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Published in:
2023 IEEE 24th International Workshop on Signal Processing Advances in Wireless Communications, SPAWC 2023 - Proceedings

DOI (link to publication from Publisher):
10.1109/SPAWC53906.2023.10304456

Publication date:
2023

Document Version
Accepted author manuscript, peer reviewed version

Citation for published version (APA):
Chen Hu, K., Fernández-Getino García, M. J., & García Armada, A. (2023). Overhead-Free Channel Estimation based on Phase-domain Injected Training for FM-OFDM. In *2023 IEEE 24th International Workshop on Signal Processing Advances in Wireless Communications, SPAWC 2023 - Proceedings* (pp. 331-335). IEEE (Institute of Electrical and Electronics Engineers). https://doi.org/10.1109/SPAWC53906.2023.10304456

Overhead-Free Channel Estimation based on Phase-domain Injected Training for FM-OFDM

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***Abstract*—Frequency-Modulated Orthogonal Frequency-Division Multiplexing (FM-OFDM) is a novel constant envelope multi-carrier waveform proposed to be exploited in high-mobility scenarios, which is a typical case to be coped in the 6G. Moreover, it also exhibits a strong robustness to phase noise and carrier frequency offsets. However, it assumes that the channel estimates are perfectly given. In this paper, phase injected training (PIT) is proposed in order to obtain accurate enough channel estimates with a zero overhead for the case of FM-OFDM, since the pilot symbols are embedded in the phase component of the data symbols. Theoretical expressions and numerical results prove the feasibility of FM-OFDM using a realistic channel estimation method.**

***Index Terms*—channel estimation, FM-OFDM, phase injected training, high-speed, overhead**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) [1] is one the most well-known and exploited multi-carrier waveforms due to its flexibility and low-complexity. However, one of the most important issues corresponds to the peak-to-average power ratio (PAPR), which reduces the efficiency of the power amplification stage due to the non-linear region of the power amplifier. The transmitted signal is significantly distorted when he high peaks of an OFDM signal are cuts-off. Constant envelope-OFDM (CE-OFDM) [2], [3] has been proposed as an alternative to the most used OFDM, which is capable of effectively reducing the high PAPR and keep all the benefits of OFDM. In order to obtain this property, an additional non-linear phase modulator and demodulator blocks are added to the traditional OFDM processing ones. Therefore, CE-OFDM is fully backward compatible with OFDM. Additionally, CE-OFDM avoids the requirement of a typical back-off of 6 dB or higher due to exhibiting a PAPR of 0 dB, and it is also able to keep the flexibility of resource scheduling at frequency domain as any OFDM-based system, where the available subcarriers can be easily allocated to different users. CE-OFDM outperforms OFDM in terms of bit error rate (BER), coverage and network capacity for some specific scenarios considered in the Sixth Generation (6G).

Recently, frequency modulated-OFDM (FM-OFDM) [4] is also proposed which is a variant of the existing CE-OFDM. The main advantage of FM-OFDM as compared to CE-OFDM corresponds to its high robustness against phase errors, i.e. produced by Doppler shifts. Therefore, this waveform is suitable for extremely high-speed scenarios. This strength is obtained by performing a phase accumulation of the OFDM symbols before feeding them to the phase modulator at the transmitter, and performing a phase differentiation after the phase demodulator at the receiver. Note that the concatenation of a phase accumulation and modulator equals to a frequency modulator, while the concatenation of a phase demodulator and differentiation corresponds to a frequency demodulator. Consequently, the FM-OFDM inherits all the benefits of CE-OFDM. However, [4] also assumed that the channel impulse response is either perfectly known or estimated by transmitting a preamble-based pilot sequence at the transmitter, typically known as pilot symbol assisted modulation (PSAM). The channel is estimated in the frequency domain before performing the demodulation, similarly to the previous works on CE-OFDM [2], [3]. Later, a technique based on phase-domain [5] has been explored to avoid the pilots.

To the best knowledge of the authors, an efficient channel estimation method has never been proposed for FM-OFDM [4]. In this work, an overhead-free channel estimation method, namely phase-domain injected training (PIT), based on [5], is proposed for FM-OFDM which is capable of circumvent the excessive overhead induced by the transmission of preamble sequences. At the transmitter, the training sequence, which consists in a phase shift per sample in the time domain, is embedded within the data symbols at the phase modulator operation. Hence, the PIT proposal is efficient in terms of bandwidth because it does not occupy any exclusive time/frequency resource, unlike PSAM. Furthermore, this proposal maintains the constant envelope property of FM-OFDM. At the receiver, a dual averaging procedure can be exploited and, thanks to the particular structure of the FM-OFDM signal, the estimated channel can be obtained. The mean squared error (MSE) of the channel estimation based on PIT is theoretically analyzed, showing how the choice of the modulation index of the data symbols and the averaging factors are crucial in order to obtain accurate enough channel estimates. Finally, the goodness of the exploitation of PIT in FM-OFDM is also evaluated via simulations, providing the numerical results of mean squared error (MSE) and bit error rate (BER).

The remainder of the paper is organized as follows. Section II introduces the system model of FM-OFDM. Section III provides the description of the proposed PIT for FM-OFDM and provides theoretical analysis of MSE. Section IV addresses the overhead issue. Section V presents several numerical results for the proposed scheme. Finally, in Section VI, the conclusions are reported.

Notation: Matrices, vectors, and scalar quantities are denoted by boldface uppercase, boldface lowercase, and normal letters, respectively. $[\mathbf{A}]_{mn}$ denotes the element in the m-th row and n-th column of $\mathbf{A}$, $[\mathbf{A}]_{:,n}$ is $\mathbf{A}$'s n-th column, and $[\mathbf{a}]_n$ represents the n-th element of $\mathbf{a}$. $\operatorname{diag}(\mathbf{a})$ denotes a diagonal matrix whose diagonal elements are formed by $\mathbf{a}$'s elements. $\Re(\cdot)$ and $\Im(\cdot)$ represent the real and imaginary part of a complex number, respectively, and $\circledast$ denotes the circular convolution operation. $|\cdot|$ is the absolute value. $\mathbb{E}\{\cdot\}$ represents the expected value of a random variable and $\mathcal{CN}(0,\sigma^2)$ represents the circularly-symmetric and zero-mean complex normal distribution with variance σ^2.

II. System Model

A bidirectional link consisting of two terminals, that could for example be an access point (AP) serving a single user terminal (UT), is considered. Both terminals can transmit and receive data symbols. The transmitter sends B contiguous FM-OFDM symbols to the receiver, and each FM-OFDM symbol is composed of K subcarriers.

Let us define the complex data vector $\mathbf{s}_b \in \mathbb{C}^K$ to be transmitted at the b-th FM-OFDM symbol $(1 \le b \le B)$ as

$$\mathbf{f}_b = \mathbf{F}_K^H \mathbf{s}_b \in \mathbb{R}^K, \quad 1 \le b \le B, \tag{1}$$

where $\mathbf{F}_K \in \mathbb{C}^{K\times K}$ is the normalized discrete Fourier transform (DFT) matrix of K-points and $\mathbf{f}_b$ corresponds to a real-valued OFDM symbol, since $\mathbf{s}_b$ satisfies the Hermitian symmetry. Then, $\mathbf{f}_b$ is further processed and goes through an accumulator and phase modulator yielding

$$\begin{gathered}[\mathbf{r}_b]_n = \sum_{i=1}^{n} [\mathbf{f}_b]_i, \quad [\mathbf{c}_b]_n = \exp\left(j2\pi\beta\, [\mathbf{r}_b]_n\right), \\ \left|[\mathbf{c}_b]_n\right| = 1, \quad 1 \le n \le K, \quad 1 \le b \le B,\end{gathered} \tag{2}$$

where β is the modulation index, $\mathbf{c}_b \in \mathbb{C}^K$ corresponds to the FM-OFDM symbol and n accounts for the time sample index. The PAPR is 0 dB because each sample of $\mathbf{c}_b$ has unit power. Similar to OFDM, a CP of L_{CP} samples is appended to each FM-OFDM symbol $\mathbf{c}_b$ yielding $\mathbf{x}_b$ in order to absorb the multi-tap channel effects. Finally, the B FM-OFDM symbols are concatenated into a single vector to be transmitted through the wireless channel.

The received signal comes from a linear convolution between the wireless channel and the transmitted signal. However after discarding the CP, it is transformed into a circular convolution of K samples, whose expression is given by

$$\mathbf{y}_b = \mathbf{h} \circledast \mathbf{x}_b + \mathbf{v}_b, \tag{3}$$

where $\mathbf{v}_b \in \mathbb{C}^K$ corresponds to the additive white Gaussian noise (AWGN) vector of each OFDM symbol, and each element is distributed as $\mathcal{CN}(0,\sigma_v^2)$. $\mathbf{h} \in \mathbb{C}^{L_{CH}}$ corresponds to the channel impulse response of L_{CH} taps. Each tap is modeled as an independent complex random variable distributed as $\mathcal{CN}(0,\sigma_\tau^2)$ $1 \le \tau \le L_{CH}$.

Later FM-OFDM symbol must be equalized in order to remove the effects of the channel. According to [2], an FDE can be used. The frequency-domain output of the DFT over $\mathbf{y}_b$ can be decomposed as

$$\tilde{\mathbf{y}}_b = \mathbf{F}_K \mathbf{y}_b = \tilde{\mathbf{H}}\tilde{\mathbf{c}}_b + \tilde{\mathbf{v}}_b \in \mathbb{C}^K, \quad \tilde{\mathbf{c}}_b = \mathbf{F}_K \mathbf{c}_b \in \mathbb{C}^K, \tag{4}$$

$$\tilde{\mathbf{H}} = \operatorname{diag}\left(\tilde{\mathbf{h}}\right) \in \mathbb{C}^{K\times K}, \quad \tilde{\mathbf{h}} = \mathbf{F}_K \mathbf{h} \in \mathbb{C}^K, \quad \tilde{\mathbf{v}}_b = \mathbf{F}_K \mathbf{v}_b \in \mathbb{C}^K, \tag{5}$$

where $1 \le n \le K$, $1 \le b \le B$, $\tilde{\mathbf{h}}$ is the channel frequency response, $\tilde{\mathbf{c}}_b$ corresponds to the b-th transmitted FM-OFDM symbol in the frequency domain and $\tilde{\mathbf{v}}_b$ accounts for the noise in frequency domain at the b-th received FM-OFDM symbol.

Hence, the channel effects can be equalized by using a one-tap equalizer in the frequency domain as

$$\widehat{\tilde{\mathbf{c}}}_b = \mathbf{Q}\tilde{\mathbf{y}}_b = \mathbf{Q}\tilde{\mathbf{H}}\tilde{\mathbf{c}}_b + \mathbf{Q}\tilde{\mathbf{v}}_b = \tilde{\mathbf{c}}_b + \tilde{\mathbf{v}}'_b \in \mathbb{C}^K, \quad 1 \le b \le B, \tag{6}$$

where $\mathbf{Q} \in \mathbb{C}^{K\times K}$ is the equalization matrix in the frequency domain. After the equalization, the symbols are converted back to the time domain using the IDFT as

$$\widehat{\mathbf{c}}_b = \mathbf{F}_K^H \widehat{\tilde{\mathbf{c}}}_b = \mathbf{c}_b + \mathbf{F}_K^H \tilde{\mathbf{v}}'_b = \mathbf{c}_b + \mathbf{v}'_b \in \mathbb{C}^{K\times 1}, \quad 1 \le b \le B, \tag{7}$$

where the noise is distributed as [6]

$$[\mathbf{v}']_n \sim \mathcal{CN}(0,\sigma_{v'}^2), \quad 1 \le n \le K, \quad \sigma_{v'}^2 = \sigma_v^2 \operatorname{tr}\left(\left(\mathbf{Q}^H\mathbf{Q}\right)^{-1}\right). \tag{8}$$

Next, a phase demodulation, based on the arctangent operation,is given by

$$\left[\widehat{\mathbf{r}}_b\right]_n = \frac{1}{2\pi\beta} \arctan\left(\frac{\Im\left\{\left[\widehat{\mathbf{c}}_b\right]_n\right\}}{\Re\left\{\left[\widehat{\mathbf{c}}_b\right]_n\right\}}\right), \;\; 1 \le n \le K, \;\; 1 \le b \le B. \tag{9}$$

Note that the instantaneous phase obtained in (9) is bounded to the $(-\pi,\pi]$ range, and any phase excursion out of this range will cause a phase ambiguity, hence degrading the overall performance. According to [2], this ambiguity may be solved by a phase unwrapper, to reconstruct the original unrestricted phase. Then, a phase differentiation operation is performed over $\widehat{\mathbf{r}}_b$ and its results are fed to the DFT as

$$\left[\widehat{\mathbf{f}}_b\right]_k = \left\{ \begin{array}{cc} \left[\widehat{\mathbf{r}}_b\right]_k & k = 1 \\ \left[\widehat{\mathbf{r}}_b\right]_k - \left[\widehat{\mathbf{r}}_b\right]_{k-1} & k > 1 \end{array} \right\}, \quad 1 \le b \le B, \tag{10}$$

$$\widehat{\mathbf{s}}_b = \mathbf{F}_K \widehat{\mathbf{f}}_b = \mathbf{s}_b + \mathbf{F}_K \boldsymbol{\xi}_b = \mathbf{s}_b + \boldsymbol{\xi}'_b, \quad 1 \le b \le B, \tag{11}$$

and the decision on complex symbols $\widehat{\mathbf{d}}_b$ can be easily obtained from $\widehat{\mathbf{s}}_b$.

III. Phase-domain Injected Training (PIT)

In this section, the proposed PIT is presented. An additional phase is injected to the data symbols ($\mathbf{r}_b$) at the phase modulator. Then, the same phase is employed in order to recover the estimated data symbols ($\widehat{\mathbf{r}}_b$). The main advantage of the proposed phase injection is its transparency in terms of allocated resources. It does not require to be exclusively allocated at any specific time-frequency resource, unlike PSAM, and hence, data-rate reduction is avoided [7] in contrast to preamble-based schemes. Our proposal does not modify the signal processing blocks of FM-OFDM and it keeps the property of constant envelope.

A. Phase-domain Injection

Let us define the training sequence $\mathbf{p}_\phi \in \mathbb{C}^K$ as

$$\left[\mathbf{p}_\phi\right]_n = \exp\left(j\phi_{n'}\right), \quad n' = \mathrm{mod}\left(n, L_p\right) + 1, \qquad (12)$$
$$1 \le n \le K, \quad K = L_p N_p,$$

where $\phi_{n'} \in \mathbb{R}$ is the phase to be injected in the n-th time sample, L_p corresponds to the number of different phases within a FM-OFDM symbol and N_p is the number of times that each $\phi_{n'}$ appears in the training sequence. Note that the number of different phases should have, at least, the same number of taps of the channel ($L_p \ge L_{CH}$) in order to allow the estimation of all of them by using the LS criterion [8].

The training sequence is injected to the data symbols at the phase modulator as

$$[\mathbf{c}_b]_n = \exp\left(j2\pi\beta\left[\mathbf{r}_b\right]_n\right)\left[\mathbf{p}_\phi\right]_n = \exp\left(j\left(2\pi\beta\left[\mathbf{r}_b\right]_n + \phi_{n'}\right)\right), \qquad (13)$$
$$n' = \mathrm{mod}\left(n, L_p\right) + 1, \quad 1 \le n \le K, \quad 1 \le b \le B.$$

The injected phases remain with the same value for all FM-OFDM symbols of one slot comprising B symbols in order to allow an averaging over these symbols. Moreover, a block of L_p different phases are also repeated N_p times within an FM-OFDM symbol, in order to allow an additional averaging over these N_p blocks, and therefore, further improving the channel estimates. At the receiver, after performing the channel estimation and phase demodulation given in (9), the injected phase is removed before carrying out the DFT operation.

After the phase injection, the constant envelope property of $\mathbf{c}_b$ is preserved. Moreover, the FM-OFDM signal can be exploited for channel estimation, thanks to the structure of the FM-OFDM signal and the injected sequence of phases. The phase of $\mathbf{c}_b$ approaches a normal distribution, whose mean is zero and variance is $\sigma_c^2 = (2\pi\beta)^2$. Therefore, making use of this statistical information, the mean value of (13) can be expressed as

$$\begin{aligned}\mathbb{E}_b\left\{\Re\left\{[\mathbf{c}_b]_n\right\}\right\} =& \mathbb{E}_b\left\{\cos\left(2\pi\beta\left[\mathbf{r}_b\right]_n\right)\right\}\cos\left(\phi_{n'}\right)\\ &- \mathbb{E}_b\left\{\sin\left(2\pi\beta\left[\mathbf{r}_b\right]_n\right)\right\}\sin\left(\phi_{n'}\right),\end{aligned} \qquad (14)$$

$$\begin{aligned}\mathbb{E}_b\left\{\Im\left\{[\mathbf{c}_b]_n\right\}\right\} =& \mathbb{E}_b\left\{\sin\left(2\pi\beta\left[\mathbf{r}_b\right]_n\right)\right\}\cos\left(\phi_{n'}\right)\\ &+ \mathbb{E}_b\left\{\cos\left(2\pi\beta\left[\mathbf{r}_b\right]_n\right)\right\}\sin\left(\phi_{n'}\right),\end{aligned} \qquad (15)$$

$$n' = \mathrm{mod}\left(n, L_p\right) + 1, \quad 1 \le n \le K, \quad 1 \le b \le B.$$

Note that the injected phases ($\phi_{n'}$, $1 \le n' \le L_p$) are deterministic values, and hence, both $\sin\left(\phi_{n'}\right)$ and $\cos\left(\phi_{n'}\right)$ are out of the expectation operation. By using some trigonometric properties, the expectation of the imaginary part is canceled since the sine function has an odd symmetry. Hence, the mean value of $\mathbf{c}_b$ over FM-OFDM symbols can be simplified as

$$\mathbb{E}_b\left\{[\mathbf{c}_b]_n\right\} = \exp\left(-\frac{\sigma_c^2}{2}\right)\exp\left(j\phi_{n'}\right) = \exp\left(-\frac{\sigma_c^2}{2}\right)\left[\mathbf{p}_\phi\right]_n, \qquad (16)$$
$$n' = \mathrm{mod}\left(n, L_p\right) + 1, \quad 1 \le n \le K.$$

It is observed that the mean value of FM-OFDM symbols is a complex number whose modulus is given by a constant value, which depends on the chosen modulation index (β), and its argument is determined by the injected phase defined in (12), while the data information is averaged out. Consequently, (16) can play a role as a training sequence to be exploited for channel estimation without spending any either time/frequency or power resources for its transmission.

B. Channel Estimation

Given $\mathbf{y}_b$, a dual averaging procedure is performed. Firstly, it is averaged in the time domain over the B contiguous FM-OFDM symbols of one slot, named inter-symbol averaging, to yield $\bar{\mathbf{y}} \in \mathbb{C}^K$ as

$$[\bar{\mathbf{y}}]_n = [\bar{\mathbf{v}}]_n + \sum_{\tau=1}^{L_{CH}}[\mathbf{h}]_\tau\left[\mathbf{p}_\phi\right]_{\mathrm{mod}(n-\tau,K)+1}[\bar{\mathbf{c}}]_{\mathrm{mod}(n-\tau,K)+1}, \qquad (17)$$

$$[\bar{\mathbf{c}}]_n = \frac{1}{B}\sum_{b=1}^{B}[\mathbf{c}_b]_n, \quad [\bar{\mathbf{v}}]_n = \frac{1}{B}\sum_{b=1}^{B}[\mathbf{v}_b]_n, \quad 1 \le n \le K, \qquad (18)$$

where $\bar{\mathbf{c}} \in \mathbb{C}^K$ and $\bar{\mathbf{v}} \in \mathbb{C}^K$ are the averaged transmitted FM-OFDM symbol and noise, respectively, over the B consecutive FM-OFDM symbols. Note that the channel impulse response and the injected phases are out of the averaging operation since they remain constant for the considered slot.

Additionally, in order to keep improving the performance of the channel estimation, an additional averaging applied to $\bar{\mathbf{y}}$ over the N_p blocks of injected phases within each FM-OFDM symbol is also performed, referred as intra-symbol averaging, to obtain $\ddot{\mathbf{y}} \in \mathbb{C}^{L_p}$ as

$$[\ddot{\mathbf{y}}]_n = \frac{1}{N_p}\sum_{u=0}^{N_p-1}[\bar{\mathbf{y}}]_{uN_p+n} = \sum_{\tau=1}^{L_{CH}}[\mathbf{h}]_\tau\exp\left(j\phi_{n'}\right)[\ddot{\mathbf{c}}]_n + [\ddot{\mathbf{v}}]_n, \qquad (19)$$

$$[\ddot{\mathbf{c}}]_n = \frac{1}{N_p}\sum_{u=0}^{N_p-1}[\bar{\mathbf{c}}]_{uN_p+n}, \quad [\ddot{\mathbf{v}}]_n = \frac{1}{N_p}\sum_{u=0}^{N_p-1}[\bar{\mathbf{v}}]_{uN_p+n}, \qquad (20)$$
$$n' = \mathrm{mod}\left(n, L_p\right) + 1, \quad 0 \le u \le N_p - 1, \quad 1 \le n \le L_p,$$

where $\ddot{\mathbf{c}} \in \mathbb{C}^{L_p}$ and $\ddot{\mathbf{v}} \in \mathbb{C}^{L_p}$ are the averaged transmitted real-valued OFDM symbol and noise, respectively, over N_p blocks of $\bar{\mathbf{y}}$. Therefore, (20) can be rewritten in matrix form as

$$\ddot{\mathbf{y}} = \left(\mathbf{P}_r + j\mathbf{P}_i\right)\mathbf{h} + \ddot{\mathbf{v}}, \qquad (21)$$

where $\mathbf{P}_r \in \mathbb{R}^{L_p \times L_p}$ and $\mathbf{P}_i \in \mathbb{R}^{L_p \times L_p}$ are two Toeplitz matrices whose their first rows are defined by

$$\mathbf{p}_{r,1} = \left[\Re\left\{[\ddot{\mathbf{c}}]_1\right\}\exp\left(j\phi_1\right) \quad \cdots \quad \Re\left\{[\ddot{\mathbf{c}}]_{L_p}\right\}\exp\left(j\phi_{L_p}\right)\right], \qquad (22)$$

$$\mathbf{p}_{i,1} = \left[\Im\left\{[\ddot{\mathbf{c}}]_1\right\}\exp\left(j\phi_1\right) \quad \cdots \quad \Im\left\{[\ddot{\mathbf{c}}]_{L_p}\right\}\exp\left(j\phi_{L_p}\right)\right], \qquad (23)$$

respectively. The rest of rows of matrices $\mathbf{P}_r$ and $\mathbf{P}_i$ are obtained by cyclic shifting (22) and (23), respectively. The noise variance is significantly reduced after the two averaging processes, which is given by $\sigma_{\ddot{v}}^2 = \sigma_v^2 / N_p B$.

Taking into account the training properties given in the previous section and applying the Law of Large Numbers [9], it is satisfied that

$$\Re\left\{[\ddot{\mathbf{c}}]_n\right\}\exp\left(j\phi_n\right) \xrightarrow{N_p B\rightarrow\infty} \exp\left(-\frac{\sigma_c^2}{2}\right)\left[\mathbf{p}_\phi\right]_n, \quad (24)$$

$$\Im\left\{[\ddot{\mathbf{c}}]_n\right\}\exp\left(j\phi_n\right) \xrightarrow{N_p B\rightarrow\infty} 0, \quad [\ddot{\mathbf{v}}]_n \in \mathbb{C} \xrightarrow{N_p B\rightarrow\infty} 0, \quad (25)$$

where $1 \leq n \leq L_p$ and it can be observed that both the noise and the imaginary part of averaged symbols ($\Im\{\ddot{\mathbf{c}}\}$) have vanished due to the fact that their mean values are zero. However, the real part of the averaged symbols ($\Re\{\ddot{\mathbf{c}}\}$) remains, and it tends to the training sequence $\mathbf{p}_\phi$, then acting as a pilot symbol that can be used for channel estimation. Taking into account (24)-(25), (21) asymptotically tends to

$$\ddot{\mathbf{y}} \xrightarrow{N_p B\rightarrow\infty} \mathbf{P}_r\mathbf{h} = \exp\left(-\frac{\sigma_c^2}{2}\right)\left(\mathbf{p}_\phi \circledast \mathbf{h}\right), \quad (26)$$

where the circular convolution is performed over L_p samples.

Considering (26), LS estimation [8] can be applied in the time domain. Assuming that the matrix $\mathbf{P}_r$ is known at the receiver (both modulus and phase), where it can be easily obtained or computed by the procedure explained in Section IV, the estimated channel can be decomposed as

$$\widehat{\mathbf{h}} = \mathbf{P}_r^{-1}\ddot{\mathbf{y}} = \mathbf{h} + \mathbf{P}_r^{-1}\left(j\mathbf{P}_i\mathbf{h} + \ddot{\mathbf{v}}\right), \quad (27)$$

where the imaginary part of the averaged symbols can be modeled as an additional interference tending to zero. The estimator given in (27) is unbiased since the averaged noise ($\mathbf{v}$) and data interference ($\mathbf{P}_i$) are zero mean random variables. Note that the LS algorithm cannot be straightforwardly applied in the frequency domain with PIT due to the fact that performing an L_p-point DFT to $\mathbf{p}_\phi$ whose length is L_p may produce some zero subcarriers, and hence, the channel frequency response at these subcarriers could not be recovered.

The MSE of the channel estimation can be defined as

$$\text{MSE} = \mathbb{E}\left\{\left|\widehat{\mathbf{h}} - \mathbf{h}\right|^2\right\} = \mathbb{E}\left\{\left|\mathbf{P}_r^{-1}\left(j\mathbf{P}_i\mathbf{h} + \ddot{\mathbf{v}}\right)\right|^2\right\}, \quad (28)$$

where it is considered that L_p coincides with the number of channel taps in order to ease the notation. By performing some matrix properties, the MSE can be expressed as

$$\begin{aligned}\text{MSE} = \frac{1}{N_p B}&\text{tr}\left(\left(\mathbf{P}_r^H\mathbf{P}_r\right)^{-1}\right)\\ &\times\left(\sigma_v^2 + \frac{1}{2}\exp\left(-2\sigma_c^2\right)\left(\exp\left(2\sigma_c^2\right) - 1\right)\right),\end{aligned} \quad (29)$$

where the value of $\text{tr}\left(\left(\mathbf{P}_r^H\mathbf{P}_r\right)^{-1}\right)$ can be pre-calculated since $\mathbf{P}_r$ is known and employed in (27). The averaged data interference ($\mathbf{P}_i$) and noise ($\ddot{\mathbf{v}}$) are uncorrelated random variables, and hence, their variance can be independently obtained.

In (29), the dual averaging procedures given in (17) and (19) are very relevant to reduce the MSE, since it inversely depends on B and N_p. Also, the reduction of the modulation index (the variance of the real-valued OFDM symbol σ_c^2) can improve the accuracy of the channel estimation. However, the performance of the data detection is worsened as the modulation index is decreased [2], [3]. Finally, the design of the training sequence $\mathbf{p}_\phi$ is also crucial to reduce the estimation error, and some optimum sequences are already given in [10].

TABLE I
SIMULATION PARAMETERS

K	1024	$\mathbf{L_p}$	64	$\mathbf{2\pi\beta}$	0.1, 0.4 & 0.7
$\mathbf{L_{CP}}$	72	$\mathbf{N_p}$	16	$\mathbf{\Delta f}$	15 KHz

IV. Analysis of the Overhead

In this section, the analysis of the required overhead for the channel estimation process between PIT and preamble-based PSAM is compared. This efficiency is a ratio between the number of data symbols transmitted and the total number of available resources in B FM-OFDM symbols.

Regarding the preamble-based approach for channel estimation, which is a particular case of PSAM, the efficiency of the system is reduced by the transmission of the preamble sequence. Meanwile, the proposed PIT does not require to transmit any reference symbols. Hence, the efficiency of the system for PSAM and the proposed PIT are given by

$$\mu_{\text{PSAM}} = \frac{\left(B - B_p\right)K}{B\left(K + L_{CP}\right)}, \quad \mu_{\text{PIT}} = \frac{K}{K + L_{CP}}, \quad (30)$$

where B_p is the number of multi-carrier symbols exclusively carrying the preamble sequences for channel estimation. Note that, the number of reference symbols to be transmitted in PSAM must be increased for high-speed scenarios in order to track the fast channel variations. Consequently, the proposed PIT significantly outperforms PSAM since no overhead is produced by injecting the training sequences for any scenario.

V. Performance Evaluation

Several numerical results are provided in order to show the performance of the proposed PIT and the preamble-based PSAM, when both schemes are implemented in FM-OFDM. A summary of the simulation parameters is given in Table I. Note that N_p is a result imposed by the chosen the numerology and the length of the channel impulse response. The channel model adopted for the simulation corresponds to the tapped delay line (TDL) model proposed by the 3GPP to evaluate 5G performance [11], specifically the urban micro-cell (UMi) scenario is chosen. Moreover, the channel coefficients, are obtained by using the TDL-A power-delay profile, which corresponds to a non-line-of-sigh (NLOS) channel model. The SNR can be defined as $\text{SNR} = \sigma_v^{-2}$, since the transmitted power and the channel gain are both normalized to one.

A. MSE Performance

Figure 1 shows the MSE comparison for the channel estimation between the proposed PIT technique and preamble-based PSAM for $B_p = 2$. Firstly, the PSAM (black line) has

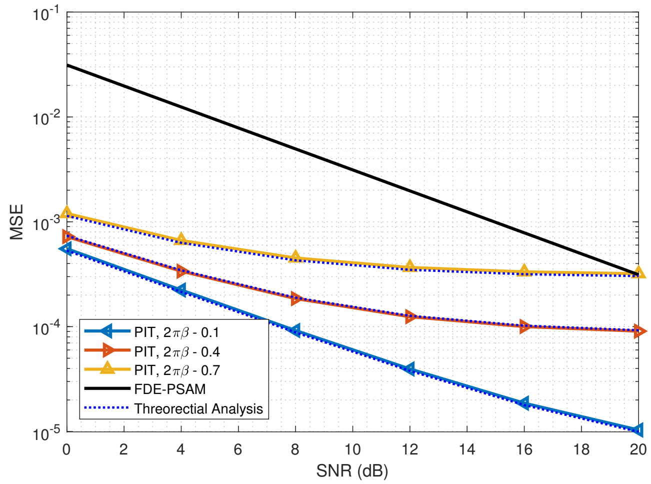

Fig. 1. MSE comparison of the estimated channel for $B = 2$.

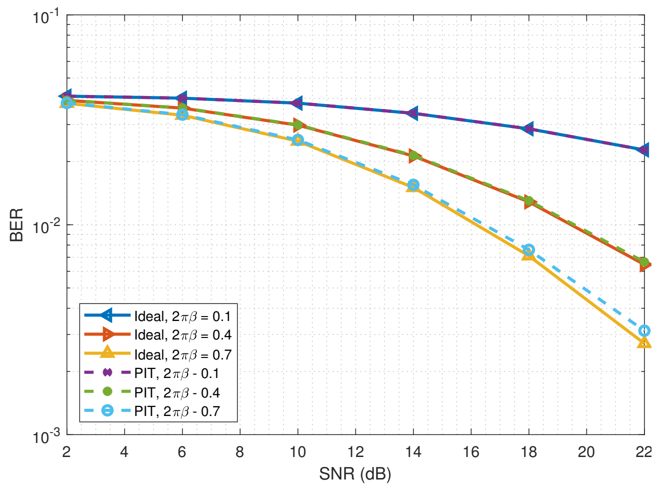

Fig. 2. BER comparison for 4-QAM and $B = 2$.

a worse performance due to the fact that it does not have an additional B and N_p averaging to reduce the noise. Moreover, even for low values of $B = 2$ and $N_p = 2$, PIT is capable of providing better results as compared to PSAM. Focusing on PIT technique, it is observed that reducing the modulation index (β) significantly improves the accuracy of the channel estimation since lower values of β will provide a higher pilot amplitude given in (24). Finally, the analytical results of the MSE for channel estimation based on PIT are obtained from (29). It can be observed that the analytical derivation of the MSE for PIT, matches with the simulation results, showing the accuracy of the theoretical expressions.

B. BER performance

The BER performance comparison for 4-QAM is illustrated in Fig. 2. In order to provide the lower bound of the performance, the case of perfect channel estimation is also plotted, which is labeled as ideal case. Since PSAM has a lower efficiency and its MSE is much worse than the proposed PIT, it is discarded in this comparison. The difference between the proposed PIT and the idealistic case is negligible, showing that PIT is able to provide accurate enough channel estimates capable of providing a similar performance in terms of BER as the idealistic case, and at the same time, avoiding the undesirable overhead generated by PSAM. Note that higher values of β will provide a better performance since it enlarges the distance of the complex symbols in the constellation.

VI. Conclusions

Channel estimation for FM-OFDM based on PIT is presented in this work. PIT is a novel concept of transmitting the training sequences embedded in the phase domain of the data symbols without spending any additional resources. The theoretical analysis of the MSE of the channel estimation has shown that the error can be reduced by increasing the averaging steps, either inter-symbol or intra-symbol, and/or reducing the modulation index of the phase modulator. Moreover, the analysis of the overhead has highlighted that no additional reference signal is required to obtain the received pilot symbols, making it very efficient in terms of data-rate, unlike the preamble-based approach. This successful combination can be deployed in the forthcoming 6G, especially for high-speed communication services, in order to provide an ubiquitous wireless experience to the world.

Acknowledgment

This work has been partially funded by the Spanish National project IRENE-EARTH (PID2020-115323RB-C33/AEI /10.13039/501100011033) and the work of K. Chen-Hu was also, in part, supported by the Villum Investigator Grant “WATER” from the Velux Foundation, Denmark.

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