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Published in:
I E E E Antennas and Wireless Propagation Letters

DOI (link to publication from Publisher):
10.1109/LAWP.2022.3174246

Publication date:
2022

Document Version
Accepted author manuscript, peer reviewed version

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LAWP.2022.3174246, IEEE Antennas and Wireless Propagation Letters

A Fast Multi-Beam Measurement Method for Millimeter-Wave Phased Arrays

Huaqiang Gao, Fengchun Zhang, Gert Frølund Pedersen, and Wei Fan

***Abstract*—Accurate knowledge of multi-beam radiation patterns of millimeter-wave (mmWave) phased arrays is key to mmWave radio system performance. The multi-beam radiation patterns can be measured by conventional antenna test ranges in a sequential manner. However, these tasks can be time-consuming and low-efficient in practice, especially for large-scale arrays with many beam-steering states. To address this problem, this article proposes a multi-beam reconstruction method, which can measure all possible steered beams in a fast and efficient manner. In the proposed method, multiple beam patterns are reconstructed synchronously by a fast element pattern retrieving method based on a few sparse measurement samples, which were obtained in a multi-probe setup or with the help of a turntable in a single probe setup. The proposed method is experimentally validated in a compact antenna testing range (CATR) setup at 28 GHz for a 4×4 mmWave phased array with an integrated antenna-in-package (AiP) design. The experimental results show that the reconstructed array multi-beam patterns by the proposed method are comparable to the conventional CATR measurement results but in a faster way, which validates the effectiveness of the proposed method.**

***Index Terms*—Antenna-in-package (AiP), antenna measurement, beam-steering, millimeter-wave (mmWave) phased array, radiation pattern.**

I. Introduction

MILLIMETER-WAVE (mmWave) phased arrays have found increasing applications in 5G communications [1]–[3]. Antenna-in-package (AiP) technology [4]–[6] has been widely adopted in mmWave phased array designs that integrate antennas and radio frequency (RF) chips into a single package with no accessible antenna connectors, which necessitates the testing of mmWave phased array AiP conducted over the air [7]. The testing requires accurate characterization of phased array performance, e.g. the antenna pattern that is key to radio system performance. The antenna radiation pattern measurements with different test ranges are well investigated in the literature, e.g. direct far-field [8], compact antenna testing range [9], plane wave generator [10], near-field [11], and mid-field methods [12].

However, some key challenges exist with the conventional methods for the radiation pattern measurement of mmWave phased arrays. The main challenge is introduced by the beam-steering capability of mmWave phased arrays. In order to test the beam-steering capability, the radiation patterns steered to multiple directions are required to be measured. The conventional methods measure the beams sequentially with a beam-lock function until all available beams are measured, e.g. 128 beams for base station (BS) applications [13]. In each beam measurement, the mechanical rotation is conducted to record the radiation pattern, which is repetitive and very time-consuming for multi-beam measurement. Besides, the antenna array pattern becomes more directive with larger array configuration, e.g. angle resolution of 1° per azimuth and elevation for an 8×8 array (even smaller for larger phased arrays), which will also increase the measurement time of each beam. The mechanical rotation for multiple beams is often very time-consuming. To achieve faster measurements, it is desired that the number of mechanical rotation movement can be significantly reduced, if not eliminated, for the multi-beam measurements.

In this article, a fast multi-beam radiation pattern measurement method is proposed for mmWave beam-steerable phased arrays. The objective is to achieve the same array radiation pattern measurement of multiple beams as the conventional methods while largely reducing the time cost. In Section II, we explain the signal model for phased array multi-beam radiation patterns and introduce the basic principle of the proposed fast multi-beam measurement method. The proposed method is experimentally validated by measurement results in Section III. Finally, Section IV concludes the article.

II. Method

A. Signal Model

The array far-field radiation pattern $s^{(b)}$ of the b-th beam can be expressed by a linear superposition of weighted array element patterns as

$$s^{(b)}(\Omega) = \sum_{n=1}^{N} w_n^{(b)} g_n(\Omega), \tag{1}$$

where $b \in [1, B]$ denotes the beam index and B is the number of beams. $\Omega = \{\theta, \phi\}$ is the far-field observation direction with the elevation angle θ and azimuth angle ϕ, respectively. g_n represents the far-field complex pattern of the n-th element in an array ($n \in [1, N]$) and N is the number of array elements. In a practical mmWave phased array with an integrated AiP design where there are no accessible element ports, the element pattern g_n is integrated into the n-th element branch in an array and thus contains the whole element branch response. $w_n^{(b)}$ denotes the beam-steering weight of the n-th

This work was supported by Huawei Technologies and InnoExplorer project funded by Innovation Fund Denmark. (Corresponding author: Wei Fan.)

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element for the b-th beam, which is implemented by phase shifters in practical phased arrays.

In the practical testing of phased arrays without the knowledge of $w_n^{(b)}$ and g_n in (1), the conventional methods directly measure the array pattern $s^{(b)}$ of the b-th beam and repeat the whole pattern measurement operation B times for B beams. Assuming the number of mechanical rotation movement K_1 in the array pattern measurement of each beam, the total number of mechanical rotation movement is $M_1 = B \times K_1$ with M_1 signal measurements. The conventional method requires large number of mechanical rotation movement for multi-beam measurements, which is very time-consuming.

The main problem of conventional methods comes from the unknown beam-steering weights and element complex patterns in the practical testing of integrated mmWave phased arrays. In conventional sub-6 GHz phased arrays with accessible element ports, the element patterns can be obtained sequentially by activating one element each time and terminating the other $N-1$ elements (i.e. in the on-off mode). However, the drawbacks of the on-off mode are two-fold for mmWave phased array AiP: 1) the on-off and all-on modes might be different in terms of element coupling and device temperature as discussed in [14], i.e. results in the on-off mode might be inaccurate for the normal working mode with all elements activated; 2) the on-off mode is not supported in some scenarios and devices [15]. Therefore, it would be desirable to obtain the element patterns in the normal all-on working mode with all N elements activated.

B. Proposed Solution

In this work, we propose a fast multi-beam measurement method for mmWave phased arrays. According to (1), the multi-beam patterns can be reconstructed once we know the complex patterns and the beam-steering weights of N array elements. The proposed method first determines both the beam-steering weights and the complex patterns of elements by measurement, then reconstructs multiple array patterns simultaneously by post-processing.

1) Determination of Beam-Steering Weights: The beam-steering weights are typically unknown for the practical testing engineers, while the desired beam-steering angles of beams are available. The beam-steering weight $w_n^{(b)}$ of the n-th element can be determined by the b-th beam as

$$w_n^{(b)} = e^{-j\frac{2\pi}{\lambda}\overrightarrow{r_n}\cdot\overrightarrow{\Omega_b'}}, \tag{2}$$

where $\Omega_b' = \{\theta_b', \phi_b'\}$ is the desired beam-steering angle of the b-th beam with elevation steering angle θ_b' and azimuth steering angle ϕ_b', respectively. $\overrightarrow{\Omega_b'} = (\cos\theta_b'\cos\phi_b', \cos\theta_b'\sin\phi_b', \sin\theta_b')$ is a unit vector corresponding to the desired beam-steering angle Ω_b'. $\overrightarrow{r_n}$ is the position vector of the n-th element and is a function of the array element spacing [14]. λ is the wavelength.

The element spacing of phased arrays under test might be also unknown in practice, which is estimated by measurements in the proposed method. A phased array with a rectangular configuration can be regarded as a uniform linear arrays (ULA) in horizontal or vertical direction. The element spacing d of a ULA is estimated by [16]

$$d = \frac{\lambda}{L\cos(\frac{\pi}{2} - \frac{\Psi}{2})}, \tag{3}$$

where L is the number of ULA elements. Ψ is the first-null beamwidth of the ULA pattern measured under the beam steered to the broadside direction. Note that the uniform element spacing is the same in both horizontal and vertical direction of rectangular arrays for 5G user equipments (UEs) where one pattern measurement in the horizontal or vertical cut is conducted for the element spacing estimation. However, the uniform element spacing in the horizontal and vertical directions might be different for 5G BSs. In this case, the pattern measurements in both horizontal and vertical cuts are required for the element spacing estimation in the corresponding direction, respectively.

2) Determination of Element Patterns: In the all-on mode of phased arrays with all N elements activated, the element complex pattern $g_n(\Omega)$ sampled in the direction Ω is retrieved by $N+1$ complex signal measurements of array pattern $s(\Omega)$. In the first measurement, the array pattern $s_0(\Omega)$ sampled in the direction Ω is recorded when the phases of beam-steering weights of elements are all set to $0°$ (i.e. $w_1 = w_2 = \cdots = w_N = 1$), as:

$$s_0(\Omega) = \sum_{n=1}^{N} g_n(\Omega). \tag{4}$$

In the next N measurements, the beam-steering weight of the n-th element is set to $180°$ (i.e. a phase inversion) while the other $N-1$ elements remain $0°$, i.e. $w_q = 1$ for $q \neq n$ while $w_q = -1$ for $q = n$. The array pattern $s_n(\Omega)$ sampled in the direction Ω is recorded when the phase weight of the n-th element is inverted, as:

$$s_n(\Omega) = \sum_{\substack{q=1\\q\neq n}}^{N} g_q(\Omega) - g_n(\Omega). \tag{5}$$

The element pattern $g_n(\Omega)$ sampled in the direction Ω is retrieved by (4) and (5) in the all-on mode as:

$$g_n(\Omega) = \frac{s_0(\Omega) - s_n(\Omega)}{2}. \tag{6}$$

To further reduce the time of element pattern retrieving, an interpolation strategy is employed on the retrieved element pattern sampled in several sparse directions. The element patterns in phased arrays are typically designed to have a wide beamwidth for good beam-steering performance, e.g. half-power beamwidth of more than $100°$ [17]–[19]. Since the element pattern is not directive, the accurate array patterns can be reconstructed with a sparse sampling. The sparse samples can be achieved by two equivalent approaches, i.e. a sparse multi-probe setup and the existing mechanical turntable setups with a large angle sampling step (i.e. few sampling angles). Assuming K_2 the number of probes in the multi-probe setup or mechanical rotation movement in the mechanical turntable setup, the proposed method requires a total of $M_2 = (N+1) \times K_2$ signal measurements for multi-beam measurements.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/LAWP.2022.3174246, IEEE Antennas and Wireless Propagation Letters

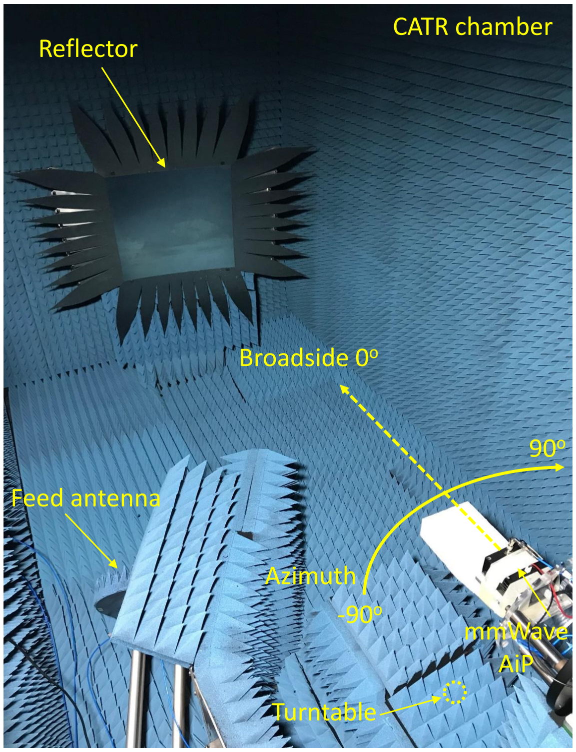

Fig. 1. A photo of measurement setup in a CATR chamber. The VNA is outside the chamber and not seen inside the chamber.

Note that the proposed method demonstrated one simple and possible example to determine the sampled element patterns in normal all-on working mode. To make this possible, the independent phase shift control of elements needs to be supported (e.g. $0°$ and $180°$). However, this is not the case for the multi-beam antennas based on Butler matrix where the phase shifts of all antenna elements are fixed simultaneously with a beam switched by selecting the corresponding feeding port [20]–[22]. There are also other ways to generate multiple beams for beam-steering antenna systems, e.g. lens antenna with multi-feeds [23] and satellite antenna with mechanical steering where the proposed method is not applicable as well. The proposed method is valid only for general phased arrays equipped with tunable phase shifters for each array element.

III. Measurement Validation

A. Measurement Setup and Procedure

A 4×4 mmWave phased array AiP detailed in [24] was utilized in the measurement validation. As shown in Fig. 1, the measurement setup is mainly composed of the AiP and a compact antenna testing range (CATR) at 28 GHz. In the CATR setup, a vector network analyzer (VNA) was employed to measure the transmission S-parameters between the feed antenna port and the AiP RF port.

In the conventional pattern measurement, AiP array patterns were directly recorded by the measured S-parameters. The AiP is rotated horizontally with the azimuth angle from $-90°$ to $90°$ with a uniform step of $1°$. The azimuth angle of $0°$ represented the AiP position shown in Fig. 1 where the broadside direction of the AiP was pointing to the CATR reflector center. The measured array patterns steered to 4 beams (i.e. $0°$, $-17°$, $-35°$, and $-59°$) were adopted as the target array patterns for comparison.

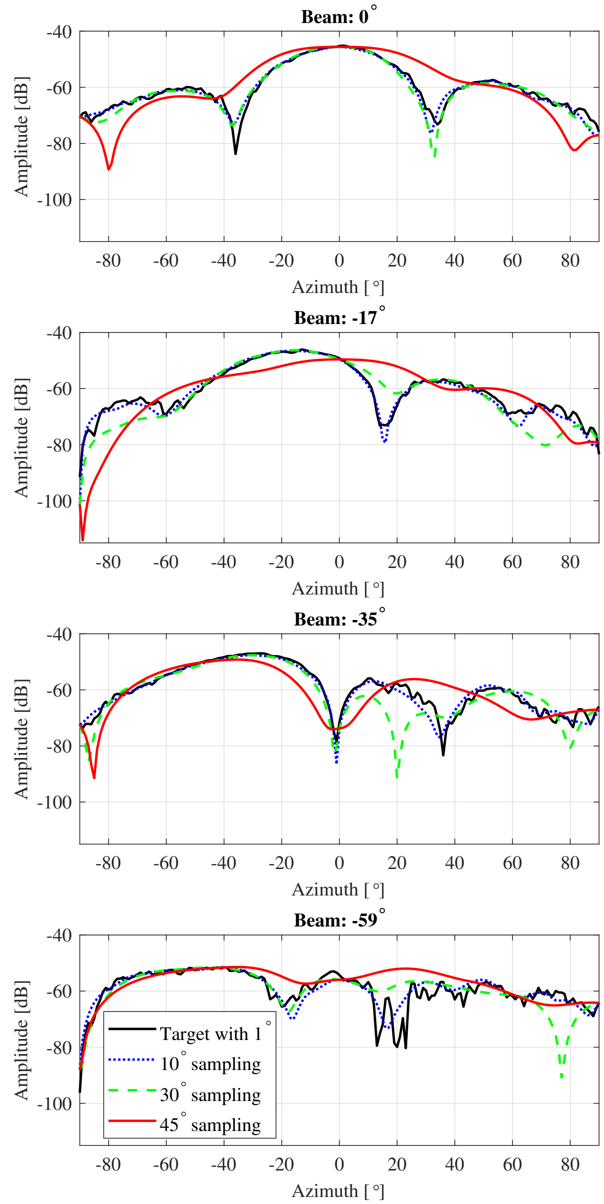

Fig. 2. Comparison of the target patterns and the reconstructed patterns using three sampling steps for 4 beams. Note that the beam of $-59°$ was the maximum beam-steering angle of the AiP employed in our practical measurements. The array power pattern at beam of $-59°$ becomes very weak and the beam cannot be formed well for this angle. The beam-steering angle range is typically limited due to the directivity of element patterns.

The proposed pattern measurement is based on the same hardware setup as the conventional measurement. For each azimuth angle of AiP with the initial phase shifts of elements set to $0°$, 17 measurements of S-parameters with 17 extra phase shift settings of elements based on Section II-B were recorded for the AiP. The measured S-parameters were used to retrieve the element pattern. Five cases of uniform sampling step are demonstrated in the proposed measurement,

TABLE I
STATISTICS OF ESS MAXIMUM AND RMS FOR FOUR SAMPLING STEPS AND FOUR BEAMS

Sampling step	Beam: 0°		Beam: −17°		Beam: −35°		Beam: −59°	
	max	rms	max	rms	max	rms	max	rms
5°	−27 dB	−40 dB	−24 dB	−34 dB	−24 dB	−35 dB	−13 dB	−25 dB
10°	−29 dB	−39 dB	−24 dB	−35 dB	−27 dB	−36 dB	−17 dB	−26 dB
15°	−28 dB	−37 dB	−24 dB	−33 dB	−24 dB	−34 dB	−14 dB	−23 dB
30°	−25 dB	−36 dB	−19 dB	−30 dB	−17 dB	−29 dB	−12 dB	−23 dB
45°	−14 dB	−22 dB	−7 dB	−18 dB	−15 dB	−22 dB	−7 dB	−17 dB

i.e. 5°, 10°, 15°, 30°, and 45°. In each case of sampling step, the sampled element patterns are first retrieved and then interpolated to a target angle resolution of conventional measurement, i.e. 1°. Finally, multiple array patterns with an angle resolution of 1° are reconstructed by superposing the interpolated element patterns and the beam-steering weights of corresponding beams.

B. Measurement Results

Taking the sampling steps of 10°, 30° and 45° for example, target and reconstructed array patterns of four beams are compared in Fig. 2. A measure of equivalent stray signal (ESS) is a well-accepted metric for antenna pattern comparison in the antenna measurement community [25], [26]. Therefore, the ESS results are used to define the pattern reconstruction errors in this article. Table I summarizes the results of ESS maximum and root mean square (rms) for each sampling step and beam direction. It can be seen that different sampling steps obtain the similar results of ESS maximum and rms for each beam, except the sampling step of 45°. The sampling step of 30° can be considered as the maximum limit for acceptable reconstruction of patterns.

To evaluate the fast multi-beam measurement advantage of the proposed method, the estimated measurement time of the conventional and proposed methods are discussed as

$$t_1 = B \times \left[\frac{\Theta}{\Delta\theta_1} \times \frac{\Phi}{\Delta\phi_1} \times (T_0 + T_1) + T_\mathrm{s} \right], \quad (7)$$

$$t_2 = \frac{\Theta}{\Delta\theta_2} \times \frac{\Phi}{\Delta\phi_2} \times [T_0 + T_1 \times (N+1)] + T_\mathrm{c}, \quad (8)$$

where t_1 and t_2 are the estimated measurement time of the conventional and proposed methods, respectively. B is the number of beams. Θ and Φ are the elevation and azimuth sampling range of interest, respectively. $\Delta\theta_1$ and $\Delta\phi_1$ are the elevation and azimuth sampling step in the conventional measurement method, respectively. $\Delta\theta_2$ and $\Delta\phi_2$ are the elevation and azimuth sampling step in the proposed measurement method, respectively. T_0 is the time of mechanically switching sampling points for both the conventional and proposed methods (≈ 0.5 s). T_1 is signal measurement time at each sampling point in the conventional method (≈ 0.01 s). In the proposed method, the signal measurement time is $T_1 \times (N+1)$ at each sampling point since $N+1$ signal measurements are recorded. T_s is the total time of electronically switching beams and resetting mechanical setups in the conventional method (≈ 10 s). T_c is the computation time for element pattern interpolation and multi-beam pattern reconstruction operations in the proposed measurement method (≈ 0.001 s).

In the measurement validation where $B = 4$, $\Theta = \Delta\theta_1 = \Delta\theta_2 = 1°$, $\Phi = 180°$, $\Delta\phi_1 = 1°$, and $N = 16$, we have $t_1 \approx 407$ s, while t_2 is estimated to be 121 s, 12 s, and 4 s with $\Delta\phi_2 = 1°$, $\Delta\phi_2 = 10°$, and $\Delta\phi_2 = 30°$, respectively. For general 3D multi-beam pattern measurement case, e.g. $\Theta = \phi = 180°$, $\Delta\theta_1 = \Delta\theta_2 = 5°$, $\Delta\phi_1 = \Delta\phi_2 = 2°$, $B = 128$, and $N = 64$, we have $t_1 \approx 59$ h and $t_2 \approx 1$ h. With a larger sampling step for the proposed method, e.g. $\Delta\theta_2 = \Delta\phi_2 = 30°$, t_2 is estimated to be 41 s.

IV. CONCLUSION AND FUTURE WORK

In this article, a fast multi-beam pattern measurement method has been proposed for mmWave phased arrays. The proposed method works for integrated phased arrays operating in the normal working mode with all elements activated. In the proposed method, array element patterns are sampled first with a sparse angle step and then interpolated to a target angle resolution of conventional measurement. A large sampling step can be adopted for element pattern interpolation, which is independent of the main lobe beamwidth of array pattern. The array patterns steered to multiple beams can be reconstructed well by the proposed method in a faster way than the conventional method. For example, the conventional and proposed measurements take around 407 s and 4 s for 4 single-cut array patterns in the measurement validation, respectively. As discussed, the measurement time will be reduced significantly by the proposed method for 3D pattern measurement of phased arrays with many beam-steering states (e.g. 128 beams).

There are two logical extensions of the proposed method for the future work. The discussion is limited to the far-field setup in the article, and it is of interest to understand how we can extend it for a near-field setup to further reduce the measurement range. Furthermore, the proposed method is based on the complex signal measurement for the element complex pattern retrieving. The element phase pattern is the relative phase of element field among all sampling points. It is possible to obtain the phase of element field relative to the composite field at each sampling point by amplitude-only measurements. However, the challenge is that the phase of composite field among all sampling points is unknown.

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