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Abstract—Operational reliability and the decision-making process of economic dispatch (ED) are closely related and important for power system operation. Consideration of reliability indices and reliability constraints together in the operation problem is very challenging due to the problem size and tight reliability constraints. In this paper, a comprehensive reliability-constrained economic dispatch model with analytical formulation of operational risk evaluation (RCED-AF) is proposed to tackle the operational risk problem of power systems. An operational reliability evaluation model considering the ED decision is designed to accurately assess the system behavior. A computation scheme is also developed to achieve efficient update of risk indices for each ED decision by approximating the reliability evaluation procedure with an analytical polynomial function. The RCED-AF model can be constructed with decision-dependent reliability constraints expressed by the sparse polynomial chaos expansion. Case studies demonstrate that the proposed RCED-AF model is effective and accurate in the optimization of the reliability and the cost for day-ahead economic dispatch.

Index Terms—Power system, operational reliability evaluation, analytical formulation, reliability-constrained economic dispatch (RCED), polynomial chaos expansion (PCE).

NOMENCLATURE

Sets and Indices

i	Index of random variables
j	Index of PCE coefficients
t	Index of discrete periods
g	Index of conventional generating units (CGUs)
s	Index of wind power scenario
k	Index of contingency
T	Time horizon of economic dispatch (ED)
Ω_g	Sets of CGU
$\Psi_{k,up}$, $\Psi_{k,down}$	Sets of the available and unavailable components corresponding to contingence k
CPs	Collocation points

Parameters

K	Contingence level
FOR	Forced outage rate
M	A large constant
$VOLL$	Value of load loss, representing the cost coefficients of demand curtailment
π_s, π_k	Probability of wind power scenario s, contingence k
$N, N_G,$ $N_W, N_s,$ N_K, N_P N_{CP}, N_{pf}	Number of random variables, CGUs, wind farms, wind power scenario, contingency, PCE coefficients for each contingency, CPs, deterministic OPF

Variables

$\mathbf{x}_t$	Vector of stochastic input variable in the analytical formulation
$\mathbf{P}_t^{\mathbf{W}}$, $\mathbf{P}_t^{\mathbf{G}}$	Vector of real-time wind power generation, day-ahead CGUs output in period t
$\boldsymbol{\xi}_{t,}$	Vector of N random variables following known probability distribution
$\zeta_{i,t}$	The i_{th} element of the input vector $\boldsymbol{\xi}_t$, a standard random variable in period t
$\boldsymbol{\xi}_{t,s,\dot{d}}$	Vector of N random variables, denoting the wind power scenario s and ED decision $\dot{d}$ in period t
$y_{k,t,s,\dot{d}}$	The demand curtailment of contingency k in period t, for wind power scenario s and ED decision $\dot{d}$
$\mathbf{Y}_{k,t}$	$N_{CP}\times1$ PCE output vector
$\mathbf{H}_{k,t}$	$N_{CP}\times N_P$ matrix with CPs of the PCE basis
h_0, h_1, h_2	PCE basis of orders 0,1, and 2
$h_{j,k,t}$	The j_{th} term of PCE basis in period t under contingency k
$\mathbf{A}_{k,t}$	$N_P\times1$ PCE coefficients vector
$a_{j,k,t}$	Coefficients of analytical formulation
$P_{w,t}^{\text{S,DA}}$	Day-ahead dispatched power output of wind farm w in period t, equal to the forecast value
$P_{g,t}^{\text{S,DA}}$	Day-ahead dispatched CGU generation
$R_{g,t}^{\text{U,DA}}/$ $R_{g,t}^{\text{D,DA}}$	The upward/downward reserves of CGU deployed in the day-ahead ED

$P_{l,t}^{DA}$	Day-ahead power flow of line l in period t
$\theta_{n,t}^{DA}$	Day-ahead phase angle of the bus n in period t
$\dot{d}$	Day-ahead controllable variables in ED decisions that affect system state analysis
$\dot{q}$	Day-ahead state variable in ED decision
$P_{w,t,s}^{RT}$	Real-time available wind power in scenario s
$P_{w,t,s}^{S,RT}$	Real-time wind power of wind farm w in the period t, scenario s
$P_{g,k,t,s}^{S,RT}$	Real-time generation of CGUs, in period t, scenario s
$P_t^{de,RT}$	Real-time total demand in period t
$\overline{P_{k,t,s,\dot{d}}^{de,RT}}$	Real-time maximum demand supply capacity of contingency k in period t, for the scenario s and ED decision $\dot{d}$
$\theta_{n,k,t,s}^{RT}$	Real-time phase angle of the bus n in period t
$P_{l,k,t,s}^{RT}$	Real-time power flow of line l in period t
z	Auxiliary binary variable
$z_{g,k}^{RT} / z_{l,k}^{RT}$	Availability of CGU g/ line l in the contingency k, $z_{g,k}^{RT}, z_{l,k}^{RT} \in \{0,1\}$
$(ENS_{k,t,s,\dot{d}}^{A})$ $ENS_{k,t,s,\dot{d}}$	(Analytical formulations of) demand curtailment in period t, contingency k and scenario s, under ED decision $\dot{d}$
$(L_{k,t,s,\dot{d}}^{A})$ $L_{k,t,s,\dot{d}}$	(Analytical formulations of) binary variable denoting whether the demand curtailment occurs in contingency k and scenario s

I. INTRODUCTION

IN the past decades, as wind power penetration is growing at a fast rate worldwide, the unpredictability and uncertainty of power systems are also increasing. One of the most important issues due to the operation decision is the reliability of the power system [1], owing to the thermal capacity of transmission lines, force outages of components, and uncertain generation.

The increasing uncertainties in power systems have given rise to a strong relationship between power system operation optimization and reliability evaluation [2]. Recently, reliability evaluation has developed a trend of embedding operation decision-making for more accurate operation strategies [3].

Operational reliability should be considered to prevent huge curtailment and make the operation decision more acceptable [1]. Many efforts have been given to model the uncertainties and solve power system scheduling problems with operational risk or reliability. Methods such as robust optimization, chance-constrained programming, and stochastic optimization are three popular approaches to deal with the security or reliability-constrained economic dispatch (RCED) under uncertainty [4].

The robust optimization based power system reliability analysis is used to find more severe contingency. For example, a robust contingency constrained unit commitment model considering the outage probability of units and transmission lines was proposed in [5]. A framework for multi-objective robust security-constrained unit commitment in the presence of wind farms and gridable vehicles was presented in [6]. A resilience-constrained day-ahead unit commitment framework

was developed for increasing the resilience of a power system exposed to an extreme weather event [7].

The chance-constrained programming (CCP) has been used to minimize the operation cost and limit the operational risk. An N-1 security and chance-constrained unit commitment model was presented to cope with wind fluctuations and component outages [8]. A chance-constrained economic model, which optimizes the generation of conventional units and the curtailment strategies of renewable energy, was proposed to minimize total operational costs and restrict operational risks [9]. CCP deals with the uncertainty by integrating the probability functions of wind power into the economic dispatch (ED). Although the confidence levels of chance constraints can be preset to accommodate the operational risk, it is difficult to capture the consequences of random failures of components in a quantitative manner. Moreover, in the context of increasing operational uncertainty, operators must continuously monitor and evaluate risk in real time [10]. This makes CCP not suitable for the operational reliability evaluation.

Another approach for operational reliability is to use the stochastic optimization (SO) with reliability indices. These indices help assess the system behavior under various load changes and renewable production. Thus, it could provide valuable insights for effective decision-making. In [11], a probabilistic framework for scheduling and dispatching generation while optimizing reserve needs was proposed. In [12], a stochastic risk-constrained framework was developed for short-term optimal scheduling of autonomous microgrids to evaluate the influence of demand response on reliability and economic issues. SO considers not only the constraints under normal operation, but also additional steady-state security constraints for each contingency [13]. To assess the system's behavior, the expected energy not supplied (EENS) of reliability indices are determined based on the real-time system states, which encompass renewable scenarios and contingencies. Then, the reliability indices can be expressed in the objective function or the constraints of SO.

The problem size of SO increases with the number of system states. This makes the analysis of massive system operational states in SO time-consuming and causes the computational burden. Several methods, such as the double cross-linked lists [14], the umbrella contingencies [15], and the universal generating functions [16], have been proposed to ease this burden by reducing the number of system states in reliability evaluation. The computational cost also can be alleviated by speeding up the state analysis and improving the SO formulation [17]. Many other methods have also been proposed, such as the Lagrange multiplier-based state enumeration [18], the unnecessary constraints removal [19], multi-parametric linear programming [20], machine learning [10], [21], neural network [22], stacked-denoising-auto-encoders-based model [23], and deep neural network [13].

However, different from the long-term reliability evaluation that is based on the units' installed capacity, the operational reliability indices are dependent on the economic dispatch (ED) decisions in the presence of uncertainties. This makes the computation complexity for operational reliability higher than

that for long-term reliability evaluation. To model the operational risk effectively and accurately, it is necessary to develop a reliability evaluation model that takes into account the decision-making process of ED as well as the uncertainty associated with wind power. Furthermore, incorporating ED decisions in operational reliability evaluation poses challenges due to the demanding computation requirements and the complicated RCED problem. The former challenge can be partly solved by scenario reduction [24]. However, the scenario reduction method may come at the cost of reduced calculation accuracy. In addition, due to the need for repeated risk evaluation every time the ED decision changes, the efficiency of calculating the indices is inevitably lowered. Therefore, the challenge lies in achieving an adaptive and accurate update of risk indices for each possible economic dispatch decision.

On the other hand, the RCED problem presents considerable difficulties. Solving an RCED problem with all the system states is not practical given the tight reliability constraints. The outage of components and wind power uncertainty results in a huge amount of system states in the RCED modeling, which significantly increases the computation complexity [3]. As the number of variables and constraints grows, the conventional method that solves the monolithic optimization problem by making all decisions simultaneously becomes impractical. Benders decomposition [25], [26] is one possible strategy to solve large-scale RCED problems with reliability indices [8]. Since reliability index EENS constraints related to the analysis results of all system states cannot be decomposed, EENS can only be applied in the objective, not the constraints of the Benders decomposition method. Besides, as problem size grows, Benders decomposition becomes computationally complex, resulting in longer computation times and increased resource requirements. In the existing RCED model, EENS is incorporated in the objective function to optimize reliability and cost. The reliability constraints of EENS are mainly utilized to evaluate the feasibility of solutions from a reliability viewpoint [1]. Efficient RCED with constraints for both EENS and loss of load probability (LOLP) has not been thoroughly explored. This leads to a disconnection between the reliability evaluation and operation optimization. These highlight the need to efficienctly assess the system behavior and optimize decisions with the constraints of reliability indices.

Thus, it would be beneficial to explore an efficient method to cope with the computation requirements of reliability-constrained operation. This paper proposes a comprehensive reliability evaluation model with an analytical formulation of operational risk/reliability evaluation to tackle the operational risk problem of power systems. The analytical formulation relating reliability indices to the ED decision and the wind power scenario is constructed based on the polynomial chaos expansion (PCE). The decision dependency between the reliability and operational decision is decoupled so that the computation complexity of reliability evaluation is significantly reduced. The complexity of RCED also can be reduced by the analytical indices. All these will make the proposed method suitable for large-scale power systems with renewable energy generation.

The contributions of this paper are summarized below.
1) Propose a reliability-constrained economic dispatch model with an analytical formulation of operational risk/reliability evaluation (RCED-AF) that enables efficient optimization of the reliability and the cost for day-ahead economic dispatch by avoiding the computationally expensive evaluation procedure.
2) Develop an operational reliability evaluation model considering the economic dispatch decision to accurately assess the system behavior under various renewable production changes. The model incorporates the day-ahead operational decisions into the reliability evaluation, in addition to the conventional assessment factors of wind power uncertainty and random failures.
3) Present a computationally efficient scheme that significantly lowers the cost of operational reliability indices calculation, while maintains the accuracy. This scheme achieves efficient update of risk indices for each ED decision by approximating the reliability evaluation procedure with an analytical polynomial function.

The remaining sections of this paper are organized as follows: Section II gives a brief introduction to the operational reliability evaluation. Section III establishes the analytical formulation of power system operational reliability evaluation. Section IV presents a mathematical formulation of RCED. Section V presents case studies. Section VI concludes and introduces recommendations for future work.

II. Operational Reliability Evaluation of Power Systems with Wind Power Penetration

Power system operational reliability evaluation can be described by the flowchart depicted in Fig. 1. Fig. 1(a) shows the reliability evaluation process, Fig. 1(b) the wind power scenario, and Fig. 1(c) the power system state analysis under ED decision. The computation procedure consists of four steps [27], [28]. i) Select a system state based on the component reliability model and massive wind power scenario, ii) Analyze the system state under ED decision, iii) Calculate the reliability indices, and iv) Apply the indices in the operation.

A. Reliability Model of Component

The component reliability model consists of the reliability model for the wind farm and power system components.

The operational reliability model of a wind farm can be represented as an equivalent multi-state wind generation provider. The multi-state wind generations are modeled by the wind power scenarios for the operational periods, which are generated by wind power forecast and forecast error in Monte-Carlo (MC) simulation. For each wind power scenario s, its probability π_s equals 1 divided by the scenario number N_s.

A two-state continuous-time Markov model is utilized to estimate the random outages of power system components. The component states act as stochastic variables, which get an up or down state in system contingency enumeration. The contingency probability π_k is settled according to the given components' forced outage rate (FOR), as (1) [29].

$$\pi_k = \prod_{i^{up} \in \Psi_{k,up}} (1 - FOR_{i^{up}}) \cdot \prod_{i^{down} \in \Psi_{k,down}} FOR_{i^{down}}. \qquad (1)$$

where $\Psi_{k,up}$, $\Psi_{k,down}$ are the sets of the available and unavailable components corresponding to contingence k, respectively.
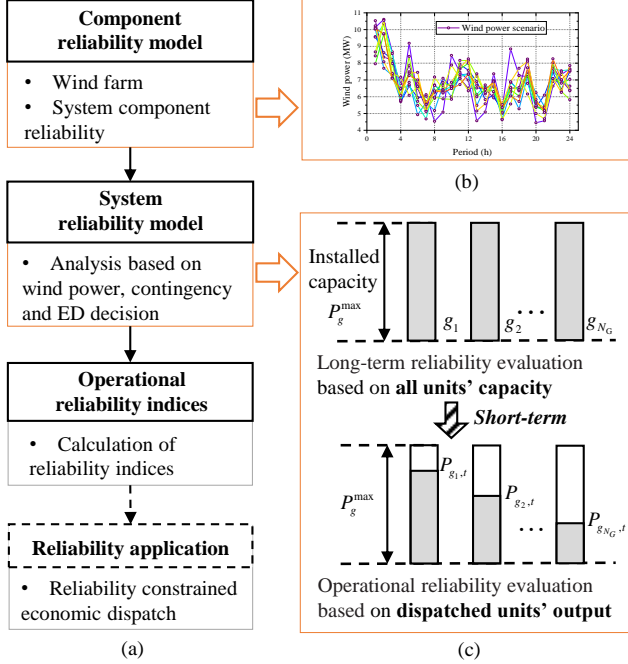

Fig. 1. Flowchart of power system operational reliability evaluations. (a) Reliability evaluation process. (b) Massive wind power scenario. (c) Power system state analysis is dependent on ED decisions.

B. Operational Reliability Model of Power Systems

The operational reliability model of the power system is based on the system state analysis at each period. The system state consists of wind power scenarios and component contingencies.

In the operation phase, as shown in Fig. 1(c), the outage capacity of each CGU equals to its scheduled output, rather than its installed capacity. This scheduled output of CGU leverages the forward knowledge of day-ahead forecasts for demand and wind power. In other words, the day-ahead ED decisions impact the system state analysis and further affect the results of operational reliability evaluation. Thus, inputs of operational system states would include not only the wind power scenarios and contingencies, but also the ED decisions. The result of the real-time system state analysis, i.e., the demand curtailment, is obtained by OPF, as Appendix A.

C. Operational Reliability Indices

The explicit reliability indices associated with the analysis of all the system states, such as *EENS* and *LOLP*, for each operational period t, can be acquired by (2)-(5).

$$EENS_{t,\dot{d}} = \Delta t \cdot \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot ENS_{k,t,s,\dot{d}}. \qquad (2)$$

$$LOLP_{t,\dot{d}} = \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot L_{k,t,s,\dot{d}}. \qquad (3)$$

$$\pi_{k,s} = \pi_k \cdot \pi_s. \qquad (4)$$

$$L_{k,s,t,\dot{d}} = \begin{cases} 0, ENS_{k,s,t,\dot{d}} \le 0 \\ 1, ENS_{k,s,t,\dot{d}} > 0. \end{cases} \qquad (5)$$

where $ENS_{k,t,s,\dot{d}}$ denotes the demand curtailment in period t for contingency k and scenario s, under a given ED decision $\dot{d}$. Δt denotes the length of period t.

In reliability evaluation, the OPF-based calculation of $ENS_{k,t,s,\dot{d}}$ is time-consuming. As there are a huge number of contingencies, multiple periods, wind scenarios, and ED decisions. Once $ENS_{k,t,s,\dot{d}}$ is analytically adaptive to both wind power scenarios and operational decisions, the computation efficiency could be improved dramatically.

III. ANALYTICAL FORMULATION FOR OPERATIONAL RELIABILITY EVALUATION

This section presents the procedure of obtaining analytical formulation for power system operational reliability evaluation. The analytical formulation is constructed based on PCE, whose basic idea is to approximate the implicit parameter-output function with a globally optimal explicit polynomial function [30].

Steps to construct the analytical formulation are: i) determine the input, ii) generate the PCE basis, iii) construct the analytical formulation for each contingency, iv) calculate the coefficients of the polynomial function, and v) combine the functions of all contingencies to find the analytical reliability indices with all of system state analysis results. Fig. 2 depicts the schematic diagram to construct the analytical formulation for contingency k. The state analysis result for contingency k is analytically approximated as a polynomial function, which correlates the demand curtailment to the wind power and ED decision.
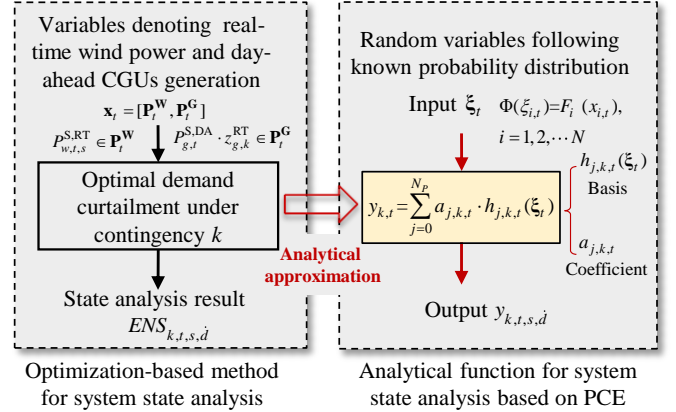
Fig. 2. Schematic diagram of analytical formulation for contingency k

A. Input of Analytical Formulation

As shown in Fig. 2, the input of the analytical formulation is expressed by standard random variables $\boldsymbol{\xi}_t = [\xi_{1,t}, \xi_{2,t}, \ldots, \xi_{N,t}]$, denoting the stochastic variables $\mathbf{x}_t = [\mathbf{P}_t^{\mathbf{W}}, \mathbf{P}_t^{\mathbf{G}}]$ of the wind power scenario $\mathbf{P}_t^{\mathbf{W}}$ and day-ahead dispatched CGUs output $\mathbf{P}_t^{\mathbf{G}}$. Note that the day-ahead dispatched CGU generation is included as the input stochastic variables for modelling the ED decisions in constructing the analytical formulation. With a standard random variable $\xi_{i,t} \in \boldsymbol{\xi}_t$ for the analytical formulation, a stochastic input variable $x_{i,t} \in \mathbf{x}_t$ can be expressed by (6) [31]. The wind power scenarios and the ED

decisions coupled with CGU outage capacity are included to model the uncertainties, i.e., $P_{g,t}^{\mathrm{S,DA}} \cdot z_{g,k}^{\mathrm{RT}} \in \mathbf{P}_t^{\mathbf{G}}$. The wind power in each period t is assumed to follow the normal distribution. The CGU generation is assumed to follow a uniform distribution within the generation limits.

$$x_{i,t} = F_i^{-1}(\Phi(\xi_{i,t})), i = 1, 2, \cdots N. \tag{6}$$

where $x_{i,t}$ denotes the i_{th} element of $\mathbf{x}_t$. F_i is the cumulative probability function of $x_{i,t}$. F_i^{-1} is the inverse function of F_i. Φ denotes the cumulative probability function of $\xi_{i,t}$. N denotes the number of random variables, and $N = N_W + N_G$.

B. Analytical Formulation and PCE Basis

In the PCE method, the outputs are represented as a weighted sum of orthogonal PCE basis functions, which are constructed based on the probability distribution of the input random variables [32]. Let $y_{k,t,s,\dot{d}}$ denotes the output of state analysis, i.e., the demand curtailment. Random variables $\xi_{t,s,\dot{d}}$ denote the possible wind power scenario s and ED decisions $\dot{d}$, following known probability distribution in period t. Then, the $y_{k,t,s,\dot{d}}$ for contingency k can be formulated by a truncated expansion of PCE.

$$y_{k,t,s,\dot{d}} = \sum_{j=0}^{N_P} a_{j,k,t} \cdot h_{j,k,t}(\xi_{t,s,\dot{d}}). \tag{7}$$

where $a_{j,k,t}$ denote the coefficient, $h_{j,k,t}(\xi_{t,s,\dot{d}})$ denote the PCE basis. There are $N_P = (N + m)! / (N! \, m!) - 1$ coefficients, with N random variables involved in PCE and maximum order m of PCE basis.

A set of one-dimensional PCE basis $\{h_{j,k,t}(\xi), j=0,1,2,\dots\}$ concerning some real positive measure should satisfy (8). Similarly, any set of multi-dimensional PCE basis is orthogonal to each other concerning their joint probability measure.

$$\int_{\mathbb{S}} h_{r,k,t}(\xi) h_{v,k,t}(\xi) d\lambda = \begin{cases} 0 & \text{if } r \neq v \\ > 0 & \text{if } r = v. \end{cases} \tag{8}$$

where λ is a probability measure defined as the cumulative probability distribution function of ξ. $\mathbb{S}$ is the support of the measure λ as shown in Table I. Each distribution is associated with a unique orthogonal polynomial.

A set of multi-dimensional PCE basis can be constructed as the tensor product of the one-dimensional PCE basis associated with each input random variable, as

$$h(\xi_t) = h(\xi_{1,t}) \otimes h(\xi_{2,t}) \otimes \cdots \otimes h(\xi_{N,t}). \tag{9}$$

where $h(\xi_{i,t})$ denotes the one-dimensional PCE basis for the ith random variable.

TABLE I
SOME TYPICAL PROBABILITY DISTRIBUTIONS AND CORRESPONDING
ORTHOGONAL POLYNOMIALS

Random variable	Orthogonal polynomial	Support
Normal	Hermite	$(-\infty, +\infty)$
Uniform	Legendre	$[-1,1]$
Gamma	Laguerre	$[0, +\infty]$
Beta	Jacobi	$[0,1]$

C. Coefficients of Analytical Formulation

The coefficients in the analytical formulation are determined by OPF and least square estimations. The procedure to calculate

the coefficients for contingency k and period t is given below.

1) Construct the PCE basis, as (9) and the typical PCE basis in Table I.
2) Choose appropriate combinations of collocation points (CPs). The CPs are finite samples of $\xi_{i,t}$ that are chosen to approximate the PCE coefficients. Every CP satisfies $CP_{i,t} = \xi_{i,t}$. The elements of CP are obtained using the union of zero and the roots of one higher-order one-dimensional polynomial basis [32]. There are two approaches to choose CPs: the efficient collocation (EC) method and the regression analysis (RA) method. The number of CPs for RA is twice the number of coefficients to be determined to reach higher accuracy.
3) Put the CPs into the coefficient matrix $\mathbf{H}_{k,t}$ of size $N_{CP} \times N_P$ (such as (25)). $\mathbf{H}_{k,t}$ consists of all the terms of the PCE basis.
4) Compute the demand curtailment for the selected CPs to obtain an output vector $\mathbf{Y}_{k,t}$, expressed as

$$\mathbf{Y}_{k,t} = (y_{1,k,t}, y_{2,k,t}, \dots, y_{Ncp,k,t}). \tag{10}$$

Note that to avoid generating inaccurate estimates of the statistical properties under strongly unsmooth conditions [33], the demand curtailment is calculated based on the maximum demand supply capacity of the power system, as in Appendix A.

5) Calculate the coefficients of the $N_P \times 1$ vector, $\mathbf{A}_{k,t}$, based on the output $\mathbf{Y}_{k,t}$ of selected CPs expressed as

$$\begin{aligned} \mathbf{A}_{k,t} &= (a_{0,k,t}, a_{1,k,t}, \dots a_{Np,k,t})^{\mathrm{T}} \\ &= (\mathbf{H}_{k,t}^{\mathrm{T}} \mathbf{H}_{k,t})^{-1} \mathbf{H}_{k,t}^{\mathrm{T}} \mathbf{Y}_{k,t}. \end{aligned} \tag{11}$$

D. Analytical Formulation of Operational Reliability Indices

Based on the PCE basis and coefficients, the analytical formulation for demand curtailment of contingency k can be constructed as

$$ENS_{k,t,s,\dot{d}}^{\mathrm{A}} = \max\{0, y_{k,t,s,\dot{d}}\}. \tag{12}$$

where $y_{k,t,s,\dot{d}}$ can be obtained from (7). Then, by taking a weighted sum of analytical formulations for all contingencies, the analytical formulation of EENS can be established as

$$EENS_{t,d}^{\mathrm{A}} = \Delta t \cdot \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot ENS_{k,t,s,\dot{d}}^{\mathrm{A}}. \tag{13}$$

and that of LOLP is

$$LOLP_{t,d}^{\mathrm{A}} = \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot L_{k,t,s,\dot{d}}^{\mathrm{A}}, \tag{14}$$

where

$$L_{k,t,s,\dot{d}}^{\mathrm{A}} = \begin{cases} 0, \sum_{i=0}^{N_P} a_{j,k,t} \cdot h_{j,k,t}(\xi_{t,s,\dot{d}}) \leq 0 \\ 1, \sum_{i=0}^{N_P} a_{j,k,t} \cdot h_{j,k,t}(\xi_{t,s,\dot{d}}) > 0. \end{cases} \tag{15}$$

E. Discussion on Probability Distribution of Wind Power Forecast Error

The day-ahead wind power can be forecasted according to the ARIMA model. The analytical formulation of power system operational reliability constructed in Subsections III-B, III-C, and III-D is under the assumption that the forecast error is

described by a specific probability distribution, e.g., normal distribution. If the probability distribution is not known or under the distributionally agnostic situations, the proposed analytical formulation still can be constructed to obtain the operation reliability indices with similar performance. The modification lies in Subsection III-B, which generates the PCE coefficients and PCE basis.

PCE coefficients and basis can be generated by applying the Stieltjes procedure on the discrete points of wind power forecast errors [32], with no knowledge of probability distribution of wind power. Note that the PCE basis and coefficients generated by the Stieltjes procedure are very close to those by the normal distribution. Once the PCE basis and coefficients are generated, the analytical formulation of power system operational reliability can be constructed following the procedure described in Subsections III-C and III-D.

IV. ANALYTICAL FORMULATION IN RELIABILITY-CONSTRAINED ECONOMIC DISPATCH

The analytical formulation of operational risk/reliability evaluation is utilized in a stochastic day-ahead RCED-AF model to enable the co-optimization of reliability and economy. The problem across 24-hour periods corresponds to a one-day time horizon, with a 1-hour dispatch time step.

A. Formulation of RCED-AF

The mathematical formulation of the presented RCED-AF problem consists of the objective function (16) and constraints (17)-(19) as follows

$$\min \underbrace{\sum_{g}^{N_G} \sum_{t}^{T} f(\dot{d}, \dot{q})}_{1_{st} \text{ stage: ED}} + \underbrace{VOLL \cdot \sum_{t}^{T} EENS_t}_{2_{nd} \text{ stage: evaluation}} \quad (16)$$

s.t.

$$\phi(\dot{d}, \dot{q}) = 0 \quad (17)$$

$$\varphi(\dot{d}, \dot{q}) \le 0 \quad (18)$$

$$g \in \Omega_g, t \in T. \quad (19)$$

The objective (16) is to minimize the total operating cost of the system, including generation and reserve costs of CGUs, and expected demand curtailment costs. The RCED problem could be split into the first-stage day-ahead ED problem and the second-stage reliability evaluation problem. The first-stage concerns variables $\dot{d}, \dot{q}$, where $\dot{d}$ is the day-ahead controllable variable that affects the result of system state analysis. $\dot{q}$ indicates the state variables. $\dot{d} = \{P_{g,t}^{S,DA}, R_{g,t}^{U,DA}, R_{g,t}^{D,DA} | \forall g, \forall t\}$, where $P_{g,t}^{S,DA}$, $R_{g,t}^{U,DA}$, and $R_{g,t}^{D,DA}$ denote the day-ahead scheduled generation, the upward, and downward reserve of the CGU g in the period t, respectively. The reserves are assumed the settled value, and the $P_{g,t}^{S,DA}$ is discussed as $\dot{d}$ in Section III. $\dot{q} = \{P_{w,t}^{DA}, P_{l,t}^{DA}, \theta_{n,t}^{DA} | \forall w, \forall l, \forall n, \forall t\}$, where $P_{w,t}^{DA}$ denotes the power output of wind farm w in the period t. $P_{l,t}^{DA}$ denotes the power flow of line l in period t. $\theta_{n,t}^{DA}$ denotes the phase angle of the bus n in the period t. The second stage concerns the state analysis of each wind power scenario and each contingency that is coupled with the first-stage variables, as in Appendix A.

The constraints (17)-(19) include power output limits of CGUs and wind farms, reserve requirements, power balance limits, power flow limit of the transmission lines, limits for the phase angles, and so on. The reliability indices $EENS_t$ and $LOLP_t$ are calculated by the second-stage problem that is dependent on the first-stage ED decision, and reliability constraints (20)-(21) are applied in the RCED.

$$EENS_t \le \overline{EENS_t}. \quad (20)$$

$$LOLP_t \le \overline{LOLP_t}. \quad (21)$$

B. Reliability Constraints of RCED-AF

In general, with a higher order of basis, higher accuracy of PCE approximation can be achieved, but the number of PCE coefficients and the computational cost also increase. For a steady-state problem, a relatively low PCE order, typically 2, is sufficient to provide the output results with acceptable accuracy [32]. Thus, a 2nd-order PCE expansion is adopted in this paper. The test results in Section V will demonstrate that the order of 2 can accurately calculate the demand curtailment and reliability indices with relatively high computational efficiency.

In this study, the sparse expression of PCE approximation is used, where the quadratic cross terms and the bi-linear terms in (9) are eliminated [34]. Then, the coefficient number N_p of the sparse 2nd-order expansions is $2N+1$.

The PCE basis for the wind power variable with normal distribution can be expressed by the Hermite basis, and that for CGU output variable with a uniform distribution is the Legendre basis. Eq. (22) gives the 1st- and 2nd-order Hermite basis, while Eq. (23) the 1st- and 2nd-order Legendre basis.

$$\begin{cases} h_0(\xi) = 1 \\ h_1(\xi) = \xi \\ h_2(\xi) = \xi^2 - 1. \end{cases} \quad (22)$$

$$\begin{cases} h_0(\xi) = 1 \\ h_1(\xi) = \xi \\ h_2(\xi) = (3 \cdot \xi^2 - 1)/2. \end{cases} \quad (23)$$

Substituting these PCE bases into the output of state analysis (7), we obtain the 2nd-order sparse PCE expressed as

$$y_{k,t,s,\dot{d}}(\xi_{t,s,\dot{d}}) = a_{0,k,t} + \sum_{j=1}^{N} a_{j,k,t} \cdot \xi_{j,t,s,\dot{d}} + \sum_{j=N+1}^{N+N_W} a_{j,k,t} \cdot$$
$$(\xi_{j-N,t,s,\dot{d}}^2 - 1) + \sum_{j=N+N_W+1}^{2 \cdot N} a_{j,k,t} \cdot (3 \cdot \xi_{j-N,t,s,\dot{d}}^2 - 1)/2. \quad (24)$$

In (24), $\xi_{i,t,s,\dot{d}}$ for $i = 1,2,...,N_W$ are the normal random variables denoting wind power, while those for $i = N_W + 1, N_W + 2, ..., N$ are the uniform variables of the dispatched generation of CGUs. Random variables $\xi_{i,t,s,\dot{d}}$ are transferred from the variables of the wind power scenario and the ED decision, according to (6).

To calculate the PCE coefficient, the CPs number N_{CP} takes twice the number N_p according to the RA method. Put the CPs of the PCE basis into the $\mathbf{H}_{k,t}$, as (25). Then, the coefficients can be obtained according to Section III-C.

$$\mathbf{H}_{k,t} = \begin{bmatrix} 1 & \xi_{1,1} & \xi_{2,1} & \cdots & \xi_{N,1} & \xi_{1,1}^2-1 & \cdots & \xi_{N_W,1}^2-1 & (3\cdot\xi_{N_W+1,1}^2-1)/2 & \cdots & (3\cdot\xi_{N,1}^2-1)/2 \\ 1 & \xi_{1,2} & \xi_{2,2} & \cdots & \xi_{N,2} & \xi_{1,2}^2-1 & \cdots & \xi_{N_W,2}^2-1 & (3\cdot\xi_{N_W+1,2}^2-1)/2 & \cdots & (3\cdot\xi_{N,2}^2-1)/2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \xi_{1,N_{CP}} & \xi_{2,N_{CP}} & \cdots & \xi_{N,N_{CP}} & \xi_{1,N_{CP}}^2-1 & \cdots & \xi_{N_W,N_{CP}}^2-1 & (3\cdot\xi_{N_W+1,N_{CP}}^2-1)/2 & \cdots & (3\cdot\xi_{N,N_{CP}}^2-1)/2 \end{bmatrix} \quad (25)$$

According to (12) and the sparse expression (24) of $y_{k,t,s,\dot{d}}$, the constraints for demand curtailment can be written as

$$\begin{cases} ENS_{k,t,s,\dot{d}}^A \geq 0 \\ ENS_{k,t,s,\dot{d}}^A \geq y_{k,t,s,\dot{d}}(\xi_{t,s,\dot{d}}). \end{cases} \quad (26)$$

It follows from (13) and (14) that constraints for reliability indices (20) and (21) are, respectively, recast as

$$EENS_{t,\dot{d}}^A = \Delta t \cdot \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot ENS_{k,t,s,\dot{d}}^A \leq \overline{EENS_t}, \quad (27)$$

$$LOLP_{t,\dot{d}}^A = \sum_{k=1}^{N_K} \sum_{s=1}^{N_S} \pi_{k,s} \cdot L_{k,t,s,\dot{d}}^A \leq \overline{LOLP_t}. \quad (28)$$

The binary variable $L_{k,t,s,\dot{d}}^A$ in (15), which is piecewise and non-convex, can be rewritten in a linear form as

$$\begin{cases} -M \cdot (1-z) < y_{k,t,s,\dot{d}}(\xi_{t,s,\dot{d}}) \leq M \cdot z \\ L_{k,t,s,\dot{d}}^A \geq -M \cdot z \\ L_{k,t,s,\dot{d}}^A \leq M \cdot z \\ L_{k,t,s,\dot{d}}^A \geq 1-M \cdot (1-z) \\ L_{k,t,s,\dot{d}}^A \leq 1+M \cdot (1-z). \end{cases} \quad (29)$$

C. Solution of RCED-AF

According to the description above, the RCED-AF model is a quadratic programming (QP) problem. The constraints for demand curtailment (12) and the reliability indices (27) and (28) can be recast as linear constraints, as long as the maximum PCE order is 1. In other words, the RCED-AF model can be converted into a linear programming (LP) problem and solved without using commercial solvers. It can be solved directly through modern QP solvers, such as Gurobi and Cplex, to obtain the solution.

The algorithm procedure of the proposed RCED-AF model is presented in Fig. 3. The procedure could be divided into two parts. The first part is to construct the analytical formulation of demand curtailment for the operational time horizon. In the second part, operational reliability indices are obtained and the RCED-AF is solved. The conventional RCED and the proposed RCED-AF are compared in Table II. Optimizing with all the wind power scenarios and contingencies leads to a high computation burden. However, with the analytical formulation, the analysis of the system state is converted from OPF into the analytical formulation. This leads to a decreased variable number and problem size. Moreover, the number of OPF N_{pf} is also independent of wind scenario number and iteration times, and then the computation time could be decreased significantly.
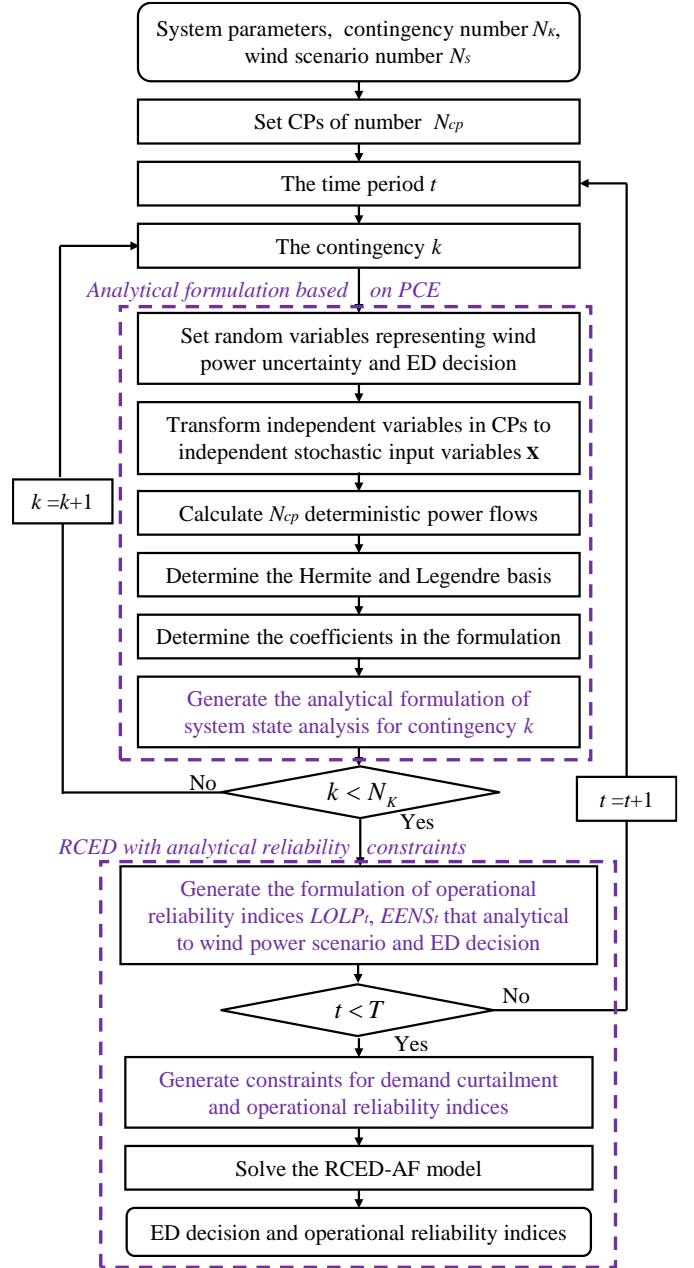

Fig. 3. Flowchart of the proposed RCED-AF model

TABLE II
COMPARISON BETWEEN CONVENTIONAL RCED AND RCED-AF

	Conventional RCED	RCED-AF
Problem type	Large-scale nonlinear programming	QP (or LP)
Reliability indices	Only in the objective (EENS)	In the objective and constraints (EENS, LOLP)
Reliability constraints	Non-convex	Polynomial formula
	For evaluating the feasibility of solutions	For optimal solutions

Solve methods	by decomposition with relaxed constraints	Solve directly or by decomposition
System state analysis	OPF based on the day-ahead ED decision	The analytical formulation for the ED decision
N_{pf}	$N_S \cdot N_K \cdot$ iteration	$N_K \cdot 2 \cdot (2N+1)$

V. CASE STUDY

In this section, we present the performance evaluation of the proposed analytical formulation and RCED-AF model. To evaluate the operational reliability, the proposed analytical approximation approach is compared with the MC method. Moreover, the accuracy and computational efficiency of the proposed RCED-AF model are verified by comparing with that of the CCP-based ED model, the Benders decomposition with relaxed reliability constraints model, and the stochastic optimization with scenario reduction model.

Three test systems are used. Case 1 considers the modified RBTS, Case 2 the IEEE RTS-79, and Case 3 the modified IEEE 118-bus system. All case studies were conducted in MATLAB on a PC with an AMD Ryzen 7 4800H 2.9-GHz processor and 16 GB RAM. The RCED-AF takes advantage of YALMIP [35] with solver GUROBI.

A. Case 1: IEEE-RBTS

The modified RBTS consists of 6 buses, 9 transmission lines, 9 CGUs, and 5 demands [36]. Two CGUs are replaced by two wind farms (WFs) on buses 1 and 2. The wind penetration is set as 30%, namely, 90 MW of total generation capacity 300 MW is from wind turbines. The *VOLL* is $3.85/kWh [36]. The electrical and reliability parameters are given in Table III. The capacity and reliability parameters are listed in Table IV. Historical sequential wind speed data is derived from NREL's Wind Integration Data Sets [37]. The curves of day-head forecast wind power and load demand are shown in Fig. 4.
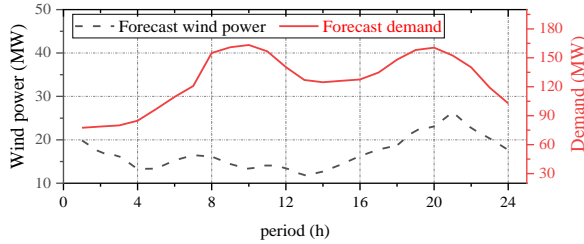

Fig. 4. The curves of day-ahead forecast wind power and load demand.

TABLE III
ELECTRICAL AND RELIABILITY PARAMETERS OF THE MODIFIED RBTS

Bus	Device	Type	Capacity (MW)	FOR	Ramp rate (MW/min)	Gen. cost ($/MWh)
1	CGU 1	thermal	40	0.030	0.4	12
1	CGU 2	thermal	40	0.030	0.4	14
1	WF 1	Wind	45	/	/	/
2	WF 2	Wind	45	/	/	/
2	CGU 3	hydro	5	0.010	2.5	1
2	CGU 4	hydro	5	0.010	2.5	1
2	CGU 5	hydro	40	0.020	20	0.5
2	CGU 6	hydro	20	0.015	10	1
2	CGU 7	hydro	20	0.015	10	0.6
2	CGU 8	hydro	20	0.015	10	0.7
2	CGU 9	hydro	20	0.015	10	0.8

TABLE IV
CAPACITIES AND RELIABILITY PARAMETERS OF RBTS TRANSMISSION LINES

Device	Capacity (MW)	FOR	Device	Capacity (MW)	FOR
line 1	85	0.0017	line 6	85	0.0017
line 2	71	0.0057	line 7	71	0.0057
line 3	71	0.0045	line 8	71	0.0011
line 4	71	0.0011	line 9	71	0.0011
line 5	71	0.0011	-	-	-

1) Accuracy and Computation Time Analysis: First, the accuracy and computation time for demand curtailment and operational reliability are investigated. Under a given ED decision and contingency, the probability distribution of the demand curtailment was obtained by the PCE and MC methods. In the MC method, the wind power scenarios are obtained by the MC simulation while the demand curtailment is obtained by the OPF.

The computation errors of PCE coefficients are compared between the RA and EC methods. The probability distribution function (PDF) curve and cumulative distribution function (CDF) curve of demand curtailment obtained by four methods, MC, 1st-order PCE (RA), 2nd-order PCE (RA), and 2nd-order PCE (EC) are shown in Fig. 5. Fig. 5(a) and 5(b) are the results under the 2-level contingency with outages of line 8 and CGU 5. Fig. 4(c) and 4(d) show the results under contingency with outages of line 1 and CGU 5. According to Fig. 5(a) and 5(b), the 2nd-order PCE and the MC methods have almost the same PDFs and CDFs of demand curtailment. It can be seen from Fig. 5(c) and 5(d) that the 2nd-order PCE (RA) method matches the MC method in PDFs well, as compared to the 1st-order PCE (RA) and 2nd-order PCE (EC) methods.

According to the demand curtailment results, the operational reliability indices are obtained. As presented in Table V, the indices of the PCE (RA) method are accurate as those of the MC method. There is nearly 0% computation error for indices LOLP under different contingency levels and wind power scenario numbers. With N_s=10, the EENS computation errors of contingency levels 1-4 are 0%, 0.14%, 0.17%, and 0.11%, respectively. Besides, with N_s =100, the EENS computation errors of contingency levels 1-4 are 0%, 0.19%, 0.15%, and 0.14%, respectively. Therefore, the PCE has reliable results for power system operational risk evaluation.

The computation times vary with the contingency level and wind power scenario number, as shown in Table VI. The time of PCE modeling increases with the number of contingencies, while it was nearly immune from the wind power scenario number. Therefore, the total computation time of analytical evaluation is less than MC when there is an increased wind power scenario number. For contingency level K=4 and wind scenario number N_s=100, the calculation time of the MC method is 1,971 s. In contrast, the total calculation time of the

proposed method is 587 s, with only 0.053 s of evaluation time (time for calling the PCE function). In other words, roughly 30,600 system states are evaluated in less than 0.1 seconds. The proposed analytical operational reliability evaluation method provides accurate and reliable results with high computation efficiency.
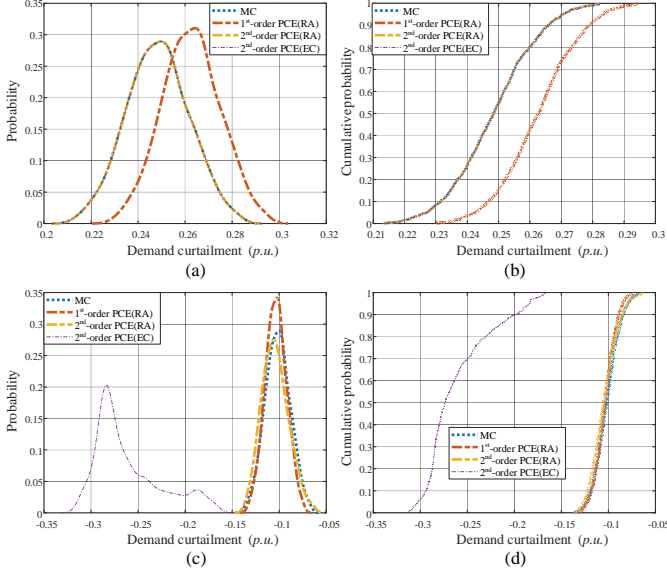

Fig. 5. Demand curtailment of PCE and MC method under contingencies. (a) PDF with outages of line 8 and CGU 5. (b) CDF with outages of line 8 and CGU 5. (c) PDF with outages of line 1 and CGU 5. (d) CDF with outages of line 1 and CGU 5.

TABLE V
COMPUTATION ACCURACY COMPARISON OF DIFFERENT CONTINGENCY LEVELS AND WIND POWER SCENARIO NUMBERS

N_s	K	LOLP-PCE	LOLP-MC	EENS-PCE (MWh)	EENS-MC (MWh)
10	1	0.100	0.100	0.839	0.839
	2	0.110	0.110	1.039	1.040
	3	0.107	0.107	0.996	0.998
	4	0.109	0.109	1.036	1.037
100	1	0.101	0.101	0.845	0.845
	2	0.107	0.107	1.029	1.031
	3	0.109	0.109	1.059	1.060
	4	0.109	0.109	1.093	1.094

TABLE VI
COMPUTATION TIME COMPARISON OF DIFFERENT CONTINGENCY LEVELS AND WIND POWER SCENARIO NUMBERS

N_s	K	PCE Basis (s)	Calling PCE function(s)	PCE total (s)	MC total (s)
10	1	4.67	0.003	4.67	1.03
	2	27.56	0.005	27.56	9.34
	3	158.74	0.008	158.75	55.87
	4	609.81	0.014	609.82	219.32
100	1	3.17	0.005	3.18	10.13
	2	28.53	0.002	28.54	104.28
	3	150.78	0.010	150.79	508.67
	4	586.76	0.053	586.81	1,971.27

2) Economy and Reliability Comparison: The reliability requirements for $\overline{EENS}$ and $\overline{LOLP}$ are 1 and 0.1, respectively.

The contingency level is 1 and the wind power scenario number is 10. Table VII shows the costs of the proposed RCED-AF model and the RCED based on Benders decomposition (RCED-BD) with relaxed reliability constraints, and chance-constrained economic dispatch (CCED) with confidential levels of 0.99 and 0.999. The RCED-BD is taken as a benchmark. The demand curtailment and operational reliability of RCED-BD and RCED-AF are evaluated along with decisions, while those of CCED are evaluated based on post-contingencies. It can be seen from Table VII that the operational costs are the same by the proposed RCED-AF and the RCED-BD models, but much higher by the CCED models. Main operation costs for the CCED models occur at the demand curtailment. Results of the operational reliability indices, LOLP and EENS, are displayed in Fig. 6. It is observed from Fig. 6 that LOLP and EENS of the RCED-AF and RCED-BD match very well. This indicates that the RCED-AF model can provide accurate optimization results. Fig. 6 also shows that both indices by CCED are higher than that by RCED. This implies that CCED models are not suitable for reliability evaluation. The RCED-AF and RCED-BD have relatively better effects on improving operational reliability.

TABLE VII
COST COMPARISON BETWEEN CASES (K=1, N_s=10)

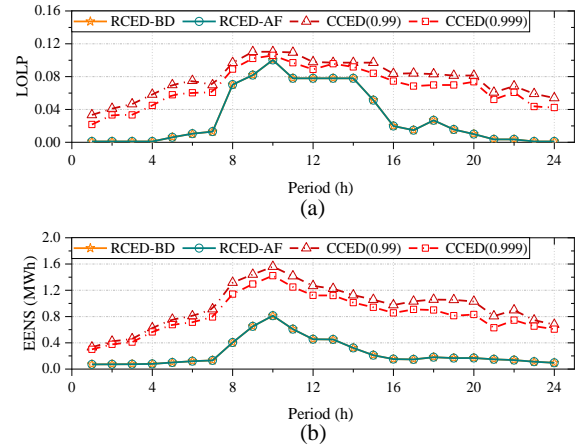| Cost Case	Total	CGU generation	CGU reserve	Demand curtailment
RCED-AF	$33,453	$10,568	$244	$22,641
RCED-BD	$33,453	$10,568	$244	$22,641
CCED (0.99)	$96,890	$8,059	$244	$88,587
CCED (0.999)	$85,696	$8,132	$244	$77,320

Fig. 6. Operational reliability indices comparison by RCED-AF, RCED-BD and CCED. (a) LOLP. (b) EENS.

3) Computation Efficiency Analysis for RCED: The number of OPF N_{pf} and computation time are shown in Table VIII, respectively. With contingency level 1 and wind power scenario number 10, the computation time of analytical formulation for 24 h is 145.11 s, while solving the RCED cost only 4.46 s. The total computation time of RCED-AF is 149.78 s, which is only 8.7% of the RCED-BD, 1,658.72 s. The total solution time is significantly reduced in the proposed RCED-AF model.

The OPF for RCED-AF is only required in the approximation

of polynomial coefficients. The number of contingencies is 19. Each contingency requires OPF of the number of collocation points N_{CP}, 46. Therefore, the number of OPFs for RCED-AF is 874. The RCED-BD, however, requires OPF for each contingency and wind power scenario, under each Benders decomposition iteration. This leads to 41,420 times OPF and a higher computation burden.

TABLE VIII
COMPUTATION EFFICIENCY COMPARISON (K=1, N_s=10)

RCED-AF		RCED-BD	
N_{pf}	Time (s)	N_{pf}	Time (s)
874	149.78	41420	1658.72

4) Reliability Constraints Analysis for RCED: Fig. 7 illustrates the demand curtailment cost and total cost corresponding to various EENS requirements. The curves show that as the reliability requirement $\overline{EENS}$ decreases, the demand curtailment cost associated with EENS increases. The total operation cost generally decreases as $\overline{EENS}$ decreases. Moreover, higher reliability requirements may not be met, as it is not feasible when $\overline{EENS}$ less than 0.8112 MWh. The lower reliability requirements can always be met as the reliability constraints are inactive for $\overline{EENS}$ greater than 0.8142 MWh. Fig. 8 illustrates the demand curtailment cost and total cost corresponding to various LOLP requirements. Similar to EENS requirements, it is not feasible when $\overline{LOLP}$ less than 0.05, and the total operation cost generally decreases as $\overline{LOLP}$ decreases. However, reducing the requirement for LOLP may not directly result in a decrease in the demand curtailment cost associated with EENS. When the $\overline{LOLP}$ is set to 0.1, the minimum demand curtailment cost is $ 22,641. Therefore, the proposed RCED-AF can help achieve reliability requirements and find the reliability boundaries by different reliability constraints.
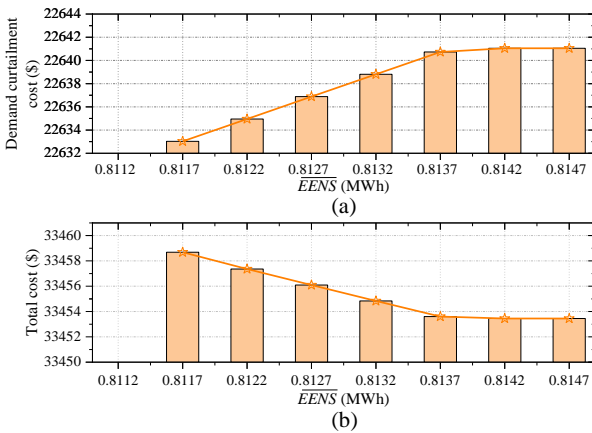

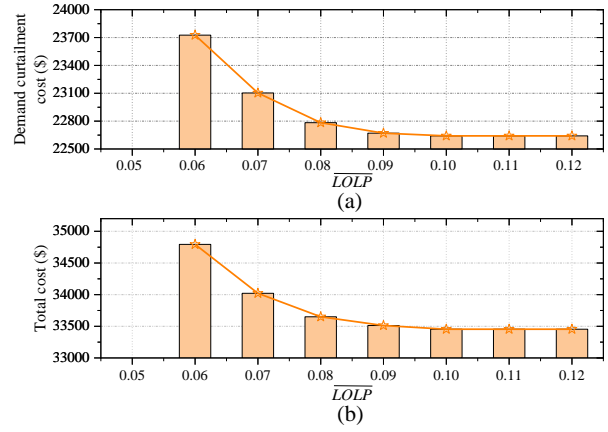Fig. 7. Operation costs under different EENS requirements. (a) Demand curtailment cost. (b) Total cost.

Fig. 8. Operation costs under different LOLP requirements. (a) Demand curtailment cost. (b) Total cost.

5) Wind Penetration Level Analysis: In this section, four wind penetration (WP) levels, 6.7%, 15.5%, 30.0%, and 46.2% are considered. The total capacity of wind farms is set as 15 MW, 30MW, 90MW, and 180 MW for WP1, WP2, WP3, and WP4, respectively. The reliability indices for various wind penetration levels are shown in Fig. 9. Note that for high-penetrate wind power systems, the CGUs operate at their minimum generation limitation. Therefore, the downward reserve requirements are removed in this section, to ensure the consumption of wind power.

The results of RCED-AF and RCED-BD match very well, which indicates that the RCED-AF model can provide accurate optimization results for different WP levels. Reliability indices LOLP and EENS of WP1 are larger than those of WP2, while WP2 is larger than WP3. This phenomenon occurs because the influence of CGUs outages diminishes with higher wind power levels, while the outages of wind turbines are disregarded. However, during some periods, such as periods 10 and 15, the indices of WP4 are larger than WP1, WP2, and WP3. The results denote that the reliability of extremely high-penetrate wind power systems, such as WP4, could be reduced due to the inherent uncertainty of wind power.
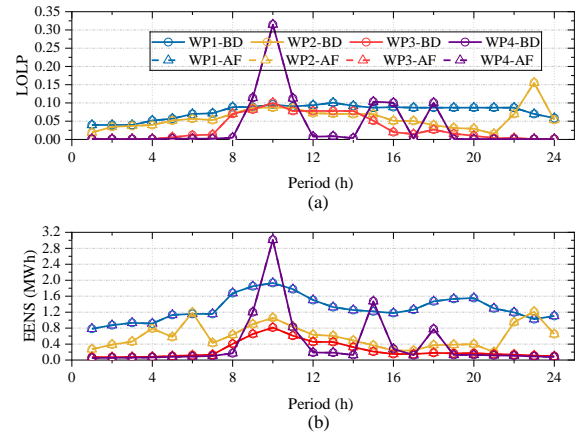

Fig. 9. Operational reliability indices under wind penetration. (a) LOLP. (b) EENS.

6) Demand Uncertainty in RCED: To model the demand uncertainty, the real-time demand scenarios $\mathbf{P}_t^D$ are incorporated into the stochastic input variables as $\mathbf{x}_t = [\mathbf{P}_t^W, \mathbf{P}_t^G, \mathbf{P}_t^D]$, assuming the real-time demand follows the normal distribution, and the standard deviation is 5% of the forecast value. According to the procedure of the RCED-AF, the operational reliability is optimized considering demand uncertainty. The total cost of the system with demand uncertainty is $ 39,613 for both RCED-BD and RCED-AF. This cost is higher compared to the system with no demand uncertainty, $32,307. The results demonstrate that power systems experience a decrease in both reliability and economic performance in the presence of demand uncertainty.

The reliability indices considering the demand uncertainty are compared in Fig. 10. The LOLP for the systems with demand uncertainty tends to be higher compared to systems with no demand uncertainty. Besides, the EENS for the systems with demand uncertainty is larger compared to those with no demand uncertainty. This is because the demand uncertainty increases the risk of demand curtailment. The computation time of RCED-BD is 3,902 s, while the proposed RCED-AF method takes 146 s.
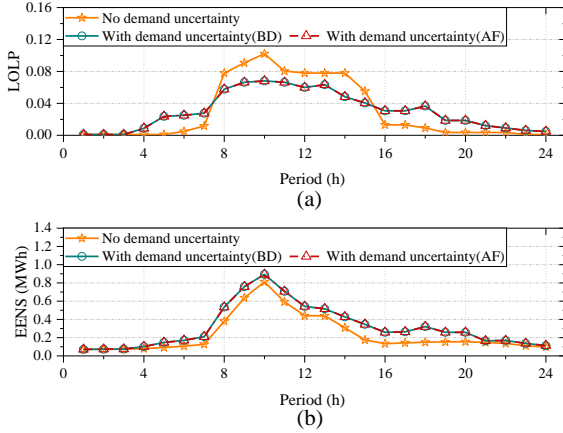
RCED-SO are shown in Table IX, respectively. The system total costs, unit generation cost, reserve cost, and demand curtailment cost of RCED-AF closely match those of RCED-BD, with relative errors of 0.005%, 0%, 0%, and 0.013% respectively. Besides, the values of RCED-SO are with relative errors of 1.25%, 0.13%, 0%, and 3.39% respectively. Therefore, the proposed RCED-AF is more accurate than the RCED-SO with scenario reduction.

The detailed operational reliability indices are shown in Fig. 11. The indices LOLP and EENS of the RCED-AF and RCED-BD match very well, which indicates that the RCED-AF model can deliver accurate optimization results. In the RCED-SO with the reduced scenario, higher errors in the indices are observed in certain periods, particularly periods 12-18 and 21-22.

TABLE IX
COST COMPARISON BETWEEN CASES

Cost Methods	Total	CGU generation	CGU reserve	Demand curtailment
RCED-AF	$988,212	$595,100	$6,357	$386,755
RCED-BD	$988,262	$595,100	$6,357	$386,804
RCED-SO	$1,000,643	$594,359	$6,357	$399,927

Fig. 10. The operational reliability indices considering demand uncertainty. (a) LOLP. (b) EENS.
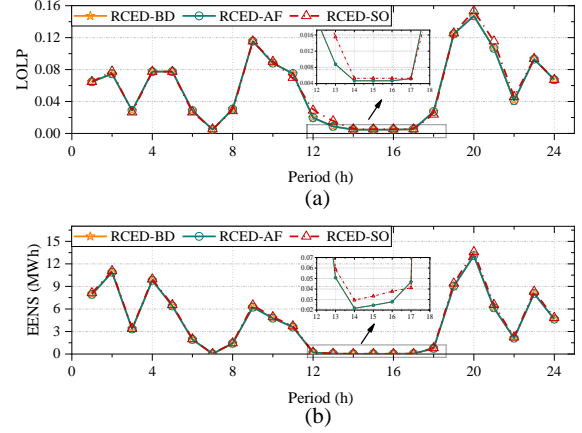
Fig. 11. Operational reliability indices of RCED-AF, RCED-BD, and RCED-SO. (a) LOLP. (b) EENS.

B. *Case 2: IEEE RTS-79 System*

The performance of the RCED-AF, RCED-BD, and stochastic optimization method (RCED-SO) with scenario reduction is compared on the IEEE RTS-79. The RTS-79 consists of 32 generating units, 33 transmission lines, and 5 transformers with an installed capacity of 3,405 MW and a peak load of 2,850 MW. Two CGUs on buses 18 and 21 are replaced by WFs with the same capacity, respectively. The reliability and operation parameters of the test system can be found in [38]. The wind scenario number is set to 50, and N_K=31 with K=1. The RCED-AF is based on the 1-order PCE (RA) to transfer the SCED-AF to LP programming. The number of wind power scenarios in RCED-SO is reduced to 10 by k-means clustering. The constraints for indices EENS and LOLP are relaxed.

1) Economy and Reliability Comparison: The system total costs of the proposed RCED-AF model, RCED-BD model, and

2) Computation Efficiency Analysis: Fig. 12 compares the computation times of RCED-AF, RCED-BD, and RCED-SO models as the number of wind power scenarios increases. With N_s=10, the total computation times of RCED-AF and RCED-SO are close but lower than that of RCED-BD. With N_s=50, the total computation time of RCED-AF is 528 s, of which the solution time is 5.53 s. In other words, the RCED-AF can be solved in seconds after constructing an analytical formulation. However, the computation time of RCED-BD is 8,889 s, and the RCED-SO is unsolved after 16,230 s calculation as out of memory. Thus, the RCED-AF significantly enhances efficiency, particularly in situations involving a large number of wind power scenarios.
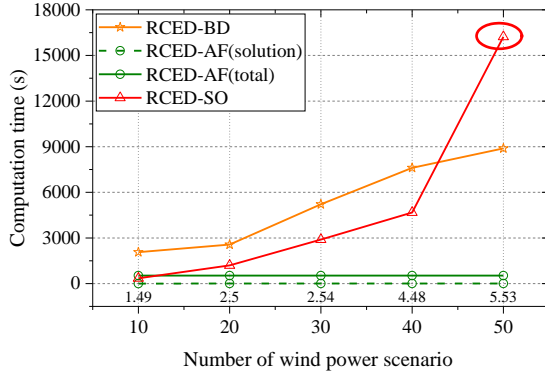
Fig. 12 The computation time of different N_s. The computation times of the proposed RCED-AF (green) are less than 10 seconds.

C. Case 3: Scalability Test on the IEEE 118-bus System

The RCED-AF is also conducted and compared with RCED-BD with relaxed reliability constraints, on the modified IEEE 118-bus system. The parameters of the test system can be found in [5]. Detailed operation data of the CGUs are derived from the previous study [39]. The CGUs at buses 10, 12, 49, 54, and 59 are replaced by wind turbines with the same capacity. The total installed capacity of CGUs and the wind are, respectively, 1,300 MW and 5,920 MW. The peak demand is 3,668 MW.

In this system, N_s=10, and N_K=50 with K=1. The system costs of the RCED-AF and RCED-BD model are shown in Table X. The system total costs, unit generation cost, reserve cost, and demand curtailment cost of RCED-AF closely match those of RCED-BD, with relative errors of 0.08%, 0.001%, 0%, and 1.85%, respectively.

The computation time of the proposed RCED-AF and that of RCED-BD are shown in Table XI. Each contingency requires OPF of the collocation points number 218. The number of OPF for RCED-AF is 10,900, while RCED-BD requires 608,000 times OPF, which significantly increases the computation cost. It is observed from Table XI that the total computation time for running RCED-AF is 1,970 s, which is 9.8% of 20,038 s for the RCED-BD.

TABLE X
COST COMPARISON BETWEEN CASES (K=1, S=10)

Cost / Case	Total	CGU generation	CGU reserve	Demand curtailment
RCED-AF	$825,208	$536,859	$252,829	$35,519
RCED-BD	$824,557	$536,865	$252,829	$34,863

TABLE XI
COMPUTATION EFFICIENCY COMPARISON

RCED-AF		RCED-BD	
N_{pf}	Time (s)	N_{pf}	Time (s)
10,900	1,970	608,000	20,038

VI. CONCLUSION

This paper presented a reliability-constrained economic dispatch model with an analytical formulation of reliability constraints to deal with the time-varying behavior of wind-integrated power systems. This model considers the day-ahead decision-making process of economic dispatch as well as the uncertainty associated with the wind power. The proposed method can be used for wind power variables with specific probability distributions and those under the distributionally agnostic situations.

Simulation results have demonstrated that the proposed analytical approximation scheme is very effective in accurately capturing the operational reliability of power systems. Moreover, the proposed RCED-AF model significantly enhances the efficiency for power system operational decision-making, particularly in situations involving a large number of wind power scenarios. Further research is needed to expand the time frame for the day-ahead optimization and to improve the computation efficiency for offline calculation of analytical formulation. In addition, other operation decisions, such as demand response, storage, and network restructure could be incorporated into the RCED-AF model to improve the system cost and reliability performance.

APPENDIX A

FULLY EXPANDED FORMULAS OF THE OPF MODEL FOR
SYSTEM STATE ANALYSIS

The real-time maximum demand supply capacity $\overline{P_{k,t,s,\dot{d}}^{\mathrm{de,RT}}}$ for contingency k can be calculated based on the wind power scenario s, under a given ED decision $\dot{d}$ at period t. The optimal demand curtailment $ENS_{k,t,s,\dot{d}}$ can be acquired by

$$ENS_{k,t,s,\dot{d}} = \max\{0, (P_t^{\mathrm{de,RT}} - \overline{P_{k,t,s,\dot{d}}^{\mathrm{de,RT}}}) - \sum_g R_{g,t}^{\mathrm{U,DA}}\}. \quad (A1)$$

The upward reserve is assumed available and its total amount equals 3% of the forecast demand plus 5% of the forecast wind generation [40].

The mathematical formulation of the system state analysis consists of the demand-supply capacity maximization objective function (A2) and the constraints (A3)-(A8).

$$\max \overline{P_{k,t,s,\dot{d}}^{\mathrm{de,RT}}} \quad (A2)$$

s.t.

$$\underline{P_g} \cdot u_{g,t} \cdot z_{g,k}^{\mathrm{RT}} \leq P_{g,k,t,s}^{\mathrm{S,RT}} \leq P_{g,t}^{\mathrm{S,DA}} \cdot u_{g,t} \cdot z_{g,k}^{\mathrm{RT}} \quad (A3)$$

$$0 \leq P_{w,k,t,s}^{\mathrm{S,RT}} \leq P_{w,t,s}^{\mathrm{RT}} \quad (A4)$$

$$C^{\mathrm{G}} \cdot P_{g,t,s}^{\mathrm{S,RT}} + C^{\mathrm{W}} \cdot (P_{w,k,t,s}^{\mathrm{S,RT}} - P_{w,k,t,s}^{\mathrm{shed}}) - C^{\mathrm{ft}} \cdot P_{l,t,s}^{\mathrm{RT}} = C^{\mathrm{de}} \cdot \overline{P_{k,t,s,\dot{d}}^{\mathrm{de}}} \quad (A5)$$

$$P_{l,k,t,s}^{\mathrm{RT}} = \Delta\theta_{l,k,t,s}^{\mathrm{RT}} / X_l \quad (A6)$$

$$\underline{P_l} \cdot z_{l,k}^{\mathrm{RT}} \leq P_{l,k,t,s}^{\mathrm{RT}} \leq \overline{P_l} \cdot z_{l,k}^{\mathrm{RT}} \quad (A7)$$

$$\underline{\theta} \leq \theta_{k,t,s}^{\mathrm{RT}} \leq \overline{\theta}. \quad (A8)$$

The real-time generation of CGUs should consider the commitment $u_{g,t}$, the day-ahead dispatch decision $P_{g,t}^{\mathrm{S,DA}}$, and the availability of CGUs in the contingency k, as (A3). Real-time wind power in scenario is limited by the available wind power, as (A4). The power balance with wind shedding $P_{w,k,t,s}^{\mathrm{shed}}$ for each bus is presented as (A5), with a bus-generator connection matrix C^{G}, bus-WPGs connection matrix C^{W},

bus-line connection matrix C^{ft}, and bus-demand connection matrix C^{de}. The power flow of lines and phase angles for each scenario is limited as (A6)-(A8).

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