Passive Fault Tolerant Control of Piecewise Affine Systems Based on H Infinity Synthesis

Gholami, Mehdi; Cocquempot, vincent ; Schiøler, Henrik; Bak, Thomas

Published in:
I F A C Workshop Series

DOI (link to publication from Publisher):
10.3182/20110828-6-IT-1002.02935

Publication date:
2011

Document Version
Accepted author manuscript, peer reviewed version

Passive Fault Tolerant Control of Piecewise Affine Systems Based on H Infinity Synthesis

M. Gholami * V. Cocquempot ** H. Schiøler * T. Bak *****

* *Department of computer science (CISS), Aalborg University, e-mail:mehdi@cs.aau.dk*
** *LAGIS-CNRS, FRE 3303, Lille1 University 59655 Villeneuve dAscq cedex, France vincent.cocquempot@univ-lille1.fr*
*** *Section of Automation and Control, Aalborg University, e-mail:{henrik, tba}@es.aau.dk*

Abstract: In this paper we design a passive fault tolerant controller against actuator faults for discretetime piecewise affine (PWA) systems. By using dissipativity theory and H_∞ analysis, fault tolerant state feedback controller design is expressed as a set of Linear Matrix Inequalities (LMIs). In the current paper, the PWA system switches not only due to the state but also due to the control input. The method is applied on a large scale livestock ventilation model.

1. INTRODUCTION

Performance of modern control systems typically relies on a number of strongly interconnected components. Component malfunctions may degrade performance of the system or even result in loss of functionality. In applications such as climate control systems for livestock buildings, this is unacceptable as it may lead to the loss of animal life. Therefore, it is desirable to develop control systems such that they are capable of tolerating component malfunctions while still maintaining desirable performance and stability properties.

Fault tolerant control (FTC) is divided generally into passive (PFTC) and active (AFTC) approaches. In AFTC, the control loop is adapted online according to information given by a fault detection and isolation (FDI) module. Generally speaking, AFTC systems are divided into three layers as proposed in Blanke et al. [2006]. The first layer is related to the control loop, the second layer corresponds to the FDI and accommodation modules and the last layer corresponds to the supervisor system. PFTC does not need any FDI or supervisor layer. In this technique the control laws are fixed and the fault is considered as a system disturbance or uncertainty. In fact, the control law is designed to preserve the system performance either in healthy or in faulty situation using robust control techniques, see Chen and Patton [1999], Qu et al. [2001], and Qu et al. [2003]. Most complex industrial systems either exhibit nonlinear behaviour or involve both discrete and continuous components. One of the modelling frameworks which is relevant for nonlinear and most classes of hybrid systems with both discrete and continuous behaviours, is piecewise affine systems (PWA). This framework has been applied in several areas, such as, switched system, Rodrigues and Boukas [2006], etc. For AFTC systems, the reader is referred to Rodrigues et al. [2006], where the authors developed an AFTC against actuator failures for discrete-time switched linear systems. In Richter et al. [2010], an AFTC approach for continuous-time PWA system subject to actuator and sensor faults is proposed. In Yang et al. [2009] a fault accommodation problem is discussed for a class of hybrid systems. A PFTC approach is presented in **??**, where a state feedback controller is designed for continuous-time PWA systems subject to actuator faults.

In Tabatabaeipour et al. [2010], a PFTC for discrete time PWA systems is presented. The approach is based on a state feedback control that is tolerant against actuator faults. The PWA systems switch only due to state variables. In this paper, we consider PFTC for the general class of discretetime PWA models whose switching sequence depends on both state and input trajectories. We use a piecewise quadratic (PWQ) Lyapunov function and H_∞ analysis in order to design a state feedback controller such that the closed loop system is asymptotically stable in healthy and in actuators failure situations. The problem is cast as a set of Linear Matrix inequalities (LMI) and solve with YALMIP/ SeDumi, see Löfberg [2004]. The H_∞ analysis is based on the passivity theory for nonlinear systems as in Cuzzola and Morari [2001].

The paper is organized as follows. Section II presents the piecewise affine model and actuator fault representation. Section III discusses H_∞ control design for PWA systems. The extension of H_∞ synthesis for fault tolerant control of piecewise affine systems is discussed in section IV. Section V is dedicated to the simulation results for the climate control system. The conclusion is presented in section VI.

2. PIECEWISE AFFINE SYSTEMS AND ACTUATOR FAULT REPRESENTATION

2.1 Piecewise Affine Systems

Consider a discrete-time piecewise affine system, $\sum_i$ as:

$$x(k+1) = A_i x(k) + B_i u(k) + a_i \qquad for \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i, \tag{1}$$

$$y(k) = Cx(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, $y(k) \in \mathcal{R}^p$ is the output. The set $\mathbb{X} \subseteq \mathbb{R}^{n+m}$ represents every possible vector $\left[x(k)^T u(k)^T\right]^T$, $\{\mathcal{X}_i\}_{i=1}^s$ denotes polyhedral regions of $\mathbb{X}$ and $a_i \in \mathbb{R}^n$ is a constant vector. Each polyhedral region is represented by:

$$\mathcal{X}_i = \{\left[x(k)^T u(k)^T\right]^T \mid F_i^x x \geq f_i^x \text{ and } F_i^u u \geq f_i^u\} \tag{3}$$

It is assumed that the regions are defined with known matrices F_i^x, F_i^u, f_i^x and f_i^u. The following notations are defined as in Cuzzola and Morari [2001]:

$$\bar{\mathcal{X}}_i = \{x(k) \mid F_i^x x \geq f_i^x\} \tag{4}$$

and

$$S_j = \{i \mid \exists x, u \text{ with } x \in \bar{\mathcal{X}}_i, [x^T \ u^T]^T \in \mathcal{X}_i\} \tag{5}$$

S_j denotes the set of all indices i such that $\mathcal{X}_i$ is a region including a vector $[x^T \ u^T]^T$ when the condition $x \in \bar{\mathcal{X}}_i$ is satisfied. $\mathscr{I} = \{1, \cdots, s\}$ is the set of indices of regions $\mathcal{X}_i$ and $\mathcal{I} = \{1, \cdots, t\}$ is the set of indices of the regions $\bar{\mathcal{X}}_j$. All possible switchings from region $\mathcal{X}_i$ to $\mathcal{X}_j$ are defined by the set $\mathcal{S}$:

$$\mathcal{S} = \{(i,j) : \ i,j \in \mathscr{I} \text{ and } \exists \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}, \begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} \in \mathbb{X} \tag{6}$$
$$\mid \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i \text{ and } \begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} \in \mathcal{X}_j\}$$

2.2 Fault Model

Actuator faults are considered. u_j is the actuator output. The partial loss of actuator can be formulated as

$$u_j^F = (1 - \alpha_j) u_j, \quad 0 \leq \alpha_j \leq \alpha_{Mj}, \tag{7}$$

where α_j is the percentage of efficiency loss of the actuator j and α_{Mj} is the maximum loss. $\alpha_j = 0$ corresponds to the nominal system, $\alpha_j = 1$ corresponds to 100% loss of the actuator and $0 \leq \alpha_j \leq 1$ corresponds to partial loss. Let us define α as

$$\alpha = diag\{\alpha_1, \alpha_2, \ldots, \alpha_m\}. \tag{8}$$

Then

$$u^F = \Gamma u, \tag{9}$$

where $\Gamma = (I_{m \times m} - \alpha)$, I is a identity matrix. Thus u^F represents the control signal that is applied in normal or faulty situation. The PWA model of the system with the fault F_i is

$$x(k+1) = A_i x(k) + B_i \Gamma_i u(k) + a_i \quad for \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i \tag{10}$$

3. H_∞ CONTROL DESIGN FOR PIECEWISE AFFINE SYSTEMS

3.1 H_∞ Performance

Consider the PWA system

$$x(k+1) = A_i x(k) + B_i u(k) + B_i^w w(k) + a_i \tag{11}$$
$$for \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i, \ x(k) \in \bar{\mathcal{X}}_j$$

$$z(k) = C_i x(k) + D_i u(k) + D_i^w w(k) \tag{12}$$

where $w(k) \in \mathbb{R}^r$ is a disturbance signal and $z(k) \in \mathbb{R}^s$ is a performance output. First, for the sake of simplicity, it is assumed that $a_i = 0$, and the control objective is to track the origin wit the initial condition $x(0) = 0$. The H_∞ performance for each integer $N \geq 0$ is written as

$$\sum_{g=0}^{N} \|z(g)\|^2 \leq \gamma^2 \sum_{g=0}^{N} \|w(g)\|^2 \tag{13}$$

whixh expresses that the H_∞ norm from th edisturbance w to the performnace output z is less than γ.

3.2 Controller Structure

Consider a piecewise linear state feedback control with the following structure

$$u(k) = K_i x(k) \ \ for \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i \tag{14}$$

where K_i is the controller gain which is designed to stabilize exponentially the closed loop PWA system. Since the index i is not a priori known, it is not possible to calculate $u(k)$. Hence, the problem is changed to the following structure

$$u(k) = K_j x(k) \ \ for \ x(k) \in \bar{\mathcal{X}}_j \tag{15}$$

It means that we do not consider a different controller in each region $\mathcal{X}_i$ with $i \in \mathscr{I}$ but a different one in each region $\bar{\mathcal{X}}_j$ with $j \in \mathcal{I}$.

Applying the control law (15) to the system (12) yields the following closed loop system:

$$x(k+1) = \mathcal{A}_{ij} x(k) + B_i^w w(k) \ for \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i, \ x(k) \in \bar{\mathcal{X}}_j \tag{16}$$

$$z(k) = \mathcal{C}_{ij} x(k) + D_i^w w(k) \tag{17}$$

where $\mathcal{A}_{ij} = A_i + B_i K_j$, $\mathcal{C}_{ij} = C_i + D_i K_j$, and $u(k) = K_j x(k)$.

Lemma 1. (Petersen [1987]) Let M, N, H be real matrices. If $H^T H \leq I$, then for every scalar $\epsilon > 0$ the following inequality hold:

$$MHN + N^T H^T M^T \leq \epsilon M M^T + \epsilon^{-1} N^T N. \tag{18}$$

Lemma 2. (Cuzzola and Morari [2001]) Consider the system (17) with zero initial condition $x(0) = 0$. If there exists a function $V(x, u) = x^T P_i x$ for $[x^T \ u^T]^T \in \mathcal{X}_i$ with $P_i = P_i^T > 0$ satisfying the dissipativity inequality

$$\forall k, \ V(x(k+1), \ u(k+1)) - V(x(k), \ u(k)) \tag{19}$$
$$< \gamma^2 \|w(k)\|^2 - \|z(k)\|^2$$

then, the H_∞ performnace condition (13) is satisfied.

Furthermore, condition (19) is fulfilled if the following matrix inequalities are satisfied

$$\forall j \in \mathcal{I}, \ \forall i \in S_j, \ \forall l \text{ with } (l,j) \in S, \ M_{l,ij} < 0. \tag{20}$$

where

$$M_{l,i,j} = \begin{bmatrix} \mathcal{A}_{ij}^T P_l \mathcal{A}_{ij} - P_i + \mathcal{C}_{ij}^T \mathcal{C}_{ij} & * \\ D_i^T \mathcal{C}_{ij} + B_i^T P_l \mathcal{A}_{ij} & B_i^T P_l B_i + D_i^T D_i - \gamma^2 I \end{bmatrix} \tag{21}$$

In the last case the system (17) is PWQ stable.

4. EXTENSION OF H_∞ SYNTHESIS FOR PASSIVE FAULT TOLERANT CONTROL OF PIECEWISE AFFINE SYSTEMS

It is assumed that the control objective is to track the reference x_r when the system is subject to fault F_i . With the change of coordinates $e = x - x_r$ the problem is transformed into the origin tracking form. In these coordinates, the system dynamics (12) subject to the fault F_i are

$$e(k+1) = A_i e(k) + B_i \Gamma_i u(k) + \tilde{B}_i^w \tilde{w}(k) \quad \begin{bmatrix} e(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i, \tag{22}$$

$$e(k) \in \bar{X}_i,$$

where $\tilde{w}(k) = \begin{bmatrix} w(k) \\ a_i + A_i x_r - x_r \end{bmatrix}$ and $\tilde{B}_i^w(k) = [B_i^w \ I]$.

The polyhedral regions are written as

$$\mathcal{X}_i = \{[e^T \ u^T]^T | \ F_i^x e \geq f_i^e \ and \ F_i^u \geq f_i^u\} \tag{23}$$

$$\bar{\mathcal{X}}_i = \{e | \ F_i^x e \geq f_i^e\} \tag{24}$$

where $f_i^e = f_i^x - F_i^x x_r$.

Applying the control law (15) to the system (22) leads to the following closed loop system:

$$e(k+1) = \mathcal{A}_{ij} e(k) + \tilde{B}_i^w \tilde{w}(k) \quad \begin{bmatrix} e(k) \\ u(k) \end{bmatrix} \in \mathcal{X}_i, \ e(k) \in \bar{X}_i, \tag{25}$$

where $\mathcal{A}_{ij} = A_i + B_i \Gamma_i K_j$, $u(k) = K_j e(k)$ and $z(k) = e(k)$.

4.1 Passive Fault Tolerant Control

Definition 1. A piecewise linear control law (15) is a passive fault-tolerant control if the closed loop system (25) is asymptotically stable and the H_∞ tracking performance is guaranteed for all $\tilde{w}(k)$. This definition is expressed in the. following theorem.

Theorem 1. The fault tolerant piecewise linear controller (15) stabilizes the system (25) whilst fulfilling the dissipativity inequality (19), if there exist symmetric matrices $Q_i = Q_i^T > 0$, invertible matrices G_i, matrices Y_i and positive scalars $\epsilon_{ij} > 0$, $i \in \mathscr{I}$, $j \in \mathcal{I}$ such that

$$\begin{bmatrix} Q_i - G_i^T - G_i & 0 & (A_i G_i + B_i Y_i)^T & G_j^T & Y_j^T \alpha_i \\ 0 & -\gamma^2 I & \tilde{B}_i^{w^T} & 0 & 0 \\ (A_i G_j + B_i Y_j) & \tilde{B}_i^w & -Q_l + \epsilon_{ij} B_i B_i^T & 0 & 0 \\ G_j & 0 & 0 & -I & 0 \\ \alpha_i Y_j & 0 & 0 & 0 & -\epsilon_{ij}^{-1} \end{bmatrix} < 0 \tag{26}$$

$$\forall j \in \mathcal{I}, \ \forall i \in S_j, \ \forall l \ with \ (l,i) \in S_{all}$$

Then the piecewise affine feedback gains are obtained by:

$$K_j = Y_j G_j^{-1} \tag{27}$$

Proof 1. Passivity inequality (19) is equivalent to:

$$(e(k)^T \mathcal{A}_{ij}^T + \tilde{w}^T(k) \tilde{B}_i^{w^T}) P_l (e(k)^T \mathcal{A}_{ij}^T + \tilde{w}^T(k) \tilde{B}_i^{w^T})^T \tag{28}$$

$$-e(k)^T P_i e(k) + e(k)^T e(k) - \gamma^2 \tilde{w}_k^T \tilde{w}_k < 0$$

which is equivalent to

$$\begin{bmatrix} \mathcal{A}_{ij}^T P_l \mathcal{A}_{ij} - P_i + I & * \\ \tilde{B}_i^{w^T} P_l \mathcal{A}_{ij} & \tilde{B}_i^{w^T} P_l \tilde{B}_i^w - \gamma^2 I \end{bmatrix} < 0 \tag{29}$$

By substituting $Q = P^{-1}$, it is obtained:

$$\begin{bmatrix} -Q_i^{-1} + I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \mathcal{A}_{ij}^T \\ \tilde{B}_i^{w^T} \end{bmatrix} Q_l^{-1} [\mathcal{A}_{ij} \ \tilde{B}_i^w] < 0 \tag{30}$$

Using Schur complement we get

$$\begin{bmatrix} -Q_i^{-1} & 0 & \mathcal{A}_{ij}^T & I \\ 0 & -\gamma^2 I & \tilde{B}_i^{w^T} & 0 \\ \mathcal{A}_{ij} & \tilde{B}_i^w & -Q_l & 0 \\ I & 0 & 0 & -I \end{bmatrix} < 0 \tag{31}$$

Pre- and post multiplying the right side of (31) by diag $\{G_j^T, \ I, \ I, \ I\}$ and diag $\{G_j, \ I, \ I, \ I\}$, substituting the value of $\mathcal{A}_{ij}$, and using the fact that $G_j^T P_j G_j \geq G_j + G_j^T - P_j^{-1}$, as in Cuzzola and Morari [2001], it is obtained that:

$$\begin{bmatrix} Q_i - G_j^T - G_j & 0 & (A_i G_j + B_i (I - \alpha_i) Y_j)^T & G_j^T \\ 0 & -\gamma^2 I & \tilde{B}_i^{w^T} & 0 \\ (A_i G_j + B_i (I - \alpha_i) Y_j) & \tilde{B}_i^w & -Q_l & 0 \\ G_j & 0 & 0 & -I \end{bmatrix} < 0 \tag{32}$$

which is equivalent to

$$\begin{bmatrix} Q_i - G_j^T - G_j & 0 & (A_i G_j + B_i Y_j)^T & G_j^T \\ 0 & -\gamma^2 I & \tilde{B}_i^{w^T} & 0 \\ (A_i G_j + B_i Y_j) & \tilde{B}_i^w & -Q_l & 0 \\ G_j & 0 & 0 & -I \end{bmatrix} - \tag{33}$$

$$\begin{bmatrix} 0 \\ 0 \\ B_i \\ 0 \end{bmatrix} [\alpha_i Y_j \ 0 \ 0 \ 0] - \begin{bmatrix} Y_j^T \alpha_i \\ 0 \\ 0 \\ 0 \end{bmatrix} [0 \ 0 \ B_i^T \ 0] < 0$$

Using Lemma 2 in Tabatabaeipour et al. [2010] with $H = -I$, it is obtained:

$$(33) \leq (*) + \epsilon_{ij} \begin{bmatrix} 0 \\ 0 \\ B_i \\ 0 \end{bmatrix} [0 \ 0 \ B_i^T \ 0] + \epsilon_{ij}^{-1} \begin{bmatrix} Y_j^T \alpha_i \\ 0 \\ 0 \\ 0 \end{bmatrix} [\alpha_i Y_j \ 0 \ 0 \ 0] \tag{34}$$

where $(*)$ is the first matrix in (33). We have $\alpha_i \leq \alpha_{M_i}$, therefore it holds that:

$$(34) \leq \begin{bmatrix} \Lambda & 0 & (A_i G_j + B_i Y_j)^T & G_j^T \\ 0 & -\gamma^2 I & \tilde{B}_i^{w^T} & 0 \\ (A_i G_j + B_i Y_j) & \tilde{B}_i^w & -Q_l + \epsilon_{ij} B_i B_i^T & 0 \\ G_j & 0 & 0 & -I \end{bmatrix} < 0 \tag{35}$$

where $\Lambda = Q_i - G_j^T - G_j + \epsilon_{ij}^{-1} Y_j^T \alpha_{M_i} \alpha_{M_i} Y_j$. With Schur complement we derive the LMI (26).

5. SIMULATION RESULTS FOR A CLIMATE CONTROL SYSTEM OF LIVE-STOCK BUILDING

The PFTC algorithm is applied to a hybrid climate control systems of a live-stock building, which was obtained during previous research, Gholami et al. [2010]. The general schematic of the large scale live-stock building equipped with hybrid climate control system is illustrated in Figure. 1. In a large scale stable, the indoor airspace is incompletely mixed; therefore it is divided into conceptually homogeneous parts called zones. Due to the indoor and outdoor conditions, the airflow direction varies between adjacent zones. Therefore, the system behavior is represented by a finite number of dynamic equations. The model is divided into subsystems as follows: Inlet model for both windward and leeward, outlet model, and stable heating system, and finally the dynamic model of temperature based on the heat balance equation. The dynamic model of the temperature turns out to be a piecewise nonlinear model. Since there is a few research on FTC of the piecewise nonlinear models, the obtained model is approximated into a discrete-time PWA system of type (12) where each nonlinear model of every polyhedral region $\mathcal{X}_i$ is approximated by a liear model. The discrete-time PWA model has 4 regions $\mathcal{X}_1, \ldots, \mathcal{X}_4$.

The piecewise-affine model of the system is derived for the following polyhedral regions of $\mathbb{X}$:

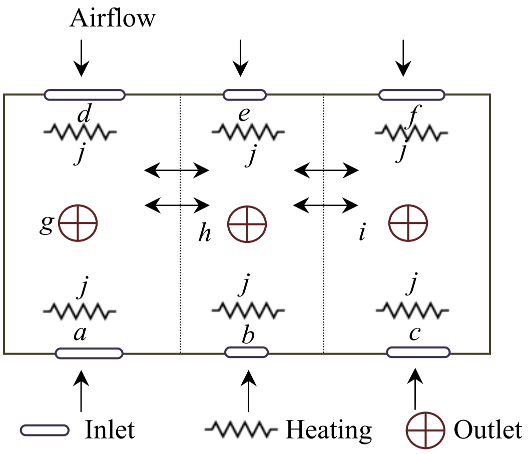

Fig. 1. The top view of the test stable

$$\mathcal{X}_1 = \{[x^T \; u^T]^T | F_1^x x + F_1^u \geq f_1, F_2^x x + F_2^u \geq f_2\}, \quad (36)$$
$$\mathcal{X}_2 = \{[x^T \; u^T]^T | F_1^x x + F_1^u < f_1, F_2^x x + F_2^u < f_2\}, \quad (37)$$
$$\mathcal{X}_3 = \{[x^T \; u^T]^T | F_1^x x + F_1^u < f_1, F_2^x x + F_2^u \geq f_2\}, \quad (38)$$
$$\mathcal{X}_4 = \{[x^T \; u^T]^T | F_1^x x + F_1^u \geq f_1, F_2^x x + F_2^u < f_2\}, \quad (39)$$

,where

$$F_1^x = [1.0817 \; -0.0457 \; -0.9938]$$
$$F_2^x = [-1.1144 \; 0.0490 \; 1.0187]$$
$$F_1^u = [0.2323 \; -0.0072 \; 0.2323 \; 0.2323 \; -0.0072 \; 0.2323 \; -0.072 \; 0.1349 \; -0.0719 \; -0.0064],$$
$$F_2^u = [-0.2558 \; 0.0074 \; -0.2558 \; -0.2558 \; 0.0074 \; -0.2558 \; 0.0742 \; -0.12 \; 0.0742 \; 0.0074],$$
$$f_1 = 0.4058, f_2 = -0.4575 \quad (40)$$

Here, the polyhedral region $\mathcal{X}_i$ is defined by two inequalities which depend on the state and input, while in (3), the $\mathcal{X}_i$ is defined by two inequalities independently based on the state or the input. As the result, it is not possible to define $\bar{\mathcal{X}}_i$ as in (4), therefore it is changed as $\bar{\mathcal{X}} = \{x \textit{ such that } x \in \mathbb{R}^n\}$. It denotes that the region $\bar{\mathcal{X}}$ for defining the controller is assumed to be common for $\mathbb{X}$.

The discrete-time PWA model is described by:

$$A_1 = \begin{bmatrix} 1.6361 & 0.0480 & -0.7716 \\ 1.5782 & 0.5522 & -0.9983 \\ 0.7747 & 0.0462 & 0.0990 \end{bmatrix}, \quad (41)$$

$$A_2 = \begin{bmatrix} 1.1145 & -0.0300 & -1.0590 \\ 1.6452 & 0.1010 & -1.4342 \\ 0.3008 & 0.0191 & -0.2324 \end{bmatrix}, \quad (42)$$

$$A_3 = \begin{bmatrix} 1.6340 & 0.0259 & -0.7150 \\ 1.5474 & 0.8335 & -1.4790 \\ 0.7674 & 0.0314 & 0.1456 \end{bmatrix}, \quad (43)$$

$$A_4 = \begin{bmatrix} 1.6274 & 0.0049 & -0.6987 \\ 1.6242 & 0.8163 & -1.4751 \\ 0.7623 & 0.0051 & 0.1640 \end{bmatrix}, \quad (44)$$

$$B_1 = \begin{bmatrix} -0.1163 & 0.0459 & -0.1163 & -0.1163 & 0.0459 & -0.1163 & 0.0018 & -0.0567 & 0.0018 & 0.0070 \\ 0.5718 & -0.3768 & 0.5718 & 0.5718 & -0.3768 & 0.5718 & -0.1518 & 0.2724 & -0.1518 & -0.0056 \\ -0.1147 & 0.0353 & -0.1147 & -0.1147 & 0.0353 & -0.1147 & 0.0022 & -0.0553 & 0.0022 & 0.0071 \end{bmatrix}, \quad (45)$$

$$B_2 = \begin{bmatrix} 0.1137 & -0.0044 & 0.1137 & 0.1137 & -0.0044 & 0.1137 & -0.0697 & 0.2883 & -0.0697 & 0.0023 \\ -0.0104 & 0.1057 & -0.0104 & -0.0104 & 0.1057 & -0.0104 & 0.0183 & 0.8276 & 0.0183 & 0.1275 \\ 0.0581 & 0.0258 & 0.0581 & 0.0581 & 0.0258 & 0.0581 & 0.0097 & 0.0939 & 0.0097 & 0.0273 \end{bmatrix}, \quad (46)$$

$$B_3 = \begin{bmatrix} -0.0677 & -0.0127 & -0.0677 & -0.0677 & -0.0127 & -0.0677 & -0.0103 & -0.0080 & -0.0103 & 0.0078 \\ 0.2031 & 0.0778 & 0.2031 & 0.2031 & 0.0778 & 0.2031 & -0.0594 & -0.0506 & -0.0594 & -0.0012 \\ -0.0697 & -0.0188 & -0.0697 & -0.0697 & -0.0188 & -0.0697 & -0.0098 & -0.0087 & -0.0098 & 0.0075 \end{bmatrix}, \quad (47)$$

$$B_4 = \begin{bmatrix} -0.0393 & -0.0380 & -0.0393 & -0.0393 & -0.0380 & -0.0393 & -0.0133 & -0.0234 & -0.0133 & 0.0086 \\ 0.0851 & 0.1683 & 0.0851 & 0.0851 & 0.1683 & 0.0851 & -0.0568 & 0.0160 & -0.0568 & 0.0029 \\ -0.0414 & -0.0434 & -0.0414 & -0.0414 & -0.0434 & -0.0414 & -0.0130 & -0.0241 & -0.0130 & 0.0085 \end{bmatrix}, \quad (48)$$

$$a_1 = \begin{bmatrix} 0.4749 \\ -0.9236 \\ 0.4214 \end{bmatrix}, \; a_2 = \begin{bmatrix} -0.0676 \\ 2.2442 \\ 0.3784 \end{bmatrix}, \quad (49)$$

$$a_3 = \begin{bmatrix} 0.2356 \\ 0.3694 \\ 0.2500 \end{bmatrix}, \; a_4 = \begin{bmatrix} 0.3510 \\ -0.5021 \\ 0.3682 \end{bmatrix}. \quad (50)$$

Here, there is no any disturbance input of type w and initial condition is considered as $x(0) = [10 \; 10 \; 10]^T$. We assume that 5 of the 6 inlets are faulty and lose 90% of their efficiency. The objective is to regulate the temperature of each zone, x around 20 ^oC. The passive fault tolerant controller based on H_∞ synthesis obtained by Theorem 1 is designed for the system using YALMIP/ SeDuMi. The LMI (26) is not feasible for $\gamma < 8$, hence it is assumed that $\gamma = 8$.

We obtain

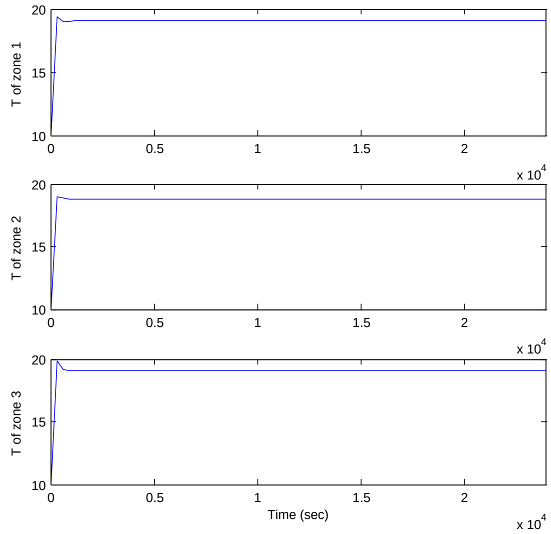

Fig. 2. Simulation results with a controller designed to tolerate 90% actuator failure for the fault-free system with $\alpha = 0$.

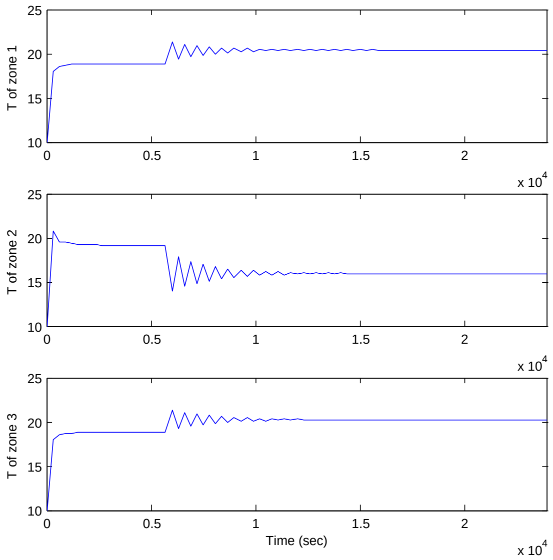

Fig. 3. Simulation results with a controller designed to tolerate 90% actuator failure for the faulty system with $\alpha = 0.9$.

$$K = 10^3 \times \begin{bmatrix} -0.0000 & -0.0000 & -0.0000 \\ -0.0000 & -0.0001 & 0.0001 \\ -0.0000 & 0.0000 & 0.0000 \\ -0.0000 & -0.0000 & -0.0000 \\ -0.0000 & -0.0001 & 0.0001 \\ 0.0034 & -0.0006 & -0.0006 \\ 0.7971 & 0.7710 & -2.5273 \\ 0.0054 & 0.0013 & -0.0020 \\ -0.7614 & -0.7644 & 2.5521 \\ -0.0462 & -0.0083 & 0.0158 \end{bmatrix} \tag{51}$$

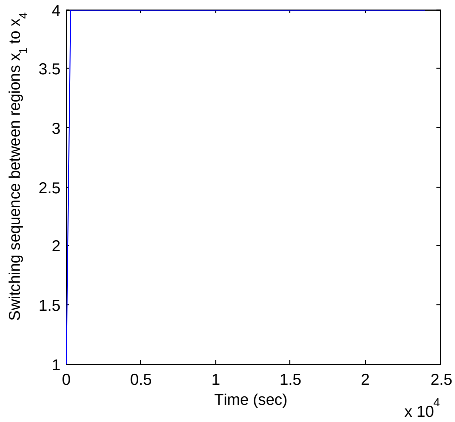

Fig. 4. Switching sequence of the closed loop system between regions $\mathcal{X}_1, \ldots, \mathcal{X}_4$ when there is no fault.

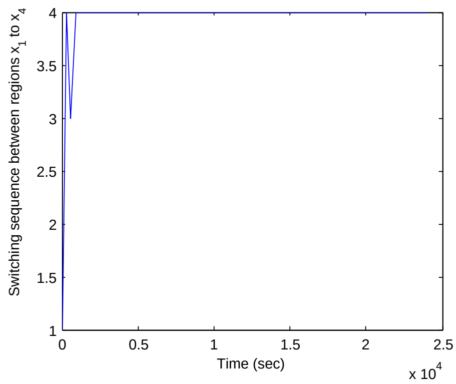

Fig. 5. Switching sequence of the closed loop system between regions $\mathcal{X}_1, \ldots, \mathcal{X}_4$ when 90% actuator failure happen at the system.

$$P_1 = \begin{bmatrix} 5.1016 & 0.0698 & -3.9781 \\ 0.0698 & 1.3854 & -0.5175 \\ -3.9781 & -0.5175 & 6.3529 \end{bmatrix} \tag{52}$$

$$P_2 = \begin{bmatrix} 4.8108 & 0.0594 & -3.7864 \\ 0.0594 & 1.3861 & -0.4458 \\ -3.7864 & -0.4458 & 6.3702 \end{bmatrix} \tag{53}$$

$$P_3 = \begin{bmatrix} 5.1644 & 0.0370 & -3.8885 \\ 0.0370 & 1.2987 & -0.4113 \\ -3.8885 & -0.4113 & 6.48669 \end{bmatrix} \tag{54}$$

$$P_4 = \begin{bmatrix} 5.1216 & 0.0509 & -4.0513 \\ 0.0509 & 1.4384 & -0.6108 \\ -4.0513 & -0.6108 & 6.1853 \end{bmatrix} \tag{55}$$

As it is obvious from Fig. 2, the fault tolerant controller regulates the temperature of each zone around $T_1 = T_2 = T_3 = 19\ ^oC$ when there is no actuator efficiency loss. The difference between the regulated temperature $T = 10\ ^oC$ and the reference $T_2 0\ ^oC$ is due to the large value of $\gamma = 8$, which leads to degradation of H_∞ performance, according to (13). In Fig. 3, it is assumed that 5 of the 10 actuators are faulty and lose 90% their efficiency at time 6000 second. As it is shown, the output of the closed loop system oscillates when the fault occurs. However the fault tolerant controller stabilizes the system with some performance degradation as $T_1 = 20.4,\ T_2 = 17,\ T_3 = 20.3\ ^oC$. The switching sequences of the fault free closed loop system as well as faulty system are given in Fig. 4 and Fig. 5.

As it was mentioned before, the fault tolerant controller is not able to regulate the temperature exactly around the reference signal due to the large value of γ. Here, the fault tolerant controller is designed for the ventilation systems of the stable where the suitable temperature for animals should stay between 16 ^oC and 21 ^oC. Therefore this performance degradation is admissible.

6. CONCLUSIONS AND FUTURE WORKS

In this paper, we derived a passive fault tolerant controller against actuator losses using a discrete-time PWA model of a piecewise nonlinear system. The PWA model switches not only based on the state but also based on the control input. The H_∞ analysis is used to design a fault tolerant controller. The stability guarantee of the closed loop system is investigated by PWQ Lyapunov function. The controller design is reformulated as a set of LMIs. The simulation confirms that the controller is able to tolerate against 90% actuator fault with performance degradation.

For the future works, the model uncertainties and noise can be considered in the FTC problem.

REFERENCES

M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Springer-Verlag, 2006.

J. Chen and R.J. Patton. *Robust model-based fault diagnosis for dynamic systems*. Kluwer Academic Publishers Norwell, MA, USA, 1999.

F.A. Cuzzola and M. Morari. A generalized approach for analysis and control of discrete-time piecewise affine and hybrid systems. *Hybrid Systems: Computation and Control*, 2034:189–203, 2001.

M. Gholami, H. Shioler, and T. Soltani, M. Bak. Multi-Zone Hybrid Model for Failure Detection of the Stable Ventialation System. In *Proceedings of Industrial Electronic Society*, 2010.

J. Löfberg. YALMIP : A toolbox for modeling and optimization in MATLAB. In *Proceedings of the IEEE Conference on Computer Aided Control Systems Design*, pages 284–289, 2004.

I. R. Petersen. A stabilization algorithm for a class of uncertain linear systems. *System and Control Letters*, 8 (4):351–357, 1987.

Zhihua Qu, C.M. Ihlefeld, Jin Yufang, and Apiwat Saengdeejing. Robust control of a class of nonlinear uncertain systems. fault tolerance against sensor failures and subsequent self recovery. In *Decision and Control, 2001. Proceedings of Decision and Control, the 40th IEEE Conference*, volume 2, pages 1472 –1478, 2001. doi: 10.1109/.2001.981102.

Zhihua Qu, Curtis M. Ihlefeld, Yufang Jin, and Apiwat Saengdeejing. Robust fault-tolerant self-recovering control of nonlinear uncertain systems. *Automatica*, 39 (10):1763 – 1771, 2003. ISSN 0005-1098. doi: DOI: 10.1016/S0005-1098(03)00181-X.

J. H. Richter, W. Heemels, N. van de Wouw, and J. Lunze. Reconfigurable control of piecewise affine systems with actuator and sensor faults: stability and tracking1. Technical report, Institute of Automation and Computer Control, Ruhr-Universität Bochum, 2010.

L. Rodrigues and E.-K Boukas. Piecewise linear h_∞ controller synthesis with application to inventory control of switched production systems. *Automatica*, 42:1245–1254, 2006.

M. Rodrigues, D. Theilliol, and D. Sauter. Fault tolerant control design for switched systems. In *2nd IFAC Conference on Analysis and design of hybrid systems, Alghero Sardinia, Italy*, 2006. 10.3182/20060607-3-IT-3902.00041.

S. Tabatabaeipour, R. Izadi-Zamanabadi, T. Bak, and A. P. Ravn. Passive fault-tolerant control of piecewise linear systems against actuator faults. *submitted to International Journal of Systems Science*, 2010.

H. Yang, B. Jiang, and V. Cocquempot. A fault tolerant control framework for periodic switched non-linear systems. *International Journal of Control*, 82(1):117–129, 2009.