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Active diagnosis of hybrid systems - A model predictive approach

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Abstract— A method for active diagnosis of hybrid systems is proposed. The main idea is to predict the future output of both normal and faulty model of the system; then at each time step an optimization problem is solved with the objective of maximizing the difference between the predicted normal and faulty outputs constrained by tolerable performance requirements. As in standard model predictive control, the first element of the optimal input is applied to the system and the whole procedure is repeated until the fault is detected by a passive diagnoser. It is demonstrated how the generated excitation signal can be used as a test signal for sanity check at the commissioning or for detection of faults hidden by regulatory actions of the controller. The method is tested on the two tank benchmark example.

I. INTRODUCTION

In a complex control system there are many components with strong interaction between them. Hence the overall system performance depends on the individual performance of components. A fault in a single component may,therefore, degrade the overall performance of the system and may even lead to unacceptable loss of system functionality. Thus fault diagnosis is of crucial importance in automatic control of complex systems.

Diagnosis methods can be divided into two main categories: active and passive. In passive diagnosis the diagnoser observes the input and output of the system and based on the observation decides whether a fault has occured or not. The input is generated by an external input or by the controller. In active fault diagnosis the diagnoser generates an input, which excites the system, to decide whether the output represents a normal or a faulty behaviour and if possible decide which faulty behaviour occured. The generated input should move the system from the operation point but at the same time not lead the system to instability or to an unacceptable performance area.

Active diagnosis is useful in the following circumstances: (i) for generation of the test signal in the commissioning phase for sanity check of the system, (ii) for faster detection of faults during normal operation, and (iii) for detection of hidden faults where, because of regulatory actions of the controller, the normal and the faulty system exhibit the same behaviour.

Typical industrial systems include both continuous and discrete components. Therefore for a precise modeling of them a hybrid system formulation is useful. Fault diagnosis

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of hybrid systems has been investigated recently, for a survey see [14] and [20]. Among these a class of approaches uses discrete/temporal abstraction of the continuous dynamics [11]. In [13], the diagnoser uses a discrete event abstraction of the system and the continuous dynamics information is used when it becomes necessary. In [20], the authors use a Petri net to make a timed abstraction of hybrid systems to deal with the discrete part of a hybrid behaviour. A model based diagnosis method based on a hybrid bond graph modelling framework is proposed in [14]. Particle filtering methods are another approach for diagnosis of hybrid systems as in [9], [8].

Most of these methods are in the area of passive diagnosis. In [6], [15], authors propose a method for active diagnosis of linear systems using an auxiliary signal for fault detection. The results of [6] are extended to nonlinear systems in [1] using linearization and also a direct optimization approach. In [10] and [16] the problem for discrete event systems is investigated. In our previous work [18], we proposed an active fault diagnosis method for linear hybrid systems in discrete time based on reach set computation for faulty and normal systems. The results are extended to automatic sensor assignment in [17]. Computation of reachable states is computationally burdensome, especially for a hybrid system. Moreover for a system with swithed inputs computation of reachable states requires explicit enumeration of all possible combinations of switches. Hence the computational effort for these systems is high [18]. The proposed method here is based on solving an optimization problem at each time. By varying the prediction horizon for the optimization problem, one can change the size of it and hence the required computational effort. Thus, it is useful for online applications. Moreover, it handles discrete inputs more efficiently by using a better heuristic search algorithm.

The structure of the proposed method is similar to that of Model Predictive Control (MPC) in which the model used for prediction is composed of a model of the normal system and models of the faulty systems. The objective function of the optimization problem is to make an observable difference between predicted outputs of the normal system and the faulty systems fulfilling constraints imposed by required performance during fault detection. The method is an online approach, the computational complexity can be managed at the cost of loosing the minium-time diagnosis property. For modeling of hybrid system we use the Mixed Logical Dynamical (MLD) framework [4], [3] which cover important classes of hybrid system. By using the MLD framework, the on-line optimization problem used for fault diagnosis will be transfomed to mixed integer linear or quadratic problem for

which there are many efficient solvers. The method is tested on the two-tank benchmark example.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section we first introduce the MLD framework and then the active diagnosis problem is formulated.

A. Mixed Logical Dynamical Systems

We use the mixed logical dynamical (MLD) framework proposed in [4] for modeling of hybrid systems. The equations describing an MLD system are as follows:

$$x(t+1) = Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \quad (1)$$

$$y(t) = Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \quad (2)$$

$$E_2 \delta(t) + E_3 z(t) \le E_1 u(t) + E_4 z(t) + E_5 \quad (3)$$

where $x \in \mathbb{R}^{n_c} \times \{0,1\}^{n_l}$ are states, $u \in \mathbb{R}^{m_c} \times \{0,1\}^{m_l}$ are the inputs, $y \in \mathbb{R}^{p_c} \times \{0,1\}^{p_l}$ are the outputs. $\delta \in \{0,1\}^{r_l}$ and $z \in \mathbb{R}^{r_c}$ are auxiliary binary and continuous variables.

The MLD framework can model classes of hybrid systems such as PieceWise Affine (PWA) systems, linear systems with piecewise linear output functions, linear systems with discrete inputs or with qualitiative outputs, bilinear systems, and finite state machines in which an LTI system generates the events [4].

Equivalence of MLD systems with other classes of hybrid systems such as PWA systems, linear complementary (LC) systems, extended linear complementary (ELC) systems, and max-min-plus-scaling (MMPS) systems under some assumptions is shown in [7].

B. Problem Formulation

A passive model-based diagnoser is a system which recieves a sequence of input/output measurements and checks the consistency of the measured I/O sequence with a given model of the normal system $\mathcal{B}_0$ and models of the system subject to different fualts , namely $\mathcal{B}_1, \ldots, \mathcal{B}_n$. The output of the diagnoser is a fault candidate index $f \in 1, \ldots, n$ such that the observed I/O sequence is consistent with the corresponding behaviour $\mathcal{B}_f$[5]. An active diagnoser is a system that generates an input sequence $U = \langle u(0), \ldots, u(m) \rangle$ and determines an occurence of fault f by observing the output sequence $Y = \langle y(0), \ldots, y(m) \rangle$.

Problem *1 (Active diagnosis problem):* Given the set $\mathcal{B} = \{\mathcal{B}_0, \ldots, \mathcal{B}_n\}$ describing behaviours of the system with no fault and subject to faults $\{f_1, \ldots, f_n\}$, find a sequence of inputs U and $i \in \{1, \ldots, n\}$ such that (U, Y) belongs only to $\mathcal{B}_i$.

If the input sequence exists, *i.e.* if the system is diagnosable then we can look for the optimal solution, where optimality can be interpreted in different senses. The proposed algorithm looks for the shortest sequence of inputs that can diagnose the system.

The main advantage of active diagnosis is when different behaviours of the system overlap, that might happen very often, see Fig. 1. The faultless behaviour and the behaviour of the system subject to the fault f_1 are diplayed by the sets $\mathcal{B}_0$ and $\mathcal{B}_1$ repectively. As long as the observed I/O pair

uniquely belongs to the set $\mathcal{B}_0$ or $\mathcal{B}_1$, such as point A or B, it can be decided whether the system is faulty or not. But if the observed pair belongs to the intersection of $\mathcal{B}_0$ and $\mathcal{B}_1$, like C , it is impossible to diagnose the fault. The main idea of the proposed algorithm is to generate an input signal to move the system from C to an area which belongs uniquely either to the set $\mathcal{B}_0$ or $\mathcal{B}_1$.

III. THE PROPOSED ALGORITHM

It is assumed that the states of the system are available or estimated by means of an observer. The observer could be of the kind proposed in [2] or the MLD estimator proposed in [3]. Using the latter yields a more unified framework. It is supposed that the initial state is in the area where the faulty behaviour and the normal behaviour overlap because otherwise the fault could be diagnosed by means of a passive diagnoser. We assume that the model of the faulty system and the normal system is given in MLD form as in (1)-(3) with subscript 0 indicating the normal system and i indicating the system equation for the system subject to fault f_i. The aim of the diagnosis is to find a minimum sequence of inputs such that the outputs based on the different dynamics becomes different from each other that is:

$$Y_i \neq Y_j \text{ for all } i,j \in \{0, \ldots, n\}, i \neq j \quad (4)$$

Moreover the difference between $y_i(k)$ and $y_j(k)$ should be observable *i.e.*

$$|y_i(T) - y_j(T)| \ge d \text{ for all } i,j \in \{0, \ldots, n\}, i \neq j \quad (5)$$

or if a relative separation is used: $|y_i(T) - y_j(T)| \ge d \cdot |y_i(T)|$, where T is the length of the sequence and d is a separation distance that is dependent on the level of noise.
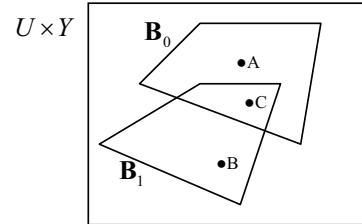

Fig. 1. System behaviour

The equations (4) and (5) aim for achieving isolability for every single fault and are very demanding. Also, one can consider the following scenarios which are less demanding:

- **Fault detection**: In fault detection we are just interested to detect if the system is working normally or is subject to any fault. In this case (4) becomes:

$$|y_0(T) - y_i(T)| \ge d \text{ for all } i \in \{1, \ldots, n\}, \quad (6)$$

- **Fault isolation for a set of faults**: In this scenario we look for a set of faults that have the same impact on the functionality of the system and also require the same fault accommodation or control reconfiguration actions, therefore, it is not important which of them should be

isolated. Moreover it could be the case that these faults can not be isolated easily and therefore we just aim at isolation of the set. Assuming that indices for these faults is given by the set $\mathcal{F}$, then (4) becomes:

$$|y_i(T) - y_j(T)| \geq d \text{ for all } i \in \mathcal{F}, j \in \{0, \ldots, n\} \backslash \mathcal{F}, \tag{7}$$

We are looking for the minimum T such that the condition (5) is satisfied. This can be formulated as an optimization problem in the following form:

$$\min_{T, \{u, \delta_i, z_i\}_0^T} \mathbf{1} \tag{8}$$

$$s.t. \begin{cases} x_i(t|t) = x(t) \\ x_i(k+1|t) = A_i x_i(k|t) + B_{1_i} u(t) \\ + B_{2_i} \delta_i(k|t) + B_{3_i} z_i(k|t) \\ y_i(k|t) = C_i x_i(k|t) + D_{1_i} u(t) \\ + D_{2_i} \delta_i(k|t) + D_{3_i} z_i(k|t) \\ E_{2_i} \delta_i(k|t) + E_{3_i} z_i(k|t) \\ \leq E_{1_i} u_i(t) + E_{4_i} z_i(k|t) + E_{5_i} \\ i = 1, \cdots, n \\ |y_i(T) - y_j(T)| \geq d, \ i, j \in \{0, \ldots, n\}, i \neq j \end{cases} \tag{9}$$

Since minimum of a constant is that constant, the above optimization problem is a constraint satisfaction problem. Optimization problem (8) can be transformed to a Mixed Integer Linear Programming (MILP) problem by introducing the following auxiliary binary variables.

$$[s_{ij1}(t) = 1] \leftrightarrow [y_i(t) - y_j(t) \leq d]$$
$$[s_{ij2}(t) = 1] \leftrightarrow [y_j(t) - y_i(t) \leq d]$$
$$s_{ij}(t) = s_{ij1}(t) \wedge s_{ij2}(t), i, j \in \{0, \ldots, n\}, i \neq j$$
$$S(t) = \wedge_{i=0}^n s_{ij}(t) \tag{10}$$

Again, the introduced variable $S(t)$ is for isolation of every single fault. For other scenarios $S(t)$ should be constructed as follows:

- **Fault detection**:

$$S(t) = \wedge_{i=1}^n s_{0i}(t) \tag{11}$$

- **Fault isolation for a set of faults**:

$$S(t) = \wedge s_{ij}(t), \text{ for all } i \in \mathcal{F}, j \in \{0, \ldots, n\} \backslash \mathcal{F} \tag{12}$$

Using the introduced auxiliary variable, (8) can be rewritten as:

$$\min_{T, \{u, \delta_i, z_i\}_0^T} S \tag{13}$$

with constraints (9) where the constraints $|y_i(T) - y_j(T)| \geq d, \ i, j \in \{0, \ldots, n\}, i \neq j$ is replaced with the corresponding mixed integer linear inequalites obtained from transforming logical propositions in (10) to equivalent mixed integer inequalities using the technique introduced in [4]. The optimization problem is similar to a minimum time optimal control problem. Given a normal model and faulty models of the system subject to the faults $\{f_1, \ldots, f_n\}$, an initial state, and a target set, we want to find the minimum T and

an input sequence $u(t), t = 1, \ldots, T$ such that $y(T)$ belongs to the target set.

We are going to solve this problem using a MPC-like structure. The structure of the proposed diagnoser is shown in Fig. 2. At each time step t, the future outputs of the system based on different models are predicted, *i.e.* $y_i(t+k|t), k = 1, \ldots, T$ and $i = 0, \ldots, n$. Having predicted outputs, $S(t+k|t)$ are predicted and then the follwing optimization problem is solved.

$$\min_{\{u, \delta_i, z_i\}_0^T} \Sigma_{k=t}^{t+T} S(k) \tag{14}$$

$$s.t. \begin{cases} x_i(t|t) = x(t) \\ x_i(k+1|t) = A_i x_i(k|t) + B_{1_i} u(t) \\ + B_{2_i} \delta_i(k|t) + B_{3_i} z_i(k|t) \\ y_i(k|t) = C_i x_i(k|t) + D_{1_i} u(t) \\ + D_{2_i} \delta_i(k|t) + D_{3_i} z_i(k|t) \\ E_{2_i} \delta_i(k|t) + E_{3_i} z_i(k|t) \\ \leq E_{1_i} u_i(t) + E_{4_i} z_i(k|t) + E_{5_i} \\ S(t) = \wedge_{i=1}^n s_i(t) i = 1, \cdots, n \end{cases} \tag{15}$$

By solving the above optimization problem the optimal input sequence is found. The first element of the sequence is applied to the system and the whole procedure is repeated at time $t+1$ again until $S(t) = 1$ which means that estimated output for different systems are different from each other. Then the output $y(t)$ is compared with the estimated outputs y_i and the fault candidate f_c is chosen as the following.

$$f_c = f_i, \ i = \operatorname*{argmin}_{i \in \{0, \ldots, n\}} |y(t) - y_i(t)| \tag{16}$$
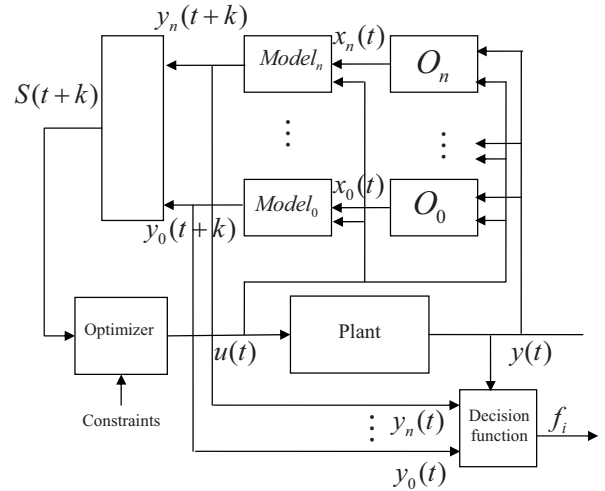

Fig. 2. Structure of the proposed method

One should note that constraints on input, outputs and states can be easily added to the optimization problem by adding them to (14) as constraints to the optimization problem. Morover we must ensure that during the fault diagnosis the system will remain in the area of tolerable

performance. This area is defined by the minimum level of control objectives and system constraints, which are required to maintain safe operation. Suppose that this area is given by some constraints on the states as a polytope $\mathcal{T} = \{x \in \mathbb{R}^n | \mathcal{P}x \leq \mathcal{M}\}$. To ensure that the system states will remain in this area during the diagnosis the following constraints are added to the optimization problem (14): $\mathcal{P}x_i(k+1|t) \leq \mathcal{M}, i = 1, \cdots, n$ and $t \leq k \leq T-1$ or in other words:

$$
\begin{cases}
\mathcal{P}A_i x_i(k|t) + \mathcal{P}B_{1_i} u(t) \\
+\mathcal{P}B_{2_i}\delta_i(k|t) + \mathcal{P}B_{3_i} z_i(k|t) \leq \mathcal{M} \\
i = \{0, \cdots, n\} \\
k = \{t, \cdots, T-1\}
\end{cases}
\tag{17}
$$

It is worthwhile to point out that soft constraints, *i.e.* constraints which can be violated, however penalized on the performance or states of the system, can be included in the optimization problem as well [4].

IV. EXAMPLE

The proposed method is tested on the two tank system shown in Fig. 3. The system consists of two cylindrical tanks with cross sectional area A. These two tanks are connected by two pipes at the bottom and at level h_v. The flows through the pipes, denoted by $Q_{12}V_{12}$ and $Q_{12}V_1$, are controlled using two on/off valves V_{12} and V_1. There is a flow Q_1 through a pump to tank 1 which is a continuous input.
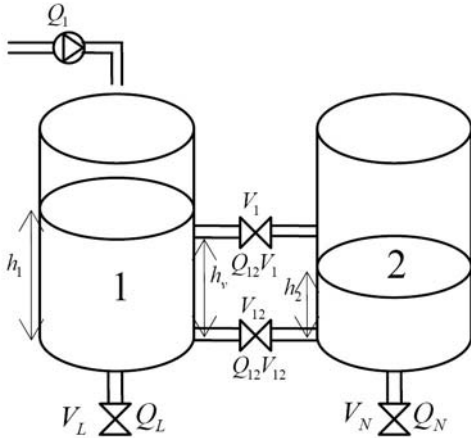

Fig. 3. Two-tank system

Dynamical equations of the system are as follows.

$$
\dot{h}_1 = \frac{1}{A}(Q_1 - Q_{12}V_{12} - Q_{12}V_1 - Q_L), \tag{18}
$$
$$
\dot{h}_2 = \frac{1}{A}(Q_{12}V_{12} + Q_{12}V_1 - Q_N), \tag{19}
$$

where h_1 and h_2 denote the levels of tanks 1 and 2 respectively. The flow $Q_{12}V_{12}$ is described by:

$$
Q_{12}V_{12} = V_{12}k_{12}sign(h_1 - h_2)\sqrt{2g|h_1 - h_2|}, \tag{20}
$$

where g is the gravity constant and k_{12} is a valve specific constant. similarly $Q_L = V_L k_L \sqrt{2gh_1}$ and $Q_N = $

$V_N k_N \sqrt{2gh_2}$. The flow through valve V_1 is given by:

$$
\begin{aligned}
Q_{12}V_1 &= V_1 k_1 sign(max\{h_v, h_1\} - max\{h_v, h_2\}) \\
&\quad \sqrt{|2g(max\{h_v, h_1\} - max\{h_v, h_2\})|} \quad (21)
\end{aligned}
$$

The MLD model of the system is derived as follows (For details see [12].). The nonlinear relation $\sqrt{x}$ is approximated by a straight line x, thus (20) becomes:

$$
Q_{12}V_{12} = V_{12}k_{12}(h_1 - h_2) \tag{22}
$$

The auxiliary continuous variable $z_{12} = V_{12}(h_1 - h_2)$ is introduced to transform the above nonlinear equation to the linear equation $Q_{12}V_{12} = k_{12}z_{12}$ with a set of mixed integer linear inequalities. For Q_N and Q_L, using the same method, we will have $Q_N = k_N z_N$ and $Q_L = k_N z_L$ where $z_N = V_N h_2$ and $z_L = V_L h_2$.

In order to transform (21) to a linear equation in the MLD framework, first we introduce the following binary variables indicating whether the level in each tank has reached h_v:

$$
[\delta_{01}(t) = 1] \leftrightarrow [h_1(t) \geq h_v] \tag{23}
$$
$$
[\delta_{02}(t) = 1] \leftrightarrow [h_2(t) \geq h_v] \tag{24}
$$

and then the term $max\{h_v, h_1\} - max\{h_v, h_2\}$ is transformed into a linear equation as $Q_{12}V_1 = k_1 z_1$, where

$$
z_1 = V_1(z_{01} - z_{02}) \tag{25}
$$
$$
z_{01} = \delta_{01}(h_1 - h_v) \tag{26}
$$
$$
z_{02} = \delta_{02}(h_2 - h_v) \tag{27}
$$

are introduced auxiliary continuous variables.

Finally, differential equations 18, 19 are discretized in time by Euler approximation $\dot{h}_i(t) \approx \frac{h_i(t+1) - h_i(t)}{T_s}$, where T_s is the sample time. The final MLD model of the system consists of two continuous states: h_1, h_2, 2 binary inputs: V_1, V_{12}, 1 continuous input: Q_1 and two continuous outputs: h_1, h_2, 2 auxialiary binary variables: δ_0, δ_1 and 5 auxiliary continuous variables: $z_{01}, z_{02}, z_1, z_N, z_L$.

V. SIMULATION RESULTS

The proposed active diagnosis method is used for sanity check of the upper valve V_1. It is assumed that the valve is stuck in the ON position. It is also assumed that at the beginning both tanks are empty *i.e.* $h_1 = h_2 = 0$. The proposed predictive method is applied to check whether the valve is faulty or normal. The variable d is assumed as 0.01 and the sample time is 10 seconds. It is assumed that the valve V_L is always closed and V_N is always open.

To obtain an MLD model of the two tanks system we use HYSDEL (hybrid system description language)[19], which is a modeling language for Discrete Hybrid Automata (DHA). Given a description of the system, HYSDEL translates it into different computational models like MLD or PWA.

The algorithm is tested on the two tank system with different prediction horizons. The size of the optimization problem (14) depends on the prediction horizon T. Figure 4 shows the result for T=3. In this case the optimization problem consists of 57 optimization variables where 30 are

continuous and 27 are binary and 261 mixed-integer linear inequalities. As it can be seen, after 6 steps the fault is detected.

Figure 5 shows the result for $T = 2$. In this case, the fault is diagnosed after 13 steps but the computational effort is less than before (38 optimization, 20 continuous and 18 binary variables and 174 mixed-integer linear inequalities).
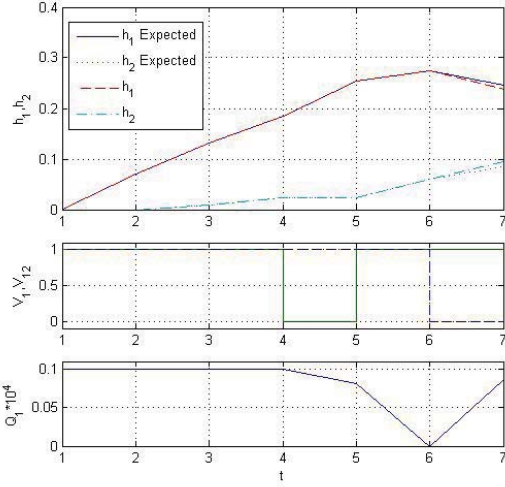

Fig. 4. Actual versus expected output of the system, Middle: Binary inputs: V_1 dashes and V_{12} solid, Bottom: Continuous input Q_1

Figure 6 shows the result for $T = 5$. In this case , the fault is diagnosed after 5 steps, but the computational effort is higher. Increasing the prediction horizon more than 5 does not yield faster diagnosis of the fault.

Table I compares the computational effort of the optimization problem required to be solved at each step and required steps to diagnose the faults. The results are based on a computer with 1.66GHz CPU and 2 GB of RAM running MATLAB 7.2. As it can be seen, a long prediction horizon result in a bigger optimization problem and hence more computational burden, but at the other hand it results in a faster diagnosis of the fault. However, increasing the prediction horizon more than a specific value (5 here) does not affect the required steps for detection ,it just increases the computational effort.

As we said in the introduction , another application of the method is when the faulty system and the normal system have the same behaviours. This situation for the two tank example is demonstrated in Fig. 7. In this example a model

TABLE I

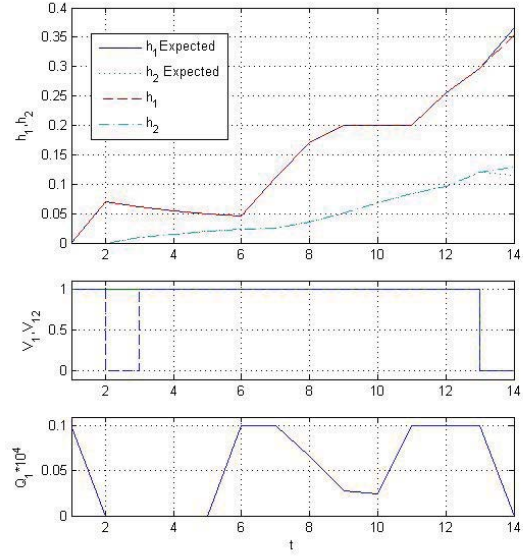| T	Steps for diagnosis	continuous variables	binary variables	Mixed integer inequalities	time a=0.0335s
2	13	20	18	174	a
3	6	30	27	261	5.26a
4	6	40	36	348	32.10a
5	5	50	45	435	655.70a
6	5	60	54	522	$1.06e^4$a

Fig. 5. Actual versus expected output of the system, Middle: Binary inputs: V_1 dashes and V_{12} solid, Bottom: Continuous input Q_1

predictive controller is designed for the two tank system. Fig. 7 shows the simulation of the closed loop system. As one can see, the control variable V_1 is manipulated such that the output of the system in the normal condition and in the faulty one is exactly the same. In this situation if a stuck ON fault happens, no passive diagnoser would be able to diagnose it, while the active diagnoser proposed here is capable of detecting this fault. Our active diagnoser was started at $t = 200$ sec. and the result witht the successful diagnosis is shown in Fig. 8.

VI. CONCLUSION

In this paper a new method for active diagnosis of hybrid systems is presented. The active diagnosis problem is reformulated as a mixed integer optimization problem using the MLD framework. At each time step an optimization problem is solved which aims at maximizing the difference between predicted outputs of the system based on model of the system subject to various faults. The first element of the optimal input sequence is applied to the system and the whole procedure is repeated until the system comes out of the area in which different models represent the same behaviour. The method can be applied online to the system. The complexity of the method is dependent on the prediction horizon of the algorithm. However, finding a short sequence for diagnosis requires a longer prediction horizon. Hence the choice of prediction horizon is a trade off between the computational complexity and the lenght of the input sequence for diagnosis.

The proposed method is not restricted to hybrid systems. It can be used for linear system or nonlinear system using linear or nonlinear programming techniques.
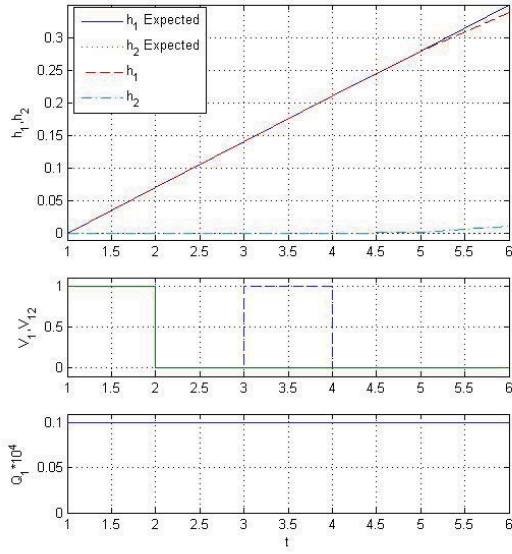
Fig. 6. Top: Actual versus expected output of the system, Middle: Binary inputs: V_1 dashes and V_{12} solid, Bottom: Continuous input Q_1
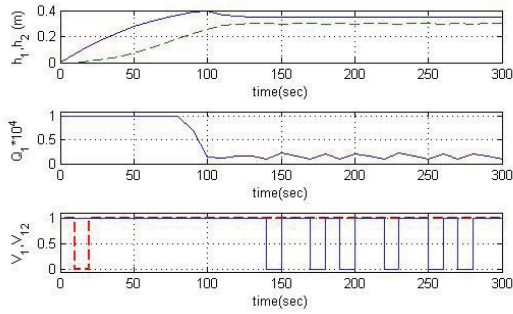

Fig. 7. Top:Actual versus expected output of the system, Middle:continuous input Q_1, Bottom:discrete inputs: V_1(dashed line), V_{12}(solid line)
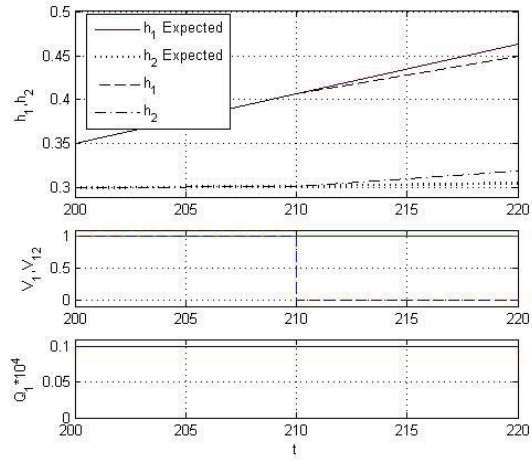

Fig. 8. Top: Actual versus expected output of the system, Middle: Binary inputs: V_1 dashes and V_{12} solid, Bottom: Continuous input Q_1

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