Selecting Actuator Configuration for a Benson Boiler

*Production Economics*

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Abstract—This paper addresses the problem of an optimal actuator configuration in an economic perspective. The objective is to minimize the economical cost of operating a given plant. Functionals encapsulating information of the business objectives given the different actuators have been established with particular focus on a boiler in a power plant operated by DONG Energy - a Danish energy supplier. The problem has been reformulated using mathematical notions from economics. The selection of actuator configuration has been limited to the fuel system which in the considered plant consists of three different fuels - coal, gas, and oil.

I. INTRODUCTION

The selection of sensors and actuators has usually depended greatly on the designer’s system knowledge, however, in recent years more focus has been placed on developing tools to aid the designer during this phase because processes are becoming more complex and difficult to assess. One such tool is the Relative Gain Array, which is used to pair inputs and outputs in a multiple input multiple output system to enable decentralized single input single output control [1, page 90].

The placement of sensors and actuators has been studied for different applications and [2] reviews methods used in the aerospace industry. More general purpose methods for selecting and placing sensors and actuators have been evaluated in [3] and [4], which include e.g. methods relying on controllability measures such as state reachability and more sophisticated methods using robust performance measures. It is also concluded in [3] that the choice of sensors and actuators dictates the expenses for hardware, implementation, operation, and maintenance.

Early reference [5] proposes a number of hierarchical structures for a process where the highest abstraction level deals with economical objectives. Optimum steady-state operations in such systems have been considered in [6] where an online iterative procedure to calculate the system set-point has been developed. However, the problem of profit optimization and actuator layout have not been considered.

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The requirements for a process control system are specified for the very top level and thus it would be natural to include the business objectives when configuring the sensor/actuator layout of a plant. An attempt of this has been presented in [7] where functionals describing the business objectives are maximized. This work will also follow this approach and utilize notions from production economics.

The goal of most companies is to maximize their profit and is treated in production economics, which under certain assumptions can be used to determine how the production plan of a company should be to maximize the profit [8]. When viewing a market from the production perspective one usually defines a number of firms and the “goods” they are capable of producing. The firms are viewed as black boxes able to transform inputs to outputs.

II. PROBLEM FORMULATION

The problem considered in this work is presented in Section II where the problem is also formulated. DONG Energy’s business objectives are described in Section III as static models in terms of the actuators considered. In Section IV the problem is solved using the static models and the results are presented. The static models are expanded in Section V to include some of the time varying parameters present. The time varying formulation is solved in Section VI and finally a discussion about the results is brought in Section VII.

A. Outline

The plant considered in this work is presented in Section II where the problem is also formulated. DONG Energy’s business objectives are described in Section III as static models in terms of the actuators considered. In Section IV the problem is solved using the static models and the results are presented. The static models are expanded in Section V to include some of the time varying parameters present. The time varying formulation is solved in Section VI and finally a discussion about the results is brought in Section VII.

I. Introduction

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The power plant considered in this paper consists of the following components:

Fuel system: The fuel system prepares the different fuels for burning, e.g. the coal mills grind the coal to small dust particles which burn quickly and efficiently.
Some characteristics of the different fuels are:

Gas is easy to control but it is an expensive fuel. Gas is more expensive of the three fuels but does have other advantages; it is possible to measure the oil flow into the boiler. However, it needs to be preheated before entering the boiler and this requires energy. This places oil between gas and coal when considering the own-consumption.

**B. Problem**

The focus of this work is to derive a mixture of the three fuels, described above, which will yield the greatest profit under consideration of the three business objectives, i.e., models of three business objectives considering the three fuels are needed.

**III. Static Plant Model**

Let \( \mathbb{R}^3_+ \) denote the set of positive elements in \( \mathbb{R}^3 \) i.e. \( \mathbb{R}^3_+ = \{ v \in \mathbb{R}^3 | v \geq 0 \} \) where the inequality is to understood coordinate wise.

The input space \( X \) is given by

\[
X = \{ v \in \mathbb{R}^3_+ | 0 \leq (v|u) \leq c \},
\]

where \((\cdot|\cdot)\) is the Euclidean inner product, and the vector \( u = (u_1, u_2, u_3) \in \mathbb{R}^3 \) with \( u > 0 \) and scalar \( c \in \mathbb{R} \) are to be determined in Section III-A. Note that \( X \) is the 3-simplex in \( \mathbb{R}^3_+ \) with vertices \( 0, (c/u_1, 0, 0), (0, c/u_2, 0), \) and \( (0, 0, c/u_3) \).

Each input

\[
x = (x_c, x_g, x_o) \in X, 
\]

\( ([kg/s], [kg/s], [kg/s]) \),

to the system describe the flow of coal, gas, and oil, respectively. In the sequel we let \( I = \{ c, g, o \} \) where the elements of the index set \( I \) refers to the three different fuels. Occasionally the identification \((c, g, o) = (1, 2, 3)\) will be used.

The output space \( Y = Y_1 \times Y_2 \times Y_3 \) is a subset of \( \mathbb{R}^3 \) where each output

\[
y = (y_c, y_g, y_o) \in Y, 
\]

\( ([MW], [MW/s], [MW]) \),

of the system describe the three objectives; efficiency, controllability, and availability, respectively. Each of these quantities contains contributions from coal, gas and oil as will be explained later.

Simple functions describing the three business objectives at steady state are derived in the following, i.e., it is assumed that the plant production is constant at a given set point.

**A. Efficiency**

The efficiency objective deals with how much electricity is produced from a certain amount of fuel. Three affine functions describing the contribution of the individual
fuels to the efficiency objective $y_e$ have been established using measurement data from two Danish power plants

$$y_{ec}(x_c) = e_c x_c + e'_c = 10.77 x_c - 1.76$$
$$y_{eg}(x_g) = e_g x_g + e'_g = 18.87 x_g + 1.85$$
$$y_{eo}(x_o) = e_o x_o + e'_o = 15.77 x_o - 0.37.$$ 

The total amount of efficiency is described by the function

$$X \rightarrow Y_1; \ y_e(x) = \sum_{i \in \mathcal{I}} y_{ei}(x_i) = (x|u) + c', \quad (2)$$

where $e = (e_c, e_g, e_o)$, $c' = \sum e'_i$, and $Y_1 = (0, 400)$. Here 400 refers to the maximum efficiency of the plant. Using the above construction the input space $X$ in (1) can now be described in more details as

$$X = \{ v \in \mathbb{R}^3_+ | 0 \leq (v|e) \leq c \},$$

with $e$ as defined above and $c = 400 - c'$, where $c'$ is a measure of the own-consumption of the power plant, i.e., the electricity used in idle running.

**B. Controllability**

The controllability objective deals with a measure of how fast the production of electricity can be changed. Allowed changes in the production is limited to a certain gradient depending on the current production ($y_e(x)$ in this work). The reason for this limit is a compliance to maximum temperature gradients in the boiler (these have not been modelled and are therefore indirectly considered this way). When running the plant in ranges $0 \mathrm{[MW]}$ to $200 \mathrm{[MW]}$ and $360 \mathrm{[MW]}$ to $400 \mathrm{[MW]}$ it is allowed to change production with $0.133 \mathrm{[MW/s]}$ independently of fuel. However, in the range $200 \mathrm{[MW]}$ to $360 \mathrm{[MW]}$ the allowed changes are dependent of which fuel is used. If coal is used it is allowed to change production with $0.267 \mathrm{[MW/s]}$ and when using oil and gas the allowed change is $0.534 \mathrm{[MW/s]}$. The changes allowed is modelled as piece-wise constant functions $h : Y_1 \rightarrow \mathbb{R}, i \in \{1, 2, 3\}$ given by

$$h_i(y_1) = \begin{cases} 0.133 & y_1 \in (0, 200) \cup (360, 400) \\ 0.267 \cdot i & y_1 \in [200, 360] \end{cases} \quad i = 1, 2$$
$$h_2 = h_3.$$

If a mixture of the three fuels are used it is assumed that the allowed change is a convex combination of the allowed change of the individual fuels. The controllability objective is modelled as these allowed changes and hence, the controllability of each actuator can be described as

$$y_{ec}(x) = \frac{y_{ec}(x_c)}{y_e(x)} h_1(y_e(x))$$
$$y_{eg}(x) = \frac{y_{eg}(x_g)}{y_e(x)} h_2(y_e(x))$$
$$y_{eo}(x) = \frac{y_{eo}(x_o)}{y_e(x)} h_3(y_e(x)).$$

Therefore, the total amount of controllability is described by the function

$$X \rightarrow Y_2; \ y_e(x) = \sum_{i \in \mathcal{I}} y_{ei}(x_i). \quad (3)$$

**C. Availability**

The availability objective can in terms of actuators be described as the amount of actuation power left. The coal system is capable of delivering coal for a production of $532 \mathrm{[MW]}$, gas $452 \mathrm{[MW]}$, and oil $480 \mathrm{[MW]}$. Thus the available reserve actuation of each actuator can be described as

$$y_{ac}(x_c) = a_c - y_{ac}(x_c) = 532 - y_{ac}(x_c)$$
$$y_{ag}(x_g) = a_g - y_{ag}(x_g) = 452 - y_{ag}(x_g)$$
$$y_{ao}(x_o) = a_o - y_{ao}(x_o) = 480 - y_{ao}(x_o).$$

The total amount of availability is described by the function

$$X \rightarrow Y_3; \ y_a(x) = \sum_{i \in \mathcal{I}} y_{ai}(x_i). \quad (4)$$

**D. Prices**

The above constructions now yields a product (or output) function $y_P$ of the system given by

$$y_P : X \rightarrow Y; \ x \mapsto (y_e(x), y_a(x), y_o(x)).$$

The cost $f_{pc}$ and earning $f_{pe}$ for the system are defined by

$$f_{pc} : X \rightarrow \mathbb{R}; \ x \mapsto (x|pc), \quad ([dkk/s]), \quad pc > 0,$$
$$f_{pe} : Y \rightarrow \mathbb{R}; \ y \mapsto (y|pe), \quad ([dkk/s]), \quad pe > 0,$$

with the (price) vectors $pc = (p_{c1}, p_{c2}, p_{c3}) = (1.2, 3.74, 6) \mathrm{[dkk/s]}$ and $pe = (p_{e1}, p_{e2}, p_{e3}) = (0.16, 247, 0.39) \mathrm{[dkk]}$ fixed. The prices used in this work corresponds to maximum market prices on the 29th of June, 2008 as will be described in Section V.

A growth in profit function can be defined by

$$X \times Y \rightarrow \mathbb{R}; \ (x, y) \mapsto (y|pe) - (x|pc),$$

which for the system yields

$$f_P : X \rightarrow \mathbb{R}; \ x \mapsto (y_P(x)|pe) - (x|pc).$$

**IV. Static Optimization**

For a given $y_1^* \in Y_1$ we wish to solve

$$\max_{x \in \mathbb{R}^3 \cap \{y_1^*\}} f_P(x), \quad (5)$$

where we note that $y_1^* \mathrm{[y_1]}$ is the 2-simplex in $\mathbb{R}^3$ with vertices $v_x^1 = (y_1^* - c')/u_1, 0, 0)$, $v_x^2 = (0, y_1^* - c')/u_2, 0)$, and $v_x^3 = (0, 0, y_1^* - c')/u_3).$ The optimal actuator configuration, $x^*$, can now be formulated by

$$x^* = \arg \max_{x \in \mathbb{R}^3 \cap \{y_1^*\}} (y_P(x)|pe) - (x|pc). \quad (6)$$

Equation (6) has been solved using MATLAB for $y_1^* \in L, L \in \{1, 2, ..., 400\}$. Figure 2 depicts the solution where
the top graph is the growth of profit and the bottom graph is the fuel configuration, both as functions of \( y_1^* \). The vertical-axis in the bottom graph should be read with the identification \((1, 2, 3) = (v_{x1}^*, v_{x2}^*, v_{x3}^*)\). As seen in the figure the optimal configuration is changed when \( y_1 \in [200, 360] \cap L \) from using only coal to using only gas. This happens because using gas in this range allows for more controllability and thus a larger growth of profit.

V. DYNAMIC PLANT MODEL

The electricity production of a power plant is not constant during the year or even during 24 hours. However, prediction of the demand for electricity 24 hours into the future makes it possible to plan production ahead of time. During this planning for the entire electrical grid a production plan is fitted to the capabilities of the individual plants, i.e. a production plan \((y_1 \text{ reference})\) is delivered to each power plant. During this planning the prices of efficiency, controllability and availability are also established. In the following these changes will be described and models of the effect will be derived.

A. Production Plan

An example of a production plan for the considered plant is depicted in Figure 3. The graph depicts the planned production from midnight the 29th of June, 2008 and 24 hour forward. As seen in the figure the production is rather low during the night but at 6 in the morning there is a steep gradient caused by the increase in consumption when people and companies start to use electricity. The production plan is coarsely modelled as piecewise affine functions which are also depicted in Figure 3.

B. Efficiency Price

The price of electricity, \( p_{E1} \), changes during the day as the demand changes, e.g., during the middle of the day when the demand is greatest the price is higher than during the early morning. The electricity price from the 29th of June 2008 is depicted in Figure 4. The figure also depicts a coarse piecewise affine model of the efficiency price which is used to describe the dynamic of the price.

C. Controllability Price

Large gradients in the production plan, as seen in Figure 3 around 6, yield a high price on controllability as it is likely that some plants are not capable of generating the gradients needed. Thus, in general the controllability price would be related to the derivative of the production plan which is depicted in Figure 5. In this work the controllability is coarsely modelled as piecewise affine functions which is depicted in Figure 5. It can be
observed that the price is high during the gradients in the morning and afternoon/evening.

![Fig. 5. Production plan gradient and controllability price during 24 hours. The approximation used in this work is depicted by the dashed graph.](image)

### D. Availability Price

No data is available to describe the availability price, $p_{E3}$. It is, however, expected that the availability price is large during high loads because a failure is more critical than during low loads. Therefore, the availability price is modelled to follow the same trends as the production plan i.e., it is high during the middle of the day and low in the morning and evening. The coarse model is depicted in Figure 6.

![Fig. 6. The availability price model during 24 hours.](image)

### E. Fuel Price

Obviously the fuel prices change over time, however, these changes are slow compared to the changes described in the previous. The time span is a matter of weeks and is therefore compared to the above roughly constant and the fuel prices given in Section III are used.

### VI. Fuel Selection in Dynamic Case

The problem in (5) is reformulated to maximize the growth of profit given the constraints described in Section V. Using these constraints necessitates some reformulation of $f_{PE}$, and thus $f_P$.

The earning $f_{PE}$ for the system will now be time dependent and defined by

$$ f_{PE} : Y \times \mathbb{R} \rightarrow \mathbb{R}; (y, t) \mapsto (yp_E(t)), \quad p_E > 0, $$

with $p_E(t) = (p_{E1}(t), p_{E2}(t), p_{E3}(t))$ as approximated in Section V-B, Section V-C, and Section V-D respectively.

The growth of profit function can then be defined by

$$ X \times Y \times \mathbb{R} \rightarrow \mathbb{R}; (x, y, t) \mapsto (yp_E(t)) - (x|p_C), $$

which for this particular system yields

$$ f_P : X \times \mathbb{R} \rightarrow \mathbb{R}; (x, t) \mapsto (yp(x)p_E(t)) - (x|p_C). $$

Furthermore, the described dynamics introduces a function describing the time evolution of the reference, $y_1(t)$. Thus the optimal fuel configuration to a given time can be formulated by

$$ x^*(t) = \arg \max_{x \in y_1^{-1}(y_1(t))} f_P(x, t). $$

As for the static optimization, the above problem is, for a given time $t_1$, linear and the solution is found at one of the vertices $v_{x^*_1(t_1)} = ((y_1^*(t_1) - c')/u_1, 0, 0)$, $v_{x^*_2(t_1)} = (0, (y_1^*(t_2) - c')/u_2, 0)$, and $v_{x^*_3(t_1)} = (0, 0, (y_1^*(t_3) - c')/u_3)$. The problem in (7) has been solved using MATLAB for each time step $t \in \{1, 2, ..., 86400\}$. Figure 7 depicts the result, where the top graph shows the growth of profit and the bottom graph depicts the optimal fuel configuration, where the identification $(1, 2, 3) = (v_{x^*_1(t)}, v_{x^*_2(t)}, v_{x^*_3(t)})$ is used. As depicted in the figure.

![Fig. 7. The growth of profit and optimal fuel configuration during 24 hours of operation.](image)
coal is used during the morning and evening hours when the production is low, which is expected from the static optimization. However, coal is also used during the middle of the day, when the value of controllability is low and thus nothing is gained by using gas as it is a more expensive fuel. Figure 8 depicts a comparison of the growth of profit during 24 hours using the optimal mixed fuel and only coal. As seen in the figure, the growth of profit is higher from 6:30-7:00 and 20:30-23:30 when using the mixed fuel.

![Comparison of Optimal Profit Growth to Coal](image)

Fig. 8. The growth of profit during 24 hours of operation using only coal.

VII. Discussion

In this work a model of three of DONG Energy’s business objectives has been formulated such that an selection between three different fuels can be performed in an optimal manner, i.e., how should the fuels be used to yield the greatest profit.

A static modelling and optimization is performed such that the optimal configuration can be found for a given production setpoint. The developed optimization method is then expanded to handle changes in prices and production reference. The result from this expansion is compared to a case where only coal is present and the use multiple fuels does increase the profit during some parts of the day.

The result from this work can be used in two ways; (online) to determine which fuels to use during the day and (offline) to determine if a plant should be instrumented with additional fuels.

However, future work should consider how the optimization can be performed when more advanced functions are used. The present method only works on linear/affine functions. One obvious expansion of the function would be inclusion of the allowed changes in form of the dynamical system description. Furthermore, it would be interesting to investigate how the fuel configuration would evolve during the day if the gas and/or oil system is capable of only e.g. 20% of production. Including environmental measures would also be very interesting and appropriate considering the focus on e.g. $CO_2$ emission.

REFERENCES