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# Reconfigurability of Piecewise Affine Systems Against Actuator Faults

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**Abstract:** In this paper, we consider the problem of reconfigurability of peicewise affine (PWA) systems. Actuator faults are considered. A system subject to a fault is considered as reconfigurable if it can be stabilized by a state feedback controller and the optimal cost of the performance of the systems is admissible. Sufficient conditions for reconfigurability are derived in terms of feasibility of a set of Linear Matrix Inequalities (LMIs). The method is implemented on a large scale livestock hybrid ventilation model which was obtained during previous research.

## 1. INTRODUCTION

Performance of modern control systems typically relies on a number of strongly interconnected components. Component malfunctions may degrade performance of the system or even result in loss of functionality. In applications such as climate control systems for livestock buildings, this is unacceptable as it may lead to the loss of animal life. Therefore, it is desirable to develop control systems that are capable of tolerating component malfunctions whilst still maintaining desirable performance and stability properties. Such controllers are called fault tolerant. Fault tolerant control (FTC) is divided generally into two categories:passive (PFTC) and active (AFTC). In PFTC, the structure of the system is fixed and pre-designed such that it can tolerate a set of faults. In AFTC, first the fault is detected using a fault detection and diagnosis (FDD) scheme. Then, based on information from the FDD the controller is re-designed or reconfigured in the case of severe faults such that the overall system stability is preserved and an acceptable performance is provided. An important step in designing an AFTC is to analyze reconfigurability of the system subject to possible faults. Reconfigurability is the ability of the system to preserve some properties, e.g. stability or performance, of the system when a fault has occurred.

Reconfigurability of linear time invariant systems is measured by controllability and observability Grammians in Frei et al. (1999). A measure for control reconfigurability of linear systems is proposed in Wu et al. (2000). The smallest second-order mode is used as a measure for reconfigurability of the system to preserve an acceptable performance in the presence of a fault. In Staroswiecki (2002), the fault tolerant property of a configuration with respect to an actuator fault is investigated. Two cases are considered. In the first case, only achieving the control objective is considered, but in the second case the control objective must be achieved and the control energy must be admissible. The method uses a Grammian based approach. This result is extended to the admissibility of a linear quadratic cost function in Staroswiecki (2003). Khelassi et al. (2009) defines reconfigurability of the system not only based on the controllability Grammian, but also based on the system reliability. While in the aforementioned methods, the reconfigurability measures are computed off-line, an online method for calculation of the controllability Grammian using input/output data is proposed in Gonzalez-Contreras et al. (2009).

The above methods are for linear systems. Most complex industrial systems either exhibit nonlinear behavior or involve both discrete and continuous components. An attractive modeling framework for such systems is the framework of piecewise affine systems (PWA). PWA systems have the capability to approximate nonlinear systems efficiently. Moreover, they arise in systems that contain PWA components such as deadzone, saturation, hysteresis, etc. This framework has been applied to several areas, such as, switched system, Rodrigues and Boukas (2006), and multi-zone climate control systems, Gholami et al. (2010).

Recofigurability of a class of linear switched systems is considered in Yang (2006). Reconfigurability is defined as the controllability of the system and an algebraic approach for reconfigurability is given. In our work, we consider reconfigurability of PWA systems against actuator faults, where only complete loss is considered. A system subject to a fault is called reconfigurable if it is not only stabilizable using a state feedback control law, but also the performance cost of the systems is admissible with any initial condition in a given bounded region. In other words, we have considered both stability and admissibility of the performance of the system as a criteria for reconfigurability. The problem is cast as the feasibility of a convex optimization problem with LMI constraints. Moreover, the optimal value of the cost function must be admissible. The optimization problem can be solved efficiently using available softwares such as YALMIP/SeDuMe or LMILAB.

The paper is organized as follows. In Section II, the PWA model and actuator faults are given. In Section III, reconfigurability is defined and sufficient conditions for reconfigurability are given. Section IV is dedicated to

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the simulation results for the climate control system. The conclusion is presented in the Section V.

## 2. PIECEWISE AFFINE SYSTEMS AND ACTUATOR FAULT MODELS

### 2.1 Piecewise Affine Systems

We consider a PWA discrete time system of the following form:

$$x(k+1) = A_i x(k) + B_i u(k) + b_i \qquad for\ x \in \mathcal{X}_i, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state and $u(k) \in \mathbb{R}^m$ is the control input. $\{\mathcal{X}_i\}_{i=1}^s \subseteq \mathbb{R}^n$ denotes a partition of the state space into a number of polyhedral regions $\mathcal{X}_i, i \in \mathcal{I} = \{1, \cdots, s\}$. Each polyhedral region is represented by:

$$\mathcal{X}_i = \{x | H_i x \leq h_i\} \tag{2}$$

The set $\mathcal{I}$ is partitioned to $\mathcal{I}_0 \cup \mathcal{I}_1$, where $\mathcal{I}_0$ denotes the index set of subsystems that contain the origin and $\mathcal{I}_1$ is the index set of the subsystems that does not contain the origin. It is assumed that $b_i = 0$ for $i \in \mathcal{I}_0$.

Each polyhedral region $\mathcal{X}_i$ can be over-approximated with a union of $l_i$ ellipsoids, i.e:

$$\mathcal{X}_i \subseteq \bigcup_{j=1}^{\ell_i} \mathcal{E}_{ij}, \tag{3}$$

where each ellipsoid is represented by the matrix $E_{ij}$ and the scalar $f_{ij}$ such that $\mathcal{E}_{ij} = \{x | \|E_{ij}x + f_{ij}\| \leq 1\}$, see Rodrigues and Boyd (2005). This approximation is used in this paper to deal with the affine term for subsystems with $i \in \mathcal{I}_1$ which helps us to cast the control problem in terms of LMIs. This approximation is more efficient for PWA slab systems where the partitioning is defined as $\mathcal{X}_i = \{x | d_i^1 \leq c_i^T x \leq d_i^2\}$. For PWA slab systems each partition $\mathcal{X}_i$ is approximated exactly by one ellipsoid with:

$$E_{il} = \frac{2c_i^T}{d_i^2 - d_i^1}, \tag{4}$$

$$f_{il} = -\frac{d_i^1 + d_i^2}{d_i^2 - d_i^1}. \tag{5}$$

All possible switchings from region $\mathcal{X}_i$ to $\mathcal{X}_j$ are represented by the set $\mathcal{S}$:

$$\mathcal{S} := \{(i,j) | x(k) \in \mathcal{X}_i, x(k+1) \in \mathcal{X}_j\} \tag{6}$$

The set $\mathcal{S}$ can be computed using reachability analysis for MLD systems, see Cuzzola and Morari (2001).

### 2.2 Fault Model

In this work, we consider actuator faults. Only complete loss of actuators is considered. Let $u_i$ denote the $i'th$ actuator and $u_i^F$ the failed $i'th$ actuator. We model a fault in an actuator as:

$$u_i^F = \delta_i u_i, \quad \delta_i \in \{0, 1\}, \tag{7}$$

where $\delta_i = 1$ presents the case of no fault in the $i'th$ actuator, and $\delta_i = 0$ corresponds to complete loss of it. We define $\Delta$ as:

$$\Delta = diag\{\delta_1, \delta_2, \ldots, \delta_m\}. \tag{8}$$

Then

$$\mathbf{u}^F = \Delta \mathbf{u}. \tag{9}$$

The PWA model of the system with the fault $f$ is given by:

$$x(k+1) = A_i x(k) + B_i \Delta^f u(k) + b_i \qquad for\ x \in \mathcal{X}_i, \tag{10}$$

## 3. STATE FEEDBACK DESIGN FOR PWA SYSTEMS

### 3.1 Piecewise Quadratic Stability

The problem of piecewise linear state feedback design is to design a state feedback of the form:

$$u(k) = K_i x(k) \quad for\ x(k) \in \mathcal{X}_i \tag{11}$$

such that the closed loop PWA system

$$x(k+1) = \mathbf{A}_i x(k) + b_i, \tag{12}$$

where $\mathbf{A}_i = A_i + B_i K_i$, is exponentially stable. The following theorem gives the conditions for stability of a Piecewise affine system.

*Theorem 1.* (Cuzzola and Morari (2001)) The system in (12) is exponentially stable if there exist matrices $P_i = P_i^T > 0$, $\forall i \in \mathcal{I}$, such that the positive definite function $V(x(k)) = x^T(k) P_i x(k), \forall x \in \mathcal{X}_i$, satisfies $V(x(k+1)) - V(x(k)) < 0$.

### 3.2 PWL Quadratic Regulator (PWLQR)

The aim of the control design problem is to design a controller of the form (11) such that it stabilizes the system and provides an upper bound on the following quadratic cost function associated with the system:

$$J = \sum_{k=0}^{\infty} x^T(k) Q_i x(k) + u^T(k) R_i u(k), \tag{13}$$

where $Q_i \geq 0$ and $R_i \geq 0$ are given weighting matrices of appropriate dimensions.

*Definition 1.* The system (1) subject to fault $f$ is called reconfigurable if there exist a state feedback control law of the form (11) which stabilizes the systems and the upper bound on the cost function (13) is admissible i.e. is less than a specified given threshold.

In the following, we derive sufficient conditions for a PWA systems to be stabilizable by a PWL state feedback controller.

*Theorem 2.* If there exist symmetric matrices $X_i = X_i^T > 0$ and matrices $Y_i$ such that:

$$\begin{bmatrix} X_i & * & * \\ (A_i X_i + B_i \Delta^f Y_i) & X_j + \mu_{il} b_i b_i^T & * \\ E_{il} X_i & \mu_{il} f_{il} b_i^T & \mu_{il}(f_{il} f_{il}^T - 1) \end{bmatrix} > 0 \tag{14}$$

$$\forall (i,j) \in \mathcal{S}, i \in \mathcal{I}_1, l = 1, \ldots, \ell_i,$$

$$\begin{bmatrix} -X_i & (A_i X_i + B_i \Delta^f Y_i) \\ (A_i X_i + B_i \Delta^f Y_i)^T & -X_j \end{bmatrix} < 0, \tag{15}$$

$$\forall (i,j) \in \mathcal{S}, i \in \mathcal{I}_0,$$

then there exist a PWL state feedback control law of the form (11) for the PWA system (10) such that the closed loop system is exponentially stable. The piecewise linear feedback gains are given by:

$$K_i = Y_i X_i^{-1} \tag{16}$$

*Proof 1.* We consider a piecewise Lyapunov candidate function of the form $V(x(k) = x(k)^T P_i x(k), P_i > 0$ for $x(k) \in \mathcal{X}_i$. The condition to be satisfied is:

$$V(x(k+1)) - V(x(k)) < 0, \quad \forall (i,j) \in \mathcal{S}. \tag{17}$$

We consider the general case where $x(k) \in \mathcal{X}_i$ and $x(k+1) \in \mathcal{X}_j$. First, we consider those switchings with $i \in \mathcal{I}_1$. To deal with the affine term, we will use the ellipsoidal approximation of regions. The equivalent of (17) for the PWA system is:

$$\begin{aligned}&[(A_i + B_i\Delta^f K_i)x(k) + b_i]^T P_j [(A_i + B_i\Delta^f K_i)x(k) + b_i] \\ &\qquad -x(k)^T P_i x(k) < 0, l = 1,\end{aligned} \tag{18}$$

which is equal to:

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \mathcal{A}_i^T P_j \mathcal{A}_i - P_i & * \\ b_i^T P_j \mathcal{A}_i & b_i^T P_j b_i \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} < 0, \tag{19}$$

where $\mathcal{A}_i = A_i + B_i \Delta^f K_i$. The ellipsoidal approximation of $\mathcal{X}_i$ can be written as:

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} E_{il}^T & * \\ f_{il}^T E_{il} & f_{il}^T f_{il} - 1 \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \leq 0, l = 1, \ldots, \ell_i, \tag{20}$$

The condition $x(k) \in \mathcal{X}_i$ is relaxed to the above approximation. Using the S-procedure, see Boyd et al. (1994), the equation (19) is satisfied if there exist multipliers $\lambda_{il} > 0$ such that :

$$(19) - \lambda_{il} \begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} E_{il}^T & * \\ f_{il}^T E_{il} & f_{il}^T f_{il} - 1 \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} < 0 \tag{21}$$

Therefore, the following matrix inequality must be satisfied:

$$\begin{bmatrix} \mathcal{A}_i^T P_j \mathcal{A}_i - P_i & \mathcal{A}_i^T P_j b_i \\ b_i^T P_j \mathcal{A}_i & b_i^T P_j b_i \end{bmatrix} - \lambda_{il} \begin{bmatrix} E_{il}^T & * \\ f_{il}^T E_{il} & f_{il}^T f_{il} - 1 \end{bmatrix} < 0, \tag{22}$$

This is equivalent to:

$$\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * \\ \lambda_{il} f_{il}^T E_{il} & \lambda_{il}(f_{il}^T f_{il} - 1) \end{bmatrix} - \begin{bmatrix} \mathcal{A}_i^T \\ b_i^T \end{bmatrix} P_j^{-1} [A_i \; b_i] > 0. \tag{23}$$

Applying Schur complement to the above equation we have:

$$\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * & * \\ \lambda_{il} f_{il}^T E_{il} & \lambda_{il}(f_{il}^T f_{il} - 1) & * \\ \mathcal{A}_i & b_i & P_j^{-1} \end{bmatrix} > 0. \tag{24}$$

Pre- and Post-multiplying the above equation with $diag\{I, \begin{bmatrix} 0 & * \\ I & 0 \end{bmatrix}\}$, we have:

$$\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * & * \\ \mathcal{A}_i & P_j^{-1} & * \\ \lambda_{il} f_{il}^T E_{il} & b_i^T & \lambda_{il}(f_{il}^T f_{il} - 1) \end{bmatrix} > 0. \tag{25}$$

Using Schur complement, it is equivalent to:

$$\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * \\ \mathcal{A}_i & P_j^{-1} \end{bmatrix} - \tag{26}$$

$$\begin{bmatrix} \lambda_{il} E_{il}^T f_{il} \\ b_i \end{bmatrix} \lambda_{il}^{-1} (f_{il}^T f_{il} - 1)^{-1} \left[\lambda_{il} f_{il}^T E_{il} \; b_i^T\right] > 0, \tag{27}$$

which is equal to:

$$\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * \\ \mathcal{A}_i & P_j^{-1} \end{bmatrix} -$$

$$\begin{bmatrix} \lambda_{il} E_{il}^T f_{il} (f_{il}^T f_{il} - 1)^{-1} f_{il}^T E_{il} & * \\ b_i (f_{il}^T f_{il} - 1)^{-1} f_{il}^T E_{il} & \lambda_{il}^{-1} b_i (f_{il}^T f_{il} - 1)^{-1} b_i^T \end{bmatrix} > 0. \tag{28}$$

Using the matrix inversion Lemma, we have:

$$(1 - f_{il}^T f_{il})^{-1} = 1 + f_{il}^T (1 - f_{il} f_{il}^T)^{-1} f_{il}. \tag{29}$$

The inequality (28) can be written as:

$$\begin{aligned}&\begin{bmatrix} P_i + \lambda_{il} E_{il}^T E_{il} & * \\ \mathcal{A}_i & P_j^{-1} \end{bmatrix} - \begin{bmatrix} \lambda_{il} E_{il}^T E_{il} & * \\ 0 & -\lambda_{il}^{-1} b_i b_i^T \end{bmatrix} + \\ &\begin{bmatrix} E_{il}^T \\ \lambda_{il}^{-1} b_i f_{il}^T \end{bmatrix} \lambda_{il} (f_{il} f_{il}^T - I)^{-1} \left[E_{il} \; \lambda_{il}^{-1} f_{il} b_i^T\right] > 0,\end{aligned} \tag{30}$$

which, by using Schur complement, is equal to:

$$\begin{bmatrix} P_i & * & * \\ \mathcal{A}_i & P_j^{-1} + \mu_{il} b_i b_i^T & * \\ E_{il} & \mu_{il} f_{il} b_i^T & \mu_{il}(f_{il} f_{il}^T - I) \end{bmatrix} > 0, \tag{31}$$

where $\mu_{il} = \lambda_{il}^{-1}$. Replacing $\mathcal{A}_i$ by $A_i + B_i \Delta^f K_i$, it is equivalent to:

$$\begin{bmatrix} P_i & * & * \\ (A_i + B_i \Delta^f K_i) & P_j^{-1} + \mu_{il} b_i b_i^T & * \\ E_{il} & \mu_{il} f_{il} b_i^T & \mu_{il}(f_{il} f_{il}^T - 1) \end{bmatrix} > 0, \tag{32}$$

Pre- and post-multiply (32) by $diag\{P_i^{-1}, I, I\}$, and defining $X_i = P_i^{-1}, Y_i = K_i P_i^{-1}$, we get (14). For subsystems that contain the origin i.e. $i \in \mathcal{I}_0$, we have $f_{il} f_{il}^T - I < 0$ which means that the LMI (14) is not feasible. For these subsystems the LMI (15) is considered and there is no need to include the region information. Therefore, the following matrix inequality must be satisfied:

$$(A_i + B_i \Delta^f K_i)^T P_j (A_i + B_i \Delta^f K_i) - P_i < 0 \tag{33}$$

Using Schur complement, the above inequality is equivalent to:

$$\begin{bmatrix} -P_i & (A_i + B_i \Delta^f K_i)^T \\ (A_i + B_i \Delta^f K_i) & -P_j^{-1} \end{bmatrix} < 0 \tag{34}$$

By pre- and post-multiplying (34) by $diag\{P_i^{-1}, I\}$, then defining $X_i = P_i^{-1}, Y_i = K_i P_i^{-1}$, we get (15). □

The above theorem only considers stability. In many situations, the system might be stabilizable but the cost of reaching to the origin from the initial state might not be admissible. To include admissibility of the upper bound on the cost function we introduce the following theorem.

*Theorem 3.* If there exist symmetric matrices $X_i = X_i^T > 0$ and matrices $Y_i$ and positive constants such that:

$$\begin{bmatrix} X_i & * & * & * & * \\ (A_i X_i + B_i \Delta^f Y_i) & X_j + \mu_{il} b_i b_i^T & * & * & * \\ E_{il} X_i & \mu_{il} f_{il} b_i^T & \mu_{il}(f_{il} f_{il}^T - 1) & * & * \\ \Delta^f Y_i & 0 & 0 & R_i^{-1} & * \\ X_i & 0 & 0 & 0 & Q_i^{-1} \end{bmatrix} > 0 \tag{35}$$

$$\forall (i,j) \in \mathcal{S}, i \in \mathcal{I}_1, l = 1, \ldots, \ell_i,$$

$$\begin{bmatrix} -X_i & * & * & * \\ (A_i X_i + B_i \Delta^f Y_i) & -X_j & 0 & 0 \\ \Delta^f Y_i & 0 & R_i^{-1} & * \\ X_i & 0 & 0 & Q_i^{-1} \end{bmatrix} < 0, \tag{36}$$

$$\forall (i,j) \in \mathcal{S}, i \in \mathcal{I}_0$$

then there exist a PWL state feedback control law of the form (11) for the PWA system (1) subject to fault $f$ such that the closed system is exponentially stable. The PWL feedback gains are given by:

$$K_i = Y_i X_i^{-1}, \tag{37}$$

and the upper bound on the cost function (13) satisfies:

$$J \leq x(0)^T X_{i_0}^{-1} x(0), \tag{38}$$

where $i_0$ is the region index for the initial condition, i.e. $x(0) \in \mathcal{X}_{i_0}$.

*Proof 2.* We consider a piecewise Lyapunov candidate function of the form $V(x(k) = x(k)^T P_i x(k), P_i > 0$ for $x(k) \in \mathcal{X}_i$. The condition to be satisfied is:

$$V(x(k+1)) - V(x(k)) + x(k)^T Q_i x(k) + \\ x(k)^T K_i^T R_i K_i x(k) < 0, \forall (i,j) \in \mathcal{S}. \tag{39}$$

The proof of stability is very similar to the previous theorem except that to deal with the term $x(k)^T Q_i x(k) + x(k)^T K_i^T R_i K_i x(k)$ we use the Schur complement two more times at the end of the proof. To prove that (38) is satisfied we sum up (39) from $k = 0$ to $k = \infty$, which results in:

$$V(x(\infty)) - V(x(0)) + \Sigma_0^{\infty}(x^T(k) Q_i x(k) + u^T(k) R_i u(k)) < 0 \tag{40}$$

Because $Q_i$ and $R_i$ are positive, hence $x(k)^T Q_i x(k) + x(k)^T K_i^T R K_i x(k) \geq 0$. Therefore, if (39) is satisfied the system is stable which means $V(x(\infty)) = 0$. As $V(x(0)) = x(0)^T P_{i_0} x(0)$. Therefore we have:

$$\sum_{k=0}^{\infty} (x^T(k) Q_i x(k) + u^T(k) R_i u(k)) < x^T(0) P_{i_0} x(0). \qquad \square$$

The upper bound found in the theorem (3) is not optimal. We are interested to minimize this cost to find a controller with the minimum cost. The upper bound of (13), could be minimized in the following way. The initial condition is considered as a random variable with uniform distribution in a bounded region $\overline{\mathcal{X}}$. Then, it is tried to minimize the expected value of the cost function. We have:

$$E(J) \leq E(tr(P_{i_0} x(0) x^T(0))) \leq \sum_{i \in \mathcal{I}} \sigma_i tr(P_i L_i), \tag{41}$$

where $L_i = E(x(0)x^T(0))$ is the expectation of $x(0)x^T(0)$ corresponding to $x(0) \in \mathcal{X}_i, i \in \mathcal{I}$ , $tr(\cdot)$ is the trace operator and $\sigma_i$ is the probability of $x(0) \in \mathcal{X}_i$. Then, the optimization problem is:

$$J^* = \min_{X_i, Y_i} \sum_{i \in \mathcal{I}} \sigma_i tr(X_i^{-1} L_i) \tag{42}$$

$$s.t. \begin{cases} (35) \\ (36) \\ X_i = X_i^T > 0, \end{cases}$$

The above optimization problem is non-convex. To convert it to a convex optimization problem , we introduce new variables $V_i, i \in \mathcal{I}$, which satisfies:

$$\begin{bmatrix} V_i & I \\ I & Z_i \end{bmatrix} \geq 0. \tag{43}$$

Using Schur complement, the above constraint is equivalent to $Z_i^{-1} \leq V_i$. Therefore, the objective function in (42), which is nonlinear in term of $Z_i$, can be converted to $\sum_{i \in \mathcal{I}} \sigma_i tr(V_i L_i)$. Consequently, the optimization problem (42) can be transformed to the following convex form:

$$J^* = \min_{X_i, Y_i, V_i, \epsilon_i} \sum_{i \in \mathcal{I}} \sigma_i tr(V_i L_i) \tag{44}$$

$$s.t. \begin{cases} (35), \\ (36), \\ (43), \\ X_i = X_i^T > 0, \end{cases}$$

In the following theorem we consider the properties for reconfigurability to be stability and admissibility of the optimal upper bound on the cost function.

*Theorem 4.* The system (1) subject to fault $f$ with respect to admissibility threshold $\overline{J}$ on the cost function (13) is reconfigurable if:

- (14) and (15) are satisfied,
- $J^* < \overline{J}$.

*Proof 3.* Satisfaction of (14) and (15) guarantees that the system is stabilizable with a PWL state feedback controller and satisfying $J^* < \overline{J}$ is equal to admissibility of the cost. Therefore, based on definition 1 the system subject to fault $f$ is reconfigurable. $\square$

## 4. EXAMPLE

The method is applied to a climate control systems of a live-stock building, which was obtained during previous research, Gholami et al. (2010). The general schematic of the large scale live-stock building equipped with hybrid climate control system is illustrated in Figure. 1. In a large scale stable, the indoor airspace is incompletely mixed; therefore it is divided into conceptually homogeneous parts called zones. In our model, there are three zones which are not similar in size. Zone 1, the one on the left, is the biggest and Zone 2, the middle one, is the smallest. Due to the indoor and outdoor conditions, the airflow direction varies between adjacent zones. Therefore, the system behavior is represented by a finite number of different dynamic equations. The model is divided into subsystems as follows: Inlet model for both windward and leeward, outlet model, and stable heating system, and finally the dynamic model of temperature based on the heat balance equation. The nonlinear model of the system is approximated by a discrete-time PWA system with 4 regions based on the airflow direction. The model of the system are derived for the following polyhedral regions:

$$\mathcal{X}_1 = \{[x^T \ u^T]^T | F_1^x x + F_1^u \geq f_1, F_2^x x + F_2^u \geq f_2\}, \tag{45}$$

$$\mathcal{X}_2 = \{[x^T \ u^T]^T | F_1^x x + F_1^u < f_1, F_2^x x + F_2^u < f_2\}, \tag{46}$$

$$\mathcal{X}_3 = \{[x^T \ u^T]^T | F_1^x x + F_1^u < f_1, F_2^x x + F_2^u \geq f_2\}, \tag{47}$$

$$\mathcal{X}_4 = \{[x^T \ u^T]^T | F_1^x x + F_1^u \geq f_1, F_2^x x + F_2^u < f_2\}, \tag{48}$$

,where

$$F_1^x = [1.0817 \ -0.0457 \ -0.9938]$$
$$F_2^x = [-1.1144 \ 0.0490 \ 1.0187]$$
$$F_1^u = [0.2323 \ -0.0072 \ 0.2323 \ 0.2323 \ -0.0072 \\ 0.2323 \ -0.072 \ 0.1349 \ -0.0719 \ -0.0064],$$
$$F_2^u = [-0.2558 \ 0.0074 \ -0.2558 \ -0.2558 \ 0.0074 \\ -0.2558 \ 0.0742 \ -0.12 \ 0.0742 \ 0.0074],$$
$$f_1 = 0.4058, f_2 = -0.4575 \tag{49}$$

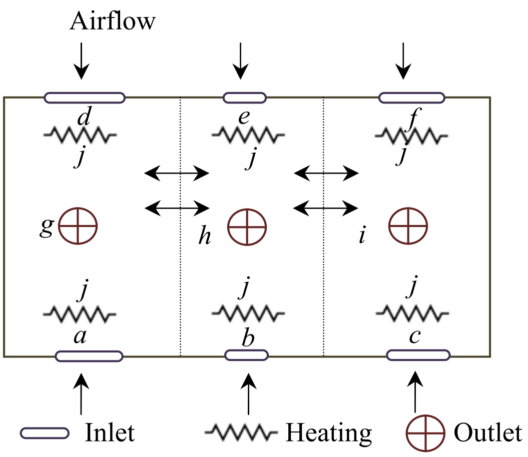


Fig. 1. The top view of the test stable

As one can see from the description of regions, they are dependent on the input and the state at $k$. But $u(k)$ is unknown and is to be calculated based on the current region. Therefore, it is impossible to calculate the current mode. To remedy this problem, instead of a PWL controller, we consider a common controller for all regions, i.e.

$$u(k) = Kx(k) \tag{50}$$

The discrete-time PWA model is described by:

$$A_1 = \begin{bmatrix} 1.6361 & 0.0480 & -0.7716 \\ 1.5782 & 0.5522 & -0.9983 \\ 0.7747 & 0.0462 & 0.0990 \end{bmatrix}, \tag{51}$$

$$A_2 = \begin{bmatrix} 1.1145 & -0.0300 & -1.0590 \\ 1.6452 & 0.1010 & -1.4342 \\ 0.3008 & 0.0191 & -0.2324 \end{bmatrix}, \tag{52}$$

$$A_3 = \begin{bmatrix} 1.6340 & 0.0259 & -0.7150 \\ 1.5474 & 0.8335 & -1.4790 \\ 0.7674 & 0.0314 & 0.1456 \end{bmatrix}, \tag{53}$$

$$A_4 = \begin{bmatrix} 1.6274 & 0.0049 & -0.6987 \\ 1.6242 & 0.8163 & -1.4751 \\ 0.7623 & 0.0051 & 0.1640 \end{bmatrix}, \tag{54}$$

$$B_1 = \begin{bmatrix} -0.1163 & 0.0459 & -0.1163 & -0.1163 & 0.0459 & -0.1163 & 0.0018 & -0.0567 & 0.0018 & 0.0070 \\ 0.5718 & -0.3768 & 0.5718 & 0.5718 & -0.3768 & 0.5718 & -0.1518 & 0.2724 & -0.1518 & -0.0056 \\ -0.1147 & 0.0353 & -0.1147 & -0.1147 & 0.0353 & -0.1147 & 0.0022 & -0.0553 & 0.0022 & 0.0071 \end{bmatrix}, \tag{55}$$

$$B_2 = \begin{bmatrix} 0.1137 & -0.0044 & 0.1137 & 0.1137 & -0.0044 & 0.1137 & -0.0697 & 0.2883 & -0.0697 & 0.0023 \\ -0.0104 & 0.1057 & -0.0104 & -0.0104 & 0.1057 & -0.0104 & 0.0183 & 0.8276 & 0.0183 & 0.1275 \\ 0.0581 & 0.0258 & 0.0581 & 0.0581 & 0.0258 & 0.0581 & 0.0097 & 0.0939 & 0.0097 & 0.0273 \end{bmatrix}, \tag{56}$$

$$B_3 = \begin{bmatrix} -0.0677 & -0.0127 & -0.0677 & -0.0677 & -0.0127 & -0.0677 & -0.0103 & -0.0080 & -0.0103 & 0.0078 \\ 0.2031 & 0.0778 & 0.2031 & 0.2031 & 0.0778 & 0.2031 & -0.0594 & -0.0506 & -0.0594 & -0.0012 \\ -0.0697 & -0.0188 & -0.0697 & -0.0697 & -0.0188 & -0.0697 & -0.0098 & -0.0087 & -0.0098 & 0.0075 \end{bmatrix}, \tag{57}$$

$$B_4 = \begin{bmatrix} -0.0393 & -0.0380 & -0.0393 & -0.0393 & -0.0380 & -0.0393 & -0.0133 & -0.0234 & -0.0133 & 0.0086 \\ 0.0851 & 0.1683 & 0.0851 & 0.0851 & 0.1683 & 0.0851 & -0.0568 & 0.0160 & -0.0568 & 0.0029 \\ -0.0414 & -0.0434 & -0.0414 & -0.0414 & -0.0434 & -0.0414 & -0.0130 & -0.0241 & -0.0130 & 0.0085 \end{bmatrix}, \tag{58}$$

$$b_1 = \begin{bmatrix} 0.4749 \\ -0.9236 \\ 0.4214 \end{bmatrix}, \ b_2 = \begin{bmatrix} -0.0676 \\ 2.2442 \\ 0.3784 \end{bmatrix}, \tag{59}$$

$$b_3 = \begin{bmatrix} 0.2356 \\ 0.3694 \\ 0.2500 \end{bmatrix}, \ b_4 = \begin{bmatrix} 0.3510 \\ -0.5021 \\ 0.3682 \end{bmatrix}. \tag{60}$$

Here, the initial condition is $x(0) = [10\ 10\ 10]^T$, and the set of the actuators of the system is $\{a, b, c, d, e, f, g, h, i, j\}$, where $a, b, c, d, e, f$ are inlets, $g, h, i$ are fans, and $j$ is the heating systems. Actuator $a, b, c, d, e, f$ respectively represent 12, 6, 12, 14, 6, 12 connected inlets. The control problem is to regulate the temperature of each zone around 19. To make notations simpler, we only write those actuator that are healthy. For example, $\{a, b, c, d\}$ means that only actuators $a, b, c,$ and $d$ are healthy and the rest are faulty. Results of the reconfigurability analysis shows that the system with more than 5 faulty actuators is not reconfigurable. It also shows that heating system, actuator $j$, should be healthy for reconfigurability of the system. In table 1, different faulty situations with 5 or 6 fault-free actuators are considered. Because of the lack of space we have just shown some cases to demonstrate the method. The first column shows the fault-free actuators and the second column shows the corresponding quadratic cost. We have only considered the cases that the system is stabilizable. The admissibility threshold of the cost is considered as 700. As it can be seen from the table, even though all the cases are stabilizable, some of them are not admissible; hence the system is not reconfigurable based on definition 1. The reconfigurable cases are boldfaced. Figure. 2 shows temperature of each zone for the fault-free system. As it is obvious the controller is able to regulate the temperature around the reference. Figure. 3 shows the output of the system for the case that the fault-free actuator set is $\{a, b, g, h, j\}$. As it can be seen, the controller is able to track the reference with some degradation in the performance which is admissible.

## 5. CONCLUSIONS AND FUTURE WORKS

We presented an approach for reconfigurability of discrete time PWA systems. Reconfigurability is defined as both stability and admissibility of the upper bound on the quadratic cost. Sufficient conditions for reconfigurability of a system subject to a fault with respect to a given threshold on the quadratic performance cost are given in terms of LMI. The upper bound is minimized by solving a convex optimization problem with LMI constraints. The approach is applied to the climate control system of a livestock building. Situations in which the system is

Table 1. Stabilizable actuator sets and associated quadratic cost

| Fault-free actuators | quadratic cost |
|---|---|
| {a,b,h,i,j} | **668.8** |
| {a,b,g,h,j} | **668.1** |
| {e,f,h,i,j} | **670** |
| {e,f,g,h,j} | **669.4** |
| {d,e,f,g,h,j} | **675** |
| {a,b,c,g,h,j} | **665** |
| {a,b,c,h,i,j} | **667.2** |
| {d,e,f,h,i,j} | **668.2** |
| {a,f,g,h,j} | 129220 |
| {d,e,f,i,j} | 271655 |
| {a,b,c,g,i,j} | 264750 |
| {d,e,f,g,j} | 261355 |
| {a,f,h,i,j} | 127770 |

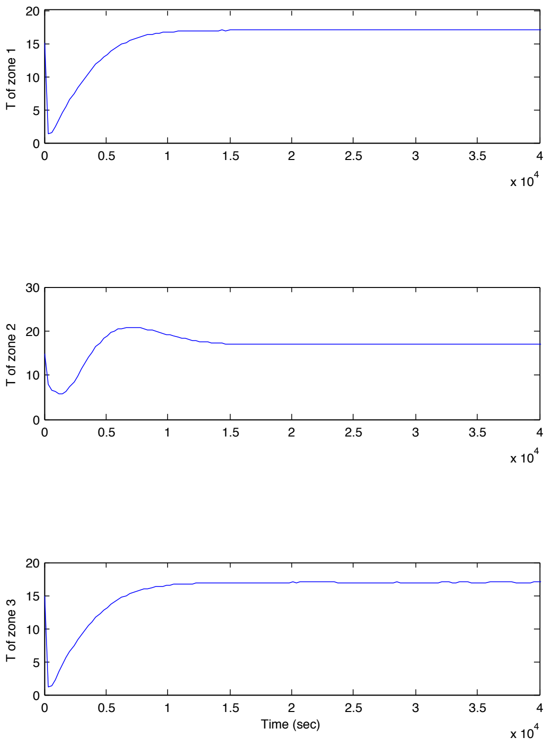


Fig. 2. Simulation results with a controller designed for the fault-free system

reconfigurable with maximum number of actuator outages are found. The simulation results demonstrates that the performance of the system is still acceptable. Future works will consider application of the method in designing an AFTC with optimal number of control laws.

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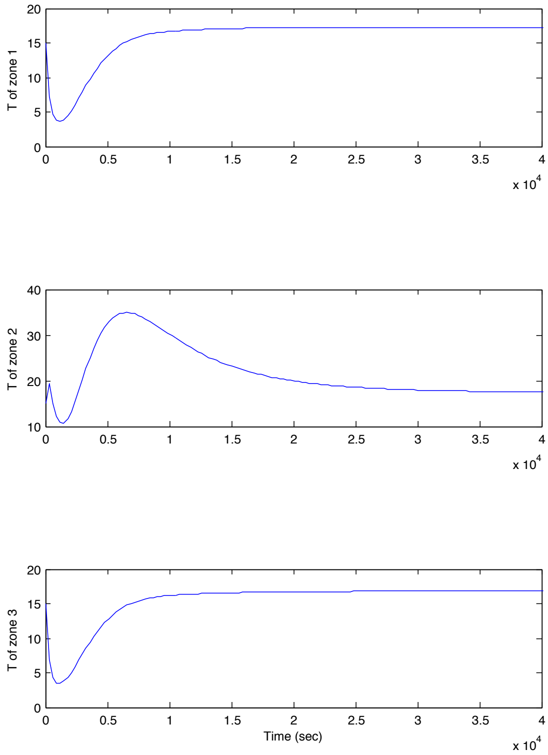


Fig. 3. Simulation results with a controller designed for the system with the fault-free actuator set:$\{a, b, g, h, j\}$

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