally used as an argument in favour of determinism, i.e., given the validity of the trilemma, D3 must be rejected if D1 and D2 are accepted. This means that everything that neither is nor will be, turns out to be impossible. In other words, if something is the case and always will be, it is necessary (i.e., it could not have been otherwise).

If we let L stand for 'it is necessary that …' and M for 'it is possible that …' D1–2 becomes rather easy to represent in terms of Prior's tense-logical formalism:

D1. $Pq \supset {\sim}M{\sim}Pq$

D2. $L(p \supset q) \supset ({\sim}Mq \supset {\sim}Mp)$

If the argument is valid, it should be possible to demonstrate the denial of D3 based on the assumption of D1 and D2. The denial of D3 can be represented in the following manner:

D3'. $(\sim p \wedge \sim Fp) \supset \sim Mp$

However, Prior suggested that two additional assumptions are needed to establish a valid argument corresponding to Diodorus' ambition:

D4. $L(p \supset HFp)$

D5. $(\sim p \wedge \sim Fp) \supset P\sim Fp$

D4 means that if something is the case, it follows that it has always been that it would be going to be the case. D5 means that if something is false and always will be false, then it has already been the case that it would always be false. D4 and D5 may be assumed to be intuitively valid in a Diodorean context, although they are not explicitly mentioned as premises of the Master Argument. Furthermore, Prior was able to refer to recent historical research showing that D4 can be found 'in ancient' writers (1967, 33) and that D5 holds if time is discrete (1967, 49).

Prior proved D3' from D1, D2, D4 and D5 in the following way (Prior 1967, 33):

1. $(\sim p \wedge \sim Fp) \supset P\sim Fp$ (D5)
2. $P\sim Fp \supset \sim M\sim P\sim Fp$ (by D1 and substitution)
3. $(\sim p \wedge \sim Fp) \supset \sim M\sim P\sim Fp$ (by 1 and 2)
4. $L(p \supset \sim P\sim Fp) \supset (\sim M\sim P\sim Fp \supset \sim Mp)$ (by D2 and substitution)
5. $L(p \supset \sim P\sim Fp)$ (D4)
6. $\sim M\sim P\sim Fp \supset \sim Mp$ (by 4 and 5)
7. $(\sim p \wedge \sim Fp) \supset \sim Mp$ (by 3 and 6)

Consequently, at least one of the premises (D1, D2, D4 and D5) must be rejected to avoid the deterministic or even fatalistic conclusion, i.e., D3' (stated in 7).

In his study of the logical problems concerning determinism Prior also considered a similar argument formulated in terms of metrical tense operators $P(n)$, i.e., "it was the case n time units ago that", and $F(n)$, i.e., "it is going to be the case in n time units that" (Prior 1967, 119):

a. $P(m)p \supset LP(m)p$ (assumption)

b. $P(m)F(m+n)p \supset LP(m)F(m+n)p$ (by a and substitution)

c. $F(n)p \supset P(m)F(m+n)p$ (by e and f)

d. $F(n)p \supset LP(m)F(m+n)p$ (by b and c)

e. $L(p \supset q) \supset (Lp \supset Lq)$ (assumption)

f. $L(P(m)F(m+n)p \supset F(n)p)$ (assumption)

g. $LP(m)F(m+n)p \supset LF(n)p$ (by e and f)

h. $F(n)p \supset LF(n)p$ (by a and g)

There are obvious similarities between this argument and the Master Argument of Diodorus. The assumptions (a) and (c) are clearly very close the premisses, D1 and D4, respectively. Furthermore, the assumption (e) is basically the same premiss as D2. In his chapter on 'Time and determinism' (1967,113 f.) Prior also discussed other versions and aspects of the Master Argument. As we shall see, all this led him to the presentation of two possible responses to the attack on the doctrine of free choice to which the Diodorean argumentation may give rise.

6. Branching time

On 3 September 1958. Saul Kripke, who was then only 17 years old, wrote a letter to Prior. Kripke had read *Time and Modality*, 'with considerable interest' (Ploug and Øhrstrøm 2012). Among other things, Kripke wanted to comment on Prior's claims regarding the modal logic we obtain from a tense-logic if we take Mp to stand for 'p is or will be the case'. In his book, Prior maintained that S4 is the modal logic that in this way comes out of his Diodorean tense-logic. Kripke demonstrated that this is wrong, and he pointed out that Prior's error has to do with an assumption regarding the notion of time assumed in his reasoning.

In his letter, Kripke argued that if we want a tense-logic corresponding to S4, a linear concept of time will be insufficient, and we should in fact base the analysis on a more complex notion of time. He wrote:

> Now, in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like—and for each possible next moment, there are several possibilities for the next moment after that. Thus, the situation takes the form, not of a linear sequence, but of a 'tree'… (see [Ploug and Øhrstrøm 2012, 374])

According to Kripke, the temporal structure is backward linear and forward branching. In his letter, he illustrated this idea in the following manner:

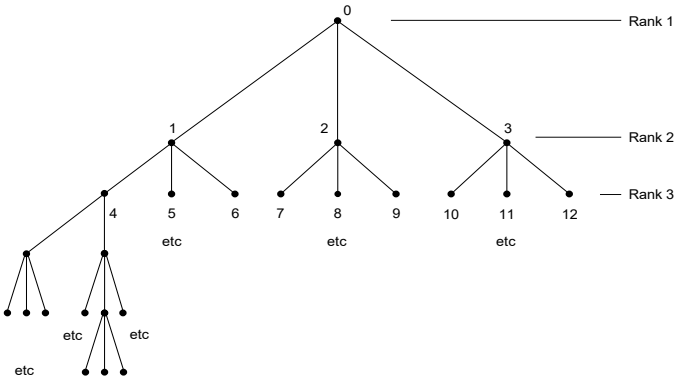

Fig. 1

Prior almost immediately accepted Kripke's idea of branching time. It is, in fact, likely that notions of this kind were well known to him. In fact, he might have known that Henri Bergson (1859–1941), in his book *Time and Free Will* (1910), had suggested a similar tree-like structure in his discussion of time and human decisions. It can even be argued that Prior already in 1957 had worked with a notion like branching time when he wrote his paper 'Opposite Number' (1957b), see (Øhrstrøm and González 2022). However, none of these earlier considerations included an account of the kind of branching time semantics suggested in Kripke's letter.

In the following years, Prior further developed the use of branching time models to give a precise account of the semantics of the tense-logical systems

he had in mind. In particular, this was important in cases in which he was unable to present the axioms of the systems.

Graphically, Prior turned Kripke's branching time diagram (Fig. 1) 90 degrees to have the future to the right and the past to the left.

One of the systems he found fascinating was the system inspired by the work of William of Ockham. He even added new elements to the branching time diagrams. First, he wanted to allow reference to time metrics according to a specific time unit (e.g. days) in the diagrams. In a draft, *Postulate Sets for Tense Logic*, written and circulated in 1965 or earlier, he also suggested a reference to 'a single designated line' in the diagram:

> In these models, the course of time (in a rather broad sense of this phrase) is represented by a line which, as it moves from left to right (past to future), continually divides into branches, so that from any given point on the diagram there is a unique route backwards (to the left; to the past) but a variety of routes forwards (to the right; to the future). In each model, there is a single designated point, representing the actual present moment; and in an Occamist model, there is a single designated line (taking one only of the possible forward routes at each fork), which might be picked out in red, representing the actual course of events. (Prior 2022f)[2]

Prior further developed this approach in a paper published the following year (Prior 1966). In this paper, Prior stated that in each Ockhamist[3] model, 'there is a single designated route from left to right, taking one direction only at each fork. This represents the actual course of events' (1966, 157). The idea is illustrated in Fig. 2, which x, y, z, t are moments, and -x-y-t- and -x-y-z- are routes (sometimes called chronicles). As indicated -x-y-t- is the designated line (chronicle). This means that z represents a possible moment at y different from the chosen one. According

[2] It appears that that Prior had an idea very close to the notion of "the thin red line" that Belnap and Green (1994) independently (re)invented and criticized almost three decades later. - I owe this observation to Alex Malpass. See (Malpass 2011).

[3] It should be noted that Prior when preparing *Past, Present and Future* changed his spelling of the name of the famous medieval logician from 'Occam' to 'Ockham'.

to the system, propositions should primarily be evaluated relative to moments belonging to the designated line, and the other lines (routes, chronicles) are used to account for statements involving modal operators. According to the Ockhamistic logic a proposition, p, is necessary if and only if 'it is beyond our power to make p false' (Prior 1966, 157), i.e., if and only if it is 'now-unpreventably' that it is true (Prior 1967, 117).

Given these ideas, Prior was able to present a formal account of the Ockhamist answer to the challenge of the Master Argument of Diodorus (conceived in the manner presented above). The Ockhamistic response consists in the rejection of the general validity of D1 (here understood as 'a' in the argument mentioned at the end of section 5). The point is that from an Ockhamistic point of view, D1 does not hold for statements formulated in the past tense about the future. To demonstrate what this means, we may consider diagram in Fig. 2, in which is obvious that $P(n)F(n+m)p$ is true at y, whereas $LP(n)F(n+m)p$ is false at y, since it was possible n time units ago that $\sim p$ would be the case *n+m time units* later, namely at z. This clearly means that D1 does not hold at y. Consequently, it is also evident that the deterministic conclusion of the Master Argument can be avoided given an Ockhamistic system.

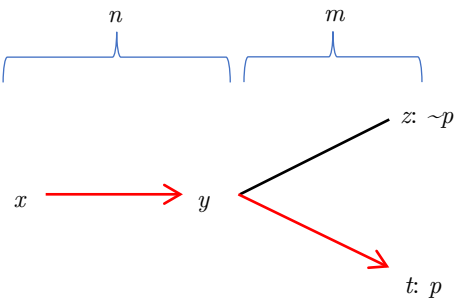

Fig. 2

For some reason, Prior decided to present a different formalisation of the Ockhamistic approach in his *Past, Present and Future* (1967). Here, there is no mention of a designated line corresponding to the actual course of events. Instead, a truth value is understood relative to a pair of a route in

the diagram and a moment belonging to the route. In fact, it turns out that we, in this way, will obtain the same theorems as according to the 1966 approach. However, the philosophical aspects of the notion of truth presented in the book clearly differ from the understanding of truth presented in 1965/66. In fact, we may speak of two different formalisations of Ockhamism: Ockhamism-1966 and Ockhamism-1967. The difference is that the former contains a reference to a designated line representing the actual course of event, whereas the latter does not contain any such reference. Historically, Ockhamism-1966 seems to be a much fairer representation of Ockham's original ideas than Ockhamism-1967. At least, it is obvious that William of Ockham held that God truly foreknows what is going to happen in the contingent future. A claim of this kind cannot even be made in terms of Ockhamism-1967.

D1 will turn out to be invalid, regardless of whether we accept Ockhamism-1966 or Ockhamism-1967. In Prior's opinion, the rejection of D1 was very problematic. He found that if something was true, we must accept this as a necessary (i.e., now-unpreventable) fact. As Prior wanted to hold on to D1 and as he wanted to avoid the fatalistic consequences of rejecting D3, he had to deny one of the premises D4–5. To solve the problem, Prior introduced a tense-logic, the so-called Peirce system, which differs from the Ockhamistic-1967 system in the use of a future operator that corresponds to the Ockhamistic *LF*. It is obvious from a diagram like Fig. 2 that this means that D4 must be rejected. It is also clear that the Peircean understanding of the future would leave no room for the idea of a designated route corresponding to the actual course of events.

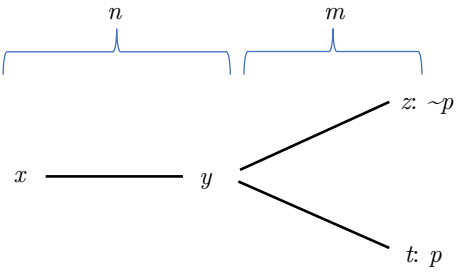

Fig. 3

From a Peircean point of view, a future tense proposition will only be true at a moment in a branching time diagram if it is true no matter which future branch we consider (Prior 1967, 128 ff.). Relative to Fig. 3 above, this means that $F(m)p$ and $F(m)\sim p$ are both false at y, whereas $\sim F(m)p$ and $\sim F(m)\sim p$ are both true at y. It has often been pointed out that it may appear counterintuitive to distinguish between $\sim F(m)\sim p$ and $F(m)p$.

Furthermore, it has been criticised that the Peirce system identifies 'it is going to be the case that …' and 'it is necessarily going to be the case that…'. Obviously, this approach seems to ignore important distinctions in natural language. On the other hand, it has been argued that if something is true about tomorrow, there must be something already now to make it true; therefore, what is going to be true must depend on some present truth. Still, it is not easy to precisely explain what a truth maker is. In fact, the discussion about truth makers can quickly become rather complicated; see e.g. (Craig 2001), (Merricks 2007) and (Tulenheimo 2020).

7. The understanding of the instants of branching time structures

Having worked with branching time structures for some years Prior wanted to give a precise account of the conceptual and ontological status of such structures and their components. How should the instants (moments) and chronicles (lines) in the branching time diagrams be understood? According to Prior, the instants and the chronicles in the diagrams should not be conceived of as objectively existing. They are nothing more than helpful constructions. In his *Past, Present and Future* (1967) and even more in his *Papers on Time and Tense* (1968), he explained how these constructions are carried out. In (1967, 79 ff. &187 ff.) he discussed the so-called world-state propositions, and in (1968, 122 ff.) Prior gave a very important account of his idea of seeing instants as a specific class of propositions. This work gave rise to an important new development of tense-logic. After Prior's death this work has been continued in the development of so-called hybrid logic, which has now grown into an important discipline that has become useful in computer science.

Prior's basic idea is that, in branching time logic, we should in fact operate with two different kinds of propositions (1968, 122 ff.). In addition to the usual tense-logical propositions formed based on atomic constants using various tense-logical operations, there is a special class of so-called instant propositions, a, b, c …., with some extremely remarkable properties.

These very special properties of the instant propositions can be presented in terms of the following three axioms where a is an arbitrary instant proposition and where p is an arbitrary proposition in the logic:

(I1) $\exists a\colon a$

(I2) $\sim L \sim a$

(I3) $L(a \supset p) \vee L(a \supset \sim p)$

It is obvious from I1 that we must extend formal language with a quantification theory that allows propositional quantification over instant propositions.

The intuitive meaning of I1–3 is rather clear. I1 simply states that there is an instant proposition that is true (right now). Actually, we might call this instant proposition *Now*. I2 states that they are all possible instants and that may be conceived as past, future or even counterfactual. I3 means that for any instant proposition, a, and any tense-logical proposition, p, either p or $\sim p$ follows necessarily from a. Intuitively, we may think of $L(a \supset p)$ as the claim that 'p is true at a'. If we substitute a with *Now* in I3, the obvious reading becomes that any tense-logical proposition, p, will be either true or false at the present moment.

It is obvious that the use of instant propositions adds significantly to the expressibility of formal language. It is also clear that the instant propositions have some very remarkable properties. In fact, it turns out that everything in the whole branching time system will follow logically from the very rich information hidden in just one instant proposition. This means that in a certain sense, the Now includes everything that has been, will be, could be true, or could have been.

8. The thin red line

Since Prior's death in 1969, many tense-logicians have discussed the notion of the future within branching time semantics. One of the first philosophers to do so was Robert P. McArthur (1974). Later, others like J.R. Lucas (1989) made important contributions to the understanding of the so-called actual future. In a sense, this was a discussion very close to Prior's early studies on Ockhamism, in which he had considered the notion of a designated line corresponding to the actual course of events. However, in a very influential paper, Nuel Belnap and Michael Green (1994) criticised the idea of what they called 'the thin red line'.

Belnap and Green (1994, 379) pointed out that, in a branching time diagram, it will not work just to assume that there is a single designated line representing the actual course of events. If we can speak of a true future at some moments, it should be possible from all moments in the diagram. All moments should be treated in the same way. In consequence, if there are thin red lines from some of the moments in the diagram, there must be a thin red line from any moment in the diagram. Belnap and Green introduced a formal solution in the following manner:

> Technically, we change *TRL* from a simple name of a history to a function, $TRL(m)$, which picks out a unique Thin Red Line for each moment, m. (Belnap and Green 1994, 380)

It seems that Belnap and Green have a powerful case here. They have also argued that a notion like the suggested *TRL* function will be inconsistent with branching time semantics. However, as we shall see this part of their argument is rather problematic.

At an arbitrary moment, m, in the diagram $TRL(m)$ will be the line including the past, present and future relative to m. Belnap and Green (1994, 380) have pointed out that m therefore will have to belong to $TRL(m)$, i.e.

(TRL1) $\quad m \in TRL(m)$

Furthermore, Belnap and Green argued that, if we want, some intuitively reasonable theorems such as $PPq \supset Pq$ and $FFq \supset Fq$ are valid; we must

make sure that *TRL* has the property of a certain kind of stability, which can be formulated in the following manner:

(TRL2) $m_1 < m_2 \supset TRL(m_1) = TRL(m_2)$

The idea is apparently that if m_2 is a moment later than m_1 then the future of m_2 must also be the future of m_1. However, if this requirement is accepted, it is easy to see that a diagram such as Fig. 3 will collapse into a linear structure. The reason is that the use of TRL2 on the combination of $y < z$ and $y < t$ will imply that $TRL(y) = TRL(z) = TRL(t)$, which means that y, z and t will all belong to the same line.

However, a defender of the idea of the thin red line cannot accept TRL2. Instead, we will have to do with the following weaker requirement:

(TRL2') $(m_1 < m_2 \wedge m_2 \in TRL(m_1)) \supset TRL(m_1) = TRL(m_2)$

This condition had in fact much earlier been suggested by Thomason and Gupta (1980). It turns out that TRL2' is enough to ensure the validity of $PPq \supset Pq$ and $FFq \supset Fq$. When this was communicated in 1996 to Nuel Belnap, the authors revised the claim they had made in (Belnap and Green 1994):

> I think you are quite right in bringing forth (2') in place of (2). This is not something that we had thought of and counts as a definite oversight on our part. Its consideration much improves the level of discussion (Personal e-mail from Nuel Belnap, 1 August 1996).

In their very influential book *Facing the Future* (2001), Belnap et al. took TRL2' into account, accepting that the TRL approach is logically possible, although they, for philosophical reasons, still found it problematic. One major formal criticism of the approach was that a semantics based on branching time diagrams with TRL functions does not include $q \supset HFq$ as a valid theorem. To verify that this is so, we consider the diagram shown in Fig. 4, assuming that q is true at j and nowhere else in the diagram. Since the assumptions mean that Fq must be false at i, it obviously follows that HFq is false at j and that the same holds for $q \supset HFq$. Consequently, $q \supset HFq$ cannot be a valid theorem if the diagram is accepted semantically.

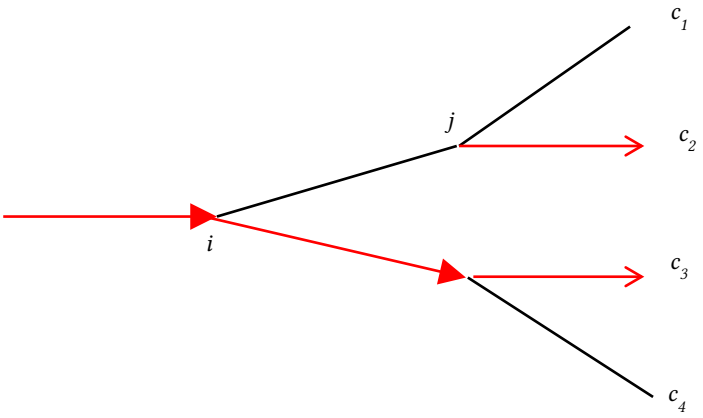

Fig. 4. The proposition q is supposed to be true at j, but nowhere else in the diagram.

It should be remembered that $q \supset HFq$ is a rather well-known and much discussed statement (see PF2 in section 4 above). As we have seen, it also played an important role in Prior's reconstruction of the Master Argument. Prior himself rejected it as invalid in general. So maybe we do not need it as a theorem. On the other hand, it is intuitively attractive, since most people will hold that if something is the case now, then it has always been going to be the case. For this reason, it may be reasonable to look for modifications to the TRL semantics according to which it is valid. One such attempt has recently been discussed by several authors (see [Øhrstrøm and Hasle 2020]). It is based on the definition of a revised TRL function for any arbitrary moment j in the branching time diagram. This function is based on the general TRL function and is defined in the following manner:

$$i<j \supset TRL_j(i) = TRL(j)$$

otherwise $TRL_j(i) = TRL(i)$

This means that $TRL_j(i)$ only differs from $TRL(i)$ for moments i that do not belong to the past of j. For moments belonging to the past of j, TRL and TRL_j will give us the same line (chronicle) in the diagram.

Furthermore, we introduce the notion of duration. Because the branching time system is backward linear, we can assume that there is a function, *back*, from TIME × N to TIME, where N is the set of natural numbers, such that *back*(*i*,*n*) is the unique instant *n* time units earlier than *i*.

Similarly, there is a function, *forward*, from TIME × TIME × N to TIME, such that *forward*(*i*,*j*,*n*) is the unique instant, *i'*, such that *back*(*i'*,*n*) = *i* and $i' \in TRL_j(i)$.

The idea is then to evaluate the tense-logical propositions relative to the moment of reference, *i*, as well as the moment we are giving priority in the actual context, *j*. Intuitively, we may think of *j* as the 'time of utterance'.

We define the truth value of a tense-logical proposition *p* at the instant *i* giving temporal priority to the instant *j*, *val*(*i*,*j*,*p*), recursively in the following way:

val(*i*,*j*,*p*)=1 iff *p* is a propositional letter assigned with the truth value 1 at the instant *i*, no matter to which moment we give temporal priority.

val(*i*,*j*,*P(n)p*)=1 iff *val*(*back*(*i*,*n*),*j*,*p*)=1.

val(*i*,*j*,*F(n)p*)=1 iff *val*(*forward*(*i*,*j*,*n*),*j*,*p*)=1.

(Negation and propositional connectives are treated in the usual manner.)

Furthermore, it is assumed that a tense-logical proposition is valid if and only if it is true at an arbitrary instant, *i'*, calculated by giving temporal priority to the same instant, *i'*. For instance, if we want to determine whether the proposition $q \supset P(n)F(n)q$ is true, we have to evaluate $val(i',i',q \supset P(n)F(n)q)$ at any moment *i'*. Let us again use *Fig. 4* as an illustration, assuming that *q* is true at *j* and that *i* is *n* time units before *j*. Clearly, $val(j,j,P(n)F(n)q) = 1$ if and only if $val(i,j,F(n)q) = 1$. To calculate the truth value of *F(n)q* at *i* giving priority to *j*, the definition means that we have to use TRL_j in the evaluation, which implies that *q* has to be evaluated along *TRL*(*j*), i.e., c_2. It follows that $val(j,j,q) = 1$ if and only if $val(j,j,P(n)F(n)q) = 1$. Consequently, it is easy to see that $q \equiv P(n)F(n)q$, and using some basic quantification theory, it is easy to verify that $q \supset HFq$ is a theorem in the system.

It is interesting that the logic of the TRL approach, and in particular the just-mentioned version of it, appears to come very close to the logic of future contingency suggested by Luis de Molina (1535–1600), who wanted to show that the doctrines of divine foreknowledge and human freedom of choice do not contradict each other (see [Øhrstrøm and Hasle 2020]).

9. Conclusion

The modern study of tense-logic and its applications includes several aspects other than those mentioned above. However, the list of topics discussed in this paper probably suffices to demonstrate that tense-logic is a very rich field. During the seven decades since Prior's first studies in the area, several ideas and theories have been developed and even more interesting questions on time and modality have been asked. Many questions are still open—both regarding the formal properties of the systems and concerning the conceptual, philosophical and sometimes even metaphysical aspects of tense-logic. What makes Prior's tense-logic so great and remarkable is that his paradigm reaches far beyond his own findings during the 15 years he got to lay the foundation of the field.

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