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Tabatabaeipour, Seyed Mojtaba; Ravn, Anders Peter; Izadi-Zamanabadi, Roozbeh; Bak, Thomas

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Active Diagnosis of MLD Systems using Distinguishable Steady Outputs

Seyedmojtaba Tabatabaeipour
Department of Electronic Systems
Aalborg University
DK-9220, Denmark
Email: smt@es.aau.dk

Anders P. Ravn
Department of Computer Science
Aalborg University
DK-9220, Denmark
Email: apr@cs.aau.dk

R. Izadi-Zamanabadi
R&D-RA-DP, Danfoss A/S
DK-6470, Denmark
Email: roozbeh@danfoss.com

Thomas Bak
Department of
Electronic Systems
Aalborg University
DK-9220, Denmark
Email: tba@es.aau.dk

***Abstract*—In active diagnosis the system is excited by a signal that aims to uncover latent errors. However, the diagnosis signal may destabilize the system, in particular in an open-loop structure, but also in a closed-loop structure, because the nominal controller is designed to stabilize the nominal system. This paper presents a method for active diagnosis of Mixed Logical Dynamical (MLD) systems where instability is avoided: The diagnoser looks for steady states of both the normal and faulty system which are reachable by the same input such that the corresponding outputs are distinguishable from each other. The input is applied to the system and the condition of the system is determined based on the output. Thus this excitation preserves stability. The method can be useful in a design phase to find a sensor allocation which guarantees diagnosability. The method is tested on the two tank benchmark example.**

I. INTRODUCTION

In a complex control system there are many components with strong interaction between them. Hence the overall system performance depends on the individual performance of components. A fault in a single component may, therefore, degrade the overall performance of the system and may even lead to unacceptable loss of system functionality. Thus fault diagnosis is of crucial importance in automatic control of complex systems.

There are two main categories of diagnosis methods: passive and active. In passive diagnosis, the input and output of the system is observed by the diagnoser. Based on the observation the diagnoser decides whether a fault has occurred or not. The input is generated by an external input or by the controller.

In Active Fault Diagnosis (AFD) the diagnoser generates an input, which excites the system, to decide whether the output represents a normal or a faulty behavior and if possible decide which fault occurred. The generated input must perturb the system from the operation point but at the same time not lead the system to instability or to an unacceptable performance.

The area of active diagnosis has attracted a lot of attentions in recent years. See papers [13], [4], [14], [12], [5], [11], [9], [15], and books [21], [3]. Most of the available methods are in open-loop configuration and for linear systems. In [19] a method for active diagnosis of hybrid system based on reachability analysis is proposed and extended for automatic sensor assignment in [17]. [18] proposes a model predictive method for active diagnosis of hybrid system using Mixed Logical Dynamical (MLD) framework. A qualitative event-based approach for active diagnosis of hybrid systems is presented in [6] where diagnosis is improved by executing or blocking controllable events. [11] and [9] present a method for active diagnosis of parametric faults in closed loop systems based on YJKB parameterization.

Stability is an important issue in the fault tolerant control systems. When a fault occurs, it takes time for the fault detection module to detect the fault and even when it is detected it needs some time to isolate and identify the fault. During this period the system is working in a faulty condition. For a closed-loop system, because the controller is designed for the nominal system the performance of the closed loop system in this period is dependent on the severity of the fault and the robustness of the nominal controller. The controlled system may become unstable in this period [22]. The faulty system may not be stabilizable with the nominal controller and the time window for an unstable system , *e.g.* double inverted pendulum, may be too small to detect and isolate the fault and then reconfigure the loop [10].

For active diagnosis the stability issue is more critical because we are exciting the system with the aim of detecting the fault. When the AFD starts the diagnosis it is not known whether the system is in the normal or the faulty condition. A stability guaranteeing method for diagnosis of additive, parametric and multiplicative faults for linear systems based on observer parameterization is proposed in [16].

In [18] a model predictive method is proposed for active diagnosis of MLD system. The problem is reformulated as a mixed integer programming problem. The objective function of the optimization problem is to make an observable difference between predicted outputs of the normal system and the faulty systems fulfilling constraints imposed by required performance during fault detection. While the computed input sequence diagnoses the fault, it may destabilize the system.

In this work a different approach is used. The system is moved from its current states to other steady states. These steady states belong to either the normal system or a faulty system which are reachable by the same input and the corresponding steady outputs are distinguishable. The fault is diagnosed based on the output measurement. Because the system is moving to steady state, regardless of its condition,

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injected input does not destabilize the system. When it is not possible to find a diagnosis signal that separates the output of the normal system from that of a faulty system, the diagnosis using separating output may not be possible. In this case this approach could be used as a pre-analysis for deciding which outputs must be measured to have the capability of diagnosis using steady outputs.

The structure of the paper is as follows. In Section 2 preliminaries and problem formulation are explained. Section 3 explains the proposed algorithm. In section 4, the method is tested on the two tank example. And finally conclusions and future investigation are discussed in Section 5.

II. Preliminaries and Problem formulation

In this section we first introduce the MLD framework and then the active diagnosis problem is formulated.

A. Mixed Logical Dynamical Systems

For modeling of hybrid systems, the mixed logical dynamical (MLD) framework proposed in [1] is used. The equations describing an MLD system are as follows:

$$x(t+1) = Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t) \quad (1)$$

$$y(t) = Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t) \quad (2)$$

$$E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4z(t) + E_5 \quad (3)$$

where $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_l}$ are states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_l}$ are the inputs, $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_l}$ are the outputs. $\delta \in \{0,1\}^{r_l}$ and $z \in \mathbb{R}^{r_c}$ are auxiliary binary and continuous variables.

The MLD framework has the capability of modeling various classes of hybrid systems such as PieceWise Affine (PWA) systems, linear systems with piecewise linear output functions, linear systems with discrete inputs or with qualitative outputs, bilinear systems, and finite state machines in which a linear time invariant system generates the events [1].

Equivalence of MLD systems with other classes of hybrid systems such as PWA systems, linear complementary (LC) systems, extended linear complementary (ELC) systems, and max-min-plus-scaling (MMPS) systems under some assumptions is shown in [7].

Using the MLD framework different problems such as optimal control, state estimation, etc. can be reformulated as a mixed-integer programming problem and then can be solved using mixed integer programming techniques.

B. Problem Formulation

In model-based passive diagnosis, the diagnoser receives a sequence of input/output measurements. A model of the normal system $\mathcal{B}_0$ and different models of the system subject to different faults, namely $\mathcal{B}_1, \ldots, \mathcal{B}_n$. are given. Then, the diagnoser checks the consistency of the measured I/O sequence with given model. As explained in [2], the output of the diagnoser is a fault candidate index $f \in 1, \ldots, n$ such that the observed I/O sequence is consistent with the corresponding behavior $\mathcal{B}_f$[2]. In this case the input is given by an external system.

The structure of an active diagnoser in depicted in Fig. 1. It consists of a generator and a diagnoser. The generator generates an input sequence $U = \langle u(0), \ldots, u(m)\rangle$ which is applied to the system and then index f is determined by the diagnoser through the observation of the applied input sequence and the output sequence $Y = \langle y(0), \ldots, y(m)\rangle$.

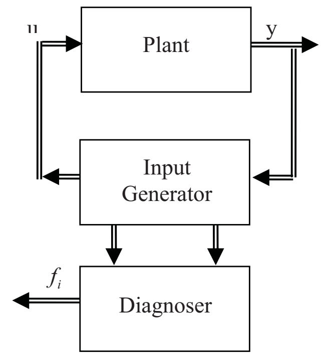

Fig. 1. Structure of an Active fault diagnoser system

The active diagnosis problem can be stated as follows:

Problem *1 (Active diagnosis problem):* Given the set $\mathcal{B} = \{\mathcal{B}_0, \ldots, \mathcal{B}_n\}$ describing behaviors of the system with no fault and subject to faults $\{f_1, \ldots, f_n\}$, find a sequence of inputs U and $i \in \{0, \ldots, n\}$ such that (U, Y) belongs only to $\mathcal{B}_i$.

If the input sequence exists, *i.e.* if the system is diagnosable then we can look for the optimal solution, where optimality can be interpreted in different senses.

The main advantage of active diagnosis is when different behaviors of the system overlap, see Fig. 2. The faultless behavior and the behavior of the system subject to the fault f_1 are in the sets $\mathcal{B}_0$ and $\mathcal{B}_1$ repectively. As long as the observed I/O pair uniquely belongs to the set $\mathcal{B}_0$ or $\mathcal{B}_1$, such as point A or B, it can be decided whether the system is faulty or not. But if the observed pair belongs to the intersection of $\mathcal{B}_0$ and $\mathcal{B}_1$, like C, it is impossible to diagnose the fault. The main idea of the proposed algorithm is to generate an input signal to move the system from C to an area which belongs uniquely either to the set $\mathcal{B}_0$ or $\mathcal{B}_1$.

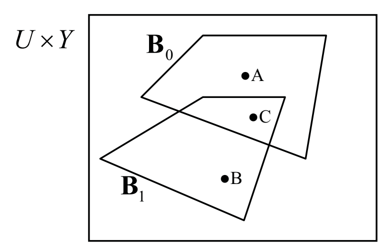

Fig. 2. Input-Output Space

III. The proposed algorithm

It is assumed that the initial states of the system are in the area in which the faulty behavior and the normal behavior overlap. If this is not the case the fault could be diagnosed by means of a passive diagnoser. It is assumed that the model of the faulty system and the normal system is given in MLD form as in (1)-(3) with subscript 0 indicating the normal system and i indicating the system equation for the system subject to fault f_i.

The diagnosis aims at finding a sequence of inputs such that the outputs based on the different dynamics becomes distinguishable from each other. In other words:

$$Y_i \neq Y_j, \quad \forall\, i, j \in \{0, \ldots, n\}, i \neq j \tag{4}$$

This difference between $y_i(k)$ and $y_j(k)$ should be observable which means:

$$|y_i(T) - y_j(T)| \geq d \texttt{ for all } i, j \in \{0, \ldots, n\}, i \neq j \tag{5}$$

or if a relative separation is used: $|y_i(T) - y_j(T)| \geq d \cdot |y_i(T)|$, where T is the length of the sequence and d is a separation distance that is dependent on the level of noise.

Satisfaction of the above constraints, (4) and (5), is actually isolation for every single fault. Isolation for every single fault is very demanding and may not be necessary. One can consider the following scenarios which are less demanding:

- **Fault detection**: In this case, the aim is to find if the system is working normally or it is faulty. We are not interested to detect which fault has occurred. Therefore (4) can be relaxed as:

$$|y_0(T) - y_i(T)| \geq d, \quad \forall\, i \in \{1, \ldots, n\}, \tag{6}$$

- **Fault isolation for a set of faults**: It is possible that a set of faults have the same impact on the functionality of the system and also require the same fault accommodation or control reconfiguration actions. Therefore it is not required to isolate these faults. Moreover it could be the case that these faults cannot be isolated easily and therefore we just aim at isolation of the set. It is assumed that indices for these faults is given by the set $\mathcal{F}$, then (4) becomes:

$$|y_i(T) - y_j(T)| \geq d \;\; \forall\, i \in \mathcal{F}, j \notin \mathcal{F}, \tag{7}$$

Note that a practical approach is to first detect the fault. Then isolate a set and then isolate a fault in this set.

Due to rich behavior of a MLD system it may have different steady states. We use this property. In this work, we are looking for steady states from systems $i, j \in \{0, \cdots, n\}$ namely, x_{s_i}, such that the corresponding output are distinguishable i.e. :

$$\left|y_{s_i} - y_{s_j}\right| \geq d, \;\; \forall\, i, j \in \{0, \ldots, n\}, i \neq j \tag{8}$$

If these steady outputs exist then the fault is diagnosable.

A steady state value for an MLD system can be obtained by solving a mixed integer problem of the following form:

$$\begin{aligned}\min_{x_s, u_s, \delta_s, z_s} & \|Q_1(y_s - y_r)\|_p + \|Q_2(x_s - x_r)\|_p + \\ & \|Q_3(u_s - u_r)\|_p + \|Q_4(\delta_s - \delta_r)\|_p + \\ & \|Q_1(z_s - z_r)\|_p \end{aligned} \tag{9}$$

$$s.t. \begin{cases} x_s = Ax_s + B_1 u_s + B_2 \delta_s + B_3 z_s \\ y_s = Cx_s + D_1 u_s + D_2 \delta_s + D_3 z_s \\ E_2 \delta_s + E_3 z_s \leq E_1 u_s + E_4 x_s + E_5 \end{cases} \tag{10}$$

,where $\|.\|_p$ is p norm, Q_i are positive definite weighting matrices. y_r, x_r, u_r, d_r, z_r are given offset vectors.

It is possible that the resulting steady state $(x_s, u_s, \delta_s, z_s)$ is not reachable. It is also possible that a steady state does not exist but cycling-steady states exist [8]. Here we assume that the steady state is reachable and that we do not have cycling-steady state behavior.

Distinguishable steady outputs, if they exist, can be found by solving the the following problem:

$$\begin{aligned}\min_{x_{s_i}, u_s, \delta_{s_i}, z_{s_i}} & \Sigma_{i=0}^{n} \|Q_{1_i}(y_{s_i} - y_r)\|_p + \|Q_{2_i}(x_{s_i} - x_r)\|_p \\ & \|Q_{3_i}(u_s - u_r)\|_p + \|Q_{4_i}(\delta_{s_i} - \delta_r)\|_p + \\ & \|Q_{5_i}(z_{s_i} - z_r)\|_p \end{aligned} \tag{11}$$

$$s.t. \begin{cases} x_{s_i} = Ax_{s_i} + B_{1_i} u_s + B_{2_i} \delta_{s_i} + B_{3_i} z_{s_i} \\ y_{s_i} = C_i x_{s_i} + D_{1_i} u_s + D_{2_i} \delta_{s_i} + D_{3_i} z_{s_i} \\ E_{2_i} \delta_{s_i} + E_{3_i} z_{s_i} \leq E_{1_i} u_{s_i} + E_{4_i} x_{s_i} + E_{5_i} \\ \left|y_{s_i} - y_{s_j}\right| \geq d \texttt{ for all } i, j \in \{0, \ldots, n\}, i \neq j \end{cases}, \tag{12}$$

where $y_r, x_r, u_r, \delta_r, z_r$, is a reference vector. Selection of this reference vector is based on the phase in which we are doing the diagnosis. If we are in the operating phase, then they are chosen equal to the current operating values. In other words, we want to find those steady states which are the closest to the current operating point and at the same time are distinguishable. If we are in the commissioning phase, they are equal to the reference signals. In other words, we are looking for those steady states which are closest to the reference signals and are distinguishable. Note that additional constraint on states and outputs could be easily handled in this formulation by adding them to the optimization constraints.

In (11), the distinguishability constraint $\left|y_{s_i} - y_{s_j}\right| \geq d$ should be written in the appropriate form. To do that, the following auxiliary binary variables are introduced:

$$\begin{gathered} [s_{ij1} = 1] \leftrightarrow [y_{s_i} - y_{s_j} \leq d] \\ [s_{ij2} = 1] \leftrightarrow [y_{s_j} - y_{s_i} \leq d] \\ s_{ij} = s_{ij1} \wedge s_{ij2}, i, j \in \{0, \ldots, n\}, i \neq j \\ S = \vee_{i=0}^{n} s_{ij} \end{gathered} \tag{13}$$

The constraints $\left|y_{s_i} - y_{s_j}\right| \geq d \texttt{ for all } i, j \in \{0, \ldots, n\}, i \neq j$ can be transformed into the equality constraint $S = 0$ and a set of mixed integer linear inequalities obtained from transforming logical propositions in (13) to equivalent mixed integer inequalities using the technique introduced in [1].

The auxiliary binary variable S as it is formulated in (13) aims at isolation of every single fault. For other scenarios S is constructed as follows:

- **Fault detection**:

$$S = \vee_{i=1}^{n} s_{0i} \tag{14}$$

- **Fault isolation for a set of faults**:

$$S = \vee s_{ij}, \quad \forall\, i \in \mathcal{F}, j \notin \mathcal{F} \tag{15}$$

Using the introduced auxiliary variable, the problem can be rewritten as:

$$\begin{aligned}\min_{x_{s_i},u_s,\delta_{s_i},z_{s_i}} \quad & \Sigma_{i=0}^{n} \|Q_{1_i}(y_{s_i} - y_r)\|_p + \|Q_{2_i}(x_{s_i} - x_r)\|_p \\ & \|Q_{3_i}(u_s - u_r)\|_p + \|Q_{4_i}(\delta_{s_i} - \delta_r)\|_p + \\ & \|Q_{5_i}(z_{s_i} - z_r)\|_p \end{aligned} \tag{16}$$

$$s.t. \begin{cases} x_{s_i} = Ax_{s_i} + B_{1_i}u_s + B_{2_i}\delta_{s_i} + B_{3_i}z_{s_i} \\ y_{s_i} = C_i x_{s_i} + D_{1_i}u_s + D_{2_i}\delta_{s_i} + D_{3_i}z_{s_i} \\ E_{2_i}\delta_{s_i} + E_{3_i}z_{s_i} \leq E_{1_i}u_s + E_{4_i}x_{s_i} + E_{5_i} \\ S = 0 \end{cases} \tag{17}$$

If the above optimization problem is feasible then there are x_{s_i} for $i = 0, \cdots, n$ such that the corresponding outputs y_{s_i} are distinguishable. Otherwise if the optimization problem in (16), (17) is infeasible, then the system is not diagnosable by this method. Having the steady state values, we can apply the steady inputs u_s to the system and based on the steady outputs decide about its condition. Assume that the actual output of the system at steady state is y_s. Then the fault candidate is f_c such that:

$$c = \underset{i \in \{0,\ldots,n\}}{\operatorname{argmin}} |y_s - y_{s_i}| \tag{18}$$

Note that in this method there is no need to estimate the states of the system and diagnosis can be done just by measuring outputs. It is possible to maximize the difference d which is used for distinguishability by adding the term $-\alpha \cdot d$ to the cost function in (16):

$$\begin{aligned}\min_{x_{s_i},u_s,\delta_{s_i},z_{s_i},d} \quad & \Sigma_{i=0}^{n} \|Q_{1_i}(y_{s_i} - y_r)\|_p + \|Q_{2_i}(x_{s_i} - x_r)\|_p \\ & \|Q_{3_i}(u_s - u_r)\|_p + \|Q_{4_i}(\delta_{s_i} - \delta_r)\|_p + \\ & \|Q_{5_i}(z_{s_i} - z_r)\|_p - \alpha \cdot d, \end{aligned} \tag{19}$$

where α is a weighting parameter.

The proposed method could be used in the design phase to decide about sensor locations to guarantee diagnosability in the steady states. Different output candidates can be considered. Then the optimization problem is solved. Feasibility of the optimization problem with the output candidate means diagnosability of the system with this method.

IV. Example

In this section, the proposed method is tested on the two tank system. The two tank system is shown in Fig. 3. The system consists of two cylindrical tanks with cross sectional area A which are connected by two pipes at the bottom and at level h_v. The flows through the pipes, denoted by $Q_{12}V_{12}$ and $Q_{12}V_1$, are controlled using two on/off valves V_{12} and V_1. There is a flow Q_1 through a pump to tank 1 which is a continuous input.

Dynamical equations of the system are as follows.

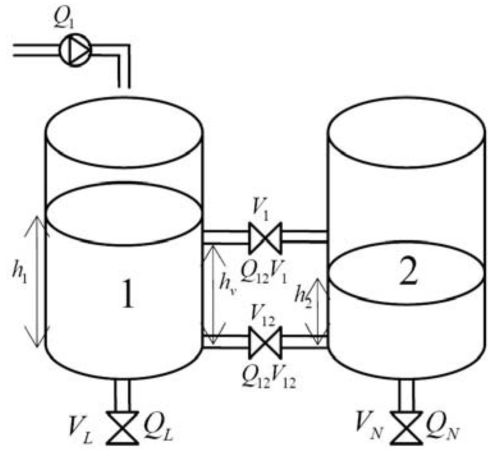

Fig. 3. Two-tank system

$$\dot{h}_1 = \tfrac{1}{A}(Q_1 - Q_{12}V_{12} - Q_{12}V_1 - Q_L), \tag{20}$$

$$\dot{h}_2 = \tfrac{1}{A}(Q_{12}V_{12} + Q_{12}V_1 - Q_N), \tag{21}$$

where h_1 and h_2 denote the levels of tanks 1 and 2 respectively. The flow $Q_{12}V_{12}$ is described by:

$$Q_{12}V_{12} = V_{12}k_{12}sign(h_1 - h_2)\sqrt{2g\,|h_1 - h_2|}, \tag{22}$$

where g is the gravity constant and k_{12} is a valve specific constant. Similarly $Q_L = V_L k_L \sqrt{2gh_1}$ and $Q_N = V_N k_N \sqrt{2gh_2}$. The flow through valve V_1 is given by:

$$\begin{aligned} Q_{12}V_1 \quad = \quad & V_1 k_1 sign(max\{h_v, h_1\} - max\{h_v, h_2\}) \\ & \sqrt{|2g(max\{h_v, h_1\} - max\{h_v, h_2\})|} \end{aligned} \tag{23}$$

The MLD model of the system is derived as follows (For details see [8].). The nonlinear relation $\sqrt{x}$ is approximated by a straight line x, thus (22) becomes:

$$Q_{12}V_{12} = V_{12}k_{12}(h_1 - h_2) \tag{24}$$

The auxiliary continuous variable $z_{12} = V_{12}(h_1 - h_2)$ is introduced to transform the above nonlinear equation to the linear equation $Q_{12}V_{12} = k_{12}z_{12}$ with a set of mixed integer linear inequalities. For Q_N and Q_L, using the same method, we will have $Q_N = k_N z_N$ and $Q_L = k_N z_L$ where $z_N = V_N h_2$ and $z_L = V_L h_2$.

In order to transform (23) to a linear equation in the MLD framework, first we introduce the following binary variables indicating whether the level in each tank has reached h_v:

$$[\delta_{01}(t) = 1] \leftrightarrow [h_1(t) \geq h_v] \tag{25}$$

$$[\delta_{02}(t) = 1] \leftrightarrow [h_2(t) \geq h_v] \tag{26}$$

and then the term $max\{h_v, h_1\} - max\{h_v, h_2\}$ is transformed into a linear equation as $Q_{12}V_1 = k_1 z_1$, where

$$z_1 = V_1(z_{01} - z_{02}) \tag{27}$$

$$z_{01} = \delta_{01}(h_1 - h_v) \tag{28}$$

$$z_{02} = \delta_{02}(h_2 - h_v) \tag{29}$$

are introduced auxiliary continuous variables.

Finally, differential equations (20), (21) are discretized in time by Euler approximation $\dot{h}_i(t) \approx \frac{h_i(t+1)-h_i(t)}{Ts}$, where T_s is the sample time. The final MLD model of the system consists of two continuous states: h_1, h_2, 2 binary inputs: V_1, V_{12}, 1 continuous input: Q_1 and two continuous outputs: h_1, h_2, 2 auxiliary binary variables: δ_0, δ_1 and 5 auxiliary continuous variables: $z_{01}, z_{02}, z_1, z_N, z_L$.

V. SIMULATION RESULTS

The proposed active diagnosis method is used for sanity check of the upper valve V_1. It is assumed that the valve is stuck in the ON position. It is also assumed that at the beginning both tanks are empty *i.e.* $h_1 = h_2 = 0$. The proposed predictive method is applied to check whether the valve is faulty or normal. The variable d is assumed as 0.02 and the sample time is 10 seconds. It is assumed that the valve V_L is always closed and V_N is always open.

To obtain an MLD model of the two tanks system we use HYSDEL (hybrid system description language)[20], which is a modeling language for Discrete Hybrid Automata (DHA). Given a description of the system, HYSDEL translates it into different computational models like MLD or PWA.

We also assume that at the commissioning phase we want to fill the tanks to $y_r = [0.3\ 0.2]'$ and also we want to do the sanity check for V_1. Therefore we look for the closest steady states to y_r such that the outputs are distinguishable. The results of the optimization problem (16) are:

$$\begin{aligned} &y_{s_0} = [0.3\ 0.235]', y_{s_1} = [0.2668\ 0.235]' \\ &Q_s = 0.1196, V_1 = 0, V_2 = 1 \end{aligned} \tag{30}$$

After finding the steady values a model predictive control is designed such that the normal system output tracks y_{s_0} with the initial states $[0\ 0]$. Figure 4 shows the result. As it can be seen by comparing the actual steady outputs with the expected values of the normal system it can be determined that the system is faulty.

As we said in the introduction, another application of the method is when the faulty system and the normal system have the same behaviors. This situation for the two tank example is demonstrated in Fig. 5. In this example a model predictive controller is designed for the two tank system to drive the system from $[0\ 0]$ to the equilibrium point $[0.2664\ \ 0.2349]$. This is a steady state for both the normal system and the faulty system.

Fig. 5 shows the simulation of the closed loop system. As one can see, the control variable V_1 is manipulated such that the output of the system in the normal condition and in the faulty one is exactly the same. In this situation if a stuck ON fault happens, no passive diagnoser would be able to diagnose it. In order to detect the fault by our method we look for separating steady state points close to the current steady state. The resulting steady values are:

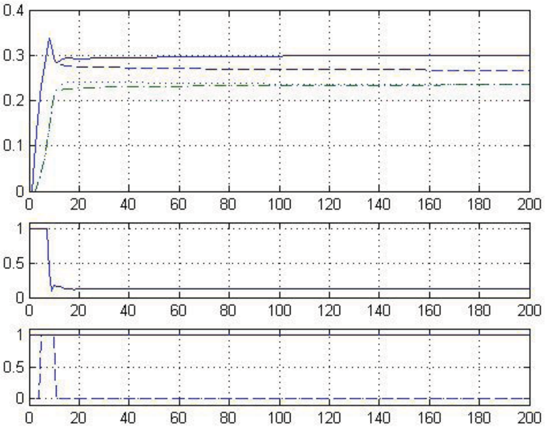

Fig. 4. Top:Actual versus expected output of the system:h_1'-', h_2'.', $\hat{h}_1$'- -', $\hat{h}_2$'.-', Middle: Continuous input Q_1, Bottom:Binary inputs: V_1 dashes and V_{12} solid

$$\begin{aligned} &y_{s_0} = [0.2667\ 0.2089]', y_{s_1} = [0.2372\ 0.2089]' \\ &Q_s = 0.1063, V_1 = 0, V_2 = 1 \end{aligned} \tag{31}$$

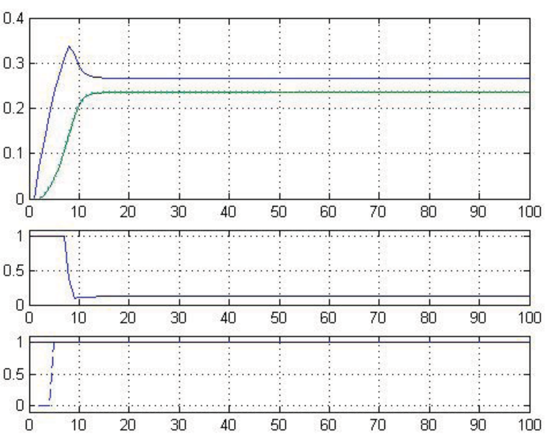

Fig. 5. Top:Actual versus expected output of the system, Middle:continuous input Q_1, Bottom:discrete inputs: V_1(dashed line), V_{12}(solid line)

The steady inputs are applied to the system and the result is shown in figure 6. As it can be seen the condition of the system is detectable by steady values of the output.

As one can see the steady value for h_2 are always the same for the faulty system and the normal system. Because the system is in the steady state, the input flow is equal to the output flow: $Q_1 = Q_N$. Q_1 is the same in both conditions. Since $Q_N = k_n\sqrt{2gh_2}$, it is obvious that in the steady state $h_{2_0} = h_{2_1}$. Therefore if we do not have measurements from h_1, it is not possible to diagnose the fault by using steady

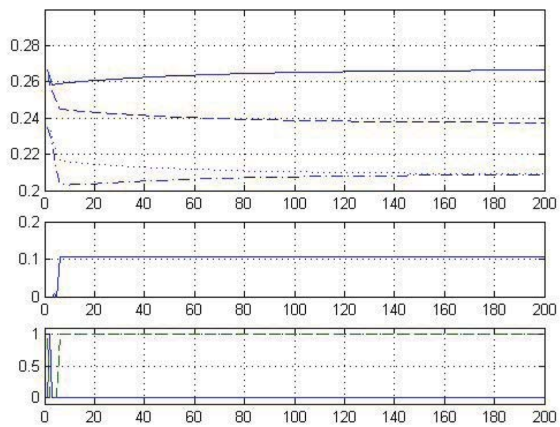

Fig. 6. Top:Actual versus expected output of the system:h_1'-', h_2'.', $\hat{h}_1$'- -', $\hat{h}_2$'.-', Middle:continuous input Q_1, Bottom:discrete inputs: V_1(dashed line), V_{12}(solid line)

values. This analysis can be used in the design phase of the system to decide where we should put sensors to be able to diagnose the fault using steady values. As it is shown in [18], it is possible to diagnose the fault while it is being perturbed from the steady values, but the problem of that method was that it may lead to instability. This method excludes the possibility of diagnosis using transient but preserves stability. A drawback of the method is that it takes a long time to reach the steady state values and therefore while it does not destabilize the system it needs a long time for diagnosis.

VI. Conclusion

In this paper a method for active diagnosis of MLD system based on analysis of steady state values of the system in normal and faulty modes is presented. The excitation obtained by this method does not destabilize the system because it moves the system to a steady state, but it is possible that there are not enough distinguishable steady output values and therefore the fault is not diagnosable using steady state values. However, this analysis method can be used in a design phase to decide about the location of sensors to guarantee diagnosability. While the method guarantee the stability during diagnosis because we should wait till the system reaches steady values the approach need a long period for diagnosis.

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