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Output SNR analysis of integrated active noise control and noise reduction in hearing aids under a single speech source scenario[1]

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Abstract

This paper analyses the output signal-to-noise ratio for a standard noise reduction scheme based on the multichannel Wiener filter and for an integrated active noise control and noise reduction scheme based on the filtered-X multichannel Wiener filter, both applied in a hearing aid framework that includes the effects of signal leakage through an open fitting and secondary path effects. In previous work, integrating noise reduction and active noise control has been shown to allow to compensate for effects of signal leakage and secondary path effects. These experimental results are now verified theoretically. The output signal-to-noise ratios are derived under a single speech source scenario. Theoretical results are then compared to simulations for a single noise source scenario and a multiple noise sources scenario.

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Abstract

This paper analyses the output signal-to-noise ratio for a standard noise reduction scheme based on the multichannel Wiener filter and for an integrated active noise control and noise reduction scheme based on the filtered-X multichannel Wiener filter, both applied in a hearing aid framework that includes the effects of signal leakage through an open fitting and secondary path effects. In previous work, integrating noise reduction and active noise control has been shown to allow to compensate for effects of signal leakage and secondary path effects. These experimental results are now verified theoretically. The output signal-to-noise ratios are derived under a single speech source scenario. Theoretical results are then compared to simulations for a single noise source scenario and a multiple noise sources scenario.

Keywords: Hearing aids, signal-to-noise ratio, active noise control, noise reduction, multichannel Wiener filter.

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1. Introduction

State-of-the-art hearing aids perform Noise Reduction (NR) in order to improve their output Signal-to-Noise Ratio (SNR) and hence to allow for a better speech understanding in background and to ease listening effort [1]. Conventionnal NR systems such as the Generalized Sidelobe Canceller (GSC) [2] or techniques based on the Multichannel Wiener Filter (MWF) [3] are commonly used.

With the advent of efficient feedback control schemes [4, 5], the usage of hearing aids with an open fitting has become more common. Whereas removing the earmold reduces the occlusion effect and improves the physical comfort [6], one major drawback is that the signal leakage through the fitting cannot be neglected anymore. Conventional NR does not take this contribution into account. Combined with the attenuation in the acoustic path between the hearing aid loudspeaker and the tympanic membrane (the so-called secondary path), the noise leaking through the fitting can override the action of the NR processing done in the hearing aid.

Active Noise Control (ANC) [7, 8, 9] is an efficient way to cancel the effect of signal leakage and generate a zone of quiet, in this case at the tympanic membrane. In the case of hearing aids, the ANC then has to be performed together with the NR in order to deliver the desired speech signal at the tympanic membrane. In previous work integrating the ANC and NR in a single set of filters, based on the so-called Filtered-x Multichannel Wiener Filter (FxMWF) [10], has been shown to allow to restore the standard NR performance [11].

In a single speech source scenario it is possible to compute the output signal-to-noise ratio (SNR) of MWF-based NR schemes [12]. In this paper, the single speech source assumption is applied to derive the output SNR of an MWF-based NR scheme in a hearing aid with an open fitting, when signal leakage and the secondary path are taken into account. The single speech source assumption is then also used to derive the output SNR the integrated ANC and NR scheme.

When the number of sound sources (speech plus noise sources) does not

exceed the number of input microphones the leakage signal can be rewritten as a linear combination of the input signals and then the output SNR can be expressed in a simple form [13]. This allows to verify the results from the simulations done in previous works [11].

When the number of sound sources is larger than the number of microphones, the leakage signal can be estimated by a linear combination of the microphone signals. Using a Mean Squared Error (MSE) estimator, and under certain assumptions, it is still possible to derive an expression of the output SNR that extends the results from [13].

The signal model for the single speech source scenario is described in Section 2. The SNR of MWF-based NR and the effects of the signal leakage and the secondary path are analysed in Section 3. The SNR of the integrated ANC and NR scheme is analysed in Section 4. The experimental results are presented in Section 5 and finally Section 6 presents the conclusions of the paper.

2. Signal model

Let M be the number of microphones (channels). The frequency-domain signal X_m for microphone m has a desired speech part X_m^s and an additive noise part X_m^n, i.e.:

$$X_m(\omega) = X_m^s(\omega) + X_m^n(\omega) \quad m \in \{1 \ldots M\} \tag{1}$$

where $\omega = 2\pi f$ is the frequency-domain variable. For conciseness, ω will be omitted in all subsequent equations.

In the sequel, superscripts s and n will also be used for other signals and vectors, to denote their speech and noise component, respectively. Signal model (1) holds for so-called "speech plus noise periods". There are also "noise only periods" (*i.e.*, speech pauses), during which only a noise component is observed.

In practice, in order to distinguish "speech plus noise periods" from "noise only periods" it is necessary to use a voice activity detector (VAD). The performance of the VAD can affect the performance of the filters. In this paper

however, as research on VAD is a vast subject of itself, it has been chosen to assume that a perfect VAD is available and to focus on the performance improvement the usage of ANC together with NR can give compared to NR alone.

The compound vector gathering all microphone signals is:

$$\mathbf{X}^T = [X_1 \ldots X_M] \tag{2}$$

An optimal (Wiener) filter $\mathbf{W}^T = [W_1 \ldots W_M]$ will be designed and applied to the signals, which minimizes a Mean Squared Error (MSE) criterion:

$$J_{\mathrm{MSE}} = \mathbb{E}\{|E|^2\} \tag{3}$$

Where $\mathbb{E}$ is the expectation operator and E is an error signal to be defined next, depending on the scheme applied. The filter output signal Z (*i.e.*, the signal to be fed in the hearing aid loudspeaker) is defined as:

$$Z = \mathbf{W}^H \mathbf{X} \tag{4}$$

where ^{H} denotes the Hermitian transpose.

The desired speech signal, as defined in [11], is arbitrarily chosen to be the (unkown) speech component of the first microphone signal ($m = 1$), up to a delay Δ. This can be written as:

$$D_{\mathrm{NR}} = \mathbf{G}_{1,\Delta}^H \mathbf{X}^s \tag{5}$$

$$\mathbf{G}_{1,\Delta}^H = [Ge^{-j\omega\Delta}|0\ldots 0] \tag{6}$$

$$= G\mathbf{e}_{1,\Delta} \tag{7}$$

where the gain G is the amplification that compensates for the hearing loss.

The autocorrelation matrices of the speech component and the noise component of the microphone signals are given by:

$$\mathbf{R}_s = \mathbb{E}\{\mathbf{X}^s \mathbf{X}^{sH}\} \tag{8}$$

$$\mathbf{R}_n = \mathbb{E}\{\mathbf{X}^n \mathbf{X}^{nH}\} \tag{9}$$

In a stationary scenario, and if the speech signal and the noise signal are assumed to be uncorrelated, $\mathbf{R}_n$ can be estimated during "noise only periods"

and $\mathbf{R}_s$ can be estimated during "speech plus noise periods" using:

$$\mathbf{R}_s = \mathbb{E}\{\mathbf{X}\mathbf{X}^H\} - \mathbb{E}\{\mathbf{X}^n\mathbf{X}^{nH}\} \tag{10}$$

In practice, the correlation matrices are not known but they are estimated recursively. The estimate of the autocorrelation matrix of the microphone signals is updated during "speech plus noise periods", using:

$$\tilde{\mathbf{R}}_x = \lambda\tilde{\mathbf{R}}_x + (1-\lambda)\mathbf{X}\mathbf{X}^H \tag{11}$$

where $\lambda \in [0\,1]$ is an exponential forgetting factor that depends on the number of past samples to be taken into account. Here $\lambda = \frac{1}{N_{\text{samples}}}$ with $N_{\text{samples}} = 20000$ samples.

The estimate of the autocorrelation matrix of the noise component of the microphone signals is updated similarly during "noise only periods", using:

$$\tilde{\mathbf{R}}_n = \lambda\tilde{\mathbf{R}}_n + (1-\lambda)\mathbf{X}\mathbf{X}^H \tag{12}$$

The estimate of the autocorrelation matrix of the speech component of the microphone signal is then given by:

$$\tilde{\mathbf{R}}_s = \tilde{\mathbf{R}}_x - \tilde{\mathbf{R}}_n \tag{13}$$

The number of sound sources (speech plus noise sources) will be denoted by Q. It is assumed in all subsequent schemes that only one speech source is present. The autocorrelation matrix of the speech component of the microphone signal ($\mathbf{R}_s$) is then rank-1 and can be rewritten as:

$$\mathbf{R}_s = P^s\mathbf{A}\mathbf{A}^H \tag{14}$$

where P^s is the power of the speech signal and $\mathbf{A}$ is the M-dimensional steering vector, which contains the acoustic transfer functions from the speech source position to the hearing aid microphones (including room acoustics, microphone characteristics, and head shadow effect).

3. MWF-based Noise Reduction

The MWF aims to minimize the squared distance between the filtered microphone signal ($\mathbf{W}^H \mathbf{X}$) and the desired speech signal ($D_{\mathrm{NR}} = \mathbf{G}_{1,\Delta}^H \mathbf{X}^s$). The corresponding MSE criterion is:

$$J_{\mathrm{MSE}} = \mathbb{E}\{|E_{\mathrm{NR}}|^2\} \tag{15}$$

$$E_{\mathrm{NR}} = \mathbf{W}^H \mathbf{X} - \mathbf{G}_{1,\Delta}^H \mathbf{X}^s \tag{16}$$

The MWF solution is given as:

$$\mathbf{W}_{\mathrm{NR}} = (\mathbf{R}_s + \mathbf{R}_n)^{-1} \mathbf{R}_s \mathbf{G}_{1,\Delta} \tag{17}$$

In the single speech source case, the Woodbury identity can be applied to compute the inverse of the pencil matrix $(\mathbf{R}_s + \mathbf{R}_n)$ [12]. The filter (17) can then be rewritten as follows:

$$\mathbf{W}_{\mathrm{NR}} = \frac{\mathbf{R}_n^{-1} \mathbf{R}_s}{\rho + 1} \mathbf{G}_{1,\Delta} \tag{18}$$

with

$$\rho = P^s \mathbf{A}^H \mathbf{R}_n^{-1} \mathbf{A} \tag{19}$$

The filter (18) is designed without taking the effects of the signal leakage and the secondary path effects into account. In Section 3.1, the output SNR performance of such a system is first computed in the absence of signal leakage and secondary path effects.

The hearing aid with an open fitting has no earmold to prevent ambient sound from leaking into the ear canal, which results in additional leakage signal l reaching the tympanic membrane. In literature this leakage signal can also be refered to as "vent-through" or "direct sound" [14][15]. No direct processing can be done on this signal, therefore its SNR is generally lower than for the signal provided by the hearing aid.

The MWF-based NR schemes ignore the so-called secondary path, *i.e.*, the propagation from the loudspeaker to the tympanic membrane. In the case of hearing aids with an open fitting, this secondary path includes the effects of the

transfer function that models the acoustic propagation from the receiver to the tympanic membrane (including the loudspeaker response itself) and the effects of the sound transfered from the ear canal to the open field through the open fitting (also know as "vent-loss"). In the context of hearing aids, this secondary path then usually acts as an attenuation. Therefore, the degradation caused by the signal leakage may be even more important when the secondary path is taken into account. Assuming that the loudspeaker characteristic is approximately linear, the secondary path can be represented by a transfer function $C(z)$.

The effects of these perturbations on the output SNR of the MWF-based NR scheme are analysed in Section 3.2 and 3.3. Figure (1) shows an MWF-based NR for a hearing aid (*i.e.*, with the secondary path and the signal leakage).

3.1. Output SNR

In the absence of signal leakage and secondary path effects the output SNR of the system is given by:

$$\begin{aligned}
\text{SNR}_{\text{NR(noLeakage)}} &= \frac{\mathbf{W}_{\text{NR}}^H \mathbf{R}_s \mathbf{W}_{\text{NR}}}{\mathbf{W}_{\text{NR}}^H \mathbf{R}_n \mathbf{W}_{\text{NR}}} \\
&= \frac{\frac{\rho^2}{\rho+1} P_{D_{\text{NR}}}}{\frac{\rho}{\rho+1} P_{D_{\text{NR}}}} \qquad (20) \\
&= \rho \qquad (21)
\end{aligned}$$

Where $P_{D_{\text{NR}}}$ is the power of the desired speech signal:

$$\begin{aligned}
P_{D_{\text{NR}}} &= \mathbf{G}_{1,\Delta}^H \mathbf{R}_s \mathbf{G}_{1,\Delta} \qquad (22) \\
&= G^2 \sigma_{s_1} \qquad (23)
\end{aligned}$$

and σ_{s_1} is the variance of the speech component in the first microphone signal $(m = 1)$:

$$\sigma_{s_1} = \mathbb{E}\{X_1^s X_1^{sH}\} \qquad (24)$$

This result is similar to the SNR formula in [12] and does not depend on the amplification gain G. However, it does not include the effects of the signal leakage and the secondary path effects.

3.2. Output SNR with signal leakage

In a hearing aid with an open fitting there is no earmold to prevent the ambient sound from leaking into the ear canal, which results in an additional leakage signal l reaching the tympanic membrane [14]. No direct processing can be done on this signal, therefore its SNR is generally lower than for the output signal provided by the hearing aid (4). In the subsequent equations, the frequency-domain leakage signal L will be used instead of the time-domain leakage signal l.

Taking the signal leakage effects into account, the output SNR of the standard NR scheme can be rewritten:

$$
\text{SNR}_{\text{NR(Leakage)}} = \frac{\mathbb{E}\{|\mathbf{W}^H\mathbf{X}^s + L^s|^2\}}{\mathbb{E}\{|\mathbf{W}^H\mathbf{X}^n + L^n|^2\}} \tag{25}
$$

$$
= \frac{\frac{\rho^2}{\rho+1}P_{D_{\text{NR}}} + \frac{\rho}{\rho+1}\alpha + P_{L^s}}{\frac{\rho}{\rho+1}P_{D_{\text{NR}}} + \frac{1}{\rho+1}\alpha + P_{L^s}} \tag{26}
$$

with

$$
P_{L^s} = \mathbb{E}\{|L^s|^2\} \tag{27}
$$

$$
P_{L^s} = \mathbb{E}\{|L^n|^2\} \tag{28}
$$

$$
\alpha = \mathbf{G}_{1,\Delta}^H\mathbf{r}_{sl} + \mathbf{r}_{sl}^H\mathbf{G}_{1,\Delta} \tag{29}
$$

$$
= G(\mathbf{e}_{1,\Delta}^H\mathbf{r}_{sl} + \mathbf{r}_{sl}^H\mathbf{e}_{1,\Delta}) \tag{30}
$$

where P_{L^s} and P_{L^s} are the power of the speech component and the noise component of the leakage signal, respectively.

The cross-correlation vector $\mathbf{r}_{sl}$ between the speech component of the microphone signals and the speech component of the leakage signal is given by:

$$
\mathbf{r}_{sl} = \mathbb{E}\{\mathbf{X}^s L^s\} \tag{31}
$$

If the speech component and the noise component of the microphone signals and the leakage signal are assumed to be uncorrelated, $\mathbf{r}_{sl}$ can be estimated using:

$$
\mathbf{r}_{sl} = \mathbb{E}\{\mathbf{X}L\} - \mathbb{E}\{\mathbf{X}^n L^n\} \tag{32}
$$

8

where $\mathbb{E}\{\mathbf{X}L\}$ is estimated during "speech plus noise periods" and $\mathbb{E}\{\mathbf{X}^n L^n\}$ and is estimated during "noise only periods".

From equations (23) and (26) one can identify the two extreme cases for the output SNR of the filter (18), when the signal leakage is taken into account, which are given by:

$$\lim_{G \to 0} \mathbf{W}_{\text{NR(Leakage)}} = \frac{P_{L^s}}{P_{L^s}} = \text{SNR}_{\text{leakage}} \tag{33}$$

$$\lim_{G \to \infty} \mathbf{W}_{\text{NR(Leakage)}} = \rho = \text{SNR}_{\text{NR(noLeakage)}} \tag{34}$$

When the amplification gain G is small, the output SNR is equivalent to the leakage SNR, *i.e.*, the NR has no effect on the signal delivered at the tympanic membrane. When the amplification gain G is large, the output SNR is equivalent to the output SNR of the NR scheme without leakage, *i.e.*, the signal leakage has no effect on the signal delivered at the tympanic membrane (see also Section 5).

3.3. Output SNR with signal leakage and secondary path effects

The MWF-based NR scheme ignores the secondary path. This secondary path usually acts as an attenuation and has an effect on the perfomance of the NR scheme. The output SNR of the MWF-based NR scheme can be rewritten to take the signal leakage effect and the effect of the secondary path into account:

$$\text{SNR}_{\text{NR(Leak+Sec)}} = \frac{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^s + L^s|^2\}}{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^n + L^n|^2\}} \tag{35}$$

$$= \frac{|C|^2 \frac{\rho^2}{\rho+1} P_{D_{\text{NR}}} + (C + C^*) \frac{\rho}{\rho+1} \alpha + P_{L^s}}{|C|^2 \frac{\rho}{\rho+1} P_{D_{\text{NR}}} + (C + C^*) \frac{1}{\rho+1} \alpha + P_{L^s}}$$

where * denotes the conjugate of a complex number.

This expression is very similar to (26) except that due to the attenuation caused by the secondary path C, the system needs a larger amplification gain G to restore the performance back to the performance of the NR scheme in the absence of signal leakage and secondary path effects (see also Section 5). In hearing aids, the amplification gain G can be design to compensate for the effects

9

of the secondary path. This solution, however, provides a fixed compensation and can become partly ineffective if the secondary path is changing over time.

In the single speech source scenario, the theory confirms the observations made in [11]. The secondary path and the signal leakage degrade the NR perfomance when the amplification gain G is small. One solution to compensate for these pertubations is to integrate ANC into the NR scheme.

4. Integrated Active Noise Control and Noise Reduction

This section first presents a frequency-domain description of the integrated ANC and NR scheme introduced in [11] and then analyses its output SNR performance. The scheme integrates NR and ANC in a single set of adaptive filters. Note that in practice, to ensure causality, the filter coefficients are computed in the frequency-domain while the filtering operation itself is performed in the time-domain (Figure 2), in a similar way as presented in [16].

The performance of feedforward ANC schemes is highly dependent on the causality of the system [7] and the impact of causality on the performance of the integrated ANC and NR scheme has been studied in [11]. In hearing aids, the causalilty margin is rather low as the acoustic propagation time from the input microphones position to the ear canal microphone position is really small. In this paper however the hearing aid processing delays (*i.e.*, Analog-to-Digital converter delays, Digital-to-Analog converter delays. . .) have been neglected such that the ANC schemes to be designed are causal. Therefore, all the subsequent theoretical expressions of the output SNR are valid only when the system is causal.

The aim of the integrated ANC and NR scheme is to minimize the squared distance between a desired speech signal ($D_{\mathrm{NR}} = \mathbf{G}_{1,\Delta}^{H}\mathbf{X}^{s}$) and the speech signal delivered at the tympanic membrane ($C^{*}\mathbf{W}^{H}\mathbf{X}^{s}$) and at the same time to minimize the residual noise at the tympanic membrane ($C^{*}\mathbf{W}^{H}\mathbf{X}^{n} + L^{n}$). Therefore the desired signal to be used here is:

$$D_{\mathrm{Int}} = \mathbf{G}_{1,\Delta}^{H}\mathbf{X}^{s} - L^{n} \tag{36}$$

and the MSE criterion to be minimized is:

$$J_{\text{MSE}} \quad = \quad \mathbb{E}\{|E_{\text{Int}}|^2\} \tag{37}$$

$$E_{\text{Int}} \quad = \quad C^* \cdot Z - D_{\text{Int}} \tag{38}$$

$$= \quad C^* \cdot \mathbf{W}^H \mathbf{X} + L^n - \mathbf{X}^s \mathbf{G}_{1,\Delta}$$

The optimal filter (FxMWF) minimizing (37) is:

$$\mathbf{W}_{\text{Int}} = \frac{\hat{C}^*}{|\hat{C}|^2}(\mathbf{R}_s + \mathbf{R}_n)^{-1}(\mathbf{R}_s \mathbf{G}_{1,\Delta} - \mathbf{r}_{nl}) \tag{39}$$

Where $\hat{C}$ is an estimate of the secondary path C and $\mathbf{r}_{nl}$ is the cross-correlation vector between the noise component of the microphone signal and the noise component of the leakage signal defined as:

$$\mathbf{r}_{nl} = \mathbb{E}\{\mathbf{X}^n L^n\} \tag{40}$$

The secondary path can be estimated off-line using classic identification methods based for example on Least Mean Square (LMS) algorithms, or on-line by adding random noise to the signal exciting the secondary path, as introduced by Eriksson et al. in [17] and later refined by Kuo et al. [18] and Zhang et al. [19].

4.1. Output SNR when the number of sources is less than or equal to the number of microphones

In [13], the output SNR of the integrated ANC and NR scheme has been derived when the number of sources (speech plus noise sources) is less than or equal to the number of microphones ($Q \leq M$). Under this assumption it can be shown that the leakage signal can be rewritten as a linear combination of the microphone signals:

$$L \quad = \quad \mathbf{P}^H \mathbf{X} \tag{41}$$

$$\mathbf{P}^T \quad = \quad [P_1 \ldots P_M] \tag{42}$$

The filter (39) then becomes:

$$\mathbf{W}_{\text{Int}(\text{Q}\leq\text{M})} = \frac{\hat{C}^*}{|\hat{C}|^2}(\mathbf{R}_s + \mathbf{R}_n)^{-1}(\mathbf{R}_s\mathbf{G}_{1,\Delta} - \mathbf{R}_n\mathbf{P}) \qquad (43)$$

In the single speech source scenario, the Woodbury identity can be applied to invert the pencil matrix $(\mathbf{R}_s + \mathbf{R}_n)$, leading to:

$$\mathbf{W}_{\text{Int}(\text{Q}\leq\text{M})} = \frac{\hat{C}^*}{|\hat{C}|^2}[\frac{\mathbf{R}_n^{-1}\mathbf{R}_s}{1+\rho}(\mathbf{G}_{1,\Delta} + \mathbf{P}) - \mathbf{P}] \qquad (44)$$

Note that this filter $W_{\text{Int}(\text{Q}\leq\text{M})}$ can be separated into two filters, as in [11]:

$$\mathbf{U} = \frac{\hat{C}^*}{|\hat{C}|^2}\frac{\mathbf{R}_n^{-1}\mathbf{R}_s}{1+\rho}\mathbf{G}_{1,\Delta} \qquad (45)$$

$$\mathbf{V} = \frac{\hat{C}^*}{|\hat{C}|^2}[\frac{\mathbf{R}_n^{-1}\mathbf{R}_s}{1+\rho}\mathbf{P} - \mathbf{P}] \qquad (46)$$

The first filter $\mathbf{U}$ is an MWF-based NR filter that compensates for the effects of the secondary path. Expression (45) is indeed very similar to (18). This can be used instead of designing the amplification gain G so that it compensates for the effects of the secondary path. If the secondary path is estimated online, the compensation is then adaptive and robust to changes of scenarios (hearing aid moves slightly, ear become partly obstructed...).

The second filter $\mathbf{V}$ is an ANC filter that estimates the noise component of the leakage signal. The output signal of $\mathbf{V}$ after filtering by the secondary path, is given by:

$$C^*\mathbf{V}^H\mathbf{X} = \frac{\mathbf{R}_n^{-1}\mathbf{R}_s}{1+\rho}(\mathbf{P}^H)\mathbf{X} - \mathbf{P}^H\mathbf{X} \qquad (47)$$

$$= L^s - L \qquad (48)$$

$$= -L^n \qquad (49)$$

Assuming that the secondary path identification error is small ($\hat{C} \approx C$), the output SNR of the system, taking both the signal leakage and the secondary

path effects into account, is given by:

$$\text{SNR}_{\text{Int}(Q \leq M)} = \frac{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^s + L^s|^2\}}{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^n + L^n|^2\}} \tag{50}$$

$$= \frac{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^s + \mathbf{P}^H \mathbf{X}^s|^2\}}{\mathbb{E}\{|C^* \mathbf{W}^H \mathbf{X}^n + \mathbf{P}^H \mathbf{X}^n|^2\}} \tag{51}$$

$$= \frac{\frac{\rho^2}{(\rho+1)^2} P_{D_{\text{NR}}} + \frac{\rho^2}{(\rho+1)^2}\alpha + \frac{\rho^2}{(\rho+1)^2} P_{L^s}}{\frac{\rho}{(\rho+1)^2} P_{D_{\text{NR}}} \frac{\rho}{(\rho+1)^2}\alpha + \frac{\rho}{(\rho+1)^2} P_{L^s}}$$

$$= \rho = \text{SNR}_{\text{NR(noLeakage)}} \tag{52}$$

The secondary path and the noise component of the leakage signal are included in the cost function of the integrated ANC and NR scheme (37). Therefore, the signal leakage has no effect on the performance of the system and the scheme delivers a constant output SNR for any amplification gain G. The SNR is then equal to the SNR of the MWF-based NR scheme in the absence of signal leakage and the secondary path effects (see also Section 5). Note also that the theoretical formula of the output SNR (52) do not depend on this vector $\mathbf{P}$. Therefore, in this case, there is no need to know $P_1 \ldots P_M$ explicitly.

4.2. Output SNR when the number of sources is larger than the numbers of microphones

When the number of sound sources is larger than the number of microphones $(Q > M)$, the leakage signal can be approximated by a linear combination of the input signals. The leakage signal can then be rewritten as:

$$L = \tilde{\mathbf{P}}^H \mathbf{X} + e_L \tag{53}$$

$$\tilde{\mathbf{P}}^T = [\tilde{P}_1 \ldots \tilde{P}_M] \tag{54}$$

where e_L is the estimation error. Here the filter $\tilde{\mathbf{P}}$ is designed to minimize the mean-square value of e_L:

$$\mathbb{E}\{|e_L|^2\} = \mathbb{E}\{|L - \tilde{\mathbf{P}}^H \mathbf{X}|^2\} \tag{55}$$

Based on this expression for the leakage signal, the MSE criterion (37) used to design the integrated filter $\mathbf{W}_{\text{Int}}$ can be rewritten as:

$$J_{\text{MSE}} = \mathbb{E}\{|C^* \cdot \mathbf{W}^H \mathbf{X} + \tilde{\mathbf{P}}^H \mathbf{X}^n + e_L^n - \mathbf{G}_{1,\Delta}^H \mathbf{X}^s|^2\} \tag{56}$$

13

The estimation error e_L is then orthogonal to the microphone signals and to the microphone signals filtered by $\tilde{\mathbf{P}}$ [20]:

$$\mathbb{E}\{Xe_L^H\} = 0 \qquad (57)$$

$$\mathbb{E}\{\tilde{\mathbf{P}}^H\mathbf{X}e_L^H\} = 0 \qquad (58)$$

If the filter $\mathbf{W}$ is varies slowly, it can be shown that e_L is also orthogonal to the microphone signals filtered by $\mathbf{W}$:

$$\mathbb{E}\{\mathbf{W}^H\mathbf{X}e_L^H\} = 0 \qquad (59)$$

The integrated filter $\mathbf{W}_{\text{Int}}$ can then be rewritten as in (43):

$$\mathbf{W}_{\text{Int(Q>M)}} = \frac{\hat{C}^*}{|\hat{C}|^2}(\mathbf{R}_s + \mathbf{R}_n)^{-1}(\mathbf{R}_s\mathbf{G}_{1,\Delta} - \mathbf{R}_n\tilde{\mathbf{P}}) \qquad (60)$$

which is (43) with $\mathbf{P}$ replaced by $\tilde{\mathbf{P}}$.

The absence of the estimation error e_L in formula (60) implies that the integrated ANC and NR scheme can compensate only for the estimated part of the leakage signal. The scheme can fully compensate for signal leakage when the number of sound sources (speech plus noise sources) is less than or equal to the number of microphones and so the estimation error e_L is zero. When the number of sources is larger than the number of microphone, the integrated ANC and NR scheme will only compensate for the estimated leakage signal ($\tilde{\mathbf{P}}^H\mathbf{X}$).

In the single speech source scenario, the Woodbury identity can be applied to invert the pencil matrix $(\mathbf{R}_s + \mathbf{R}_n)$ in (60), leading to an expression similar to (44):

$$\mathbf{W}_{\text{Int(Q>M)}} = \frac{\hat{C}^*}{|\hat{C}|^2}[\frac{\mathbf{R}_n^{-1}\mathbf{R}_s}{1+\rho}(\mathbf{G}_{1,\Delta} + \tilde{\mathbf{P}}) - \tilde{\mathbf{P}}] \qquad (61)$$

Assuming that the secondary path identification error is small ($\hat{C} \approx C$), the

output SNR of the system is given by:

$$\text{SNR}_{\text{Int(Q>M)}} = \frac{\mathbb{E}\{|C^*\mathbf{W}^H\mathbf{X}^s + L^s|^2\}}{\mathbb{E}\{|C^*\mathbf{W}^H\mathbf{X}^n + L^n|^2\}} \tag{62}$$

$$= \frac{\mathbb{E}\{|C^*\mathbf{W}^H\mathbf{X}^s + \tilde{\mathbf{P}}^H\mathbf{X}^s + e_L^s|^2\}}{\mathbb{E}\{|C^*\mathbf{W}^H\mathbf{X}^n + \tilde{\mathbf{P}}^H\mathbf{X}^n + e_L^n|^2\}} \tag{63}$$

$$= \frac{\frac{\rho^2}{(\rho+1)^2}(P_{D_{\text{NR}}} + \alpha + \tilde{\mathbf{P}}^H R^s \tilde{\mathbf{P}}) + E_{e_L^s}}{\frac{\rho}{(\rho+1)^2}(P_{D_{\text{NR}}} + \alpha + \tilde{\mathbf{P}}^H R^s \tilde{\mathbf{P}}) + E_{e_L^n}}$$

where $E_{e_L^s}$ and $E_{e_L^n}$ are the power of the speech component and the noise component in the estimation error e_L, respectively:

$$E_{e_L^s} = \mathbb{E}\{|e_L^s|^2\} \tag{64}$$

$$E_{e_L^n} = \mathbb{E}\{|e_L^n|^2\} \tag{65}$$

When the number of sound sources (speech plus noise sources) is less than or equal to the number of microphones, the estimation error e_L is zero. Formula (63) then reduces to (52). In this case, the integrated ANC and NR scheme delivers a constant output SNR equal to the output SNR of the MWF-based NR in the absence of signal leakage and the secondary path effects. This confirms what was shown in the previous section.

When the number of sources is larger than the number of microphones, the estimation error e_L is not zero in general. One can then again identify the two extreme cases for the output SNR of the integrated ANC and NR scheme:

$$\lim_{G \to 0} \mathbf{W}_{\text{Int(Q>M)}} = \frac{\tilde{\mathbf{P}}^H R^s \tilde{\mathbf{P}} + E_{e_L^s}}{\tilde{\mathbf{P}}^H R^s \tilde{\mathbf{P}} + E_{e_L^n}} \tag{66}$$

$$\lim_{G \to \infty} \mathbf{W}_{\text{Int(Q>M)}} = \rho = \text{SNR}_{\text{NR(noLeakage)}} \tag{67}$$

As the number of sources grows, the power of the estimation error e_L is expected to grow also. The extreme output SNR case for low amplification then becomes:

$$\lim_{(G,Q)\to(0,\infty)} \mathbf{W}_{\text{Int(Q>M)}} = \frac{E_{e_L^s}}{E_{e_L^n}} = \text{SNR}_{e_L} \tag{68}$$

The integrated ANC and NR scheme then tends to behave more like an MWF-based NR under signal leakage effects as it only compensate for the secondary path effects (see also Section 5).

5. Experimental results

The MWF-based NR and the integrated ANC and NR scheme presented in Section 3 and in Section 4, respectilvely, have been tested experimentally and their performance has been compared to the theoretical performance.

5.1. Experimental setup

The simulations were run on acoustic path measurements obtained with a CORTEX MK2 manikin head and torso equipped with artificial ears and a two-microphones behind-the-ear (BTE) hearing aid. The sound sources (FOSTEX 6301B loudspeakers) were postioned at 1 meter from the center of the head. The speech source was located at $0°$ and the noise sources at $30°$, $90°$, $270°$ and $330°$. The BTE was worn on left ear, facing the noise source at $270°$. Commercial hearing aids currently do not have an ear canal microphone, therefore the artificial ear eardrum microphone is used here to generate the error signal. The tests were run on 22 seconds long signals. The speech was composed of three sentences from the HINT database [21] concatenated with silence periods. The noise was the multitalker babble from Auditec [22]. All the signals were sampled at 16kHz.

The filter length is set to $N = 128$, and the NR delay is set to half of the NR filter length ($\Delta = 64$). The secondary path $C(z)$ is estimated off-line using an identification technique based on the Normalized Least Mean Squares (NLMS) algorithm. The length of the estimated path $\hat{C}(z)$ is set to $L = 32$.

The (BTE microphone) input SNR is often used as a reference measure in standard NR schemes. In this case, as the integrated ANC and NR scheme also perfoms ANC, the leakage signal SNR, which can also be considered as the SNR when the hearing aid is turned off, is taken as a reference. The intelligibility-weighted signal-to-noise ratio improvement [23] is used here as a performance measure, which is defined as

$$\Delta\text{SNR}_{\text{intellig}} = \sum_i I_i(\text{SNR}_{\text{i,out}} - \text{SNR}_{\text{i,leak}}) \tag{69}$$

where I_i is the band importance function defined in [24] and $\text{SNR}_{i,\text{out}}$ and $\text{SNR}_{i,\text{leak}}$ represent the output SNR and the leakage SNR (in dB) of the ith band, respectively.

Two experiments are presented here. The first experiment shows the output $\Delta\text{SNR}_{\text{intellig}}$ of the MWF-based NR and the integrated ANC and NR scheme when only the noise source at $270°$ is active. The second experiment aims at showing the effect of the number of sources (speech plus noise sources) on the output $\Delta\text{SNR}_{\text{intellig}}$ of the integrated ANC and NR scheme.

5.2. Output SNR in a single noise source scenario

To evaluate the effects of signal leakage and the secondary path effects, the microphone signals are first filtered by the MWF-based NR scheme and then filtered by the integrated ANC and NR scheme. Depending on which disturbance is being tested, the output signal can then be filtered by the secondary path model $C(z)$ and/or the leakage signal can be added. The system is calibrated so that the reference SNR (leakage signal SNR), is equal to 0dB.

Figure 3 presents the theoretical output $\Delta\text{SNR}_{\text{intellig}}$ of the MWF-based NR with signal leakage, with signal leakage and secondary path effects and the theoretical output $\Delta\text{SNR}_{\text{intellig}}$ of the integrated ANC and NR scheme, all are based on the formulas derived in Sections 3 and 4. The theoretical $\Delta\text{SNR}_{\text{intellig}}$ is presented for an amplification gain varying from 0dB to 20dB. The amplification gain G is flat, *i.e.*, it takes the same value for all frequency bands. The system is calibrated so that for $G = 0$dB, for a source at $0°$, the leakage signal and the signal fed in the loudspeaker have equal full-band power.

When the signal leakage is the only disturbance considered, the degradations induced by the signal leakage remain small even for reasonably small amplification gain G (down to 8dB (Figure 3)). However, introducing both the signal leakage and the secondary path the degradations are significant for amplification gains up to at least 20dB. The integrated ANC and NR scheme delivers a constant output $\Delta\text{SNR}_{\text{intellig}}$ for any amplification gain G. This confirms the results presented in [11].

17

Figure 4 shows the simulated output $\Delta\text{SNR}_\text{intellig}$ for the MWF-based NR (with signal leakage only and with signal leakage and secondary path effects) and the simulated output $\Delta\text{SNR}_\text{intellig}$ for the integrated ANC and NR scheme. The amplification gain G varies from 0dB to 20dB. The performance of the MWF-based NR under the different disturbances is similar to the theoretical performance. The integrated ANC and NR scheme delivers an almost constant output $\Delta\text{SNR}_\text{intellig}$ which also confirms the theoretical results, *i.e.*, that the integrated ANC and NR scheme allows to restore the NR performance and deliver a constant output $\Delta\text{SNR}_\text{intellig}$ for any amplification gain G in the scenario where the number of sources is less than or equal to the number of microphones.

5.3. Output SNR in a multiple noise sources scenario

To evaluate the effects of the number of sound sources (speech plus noise sources) on the performance of the integrated ANC and NR scheme, multiple noise sources are used to compose the input signals. When only one noise source is active, the noise source at 270° (facing the hearing aid) is used. Then the noise source at 90° is added for the 2 sources scenario, the noise source at 330° is added for the 3 sources scenario and finally the noise source at 30° is added for the 4 sources scenario.

As explained in Section 4.2 the number of sound sources (speech plus noise sources) has an impact on the performance of the integrated ANC and NR scheme. The number of noise sources similarly has an impact on the performance of MWF-based NR schemes (*i.e.*, ρ depends on the number of noise sources). In order to observe the effects of the number of sound sources (speech plus noise sources) on the ANC part of the integrated ANC and NR schemes it is more convenient to look at the normalized output SNR improvement:

$$\overline{\Delta\text{SNR}_\text{intellig}} = \frac{\Delta\text{SNR}_\text{intellig}}{\Delta\rho_\text{intellig}} \tag{70}$$

Where $\Delta\rho$ is the intelligibility-weighted signal-to-noise ratio improvement for the MWF-based NR scheme when no perturbation is taken into account. It is defined as follows:

18

$$\Delta\rho_{\text{intellig}} = \sum_i I_i(\rho_i - \text{SNR}_{i,\text{leak}}) \qquad (71)$$

Where ρ_i represents the output SNR (in dB) of the ith band for the MWF-based NR scheme with no perturbation.

Figure 5 shows the simulated output $\Delta\text{SNR}_{\text{intellig}}$ performance of the integrated ANC and NR scheme for an amplification of 0dB (where the integrated ANC and NR scheme acts as an ANC filter that estimates the noise component of the leakage signal) and an amplification of 20dB (where the integrated ANC and NR scheme acts as an MWF-based NR with no pertubations) and for a number of noise sources varying from 1 to 4. For an amplification gain of 20dB the output $\Delta\text{SNR}_{\text{intellig}}$ decreases when the number of noise sources increases. This shows the effect of the number of noise sources on the NR part of the integrated ANC and NR scheme. For an amplification gain of 0dB the degradation of the output $\Delta\text{SNR}_{\text{intellig}}$ introduced by the number of noise sources is related to the fact that the integrated ANC and NR scheme can only compensate for an estimate of the leakage signal when the number of sources becomes larger than the number of microphones (as explained in Section 4.2).

Figure 6 shows the normalized simulated output SNR improvement ($\overline{\Delta\text{SNR}_{\text{intellig}}}$) of the integrated ANC and NR scheme, for an amplification varying from 0dB to 20dB and for scenarios with 1 to 4 noise sources. For large amplification gains, the integrated ANC and NR scheme deliver the same output $\overline{\Delta\text{SNR}_{\text{intellig}}}$ as the MWF-based NR in absence of signal leakage and secondary path effects for any number of noise sources. For small amplification gains, when the number of noise sources increases, the output SNR decreases. The ANC can compensate only for the estimated part of the leakage signal and the integrated ANC and NR scheme tends to behave as a MWF-based NR compensating for the secondary path. This can be seen on Figure 7 where the simulated output $\Delta\text{SNR}_{\text{intellig}}$ performance of the integrated ANC and NR scheme and of the MWF-based NR are plotted for the 4 noise sources scenario. The integrated ANC and NR scheme then tends to behave as the MWF-based NR under signal leakage effects

19

(*i.e.*, when the effects of the secondary path are compensated for).

6. Conclusions

For a single speech source scenario, it can be shown theoretically that the signal leakage and the secondary path effects degrade the performance of a standard NR based on MWF, especially so when a small amplification G is applied in the hearing aid. This confirms previous observations and simulations results as well as the need, with increased usage of hearing aids with open fitting, of a scheme that compensates for signal leakage.

It has been shown in previous work that integrating NR and ANC in a single set of filters allows to compensate for the signal leakage and the secondary path effects. When the number of sound sources (speech plus noise sources) is less than or equal to the number of microphones the leakage signal can be written as a linear combination of the microphone signals. It is then possible to derive a simple formula for the output SNR of the integrated ANC and NR scheme. The integrated ANC and NR scheme then allows to restore the NR performance and delivers a constant output SNR for any amplification gain G as in absence of signal leakage and secondary path effects.

When the number of sound sources is larger than the number of microphones, it is possible to rewrite the leakage signal as the sum of a linear combination of the microphone signals and an estimation error. The integrated ANC and NR scheme can then compensate only for the estimated part of the leakage signal. When the number of sound sources grows so does the estimation error, and so the integrated ANC and NR scheme tends to behave like an MWF-based NR, which also compensantes for the secondary path effect. Therefore, it still improves the output SNR performance compared to a standard MWF-based NR.

Acknoledgements

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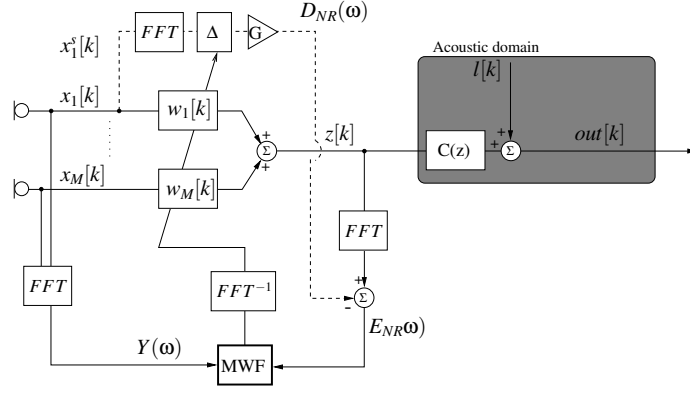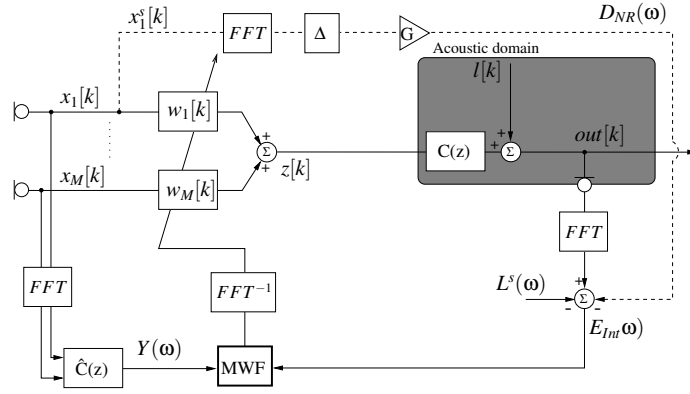
Figure 1: MWF-based NR in hearing aids framework

Figure 2: Integrated ANC and NR in the hearing aids framework
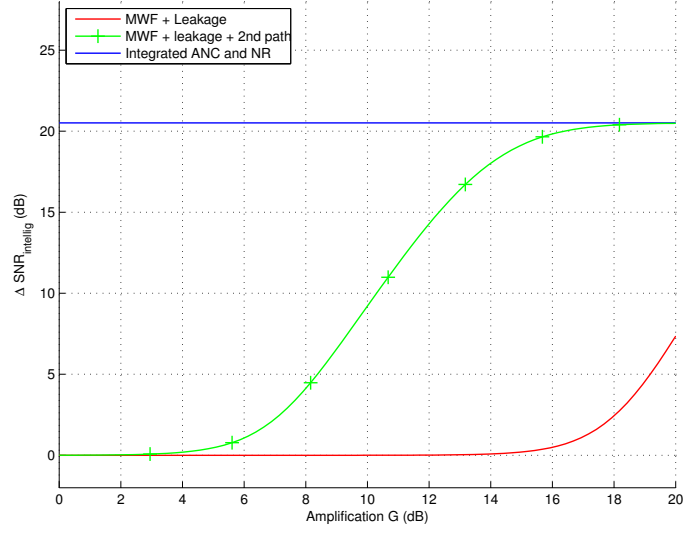
24

Figure 3: Theoretical output SNR for the MWF-based NR scheme and for the integrated ANC and NR scheme
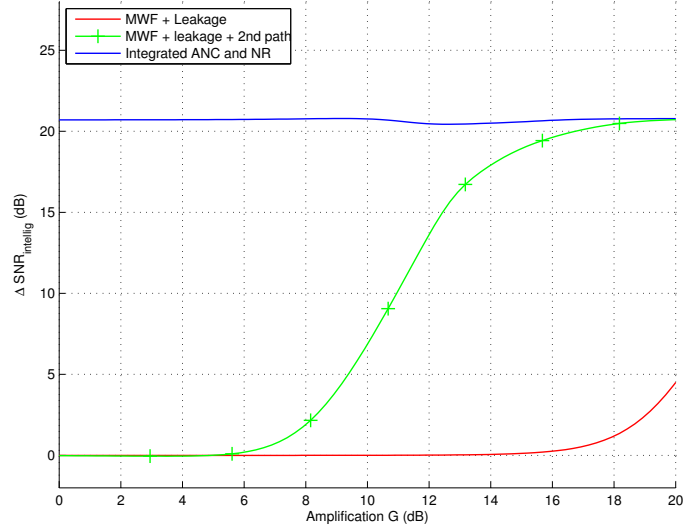

Figure 4: Simulated output SNR for MWF-based NR scheme and for the integrated ANC and NR scheme (noise source at $270°$)
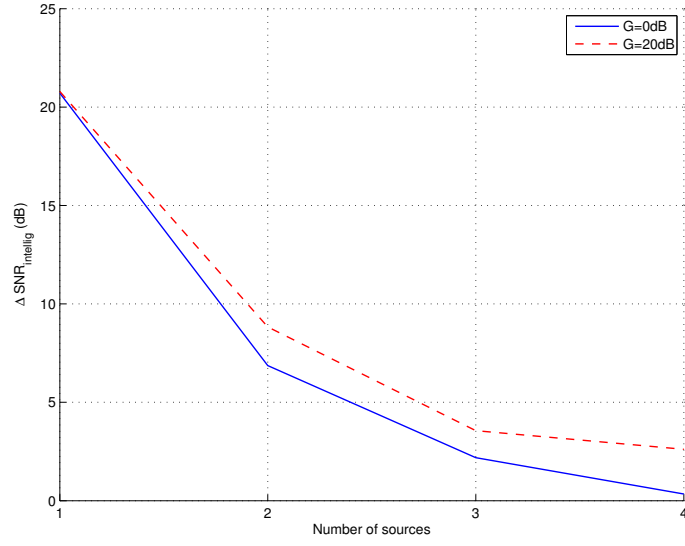
Figure 5: Simulated output SNR for the integrated ANC and NR scheme depending on the number of noise sources ($G = 0$dB and $G = 20$dB)
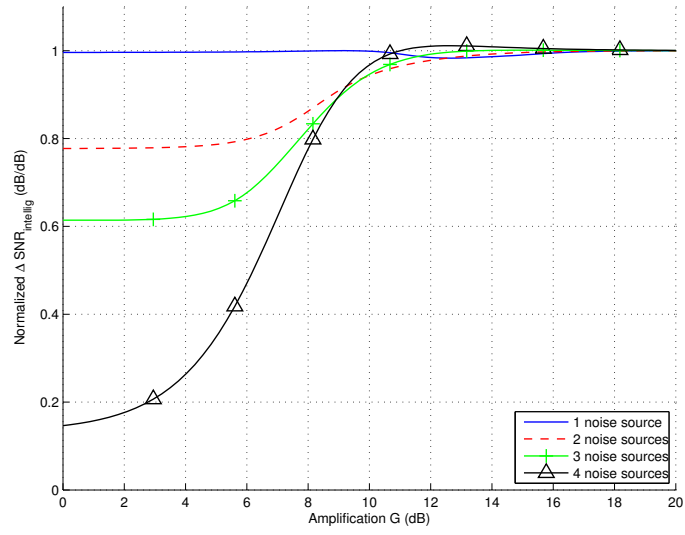

Figure 6: Normalized simulated output SNR ($\frac{10\log_{10}\mathrm{SNR}}{10\log_{10}\rho}$) for the integrated ANC and NR scheme (noise source at $270°$, $90°$, $330°$ and $90°$)
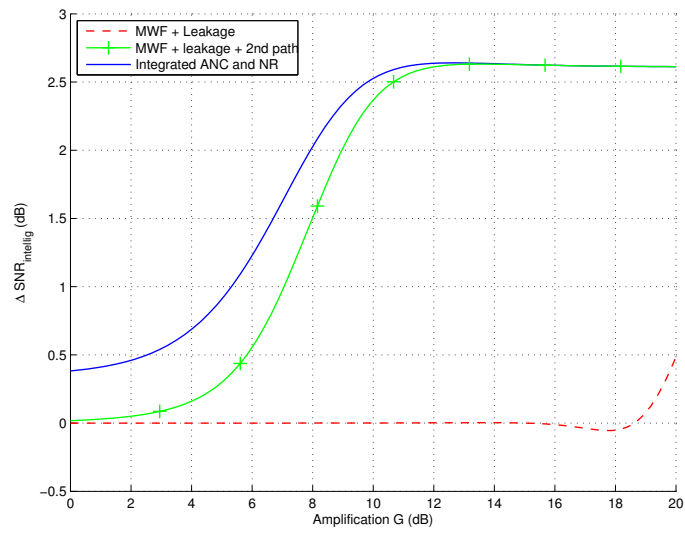
Figure 7: Simulated output SNR for MWF-based NR scheme and for the integrated ANC and NR scheme (noise source at $30°$, $90°$, $270°$ and $330°$)