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Structured and Intuitive Phasor Transmission and Scattering Equations

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***Abstract*—Two versions of phasor transmission equations derived using power wave theory are presented. These equations aim at having similar structure as the well-known Friis' transmission equation and thus being easy to apply whenever the phase shift in the wireless link cannot be omitted. All versions are arranged so that they intuitively follow the flow of electromagnetic energy from the transmitter all the way to the receiver. In addition, two versions of phasor scattering equations using the same path loss and antenna quantities are introduced. These are intended to help with simplified analysis of communication channels with unintentional or intentional scatterers such as the reconfigurable intelligent surfaces.**

***Index Terms*—Antenna theory, Radiowave propagation**

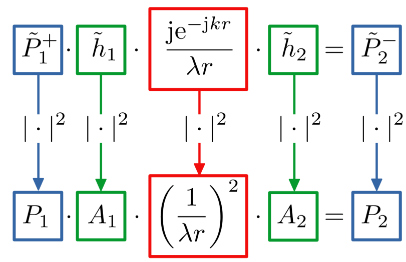

Fig. 1. Phasor transmission equation (top) and Friis' equation (bottom) where antennas are characterized by complex transfer functions and realized effective areas, respectively.

I. Introduction

Friis' transmission equation [1] is without doubt one of the most essential pieces of knowledge in wireless communication engineering. Its popularity is largely due to its structure, which naturally follows the flow of electromagnetic energy along the communication channel. Factors of the formula may be identified with distinct parts of the wireless link, and several useful quantities, such as the total radiated power, equivalent isotropic radiated power, or power density can be obtained as sub-expressions of the equation.

The drawback of the original Friis' equation is that it operates with power quantities, which do not satisfy the superposition principle. Therefore, it is not suitable for analyzing wireless links with multiple transmitters or receivers (including electrically large arrays for massive MIMO), or with multipath propagation, where different paths contribute with different phase shifts. Moreover, it is not applicable to wireless links with ultrawideband pulses that require time domain analysis, and for which various versions of time-domain transmission equations have been proposed. The need for phase in the transmission equation has been addressed by various alternatives to Friis' equation using phasors, with antennas described by either a transfer function or a complex effective length [2]. An interesting finding followed, namely that transmitting and receiving antennas contribute differently to the wireless link, as a consequence of the Lorentz' reciprocity theorem, resulting in an asymmetric transmission equation [3].

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In our recent paper, we have proposed phasor alternatives to Friis' transmission equation that preserve the symmetry and modularity of the original [4]. The equations employ complex effective length vectors, both original and normalized, and a newly proposed field gain, serving as a phasor counterpart to the commonly used power gain. Voltage and field waves are used as sub-expressions of the equations and intermediate quantities along the communication channel.

In this paper, which is a contribution to the *Electromagnetic Education* session, we propose a symmetric phasor transmission equation that is further simplified by using power wave theory [5]. We demonstrate that the proposed equation is structured in a similar way as the Friis' original, i.e. it is symmetric and modular (see Fig. 1), and its application to practical problems is thus very intuitive, which is of great help when teaching new generations of wireless engineers. Both transfer function and field gain versions of the equation have been derived.

In addition, two versions of phasor scattering equations are proposed, following the same symmetric and modular approach. These should help analyzing wireless communication channels with scatterers, either unintentional or intentional such as the reconfigurable intelligent surfaces (RIS) [6].

II. Transmission Equation

Let us assume that all field and circuit quantities are rms phasors with the $\mathrm{e}^{\mathrm{j}\omega t}$ convention, with frequency f in $\omega = 2\pi f$, and wavelength λ. Also, the transmitting (TX) and receiving (RX) antennas separated by distance r are placed in free space described by propagation constant k, both are fed with single mode transmission lines (such as coaxial cables), and there is a polarization match between them.

Although the original paper of Friis [1] does not display it in this form, the classical version of Friis' transmission equation can be written as

$$P_1 \cdot A_1 \cdot \left(\frac{1}{\lambda r}\right)^2 \cdot A_2 = P_2 \tag{1}$$

The arrangement of terms in (1) has the advantage that it visually follows the flow of electromagnetic energy through the wireless link from left to right. Input power P_1 feeds the TX antenna with effective area A_1, which radiates the electromagnetic energy through free space with path loss $(1/\lambda r)^2$, until it arrives at the RX antenna with effective area A_2, which then produces power P_2 at its output. The effective areas are considered as realized, i.e. containing any mismatch effects between the antenna and its feeder. In addition, the subexpression of (1) to the left of A_2 is equal to the power density of the propagating plane wave at distance r from the TX antenna, right before it arrives at the RX antenna.

In many situations, however, it is necessary to know also the phase of the involved signals, such as in multipath propagation or with multiple TX and RX. This can be solved by using a phasor transmission equation, of which one possible form can be [4]

$$\tilde{P}_1^+ \cdot \tilde{h}_1 \cdot \frac{\mathrm{j}\mathrm{e}^{-\mathrm{j}kr}}{\lambda r} \cdot \tilde{h}_2 = \tilde{P}_2^- \tag{2}$$

Here, $\tilde{P}_1^+$ is the power wave incoming (+) from TX to the terminals of the TX antenna and $\tilde{P}_2^-$ is the power wave outgoing (−) from the RX antenna terminals to RX. The TX and RX antennas are characterized by complex values $\tilde{h}_1$ and $\tilde{h}_2$, respectively. This value can be called a *transfer function* of an antenna [7], or, alternatively, a *normalized effective length* [4], since its dimension is meters and it is proportional to the traditional effective length h by

$$\tilde{h} = \frac{h}{2}\sqrt{\frac{\eta_0}{Z_0}} \tag{3}$$

where η_0 is the free-space wave impedance and Z_0 is the reference impedance for the power waves at the antenna terminals. Just like the effective areas in (1), also the transfer function contains the mismatch effects of the antenna and its feeder.

The elegance of (2) lies in its one-to-one relationship with the Friis' equation (1) as shown in Fig. 1. Each term of the Friis' equation can be obtained simply as a square of the magnitude of its counterpart in the phasor equation, and the wireless link is again intuitively traversed from left to right following the multiplication.

Furthermore, similarly to (1), the subexpression of (2) to the left of $\tilde{h}_2$ is equal to the normalized electric field of the plane wave incident on the RX antenna $\tilde{E}_2 = E_2/\sqrt{\eta_0}$, where E_2 is the incident plane wave electric field. From Fig. 1 it follows that the power wave-like quantity $\tilde{E}_2$ has the familiar simple relation to the power density $S_2 = |\tilde{E}_2|^2$. The mentioned subexpression can, for example, be conveniently used to determine the transfer function $\tilde{h}_1$ from measured electric field when the input power wave is known.

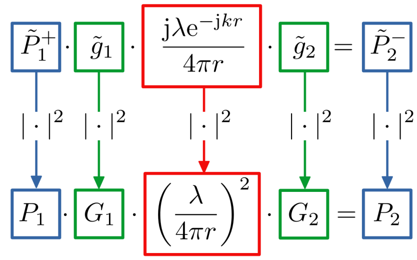

Fig. 2. Phasor transmission equation (top) and Friis' equation (bottom) where antennas are characterized by field gains and realized gains, respectively.

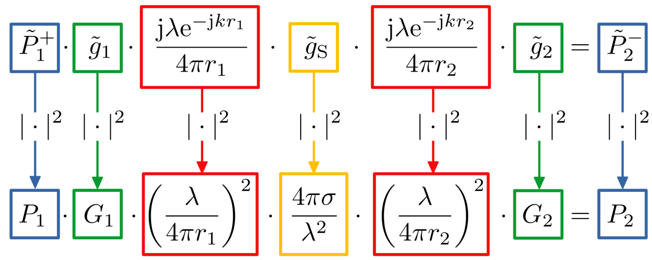

Fig. 3. Phasor scattering equation (top) and symmetrized radar equation (bottom) where the scatterer is characterized by the complex scattering field gain and RCS, respectively.

Another form of Friis' equation is also common in practice, using realized power gains G_1 and G_2 instead of realized effective areas [8]. Also this form can have a phasor version with one-by-one counterparts to each of the terms, as demonstrated in Fig. 2. The counterpart to the realized power gain G is the *field gain* $\tilde{g}$ defined in [4] and equivalent to another form of transfer function introduced in [9].

III. Scattering Equation

If the wireless link contains a scatterer, as in the case of a radar target or RIS, the power received at RX is expressed using the radar range equation [8], in its simplest form omitting the polarization mismatch and reshuffling the terms to again follow the signal path:

$$P_1 \cdot G_1 \cdot \frac{1}{4\pi r_1^2} \cdot \sigma \cdot \left(\frac{\lambda}{4\pi r_2}\right)^2 \cdot G_2 \;=\; P_2 \tag{4}$$

Here, r_1 and r_2 are the distances of TX and RX from the scatterer, respectively, and σ is the radar cross section (RCS) of the scatterer. What is not very satisfying on (4) is the asymmetry of the equation, in that it uses two different types of path loss: the spherical power spreading for the first segment from TX to the scatterer and the path loss term from Friis' equation in Fig. 2 for the second segment from the scatterer to RX.

Figure 3 presents a symmetrized radar equation in the same style as Fig. 2 together with the proposed phasor version. To ensure a simple conversion, the scatterer is characterized by a complex scattering field gain $\tilde{g}_\mathrm{S}$. This dimensionless quantity

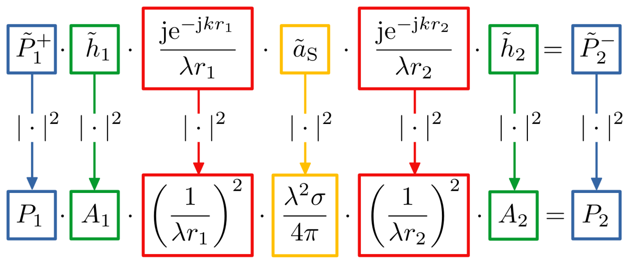

Fig. 4. Phasor scattering equation (top) and symmetrized radar equation (bottom) where the scatterer is characterized by the complex scattering area and RCS, respectively.

is equivalent to a product of two field gains, as if the scatterer consisted of two interconnected antennas characterized by these field gains.

Another version of the scattering equation is also possible, with path loss and antenna characterization as in Fig. 1. This version is shown in Fig. 4 and features a complex scattering area $\tilde{a}_\mathrm{S}$ as the phasor counterpart to RCS. This quantity has a dimension of square meters and is equivalent to a product of two transfer functions (or normalized effective lengths), again as if the scatterer consisted of two interconnected antennas characterized by the respective transfer functions.

IV. Discussion and Conclusion

Two versions of phasor transmission equations and two versions of phasor scattering equations have been introduced. The equations are intentionally arranged in a structured way that allows to visually follow the flow of electromagnetic energy through distinct parts of the communication channel. Unlike the recently proposed antenna equation [10], the present approach aims at symmetric arrangement with the TX and RX antennas characterized by the same quantity, in the best tradition of the well-established Friis' transmission equation. Besides, the notation of the involved phasor quantities utilizes a simple accent, the tilde (˜), which, thanks to its shape ("wave"), gives a hint that the particular quantity is based on power wave theory. Overall, these properties should help with simplified analysis of wireless links, where phase cannot be omitted from the calculations, such as in fading scenarios both with or without scatterers.

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