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Performability Measure for a Power Plant

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***Abstract*— This paper considers the problem of economical optimization of the power production in a power plant capable of utilizing three different fuel systems. The considered fuel systems are coal, gas, and oil; each has certain advantages and disadvantages e.g. gas is easier to control than coal but it is more expensive. A profit function is stated and an analysis of the optimal fuel configuration is performed based on the Hamiltonian from the maximum principle. The analysis leads to the introduction of a performability measure, which, when the value is above a confidence threshold, indicates that a change of fuel system usage is beneficial. That is, the performability measure determines when an increase of performance is possible.**

I. Introduction

The ultimate goal of a company is to maximize profit and therefore, a monetary optimization functional is in the focus of this work. Our approach has an added advantage that the different objectives (with different units) are mentally easy to asses against each other, i.e., the designer should try to fulfill the (those) objective(s) that yields the highest profit. In this work a power plant capable of using three different fuel systems is considered. The fuel systems considered in this work consist of a coal system, a gas system, and an oil system. Each of the fuel systems has certain advantages and disadvantages. Coal is inexpensive but usage of coal imposes some dynamical restrictions of how fast it is possible to change the production; the coal is first grinded in coal mills before the dust is burned in the furnace. When using gas or oil, on the other hand, the production is allowed to be changed faster as the fuel flow can be measured directly, which makes it easy to control the production. Gas and oil are, however, expensive fuels and thus for a given production/demand of electricity they deliver a lower profit (a more detail description of the different fuels and the advantages can be found in [1] and [2]).

Traditional thermal power plants, i.e., coal, gas, or oil fired power plants, have been studied in detail e.g. in [3]. A thermal power plant basically functions by burning a fuel in the boiler which evaporates water to steam under high pressure. The stream then drives a turbine generating electrical power which is delivered to the electrical grid. A first principle model of a thermal power plant has been developed in [4], where the considered fuel is coal dust which arises from a number of coal mills grinding the raw coal. The detailed model in [4] was used to establish an observer for the flow of coal into the boiler to improve the control of the coal mills. Simpler models for system control are presented in [5], where the different methods for changing the output from the complete Danish portfolio of DONG Energy[1] are described. Controlling this output can, e.g., be done by changing the fuel flow into the furnace; an effect which can be modeled by 3rd order dynamics.

Two of DONG Energy's four business objectives are considered. They deal with Controllability and Efficiency.

Controllability: is a measure of how well and how fast a power plant can be controlled to a given reference.

Efficiency: is a measure of how efficient the fuel is converted into electricity, i.e., given an amount of fuel how much electricity is produced.

The monetary value of these objectives is established using the price data available at Nord Pool [6] and in collaboration with DONG Energy. In Figure 1 the price of electricity during 24 hours is illustrated for the period from Marts 28th to April 26th, 2009.

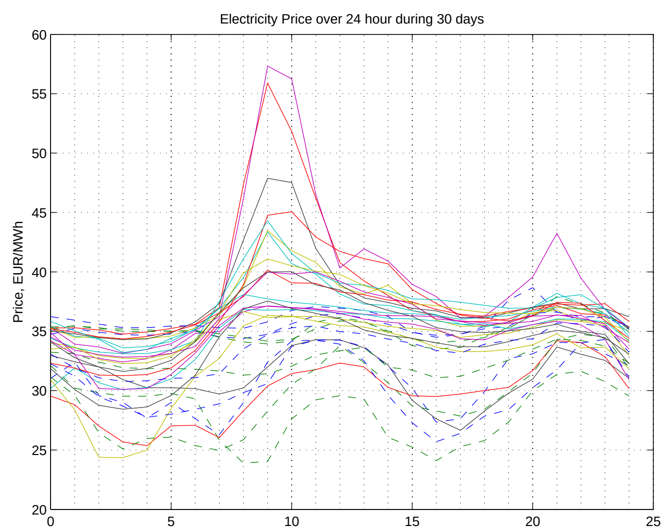

Fig. 1. The efficiency price over 24 hours from Marts 28th to April 26th 2009, where each day is depicted by a new graph. The dashed graphs illustrates the data for weekends and solids are the weekdays. The data used to generate this plot has been found on www.nordpool.dk

A prognosis of the next day's electricity consumption

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[1]DONG Energy is a Danish energy supplier

is established by Energinet.dk [7] which are responsible for the operation of the electrical grid in Denmark. The estimated electricity consumption in an area (e.g. West Denmark) is balanced by the desired energy production, which is shared between the different electricity producers in accordance with the bids on Nord Pool. Thereby a production plan is generated for each producer. The production plan used in this work is an approximation of data which has been delivered by DONG Energy. This is depicted in Figure 2, where the solid is the true data and the dashed is the approximation.

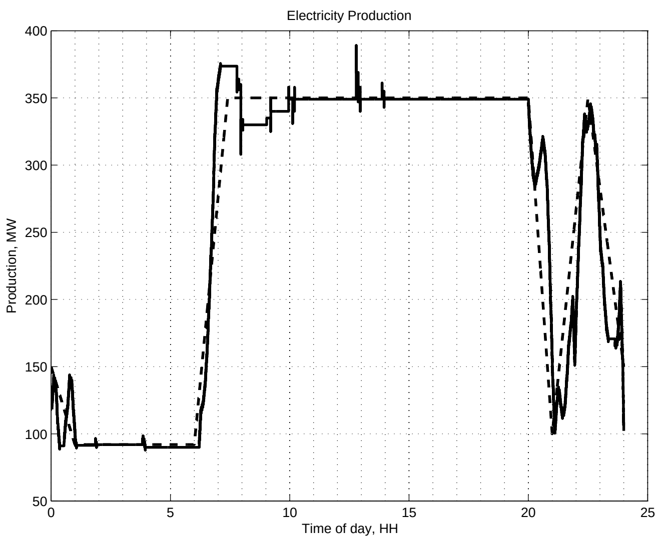

Fig. 2. The production during June 29th, 2008. The data used to generate this plot has been provided by DONG Energy (solid) and the approximation used in this work (dashed).

Similar ideas have been addressed in [8], where the data from Nord Pool has been used to schedule the usage of hydro power plants in Norway. In this work the production plan for the current day has been fulfilled while maximizing the profit of the hydro plant.

The high level control and planning structure of a power plant is illustrated in Figure 3 where solid illustrates the current configuration and dashed indicates the additions proposed by this work.

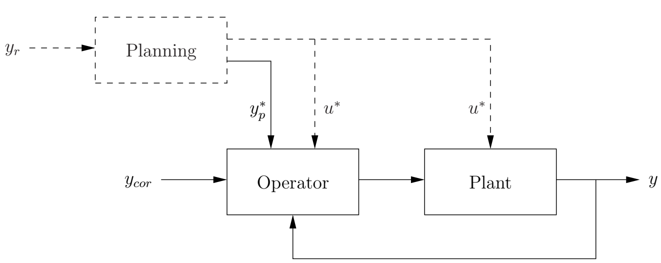

Fig. 3. Illustration of the high level control and planning structure of a power plant. The dashed indicates the addition proposed by this work.

A production plan (or reference), y_r, for the next 24 hours operation is provided to the power plant. Traditionally this production plan is delivered to the operator who controls the plant. The operator then controls the fuel flow into the plant such that the electricity prescribed by the production plan is generated. As the predictions used to generate the production plan do not necessarily fit the real life demands exactly, a correction is needed. This correction is handled by the electrical grid responsible and the correction signal, y_{cor} is fed to the operator, which adjusts the plant production accordingly (for further details see [9] , [10], and [11]).

In this work we introduce an additional planning level in the operation of the power plant before the production plan is delivered to the operator (see Figure 3). The planning level selects an optimal actuator configuration, y_p^*, for the particular production plan according to the business objectives (this is illustrated by the “Planning” block in Figure 3). The planning consists of selecting which actuator systems should be used, and a reference for these actuator systems, u^*, is delivered to the operator or directly to the actuator.

Pontryagin’s maximum principle [12] is used to derive a optimal strategy for selecting the active actuator configuration and a controller generating the control signal for each fuel system is designed using feedback linearization and reference tracking [13]. The problem posed in this work would conventionally be solved using numeric optimization or model predictive control. The numeric optimization method generates input signals offline and thus it is not possible to follow the production reference if a correction to the production plan is made online. Model predictive control, on the other hand, could take this into account but it is computational heavy as an optimization problem needs to be solve for each time step.

The proposed method combines the two methods by offline generating an optimal fuel configuration strategy which is used to produces the input signal for the plant online. This allows for small changes of the production reference online. The proposed strategy could approximately double the profit of the plant compared to a power plant using only coal. This is shown by using historic data from June 29th, 2008.

A. Outline

A model of the power plant considered in this work is presented in Section II, also models of the business objectives and the optimization problem are described inhere. In Section III Prontryagin’s maximum principle is applied to the optimization problem and switching signals is defined which indicates when a switch between three different fuel systems is beneficial. Furthermore, in Section IV a control strategy is proposed which, for given an active fuel system, tracks the production reference. In combination Section III and Section IV gives a profit maximizing strategy for the power plant. Finally in Section V the results are compared to other work and a discussion of the results is given including some suggestions for future work.

II. Problem Statement

In this section the models from our previous work will be recalled and then the optimization problem will be presented. The complete dynamics of the three different fuel systems of the power plant considered in this work is given by

$$\begin{aligned}\dot{\boldsymbol{z}}(t) &= \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{x}(t) &= \boldsymbol{C}\boldsymbol{z}(t),\end{aligned} \tag{1}$$

where $\boldsymbol{z}$ is the state, $\boldsymbol{x}$ is the flow of the three fuel systems, $\boldsymbol{u}$ is the control input, and the matrices are given by

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_c & \boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & \boldsymbol{A}_g & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x3} & \boldsymbol{A}_o \end{bmatrix}, \ \boldsymbol{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ h_{i_1} & h_{i_2} & h_{i_3} \end{bmatrix},$$
$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_c & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{B}_g & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{B}_o \end{bmatrix}, \ \boldsymbol{B}_i = \begin{bmatrix} 0 \\ 0 \\ h_{i_0} \end{bmatrix},$$
$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} \\ \boldsymbol{0}_{1x3} & \boldsymbol{C}_1 & \boldsymbol{0}_{1x3} \\ \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} & \boldsymbol{C}_1 \end{bmatrix}, \ \boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

with $h_{i_j}, i \in \mathcal{I} = \{c, g, o\}$, constants describing the dynamics of the three fuel systems. The h_{i_j}'s are obtained from transfer functions of the form

$$H_i(s) = \frac{1}{(\tau_i s + 1)^3},$$

where $\tau_i, i \in \mathcal{I}$, is 90, 60, and 70, respectively. See [14] for further comments on the above quantities.

Two business objectives, efficiency[2] and controllability, are considered in this work. The efficiency objective, $y_e(\boldsymbol{z})$, is modeled as

$$y_e(\boldsymbol{z}) = \boldsymbol{\gamma}^T \boldsymbol{Q}\boldsymbol{z} + \boldsymbol{\gamma}^T \boldsymbol{b},$$

where

$$\begin{aligned}\boldsymbol{Q} &= \operatorname{diag}(\boldsymbol{e}_x)\boldsymbol{C}, \quad \boldsymbol{e}_x = (10.77, 18.87, 15.77),\\ \boldsymbol{b} &= (-1.76, 1.85, -0.37), \quad \boldsymbol{\gamma} = (1, 1, 1),\end{aligned}$$

with $\boldsymbol{C}$ as in (1). In physical terms the vector $\boldsymbol{e}_x$ is a conversion factor from mass flow to energy flow for each of the three fuels. The vector $\boldsymbol{b}$ is a constant energy usage for each of the fuel system, i.e., the gas system generate electricity by lowing the gas pressure through a turbine and the coal and oil system require energy to grind the coal and heat the oil, respectively. The values of both $\boldsymbol{e}_x$ and $\boldsymbol{b}$ has been determined from measurement data provided by DONG Energy.

The controllability objective, $y_c(\boldsymbol{z}, t)$, is modeled as

$$y_c(\boldsymbol{z}, t) = \boldsymbol{\vartheta}(t)\boldsymbol{z} + \zeta(t),$$

[2]Efficiency is often also referred to as production but as it depends on the efficiency of the power plant and fuel system this notation will be used in this work.

where

$$\boldsymbol{\vartheta}(t) = \begin{cases} 0 & y_r(t) \in S_1 = \{s \in \mathbb{R} | 0 \leq s \leq 200\} \\ \frac{\boldsymbol{\xi}^T \tilde{\boldsymbol{Q}}}{y_r(t)} & y_r(t) \in S_2 = \{s \in \mathbb{R} | 200 < s < 360\} \\ 0 & y_r(t) \in S_3 = \{s \in \mathbb{R} | 360 \leq s \leq 400\}, \end{cases}$$

$$\zeta(t) = \begin{cases} 0.133 & y_r(t) \in S_1 \\ \frac{\boldsymbol{\xi}^T \boldsymbol{b}}{y_r(t)} & y_r(t) \in S_2 \\ 0.133 & y_r(t) \in S_3, \end{cases}$$

with $\boldsymbol{\xi} = (0.267, 0.534, 0.534)$, $y_r(t)$ denoting the reference signal. The sets S_1, S_2, and S_3 describes different operating regions of the power plant, where the value of controllability objectives for each fuels are different, i.e., $\boldsymbol{\xi}$ is the maximum value of each of the fuels when operation in the range $(200\,MW, 360\,MW)$ and in region S_1 and S_2 the value is 0.133 regardless of the fuel in use.

The functions in the above have in this work been approximated by piecewise affine functions. Now the growth of profit, $g_p(\boldsymbol{z}, t)$, is formulated as in [14], i.e.,

$$\begin{aligned} g_p(\boldsymbol{z}, t) &= \underbrace{p_{R1}(t) y_e(\boldsymbol{z}) + p_{R2}(t) y_c(\boldsymbol{z}, t)}_{\text{income}} - \underbrace{\boldsymbol{p}_C^T \boldsymbol{x}}_{\text{expense}} \\ &= \boldsymbol{\Theta}(t)\boldsymbol{z} + \varphi(t), \end{aligned}$$

where

$$\begin{aligned}\boldsymbol{\Theta}(t) &= p_{R1}(t)\boldsymbol{\gamma}^T \boldsymbol{Q} - \boldsymbol{p}_C^T \boldsymbol{C} + p_{R2}(t)\boldsymbol{\vartheta}(t),\\ \varphi(t) &= p_{R1}(t)\boldsymbol{\gamma}^T \boldsymbol{b} + p_{R2}(t)\zeta(t),\end{aligned}$$

with $\boldsymbol{p}_C$ a vector which entries are the price of the different fuels and p_{R1} and p_{R2} the prices imposed on the two business objectives, efficiency and controllability, respectively. Further description and explanation of the above quantities can be found in [1], [15], [2], and [14].

We can state the problem as an optimal control problem, i.e.

$$\begin{aligned} \max_{\boldsymbol{u} \in \mathcal{U}} \quad & \int_0^T g_p(\boldsymbol{z}, t) dt \\ \text{subject to} \quad & \dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t), \ \boldsymbol{z}(0) = \boldsymbol{z}_0 \end{aligned} \tag{2}$$

where $\boldsymbol{z}_0 = (13.95, \boldsymbol{0})$ and the input space, U, is defined as

$$U = \{\boldsymbol{u} \in \mathbb{R}^3_+ | \boldsymbol{e}_u^T \boldsymbol{u} \leq c_u\},$$

with $\boldsymbol{e}_u = (10.77, 18.87, 15.77)$ and $c_u = 400 - \boldsymbol{\gamma}^T \boldsymbol{b}$ as in [14]. This ensures that the flow of coil, gas, and oil is within the constraints of the plant at steady state. In Section IV an additional constraint for the efficiency output, $y_e(\boldsymbol{z})$, is imposed as the output should track the reference, $y_r(t)$.

III. Optimization

In the following section the problem in (2) without reference tracking is solved by means of Pontryagin's maximum principle, i.e., necessary conditions are deduced to obtain optimal solution candidates.

The Hamiltonian for the problem is given by

$$\begin{aligned} H(\boldsymbol{z},\boldsymbol{u},\boldsymbol{\lambda},t) &= g_p(\boldsymbol{z},t) + \boldsymbol{\lambda}^T(\boldsymbol{A}\boldsymbol{z}+\boldsymbol{B}\boldsymbol{u}) \\ &= \boldsymbol{\Theta}(t)\boldsymbol{z} + \boldsymbol{\varphi}(t) + \boldsymbol{\lambda}^T\boldsymbol{A}\boldsymbol{z} + \boldsymbol{\lambda}^T\boldsymbol{B}\boldsymbol{u}, \end{aligned} \quad (3)$$

and thus the adjoint equation is

$$\begin{aligned} \dot{\boldsymbol{\lambda}}(t) &= -\frac{\partial H(\boldsymbol{z}(t),\boldsymbol{u}(t),\boldsymbol{\lambda}(t),t)}{\partial \boldsymbol{z}} \\ &= -\boldsymbol{\Theta}(t)^T - \boldsymbol{A}^T\boldsymbol{\lambda}(t). \end{aligned} \quad (4)$$

Now assume that $\boldsymbol{u}^*(t)$ solves (2) and let $\boldsymbol{z}^*(t)$ be the associated optimal state obtained by solving (1) with $\boldsymbol{z}_0$ as the initial condition.

The pontryagin's maximum principle then yields the following point-wise maximization of (3)

$$\begin{aligned} H(\boldsymbol{z}^*(t),\boldsymbol{u}^*(t),\boldsymbol{\lambda}(t),t) &= \max_{\boldsymbol{u}\in U} H(\boldsymbol{z}^*(t),\boldsymbol{u},\boldsymbol{\lambda}(t),t) \\ &= \varphi(t) + (\boldsymbol{\Theta}(t)+\boldsymbol{\lambda}(t)^T\boldsymbol{A})\boldsymbol{z}^*(t) + \max_{\boldsymbol{u}\in U}\boldsymbol{\sigma}(t)^T\boldsymbol{u}, \end{aligned}$$

where $\boldsymbol{\sigma}(t) = \boldsymbol{B}^T\boldsymbol{\lambda}(t)$ and $\boldsymbol{\lambda}(t)$ is the solution to (4) fulfilling the transversality condition $\boldsymbol{\lambda}(T) = \boldsymbol{0}$. Hence the optimal input is located at the boundary of the feasible region of the input set U as this is maximization of a linear problem.

Since the adjoint equation (4) does not depend on the state we obtain an explicit expression of the switching function, $\boldsymbol{\sigma}(t)$ - see Figure 4. Thus it is possible to obtain the optimal fuel configuration explicit by

$$\boldsymbol{u}^*(t) = \arg\max_{\boldsymbol{u}\in V}\boldsymbol{\sigma}(t)^T\boldsymbol{u} \quad (5)$$

where V is the set of vertices in the input set U. The optimal fuel is depicted in Figure 5, where the identification 1 (coal), 2 (gas), and 3 (oil) should be used on the 2nd axis.

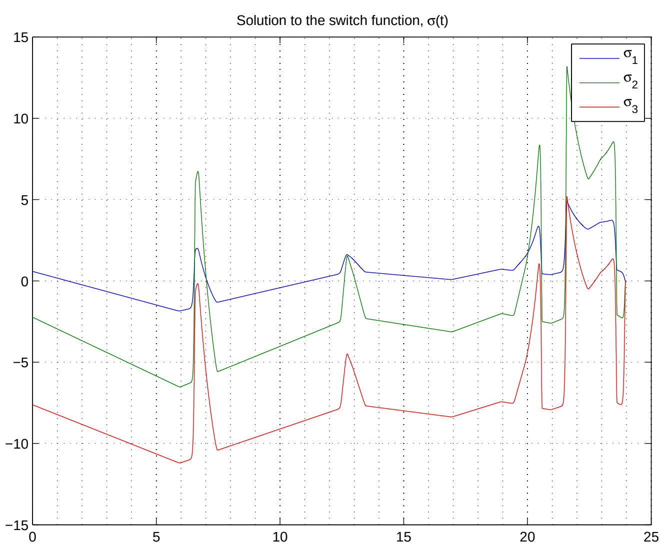

Fig. 4. Graph of the switch function, $\boldsymbol{\sigma}(t) = \boldsymbol{B}^T\boldsymbol{\lambda}(t)$, obtained by solving the adjoint equation using the transversality condition $\boldsymbol{\lambda}(T) = \boldsymbol{0}$.

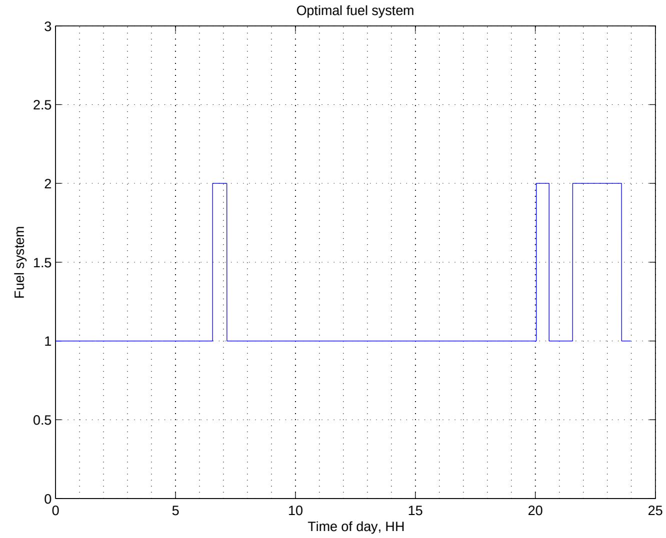

Fig. 5. The optimal fuel usage during June 29th, 2008. The identification 1=coal, 2=gas, and 3=oil should be used in the 2nd axis.

Remark that the optimal solution given by (5) does not in itself give an optimal strategy for the power plant operation as no reference tracking is included.

IV. Reference Tracking

In this section we include tracking of the reference, $y_r(t)$, in the optimal control problem in (2).

For each of the different fuel systems a reference tracking controller is constructed. Let $\boldsymbol{e} = (e_1, e_2, e_3)$ be the tracking error defined by

$$\begin{aligned} e_1 &= y_e - y_r = \gamma^T\boldsymbol{Q}\boldsymbol{z} + \gamma^T\boldsymbol{b} - y_r \\ e_2 &= \dot{e}_1 = \gamma^T\boldsymbol{Q}\boldsymbol{A}\boldsymbol{z} - \dot{y}_r \\ e_3 &= \dot{e}_2 = \gamma^T\boldsymbol{Q}\boldsymbol{A}^2\boldsymbol{z} - \ddot{y}_r \end{aligned}$$

where the fact $\gamma^T\boldsymbol{Q}\boldsymbol{B} = \boldsymbol{0}$ has been used in the equations for e_2 and e_3. The error dynamics can thus be written as

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= \gamma^T\boldsymbol{Q}\boldsymbol{A}^3\boldsymbol{z} + \gamma^T\boldsymbol{Q}\boldsymbol{A}^2\boldsymbol{B}\boldsymbol{u} - \dddot{y}_r, \end{aligned}\right\} \quad (6)$$

hence by introducing the auxiliary control input

$$v = \gamma^T\boldsymbol{Q}\boldsymbol{A}^3\boldsymbol{z} + \gamma^T\boldsymbol{Q}\boldsymbol{A}^2\boldsymbol{B}\boldsymbol{u} - \dddot{y}_r, \quad (7)$$

the error dynamics given by (6) becomes a triple integrator with v as an input. A feedback controller $\boldsymbol{K}$, for this system, is then designed such that $v = \boldsymbol{K}\boldsymbol{e}$ drives the error to zero. The controller $\boldsymbol{K}$ has in this work been designed using pole placement yielding $-0.0032, -0.00020 \pm 0.0054i$ as closed loop poles.

Substituting $\boldsymbol{K}\boldsymbol{e}$ for $\boldsymbol{v}$ and solving for $\boldsymbol{u}$ in (7) we obtain, for each time t, a set of feasible inputs, $U(t)$, defined by

$$U(t) = \left\{\boldsymbol{u}\in U | \gamma^T\boldsymbol{Q}\boldsymbol{A}^2\boldsymbol{B}\boldsymbol{u} = \dddot{y}_r(t) - \gamma^T\boldsymbol{Q}\boldsymbol{A}^3\boldsymbol{z}(t) - \boldsymbol{K}\boldsymbol{e}(t)\right\},$$

and guarantees tracking of the reference, $y_r(t)$.

In summary, if the optimal control problem given by (2) is to be solved with the additional constraint of reference tracking one needs to replace the input set U by the time varying $U(t)$.

Due to computational complexity the above has been simplified to a case of only one active fuel at the time as suggested by Figure 5. In this case $U(t)$ becomes a singleton which is easily calculated. The resulting input is depicted in Figure 6, where coal is used during most of the day, but gas is used during the periods with a high price on controllability. Furthermore, during the switching between fuels the reference is closely followed which is depicted in Figure 7. Thus, the control signal

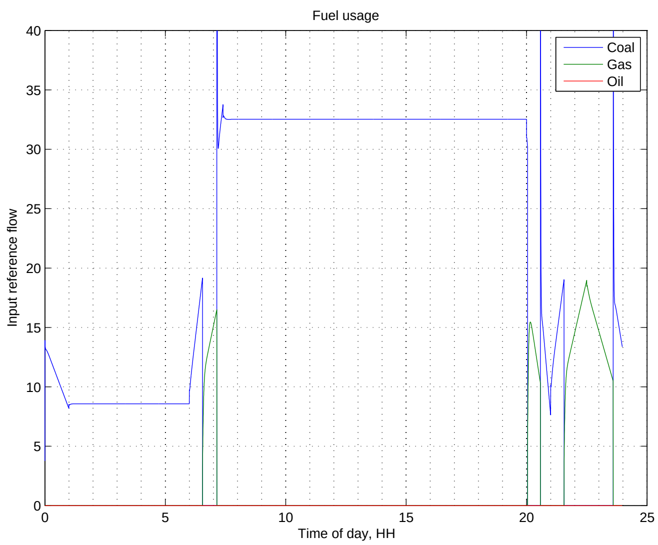

Fig. 6. The result of the reference tracking using the optimal fuel as suggested by the switch function, i.e., graphs of the control signal for the different fuel systems are depicted.

generated from (7) does allow for reference tracking during fuel configuration switching.

The important part of this work is, however, profit maximization and to evaluate this above strategy it is compared to a case where only the coal system is used. The tracking controllers used in the two cases are identical. The accumulated profit from the optimal strategy is depicted in Figure 8 along with the profit during the day of using only the coal system. The profit of the two case are identical until 6:30 there the optimal configuration is changed to gas (see Figure 5). After 6:30 the growth of profit of the two cases are similar until 20:00 where the gas system is used again and the two graphs drift further apart. Thus, a plant capable of using multiple fuels (in particular coal and gas) is beneficial when considering the economics of operating the plant.

V. Discussion

A continuous solution for the problem in this work has been developed in [16], where the tracking of the reference was included in objective function as a

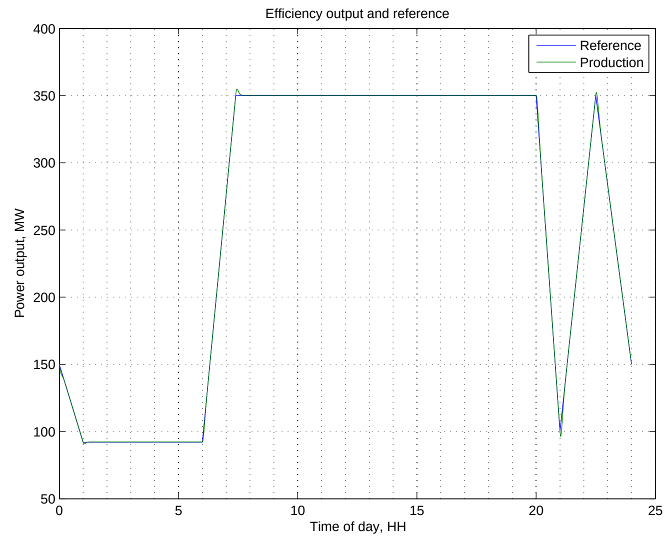

Fig. 7. Reference tracking of $y_r(t)$ during June 29th, 2008 using the proposed strategy with switching between fuel systems.

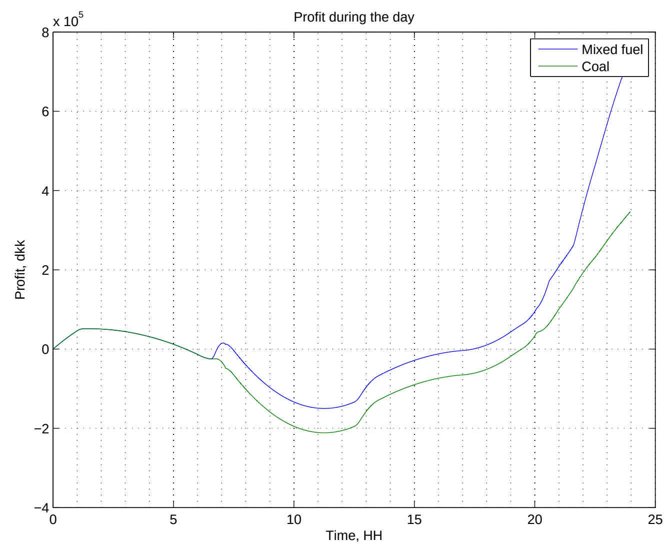

Fig. 8. The profit during June 29th, 2008 with the proposed solution of mixing fuels compared to the profit of a plant capable of using only coal.

quadratic error term, i.e., the maximization problem became quadratic and was formulated as

$$\begin{aligned} \max_{\boldsymbol{u}\in\mathcal{U}} \quad & \int_0^T f(\boldsymbol{z},t)dt \\ \text{subject to} \quad & \dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t), \end{aligned}$$

where

$$\begin{aligned} f(\boldsymbol{z},t) &= g_p(\boldsymbol{z},t) - \beta_q t_e(\boldsymbol{z},t) \\ &= -\boldsymbol{z}^T\tilde{\boldsymbol{Q}}\boldsymbol{z} + 2\tilde{\boldsymbol{q}}(t)^T\boldsymbol{z} + \tilde{\varphi}(t) \end{aligned}$$

with t_e the quadratic tracking error,

$$\tilde{\boldsymbol{Q}} = \beta_q \boldsymbol{Q}^T \boldsymbol{\gamma}\boldsymbol{\gamma}^T \boldsymbol{Q},$$
$$\tilde{\boldsymbol{q}}(t)^T = \frac{1}{2}\boldsymbol{\Theta}(t) + \beta_q y_r(t)\boldsymbol{\gamma}^T \boldsymbol{Q},$$
$$\tilde{\varphi}(t) = \varphi(t) - \beta_q y_r(t)^2,$$

and β_q a positive weighting factor.

When Prontryagin's maximum principle is applied to the problem above, the adjoint equation will depend on the system state, $\boldsymbol{z}$ and thus it is a two-value boundary problem which is difficult to solve. In [16] an iterative approach is suggested, where the initial state trajectory is found by converting the problem to discrete time and then using discrete optimization to the problem. The adjoint and state equations are, thereafter, solved successively. The procedure converges for this problem, however, it is quite complex and computational heavy. The result from [16] is similar to the the result obtained in this paper both in terms of fuel usage and profit during the day. Therefore, the approach in this work does have a large advantage when considering the computational burden and in particular as the result is almost identical.

Some immediate improvements are possible and a subject for future investigation would be the inclusion of the varying input set given by the tracking constraint in the optimization.

In conclusion, this work proposes a strategy for controlling a power plant such that profit is maximized and with less computational complexity than previous results. Furthermore, the improvement in profit compared to a coal only power plant is substantial as it is possible to approximately double the profit.

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