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# Outage Performance in Cognitive Radio Systems with Opportunistic Interference Cancelation

Rocco Di Taranto and Petar Popovski

*Abstract*—**In this paper, we investigate the problem of spectrally efficient operation of a cognitive radio, also called secondary spectrum user, under an interference from the primary system. A secondary receiver observes a multiple access channel of two users, the secondary and the primary transmitter, respectively. The secondary receiver applies Opportunistic Interference Cancelation (OIC) and Suboptimal Opportunistic Interference Cancelation (S-OIC) thus decoding the primary signal when such an opportunity is created by the rate selected at the primary transmitter and the power received from the primary transmitter. First, we investigate how the secondary transmitter, when using OIC and S-OIC for fixed transmitting power, should select its rate in order to meet its target outage probability under different assumptions about the channel-state-information available at the secondary transmitter. We study three different cases and for each of them identify the region of achievable primary and secondary rates. Second, we determine how the secondary transmitter should select its transmitting power not to violate the target outage probability at the primary terminals. Our numerical results show that the best secondary performance is always obtained when the secondary transmitter knows the instantaneous channel-state-information toward the intended receiver. We also evaluate the degradation in terms of achievable rate at the secondary receiver when it uses suboptimal decoding (S-OIC rather than OIC) and the interplay between the allowed power at the secondary transmitter (which depends on the target outage probability at the primary receiver) and the decodability at the secondary receiver.**

*Index Terms*—**Cognitive radio, dynamic spectrum sharing, (suboptimal) opportunistic interference cancelation.**

## I. Introduction

The main idea behind the concept of Cognitive Radio (CR) [1] is to allow *secondary usage* of a spectrum licensed to another, primary spectrum user. If a CR device uses certain communication resource concurrently with the primary system, then it should use transmit power that will guarantee acceptable interference to the primary system. On the other hand, a cognitive (secondary) receiver needs to operate under interference of a primary system. Such interference is commonly treated as noise, but information-theoretic approaches provide more sophisticated treatment of interference. In [2], the primary system provides the secondary system with the primary messages in a non-causal manner. A more practical assumption is that the secondary system knows only the primary codebooks, but not the messages. With this assumption, which does not deteriorate the security in the primary system (security is relying on encryption at the higher layers), the secondary may be able to decode and cancel the primary interference. In [3] authors prove that, in case of a high rate (undecodable) non-interactive interferer, it is not possible to do better than treating the interference as additional power constrained noise, even when its codebook is known. In our previous work [4] we have shown that for spectrally efficient operation, the secondary system should apply *Opportunistic Interference Cancelation* (OIC). The secondary receiver (SRX) receives the signal from the secondary transmitter (STX) along with the interference from the primary transmitter (PTX): The term "opportunistic" stands for the fact that the decodability of the primary signal depends on its rate as well as its power level at SRX. If STX knows, as it is assumed in [4], the channel state information (CSI) for both the desired and the interfering links, then STX can select the highest possible secondary rate without provoking outage at SRX.

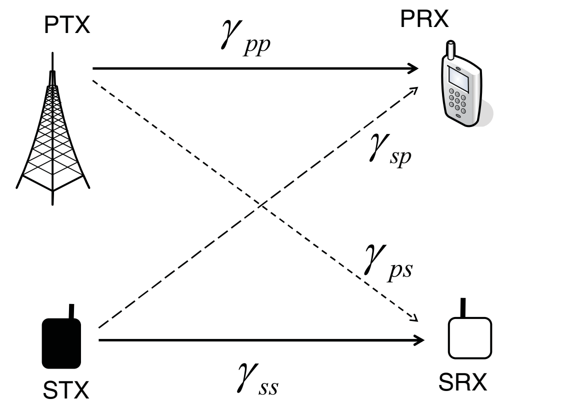


Fig. 1. The target scenario in which a primary transmitter (PTX) serves a primary receiver (PRX). A secondary receiver (SRX) is in the coverage area of the primary transmitter and thus experiences interference from PTX in addition to the desired signal from the secondary transmitter STX.

In a more practical scenario depicted in Fig. 1 the premise that STX always knows the instantaneous channel state information ($\gamma_{ss}$, $\gamma_{pp}$, $\gamma_{ps}$ and $\gamma_{sp}$) is disputable and generally not accepted: Therefore in this paper we study how STX should select *a)* its transmitting power not to violate the target outage probability at the primary terminals and *b)* the secondary transmitting rate when using *Opportunistic Interference Cancelation* under different assumption about the available CSI[1]. This paper also introduces and analyzes *Suboptimal OIC* (S-

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[1] In this paper we use the terms channel-state-information (CSI) and signal-to-noise-ratio (SNR) interchangeably.

OIC). With this technique SRX has not the ability to decode parts of the secondary signal in successive instants of time (as it is done for example with superposition coding [4]). With S-OIC SRX can either decode the whole secondary signal by the treating the primary interference as noise, or decode the whole primary signal by treating the secondary signal as noise. As a consequence a shrinking of the region of maximum achievable rates is determined (details are in next sections), but at the same time it results in simpler implementation at the cognitive users. It should be noted that the multiple-access channel studied here is different from the conventional two-user multiple-access channel, in fact here the primary legacy system acts as in the absence of secondary system, e.g., its rate $R_p$ is fixed irrespective of the secondary channel. As a consequence, the outage at SRX in our scenario is caused by two independent events, that is, the instantaneous values of the primary interfering and secondary direct channel gains. In a different context the application and feasibility of the OIC technique has been also investigated. For example, authors in [8] study a decentralized resource allocation strategy for the multi-carrier-based multiuser communication system where two coexisting users independently and sequentially update transmit power allocations over parallel sub-carriers to maximize their individual transmit rates.

The goal of this paper is twofold. First, we investigate how the secondary system (using both OIC and S-OIC techniques described in detail in next sections) should select its rate in order to meet the target outage probability under different assumptions about the CSI available to the secondary transmitter. The results provide an interesting insight into the impact of the decodable interference: The knowledge of the interfering codebooks should motivate the CR to select higher (optimistic) transmission rates, even if the instantaneous CSI is not known at the transmitter. Second, we determine how the secondary transmitter should select its transmitting power not to violate the target outage probability at the primary terminals and we show the interplay between the allowed power at the secondary transmitter and the decodability at the secondary receiver.

This paper is organized as follows. In Section II we define our target scenario and system model. In Section III and IV we investigate how the secondary transmitter (STX), when using OIC and S-OIC respectively, should select its rate in order to meet its target outage probability under different assumptions about the CSI available at STX. In Section V we calculate the permissible power in the secondary system for fixed outage probability in the primary system. In Section VI we present numerical results. Section VII concludes the paper.

## II. System Model

We consider the cognitive radio network shown in Fig. 1. PTX communicates with one primary terminal (PRX). The cognitive link consists of STX and SRX. STX is aware about the surrounding PRX. Secondary transmission and the relative interference toward PRX are tolerated as long as rights of primary users are not harmed. Primary receivers do not apply interference cancelation and therefore they do not need to know the secondary codebooks. We have two main concerns in this paper: *a)* to determine the maximal transmitting power at STX (for fixed outage at PRX), and *b)* to improve the communication performance in the secondary system (for fixed transmitting power at STX). We consider secondary communication under downlink interference from the primary system, since in that case a PRX is likely in a close proximity with respect to STX, which decreases the allowed transmitting power of STX and thus improvement of the secondary spectral efficiency is of paramount importance.

In Fig. 1, $\gamma_{ss}$ and $\gamma_{pp}$ denote, respectively, the instantaneous SNR in the links STX-SRX and PTX-PRX. $\gamma_{sp}$ ($\gamma_{ps}$) denotes the instantaneous SNR in the interfering link from STX (PTX) to PRX (SRX). PTX serves PRX in *scheduling epochs*. In each epoch, PTX uses a fixed transmission rate $R_p$ in the downlink. In absence of the interference, the signal received at PRX is given by

$$y_{pp} = h_{pp}x_p + z_p, \tag{1}$$

where $x_p$ is the signal sent by PTX, normalized as $E[|x_p|^2] = 1$, $z_p$ is the Gaussian noise at PRX with variance $E[|z_p|^2] = \sigma^2$. The complex value $h_{pp}$ is the channel gain between PTX and PRX and the instantaneous SNR at PRX is determined as:

$$\gamma_{pp} = \frac{|h_{pp}|^2}{\sigma^2}. \tag{2}$$

We assume a block-fading model [5], in which the instantaneous SNR is constant during the whole packet transmission. Considering normalized bandwidth, the achievable transmission rate is given by:

$$R_p = \log_2(1+\beta_p) = C(\beta_p), \tag{3}$$

where $\beta_p$ is the minimal SNR required to decode $R_p$. If the achievable instantaneous rate is lower than $R_p$, i.e., if $\beta_p > \gamma_{pp}$, then outage occurs. Let the maximal allowed outage probability be $\alpha_0$. If PRX has a probability of outage $\alpha < \alpha_0$, then it has an *outage margin* and can receive additional interference from the secondary transmission without violating the target operation regime of the primary system.

In our model we do not consider the effect of shadowing and the average SNR received by the PRX depends on the distance between PTX and PRX, denoted by $l$. Since PTX is likely to be mounted at a high location, thus having a line-of-sight towards PRX, we assume that the fading between PTX and PRX has a Ricean distribution, such that the instantaneous SNR $\gamma_{pp}$ at the PRX has the following distribution [6]:

$$p_{\gamma_{pp}}(l) = \frac{1}{\overline{\gamma}_{pp}(l)} e^{-\left(\frac{x}{\overline{\gamma}_{pp}(l)}+K\right)} I_0\left(2\sqrt{\frac{Kx}{\overline{\gamma}_{pp}(l)}}\right), \tag{4}$$

where $\overline{\gamma}_{pp}(l)$ is the mean of the diffuse component for a terminal at distance $l$, which is Rayleigh distributed. $K$ is the Ricean factor, i.e., the ratio between the mean power of the line-of-sight (LOS) and the diffuse component.

Let $P_{out}(l)$ denote the outage probability for a PRX at distance $l$ from PTX, for given transmission rate $R_p$. The outage probability experienced by any primary terminal (PRX) should be $P_{out}(l) \leq \alpha_0$, for any $l \leq L$, where $L$ is the distance at the edge of the primary coverage area. Clearly, the highest outage probability will be experienced by the primary terminals located at distance $L$. For given maximal outage probability $\alpha_0$ and given Ricean factor $K$, the average diffuse component $\overline{\gamma}_{pp}(L)$ at the edge of the cell can be determined by setting:

$$P_{out}(L, R_p) = \alpha_0, \tag{5}$$

which means that the terminals at the edge will have zero outage margin and cannot experience any additional interference. On the other hand, if the primary terminal is at distance $l < L$, then it can stand additional interference from a secondary transmitter (STX). Although here we use a single PRX to determine the allowed power levels in the secondary system the generalization to multiple PRXs is straightforward and the analysis presented here can be readily used: In that case the power constraint must be calculated with respect to each PRX and the strictest power constraint has to be taken into account.

The secondary transmitter STX has a single transmit antenna. The secondary signal transmitted is $\sqrt{P_s}x_s$, where $E[x_s]^2 = 1$ and $P_s$ is the average power. In this condition, the interfered signal at the primary receiver can be written as:

$$y_{pp} = h_{pp}x_p + h_{sp}\sqrt{P_s}x_s + z_p, \tag{6}$$

where $h_{sp}$ is the channel coefficient between STX and PRX, which experiences Rayleigh fading [6] (deployment scenario for cognitive radio is urban indoor/outdoor environment, without line-of-sight). Following [6]-[7], the instantaneous SNR $\gamma_{sp}$ at PRX for the signal sent by STX is exponentially distributed with average value:

$$\overline{\gamma}_{sp} = \frac{P_s h_{sp}}{\sigma^2}. \tag{7}$$

Assuming that STX transmits with Gaussian codebooks, the instantaneous achievable rate in the primary system is:

$$R_p^{max} = C\left(\frac{\gamma_{pp}}{1+\gamma_{sp}}\right). \tag{8}$$

The secondary receiver (SRX) has a single antenna. $h_s$ denotes the channel coefficient between STX and SRX, and experiences Rayleigh fading. The SNR of the signal transmitted by STX to SRX is $\gamma_{ss} = \frac{P_s h_s^2}{\sigma^2}$. The noise variance at SRX also contains the interference that the secondary system experiences from primary.

The rate $R_p$, selected according to the requirements and channel conditions of the primary users, is known by the secondary users since we assume that the secondary can read the protocol header of the primary system and learn which primary user is served and at which rate. In this context, the primary system makes provision for secondary spectrum usage by allowing certain interference margin and outage probability at PRX. That is, if the channel towards PRX has SNR equal to $\gamma$, then $R_p < \log_2(1+\gamma)$ and additionally there is a certain outage probability at PRX which is deemed acceptable. The secondary transmitters are aware of this interference margin and target outage probability at PRX and select their transmit power in order not to surpass them.

$R_s$ denotes the transmission rate at STX. During a scheduling epoch, the SNRs are constant (block fading model): In each new epoch, we assume that the channel on the links STX-SRX and PTX-SRX fade independently according to a Rayleigh distribution, and the averages of $\gamma_{ss}$ and $\gamma_{ps}$ are denoted by $\bar{\gamma}_{ss}$ and $\bar{\gamma}_{ps}$, respectively. We assume that SRX always knows the instantaneous $\gamma_{ss}$ and $\gamma_{ps}$ (by listening at

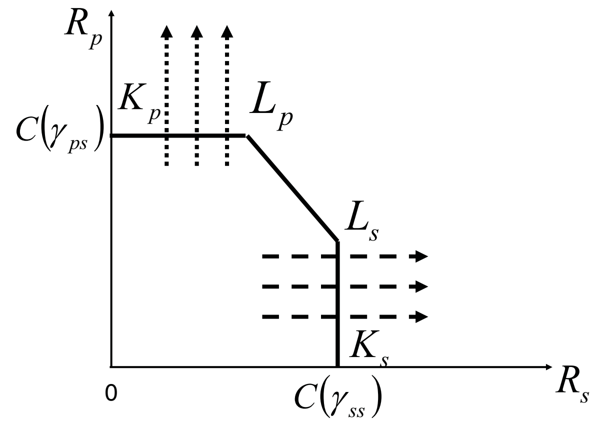


Fig. 2. The region of achievable rate pairs $\mathcal{R} = (R_s, R_p)$, in a two-user multi-access channel.

the beacon signal from PTX and STX respectively for each channel realization) and their statistics $\bar{\gamma}_{ss}$ and $\bar{\gamma}_{ps}$ (obtained by listening at primary and secondary transmission for a sufficiently long period of time). Then SRX reports part of the collected information to STX which can then select the secondary rate $R_s$ accordingly. It should be noted that, if at least one of the instantaneous SNRs $\gamma_{ss}$ and $\gamma_{ps}$ is not reported to the to STX, then there is always a non-zero outage probability at SRX in all the cases studied in this paper. Our goal in next sections is to see what is the value of reporting instantaneous and/or average channel state information of secondary direct and/or primary interfering links from SRX (which is anyway always assumed to know all of them) to STX.

## III. Opportunistic Interference Cancelation: Outage Analysis under Incomplete CSI

The transmissions of PTX and STX are assumed synchronized at SRX, such that SRX observes a multiple access (MA) channel [6] [9]. When using *Opportunistic Interference Cancelation* (OIC) SRX can reliably decode both primary and secondary signal if the rate pair $(R_s, R_p)$ is within the capacity region of the MA channel, see Fig. 2:

$$R_s \leq \log_2(1+\gamma_{ss}) \tag{9}$$

$$R_p \leq \log_2(1+\gamma_{ps}) \tag{10}$$

$$R_p + R_s \leq \log_2(1+\gamma_{ss}+\gamma_{ps}) \tag{11}$$

It should be noted that (9)-(11) simply identify the regions of achievable primary and secondary rates; the actual techniques to implement in order to achieve these maximum rates (for example by using superposition coding [4])) are outside of the scope of this paper. When at least one of the *instantaneous* SNRs $\gamma_{ss}, \gamma_{ps}$ is not known at STX (i.e., it is not reported from SRX, which always knows them, to STX), then it is not guaranteed that $R_s$ is selected such that the rate pair $\mathcal{R} = (R_s, R_p)$ is in the capacity region of the MA channel. In the sequel, we consider three different cases of incomplete CSI at STX and for each of them identify the regions of achievable primary and secondary rates. It is important to stress that when at least one among $\gamma_{ss}$ and $\gamma_{ps}$ is not known at STX, multiuser decoding at SRX is needed to reach the achievable primary

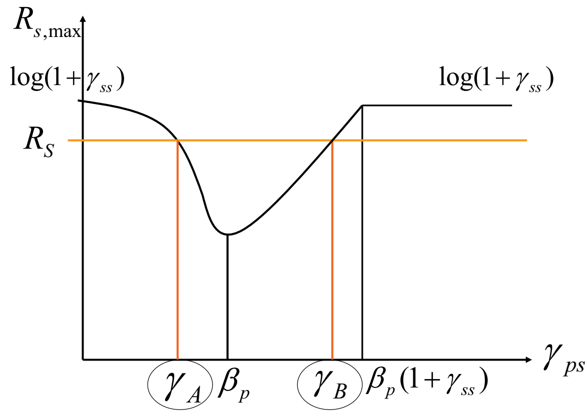


Fig. 3. Maximum achievable secondary rate in function of $\gamma_{ps}$, for fixed $\gamma_{ss}$.

and secondary rates in (9)-(11), because superposition coding as implemented in [4] cannot be used in absence of knowledge of both instantaneous channel state information.

### A. $\gamma_{ps}$ is known and $\gamma_{ss}$ is not known

In this case, the multi access region in Fig. 2, varies along the abscissa depending on the instantaneous value of $\gamma_{ss}$, which is unknown to STX. Here we want to determine the outage probability at SRX, when STX, knowing instantaneous $\gamma_{ps}$ and average $\bar{\gamma}_{ss}$, selects rate $R_s$.

We distinguish two cases depending on the decodability of the primary signal at SRX. The first case is if $\gamma_{ps} < \beta_p$, which implies that the primary is not decodable at SRX and needs to be treated as a noise. Then the outage probability for a given secondary rate $R_s$ is:

$$\Pr\left(\log_2(1+\frac{\gamma_{ss}}{1+\gamma_{ps}}) < R_s\right) = 1-\exp\left[(-2^{R_s}-1)(1+\gamma_{ps})\right]. \quad (12)$$

It is worth noting that the integration (12) is performed only along $\gamma_{ss}$ because $\gamma_{ps}$ is assumed to be an instantaneously known random variable. Let us now consider the case $\gamma_{ps} \geq \beta_p$ and the primary signal is decodable. The maximal achievable secondary rate is:

$$R_{s,\max} = \min\left(\log_2(1+\gamma_{ss}), \log_2(\frac{1+\gamma_{ss}+\gamma_{ps}}{1+\beta_p})\right), \quad (13)$$

and for known $\gamma_{ps}$, it is a function of $\gamma_{ss}$. This follows from (9) and (11) when we consider the case of fixed $R_p$. For a given secondary rate $R_s$, we determine the minimal $\gamma_{s0}$, such that (13) is satisfied by putting $\gamma_{ss} = \gamma_{s0}$. Then outage occurs whenever the instantaneous $\gamma_{ss} < \gamma_{s0}$, which is found as

$$\Pr(\gamma_{ss} < \gamma_{s0}) = 1 - \exp\left(-\frac{\gamma_{s0}}{\bar{\gamma}_{ss}}\right). \quad (14)$$

### B. $\gamma_{ss}$ is known and $\gamma_{ps}$ is not known

If only $\gamma_{ps}$ is unknown *at STX*, the MA capacity region changes due to the "vertical movement" of the capacity region on Fig. 2. For known $\gamma_{ss}$, the maximal achievable secondary rate $R_{s,\max}(\gamma_{ps})$ is a function of $\gamma_{ps}$, plotted on Fig. 3. In absence of any interference we have

$$R_{s,\max}(\gamma_p = 0) = \log_2(1+\gamma_{ss}). \quad (15)$$

In the region where $0 < \gamma_{ps} < \beta_p$, the primary signal cannot be decoded at SRX and it is treated as noise at SRX, such that

$$R_{s,\max}(\gamma_{ps}) = \log_2\left(1 + \frac{\gamma_{ss}}{1+\gamma_{ps}}\right). \quad (16)$$

When $\gamma_{ps}$ grows beyond $\beta_p$, the primary becomes decodable at SRX. We first consider the interval $\beta_p \leq \gamma_{ps} \leq \beta_p(1+\gamma_{ss})$, where each $\gamma_{ps}$ is represented as $\gamma_{ps} = \beta_p(1+\alpha\gamma_{ss})$ with $0 \leq \alpha \leq 1$. It can be shown that, in this interval the maximal achievable rate $R_s$ is found by considering the rate pair $(R_s, R_p)$ that lies on the diagonal (slope -1) border of the capacity region on Fig. 2, such that:

$$\begin{aligned} R_{s,\max}(\gamma_{ps}) &= \log_2(1+\gamma_{ss}+\gamma_{ps}) - R_p \\ &= \log_2\left(1+\gamma_{ss}\frac{1+\alpha\beta_p}{1+\beta_p}\right). \end{aligned} \quad (17)$$

Finally, if $\gamma_{ps} > \beta_p(1+\gamma_{ss})$, then the primary signal can be decoded by treating the secondary signal as a noise, such that primary is completely canceled and the maximal secondary rate becomes independent of $\gamma_{ps}$:

$$R_{s,\max}(\gamma_{ps}) = \log_2(1+\gamma_{ss}) \quad (18)$$

We use Fig. 3 to determine the outage probability in this case. If $R_s$ is less than the minimum of the function $R_{s,\max}(\gamma_{ps})$, i. e. $R_s < \log_2\left(1+\frac{\gamma_{ss}}{1+\beta_p}\right) = \mu$, then the outage probability is zero, regardless of $\gamma_{ps}$. Conversely, if $R_s > \log_2(1+\gamma_{ss})$, then the outage probability is one, regardless of $\gamma_{ps}$. If $R_s$ is selected to be in the interval $\mu \leq R_s \leq \log_2(1+\gamma_{ss})$, then it is seen from Fig. 3, that the actual secondary rate intersects $R_{s,\max}(\gamma_{ps})$ in two points whose abscissas correspond to $\gamma_A$ and $\gamma_B$: the outage probability is given by the integral of the probability density function (pdf) of $\gamma_{ss}$ (which is Rayleigh distributed) between $\gamma_A$ and $\gamma_B$. Defining $\beta_s = 2^{R_s} - 1$, $\gamma_A$ and $\gamma_B$ can be determined in closed form as follows:

$$\gamma_A = \frac{\gamma_{ss}-\beta_s}{\beta_s}, \quad \gamma_B = \beta_p(1+k\gamma_{ss}), \quad (19)$$

where $k = \frac{\beta_s(1+\beta_p)-\gamma_{ss}}{\gamma_{ss}\beta_p}$.

### C. Both $\gamma_{ss}$ and $\gamma_{ps}$ are not known

Here we want to determine the outage probability at SRX, when STX, knowing only average $\bar{\gamma}_{ps}$ and $\bar{\gamma}_{ss}$, selects rate $R_s$. For easier notation, we introduce $\beta_s$ that corresponds to the selected rate as

$$R_s = \log_2(1+\beta_s). \quad (20)$$

For given values of $\beta_s$ and $\beta_p$, Fig. 4 depicts the region $(\gamma_{ss}, \gamma_{ps})$, patterned with vertical lines, that renders the secondary signal undecodable at SRX. We explain the shape of this undecodability region by considering three intervals for $\gamma_{ss}$. Note that, for each value of $\gamma_{ss}$, we can plot the function $R_{s,\max}(\gamma_p)$, as on Fig. 3.

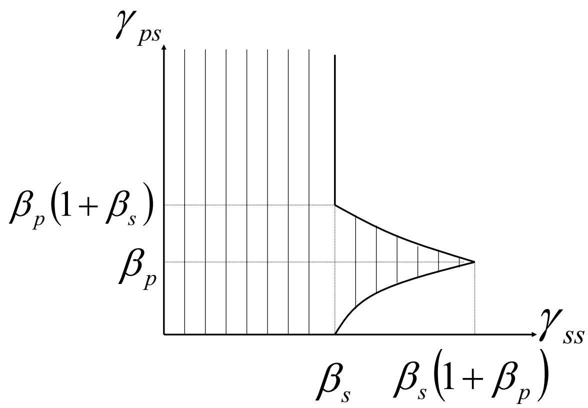


Fig. 4. Outage probability region: the region patterned with vertical lines renders the secondary signal undecodable at SRX.

If $\gamma_{ss} < \beta_s$, then, for each $\gamma_{ss}$, the function $R_{s,\max}(\gamma_{ps})$ lies below the line $R_s = \log_2(1+\beta_s)$, such that the outage probability is one, regardless of $\gamma_{ps}$. In the interval $\beta_s \leq \gamma_{ss} \leq \beta_s(1+\beta_{ps})$, it can be shown that for each fixed value of $\gamma_{ss}$, the function $R_{s,\max}(\gamma_{ps})$ has two intersecting points with the line $R_s = \log_2(1+\beta_s)$, i. e., $R_{s,\max}(\gamma_{ps}) = R_s$ for $\gamma_{ps} = \gamma_A$ and $\gamma_{ps} = \gamma_B$, as depicted on Fig. 3. Note that $\gamma_A$ and $\gamma_B$ are functions of $\gamma_{ss}$ and they approach each other as $\gamma_{ss}$ grows towards $\beta_s(1+\beta_p)$. In this interval, for fixed $\gamma_{ss}$, outage occurs if $\gamma_A < \gamma_{ps} < \gamma_B$. Finally, for each $\gamma_{ss} \geq \beta_s(1+\beta_p)$, the function $R_{s,\max}(\gamma_{ps})$ lies below the line $R_s = \log_2(1+\beta_s)$, such that the outage probability is zero, regardless of $\gamma_{ps}$.

The integral of the probability density function (pdf) of $\gamma_{ss}$ over the two-dimensional region patterned with vertical lines in Fig. 4 cannot be solved in closed form; therefore we evaluate the secondary outage probability numerically in Section VI. It is worth noting that a two-dimensional integration is needed here because not $\gamma_{ss}$ nor $\gamma_{ps}$ instantaneous are assumed to be known.

## IV. Sub-Optimal Opportunistic Interference Cancelation: Outage Analysis under Incomplete CSI

In this section, we consider the case where secondary users apply *Suboptimal Opportunistic Interference Cancelation* (S-OIC): Secondary system has not the ability to use transmission strategies, for example based on superposition coding [4] (where the secondary signal is split in the sum of two successively decoded components), that allows to reach all the maximum rate pairs showed in Fig. 2. We assume that SRX instead of using complex multiuser decoding, applies suboptimal successive interference cancelation and, depending on the primary and secondary rate and on the instantaneous channel gains shown in Fig. 1, *a)* it can decode the whole secondary signal by treating the primary signal as noise or *b)* it can first decode and cancel the primary signal by treating the secondary signal as noise and, second, decode the whole secondary signal in absence of any interference. It follows that SRX can reliably decode both primary and secondary signal if the rate pair $(R_s, R_p)$ is within the capacity region shown in Fig. 5. Note that this capacity region is not convex as we cannot force the primary system to do time sharing for example. The shape of the capacity region can be explained as follows. If $0 \leq R_p \leq C\left(\frac{\gamma_{ps}}{1+\gamma_{ss}}\right)$, primary signal can be decoded at SRX even in presence of the secondary signal. Therefore, first, primary signal is decoded and canceled; then, since $R_s \leq C(\gamma_{ss})$, secondary signal can be decoded in absence of any interference. This means that, if $0 \leq R_p \leq C\left(\frac{\gamma_{ps}}{1+\gamma_{ss}}\right)$, the maximum decodable secondary rate equals $C(\gamma_{ss})$. On the other hand, if $R_p > C\left(\frac{\gamma_{ps}}{1+\gamma_{ss}}\right)$ the whole primary signal cannot be decoded in presence of the secondary signal. So, secondary signal should be first decoded, and this can happen successfully only if $R_s < C\left(\frac{\gamma_{ss}}{1+\gamma_{ps}}\right)$. Similar considerations hold for the rate regions defined by $0 \leq R_s \leq C\left(\frac{\gamma_{ss}}{1+\gamma_{ps}}\right)$ and $R_s > C\left(\frac{\gamma_{ss}}{1+\gamma_{ps}}\right)$.

When at least one of the instantaneous SNRs $\gamma_{ss}, \gamma_{ps}$ is not known at STX (i.e., not reported from SRX), then it is not guaranteed that $R_s$ is selected such that the rate pair $\mathcal{R} = (R_s, R_p)$ is in the capacity region shown in Fig. 5. In the sequel, we consider three different cases of incomplete CSI.

### A. $\gamma_{ps}$ is known and $\gamma_{ss}$ is not known

In this case, the multi access region in Fig. 5, varies along the abscissa depending on the instantaneous value of $\gamma_{ss}$, which is unknown to STX. Here we want to determine the outage probability at SRX, when STX, knowing instantaneous $\gamma_{ps}$ and average $\bar{\gamma}_{ss}$, selects rate $R_s$ and uses *Suboptimal Opportunistic Interference Cancelation* (S-OIC).

We distinguish two cases depending on the decodability of the whole primary signal at SRX. The first case is if $\gamma_{ps} < \beta_p(1+\gamma_{ss})$, which implies that the whole primary signal is not decodable at SRX in presence of the secondary signal and needs to be treated as a noise when SRX decodes the secondary signal. Then the outage probability for a given secondary rate $R_s$ is:

$$\Pr\left(\log_2(1+\frac{\gamma_{ss}}{1+\gamma_{ps}}) < R_s\right) = 1-\exp\left[(-2^{R_s}-1)(1+\gamma_{ps})\right]. \quad (21)$$

It is worth noting that the integration (21) is performed only along $\gamma_{ss}$ because $\gamma_{ps}$ is assumed to be an instantaneously known random variable. The second case is with $\gamma_{ps} \geq \beta_p(1+\gamma_{ss)})$ when the whole primary signal is decodable even in presence of the secondary signal (treated as interference). The primary signal is decoded and canceled and therefore the outage probability for a given secondary rate $R_s$ can be written as follows:

$$\Pr\left(\log_2(1+\frac{\gamma_{ss}}{1+\gamma_{ps}}) < R_s\right) = 1-\exp\left[(-2^{R_s}-1)\right]. \quad (22)$$

### B. $\gamma_{ss}$ is known and $\gamma_{ps}$ is not known

If only instantaneous $\gamma_{ps}$ is unknown *at STX*, the MA capacity region changes due to the "vertical movement" of the capacity region on Fig. 5. For known $\gamma_{ss}$, the maximal achievable secondary rate $R_{s,\max}(\gamma_{ps})$ is a function of $\gamma_{ps}$, plotted on Fig. 6. In absence of any interference we have

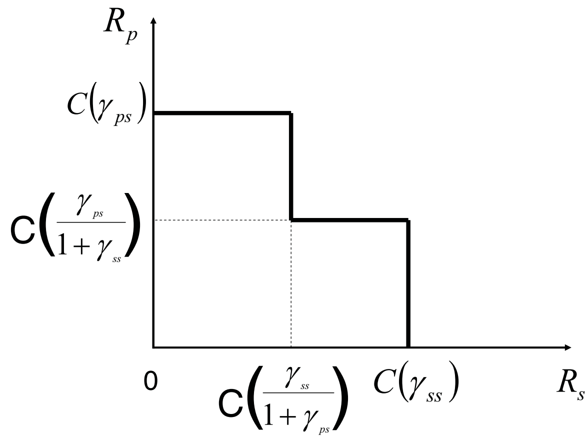


Fig. 5. The region of achievable rate pairs with suboptimal OIC $\mathcal{R} = (R_s, R_p)$, in a two-user multi-access channel.

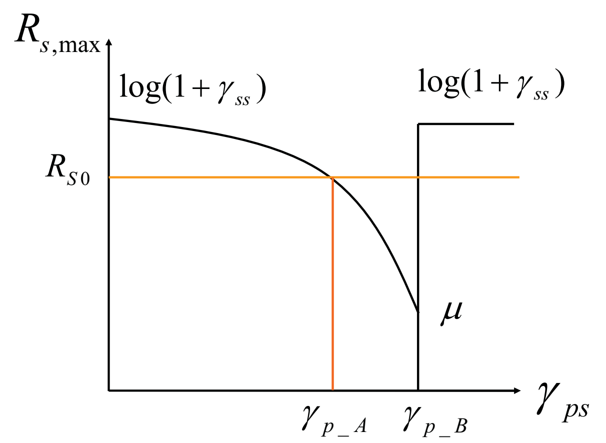


Fig. 6. Maximum achievable secondary rate in function of $\gamma_{ps}$, for fixed $\gamma_{ss}$ (with suboptimal OIC).

$$R_{s,\max}(\gamma_{ps} = 0) = \log_2(1 + \gamma_{ss}). \tag{23}$$

In the region where $0 < \gamma_{ps} < \beta_p(1+\gamma_{ss})$, the whole primary signal cannot be decoded at SRX and it is always treated as noise at SRX, such that

$$R_{s,\max}(\gamma_{ps}) = \log_2\left(1 + \frac{\gamma_{ss}}{1+\gamma_{ps}}\right). \tag{24}$$

Finally, if $\gamma_{ps} > \beta_p(1 + \gamma_{ss})$, then the whole primary signal can be decoded even by treating the secondary signal as a noise, such that primary is completely canceled and the maximal secondary rate becomes independent of $\gamma_{ps}$:

$$R_{s,\max}(\gamma_{ps}) = \log_2(1 + \gamma_{ss}) \tag{25}$$

We use Fig. 6 to determine the outage probability in this case. If $R_s$ is less than the minimum of the function $R_{s,\max}(\gamma_{ps})$, i. e. $R_s < \log_2\left(1 + \frac{\gamma_{ss}}{1+\beta_p(1+\gamma_{ss})}\right) = \mu$, then the outage probability is zero, regardless of $\gamma_{ps}$. Conversely, if $R_s > \log_2(1 + \gamma_{ss})$, then the outage probability is one, regardless of $\gamma_{ps}$. If $R_s$ is selected to be in the interval $\mu \leq R_s \leq \log_2(1 + \gamma_{ss})$, then it is seen from Fig. 6, that the actual secondary rate intersects $R_{s,\max}(\gamma_{ps})$ in two points whose abscissas correspond to $\gamma_A$ and $\gamma_B$: the outage probability is given by the integral of the probability density function (pdf) of $\gamma_{ss}$ (which is Rayleigh distributed) between $\gamma_A$ and $\gamma_B$. Defining $\beta_s = 2^{R_s} - 1$, $\gamma_A$ and $\gamma_B$ can be determined in closed form as follows:

$$\gamma_A = \frac{\gamma_{ss} - \beta_s}{\beta_s}, \quad \gamma_B = \beta_p(1 + \gamma_{ss}). \tag{26}$$

### *C. Both $\gamma_{ss}$ and $\gamma_{ps}$ are not known*

Here we want to determine the outage probability at SRX, when STX, knowing only average $\overline{\gamma}_{ps}$ and $\overline{\gamma}_{ss}$, selects rate $R_s$ and applies suboptimal OIC (S-OIC). For easier notation, we introduce $\beta_s$ that corresponds to the selected rate as

$$R_s = \log_2(1 + \beta_s). \tag{27}$$

For given values of $\beta_s$ and $\beta_p$, Fig. 7 depicts the region $(\gamma_{ss}, \gamma_{ps})$, patterned with vertical lines, that renders the secondary signal undecodable at SRX. We explain the shape of this undecodability region by considering three intervals for $\gamma_{ss}$. Note that, for each value of $\gamma_{ss}$, we can plot the function $R_{s,\max}(\gamma_{ps})$, as on Fig. 6.

If $\gamma_{ss} < \beta_s$, then, for each $\gamma_{ss}$, the function $R_{s,\max}(\gamma_{ps})$ lies below the line $R_s = \log_2(1 + \beta_s)$, such that the outage probability is one, regardless of $\gamma_{ps}$. In the interval $\beta_s \leq \gamma_{ss} \leq \beta_s(1 + \beta_p)$, it can be shown that for each fixed value of $\gamma_{ss}$, the function $R_{s,\max}(\gamma_{ps})$ has two intersecting points with the line $R_s = \log_2(1 + \beta_s)$, i. e. $R_{s,\max}(\gamma_{ps}) = R_s$ for $\gamma_{ps} = \gamma_A$ and $\gamma_{ps} = \gamma_B$, as depicted on Fig. 6. Note that $\gamma_A$ is function $\gamma_{ss}$ and approaches $\gamma_B$ as $\gamma_{ss}$ grows towards $\beta_s(1 + \beta_p)$. In this interval, for fixed $\gamma_{ss}$, outage occurs if $\gamma_A < \gamma_{ps} < \gamma_B$. Finally, for each $\gamma_{ss} \geq \beta_s(1 + \beta_p)$, the function $R_{s,\max}(\gamma_{ps})$ lies below the line $R_s = \log_2(1 + \beta_s)$, such that the outage probability is zero, regardless of $\gamma_{ps}$.

The integral of the probability density function (pdf) of $\gamma_{ss}$ over the two-dimensional region patterned with vertical lines in Fig. 7 cannot be solved in closed form; therefore we evaluate the secondary outage probability numerically in Section VI.

## V. Permissible Power Level in the Secondary System

So far we have assumed that the maximal secondary power is somehow selected, and starting from that assumption, we have determined the rate selection at STX based on the available knowledge of the secondary direct ($\gamma_{ss}$) and primary interfering ($\gamma_{ps}$) channel state information. In this section we introduce a model that accounts for the interference between secondary and primary systems and provides criterion to choose secondary power based on the target performance of the primary system, i.e., maximal allowed degradation. Following our approach in [10], in this section we calculate the power $P_s$ that the secondary system should choose in order not to violate the target performance in the primary system. For this purpose we need to make an additional assumption: The secondary system knows *a)* the average value of the diffuse component in the primary system $\overline{\gamma}_{pp}$ and *b)* the

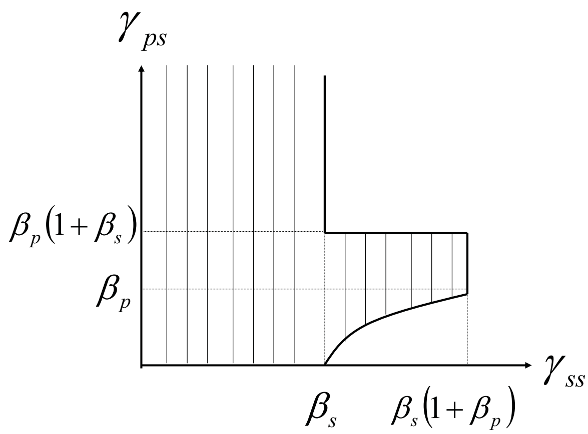


Fig. 7. Outage probability region: the region patterned with vertical lines renders the secondary signal undecodable at SRX (with suboptimal OIC).

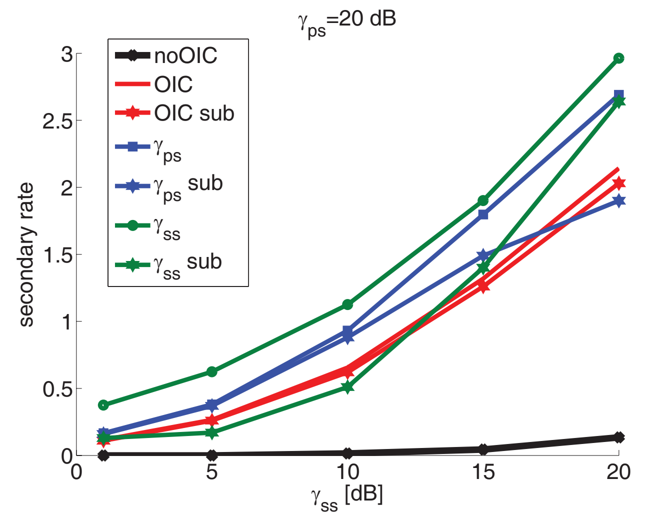


Fig. 8. Maximum (averaged) allowed secondary rate in function of $\gamma_{ss}$ ($\overline{\gamma_{ps}} = 20dB$, $\xi = 0.1$).

average of the diffuse component in the secondary-to-primary interfering link $\overline{\gamma}_{sp}$. On the one hand, the value of $\overline{\gamma}_{sp}$ can be inferred by listening to the uplink transmission of PRX. On the other hand, the determination of $\overline{\gamma}_{pp}$ requires either explicit signaling from PRX to STX or another indirect way of knowing which can be, for example, by having STX overhear the transmissions of PTX and based on the ACK/NACK sent by PRX, assess the outage probability, say $P_{out}$, at PRX in absence of interference. Assuming that the Ricean factor $K$ is known a priori, $P_{out}$ has a one-to-one correspondence with $\overline{\gamma}_{pp}$ and can be therefore estimated.

STX uses a single antenna. An outage at the primary system occurs when:

$$R_p > C\left(\frac{\gamma_{pp}}{1+\gamma_{sp}}\right) \Longleftrightarrow r_p > \frac{\gamma_{pp}}{1+\gamma_{sp}}. \quad (28)$$

After transforming, the probability of outage can be written as follows:

$$\Pr\left(\gamma_{sp} \geq \frac{\gamma_{pp} - r_p}{r_p}\right) = \int_0^\infty \Pr\left(\gamma_{sp} \geq \frac{x - r_p}{r_p}\right) p_{\gamma_{pp}}(x)dx = \quad (29)$$

$$= \int_0^{r_p} \frac{1}{\overline{\gamma}_{pp}} e^{-\left(\frac{x}{\overline{\gamma}_{pp}}+K\right)} I_0\left(2\sqrt{\frac{Kx}{\overline{\gamma}_{pp}}}\right)dx + \quad (30)$$

$$\int_{r_p}^\infty \frac{1}{\overline{\gamma}_{pp}} e^{-\left(\frac{x-r_p}{r_p\overline{\gamma}_{sp}}+\frac{x}{\overline{\gamma}_{pp}}+K\right)} I_0\left(2\sqrt{\frac{Kx}{\overline{\gamma}_{pp}}}\right)dx,$$

where in the first integral we use $\Pr(\gamma_{sp} \geq \frac{x-r_p}{r_p}) = 1$ for $x \leq r_p$. In the next section, the integrals above are evaluated numerically in order to find the mean $\overline{\gamma}_{sp}$ of the permissible power of the secondary transmitter for a fixed probability of outage at PRX.

## VI. Numerical Evaluation

### A. OIC and S-OIC

In this subsection, we evaluate the maximum secondary rate $R_s$ that has predetermined outage probability at SRX. We consider OIC and S-OIC for the three different cases of CSIT available at STX, described in Section III-A, III-B, III-C and Section IV-A, IV-B, IV-C. As a reference, we consider the case in which averages $\overline{\gamma}_{ps}, \overline{\gamma}_{ss}$ are known, but no OIC nor S-OIC is applied. It is worth noting that we do not consider the case where both $\gamma_{ps}$ and $\gamma_{ss}$ instantaneous are known because in that case STX can select a rate $R_s$ always decodable at SRX, i.e., outage is always zero. We assume that the channels on the links STX-SRX and PTX-SRX fade independently according to a Rayleigh distribution with averages $\overline{\gamma}_{ss}$ and $\overline{\gamma}_{ps}$, which are parameters in our simulations.

Fig. 8 shows the maximum allowed secondary rate $R_s$ (averaged over a large number of independent channel realizations in the links STX-SRX and PTX-SRX) that has outage probability equal to $\xi = 0.1$. We have set the parameter $\overline{\gamma}_{ps} = 20$ dB. As expected, $R_s$ increases with $\overline{\gamma}_{ss}$ in all the cases. As it can be seen the secondary rate is sensibly higher with OIC and S-OIC even in the case where STX knows only the average $\overline{\gamma}_{ps}$ and $\overline{\gamma}_{ss}$: the beneficial effects of OIC and S-OIC are not lost even if SRX does not report to STX any of the instantaneous channel gains. When STX knows either the instantaneous $\gamma_{ss}$ or $\gamma_{ps}$ the maximum $R_s$ with OIC is further improved: SRX has an additional information and exploits it to maximize its average rate. It is also important to notice that knowing the instantaneous $\gamma_{ss}$ when using OIC always gives the highest secondary rate compared to knowledge of instantaneous $\gamma_{ps}$.

Fig. 8 shows also the degradation in maximum achievable secondary rate when secondary system applies S-OIC rather than OIC. As it can be seen when only average $\overline{\gamma}_{ps}$ and $\overline{\gamma}_{ss}$ are known, there is a minimal difference in performance between OIC and S-OIC. The situation is different when considering the cases with known instantaneous $\gamma_{ss}$ or $\gamma_{ps}$: The performance of S-OIC is sensibly degraded, especially for high values of $\bar{\gamma}_{ss}$ and $\overline{\gamma}_{ps}$, respectively. This can be explained by comparing Fig. 2 and Fig. 5: For fixed $\gamma_{ss}$ and $\gamma_{ps}$, the region of achievable rates with S-OIC corresponds exactly to that with OIC, except for the right-upper-triangle region. Whenever SRX selects a rate $R_s$ in the upper-right-triangle region, $R_s$ is achievable with OIC but not with S-

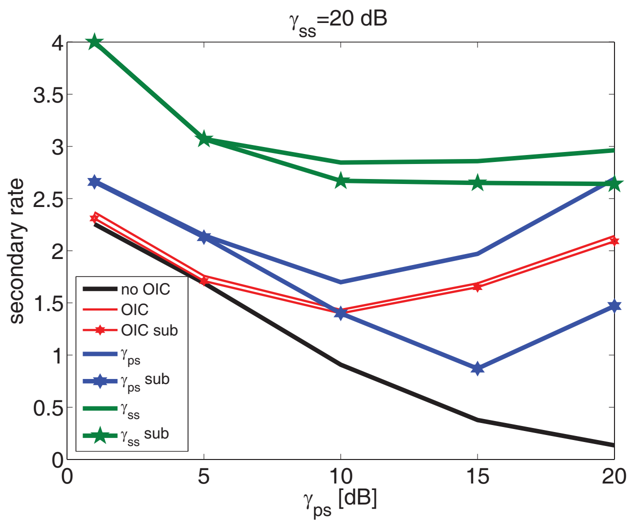


Fig. 9. Maximum (averaged) allowed secondary rate in function of $\gamma_{ps}$ ($\overline{\gamma_{ss}} = 20dB$, $\xi = 0.1$).

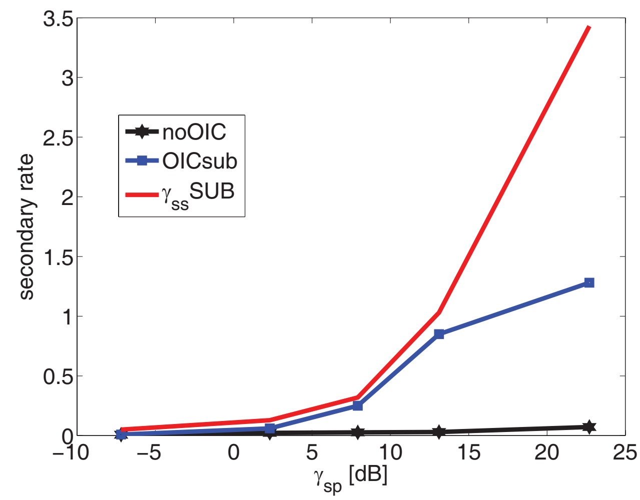


Fig. 10. Maximum (averaged) allowed secondary rate in function of $\overline{\gamma}_{sp}$ ($\xi = 0.1$).

OIC. As a consequence, the higher the probability that the rate selected by SRX is in the upper-right-corner region, the higher the difference in performance between OIC and S-OIC. When none of the instantaneous channel gains is known at STX, the probability that the instantaneous channel gains will have values such that the selected rate falls in the upper-right corner region of the MA channel is very low. When one of the two channel gains are known at STX, STX knows one of the extreme values of the region of achievable rate region along the x-axis or y-axis (depending whether it knows $\gamma_{ss}$ or $\gamma_{ps}$ it knows $C(\gamma_{ss})$ or $C(\gamma_{ps})$). With OIC STX exploits this information in order to select a rate which falls with higher probability in the rate region shown in Fig. 2. On the contrary this information is not much useful when applying S-OIC because the region of maximum achievable rate shown in Fig. 5 is smaller as previously discussed.

Fig. 9 shows the average value of the maximal allowable $R_s$ for different values of $\gamma_{ps}$ when $\xi = 0.1$ and $\overline{\gamma}_{ss} = 20dB$. It is interesting to notice that, as expected, the maximum averaged $R_s$ (for all the three cases with OIC) has a minimum in correspondence of $\overline{\gamma}_{ps} = 10$. This happens because the primary signal is not decodable at SRX if $\gamma_{ps} < \beta_p$: $R_s$ decreases (on average) for increasing values of $\overline{\gamma}_{ps}$ below $\beta_p$ . On the contrary, when $\overline{\gamma}_{ps}$ is on average larger than $\beta_p$ the primary signal can be decoded and this explains why the secondary rate increases. Similar considerations when comparing OIC and S-OIC.

### *B. Permissible power in the secondary system*

In this subsection, we evaluate the maximum power allowed in the secondary system, when it coexists with a primary system with a given target outage probability. We assume that the channel in the primary link PTX-PRX fades according to a Ricean distribution, with a factor $K$. The average SNR of the diffuse component from the primary signal measured at PRX is denoted by $\overline{\gamma}_{pp}$. The channel in the interfering link STX-PRX is assumed to be Rayleigh-distributed, with average SNR $\overline{\gamma}_{sp}$. By solving numerical integral in (30) we have obtained the mean of the diffuse component in the Rayleigh fading secondary link $\overline{\gamma}_{sp}$ against the mean of the diffuse component in the Ricean fading primary link $\overline{\gamma}_{pp}$ (Ricean factor $K = 10$), for a fixed target outage probability at PRX ($\alpha = 0.01$). Our numerical results confirm that $\overline{\gamma}_{sp}$ increases with $\overline{\gamma}_{pp}$: This happens because when, on average, the SNR in the primary link increases, secondary user can transmit with higher power (and this results in both higher $\overline{\gamma}_{sp}$ and $\overline{\gamma}_{ss}$), for fixed outage probability at PRX.

Let us notice that the average value of the diffuse component in PTX-SRX link, $\overline{\gamma}_{ps}$, although proportional to the average value of the diffuse component in the primary link PTX-PRX, $\overline{\gamma}_{pp}$ (increasing the transmitting power at PTX certainly increases the quality in these two links), is in general different for a proportionality factor. For simplicity we ignore this proportionality factor here, i.e., we assume $\overline{\gamma}_{pp} = \overline{\gamma}_{ps}$. We can note that this corresponds to the case where SRX and PRX are at the same position in the cell.

With this assumption, in Fig. 10 we plot the maximum rate $R_s$ at STX, for fixed outage at the secondary receiver ($\xi = 0.1$). The x-axis shows $\overline{\gamma}_{ss}$ which previously has been numerically obtained in correspondence of different values of $\overline{\gamma}_{pp}$ (i.e., by solving (30). So in the x-axis, $\overline{\gamma}_{ss} = f(\overline{\gamma}_{pp}) = f(\overline{\gamma}_{ps})$, because we have assumed that $\overline{\gamma}_{pp} = \overline{\gamma}_{ps}$. These results can be interpreted as follows: If the primary power increases while the primary rate $R_p$ is kept constant there are two reasons for improving the performance of the secondary system in terms of rate $R_s$: (a) increased power at STX (as it is shown in Fig. 10) and (b) easier decoding of the primary signal at SRX (due to the higher primary SNR at SRX). In Fig. 10 we have plotted results relative to three cases *(a)* no OIC, *b)* S-OIC with unknown $\gamma_{ps}$ and $\gamma_{ss}$, *c)* S-OIC with unknown $\gamma_{ps}$, but known $\gamma_{ss}$) that do not require instantaneous knowledge about $\gamma_{ps}$. Nevertheless, numerical results confirm that similar tendencies are obtained when assuming known $\gamma_{ps}$ at STX.

## VII. Conclusions and Future Work

In this paper we have investigated how the outage probability in the secondary system changes when various types of channel state information are reported from the secondary receiver to the secondary transmitter. We have studied three different cases, and for each of them we have identified the region of achievable primary and secondary rates. Our simulation results have shown that the best secondary performance is always obtained when the secondary transmitter knows the instantaneous channel gain toward the intended receiver. Moreover, we have studied how the secondary transmitter should select its transmitting power not to violate the target outage probability at the primary receiver and we have shown the interplay between the allowed power at the secondary transmitter and decodability at the secondary receiver.

Our study opens a large number of items which deserve future investigations. In this work it is assumed that the secondary receiver first estimates various channel state information and then communicates them to the secondary transmitter which accordingly selects its power and rate. An interesting direction is to further study the impact of outdated channel state information or delay in the selection of the secondary rate and transmitting power.

## ACKNOWLEDGEMENT

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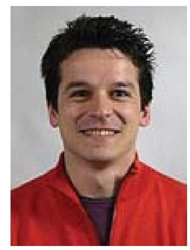

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