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## **Heat-And-Mass Transfer Relationship to Determine Shear Stress in Tubular Membrane Systems**

Ratkovich, Nicolas Rios; Nopens, Ingmar

*Published in:*  
International Journal of Heat and Mass Transfer

*DOI (link to publication from Publisher):*  
[10.1016/j.ijheatmasstransfer.2012.06.035](https://doi.org/10.1016/j.ijheatmasstransfer.2012.06.035)

*Publication date:*  
2012

*Document Version*  
Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Ratkovich, N. R., & Nopens, I. (2012). Heat-And-Mass Transfer Relationship to Determine Shear Stress in Tubular Membrane Systems. *International Journal of Heat and Mass Transfer*, 55(21-22), 6152–6162. <https://doi.org/10.1016/j.ijheatmasstransfer.2012.06.035>

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Elsevier Editorial System(tm) for International Journal of Heat and Mass Transfer  
Manuscript Draft

Manuscript Number:

Title: Heat-And-Mass Transfer Relationship to Determine Shear Stress in Tubular Membrane Systems

Article Type: Full Length Article

Keywords: Membrane bioreactor; heat-and-mass transfer analogy; shear stress; Sherwood number; empirical model.

Corresponding Author: Mr. Nicolas Ratkovich, Ph.D

Corresponding Author's Institution:

First Author: Nicolas Ratkovich, Ph.D

Order of Authors: Nicolas Ratkovich, Ph.D; Pierre Berube, Professor; Ingmar Nopens, Professor

Abstract: The main drawback of Membrane Bioreactors (MBRs) is the fouling of the membrane. One way to reduce this fouling is through controlling the hydrodynamics of the two-phase slug flow near the membrane surface. It has been proven in literature that the slug flow pattern has a higher scouring effect to remove particulates due to the high shear rates and high mass transfer between the membrane surface and the bulk region. However, to calculate the mass transfer coefficient in an efficient and accurate way is not straightforward. Indeed, for accurate determination, numerous complex experimental measurements are required. Therefore, this work proposes an alternative method that uses already existing heat transfer relationships for two phase flow and links them through a dimensionless number to the mass transfer coefficient (Sherwood number) to obtain an empirical relationship which can be used to determine the shear stress.

# Heat-And-Mass Transfer Relationship to Determine Shear Stress in Tubular Membrane Systems

N. Ratkovich<sup>a,\*</sup>, P.R. Berube<sup>b</sup>, I. Nopens<sup>a</sup>

<sup>a</sup> BIOMATH - Department of Applied Mathematics, Biometrics and Process Control, Ghent University, Coupure Links 653, B-9000, Ghent, Belgium.

<sup>b</sup> Department of Civil Engineering, The University of British Columbia, 6250 Applied Science Lane, Vancouver, BC, V6T 1Z4, Canada.

\* Corresponding author: Tel: +32 9 264 59 35, Fax: +32 9 264 62 20, E-mail:

[nriosrat@biomath.ugent.be](mailto:nriosrat@biomath.ugent.be)

## ABSTRACT

The main drawback of Membrane Bioreactors (MBRs) is the fouling of the membrane. One way to reduce this fouling is through controlling the hydrodynamics of the two-phase slug flow near the membrane surface. It has been proven in literature that the slug flow pattern has a higher scouring effect to remove particulates due to the high shear rates and high mass transfer between the membrane surface and the bulk region. However, to calculate the mass transfer coefficient in an efficient and accurate way is not straightforward. Indeed, for accurate determination, numerous complex experimental measurements are required. Therefore, this work proposes an alternative method that uses already existing heat transfer relationships for two phase flow and links them through a dimensionless number to the mass transfer coefficient (Sherwood number) to obtain an empirical relationship which can be used to determine the shear stress.

## Keywords

Membrane bioreactor, heat-and-mass transfer analogy, shear stress, Sherwood number, empirical model

## 1. INTRODUCTION

Bearing in mind the more stringent effluent quality standards imposed by the EU Water Framework Directive (EU-WFD), wastewater treatment efficiencies need to be improved. These improvements can be achieved both in terms of biological removal efficiency as well as in the sludge-water separation step. For the last step two types of technologies exist, the Conventional Activated Sludge (CAS) systems where the separation is brought about by gravity and the Membrane Bioreactors (MBR) where the separation is achieved by filtration. The last one has proven to be a good alternative to achieve high effluent quality compared to the CAS system. A common problem encountered with MBR systems is the fouling of the membrane resulting in a need for its frequent cleaning and replacement [1]. Membrane fouling is the main bottleneck of

full-scale application of membrane bioreactors (MBRs) and has restricted its market breakthrough due to the reduction of productivity and increased maintenance and operational cost. Literature has shown that, next to the composition of the sludge, the hydrodynamics near the membrane surface play an important role. In search for better control of fouling, literature has focused on the determination of the fouling constituents. However, it has been shown that the hydrodynamics near the membrane surface play an as important role. To reduce the fouling on the membrane air is often introduced in the sludge flow to create a gas-liquid two-phase cross-flow, to increase the surface shear stress to remove foulants that are already attached and to increase the mass transfer between the cake layer and the bulk region [2]. However, the governing mechanisms are not yet completely understood, which results in a trial and error approach to optimize hydrodynamic control of fouling

Due to the complexity involved in mass transfer measurements for two-phase flows, some studies have focused on developing relationships between heat and mass transfer. This is possible because of the analogies between heat and mass transfer models in dimensionless form which are based on the transport of momentum, mass, heat and energy, and more specifically in the Lewis number ( $Le$ ). The latter is a dimensionless number defined as the ratio of thermal to mass diffusivity [3,4]:

$$\frac{Sh}{Nu} = \left( \frac{Sc}{Pr} \right)^{\frac{1}{3}} = Le^{\frac{1}{3}} \quad (1)$$

where  $Sh$ ,  $Nu$ ,  $Sc$  and  $Pr$  are the Sherwood, Nusselt, Schmidt and Prandtl numbers respectively. They are defined in Tab. 1. In Tab. 1,  $h$  is the convection coefficient,  $d$  is the tube diameter,  $k_c$  is the thermal conductivity,  $c_p$  is the specific heat,  $\mu$  is the viscosity,  $k_m$  the mass transfer coefficient,  $D_f$  is the diffusion coefficient and  $\rho$  is the density.

These kinds of analogies are commonly used in cases where it is easier to obtain heat transfer data rather than mass transfer data. The work presented here focuses on a better understanding of the mass transfer coefficient near the membrane surface using a heat transfer analogy for two-phase slug flow for side-stream MBR.

## 2. MATERIAL AND METHODS

### 2.1 Description of the setup

A description of the setup that was used to collect shear stress information is given in Fig. 1. A plexiglas tube with a length of 2 m and an inner diameter of 9.9 mm was used. This tube is similar in geometry to the airlift tubular membranes of interest. A flow cell, located in the middle of the plexiglas tube (1 m) has two electrochemical shear probes, which are used to measure

surface shear stresses. A temperature controlled water bath (20°C) is used to keep the temperature of the electrolyte solution flowing through the system constant. A peristaltic pump (Masterflex LS, USA) is used to recirculate the electrolytic solution from the gas-liquid separator tank to the plexiglass tube at controlled liquid flow rates. Two flow meters (Cole-Parmer, N082-03, USA) are used to monitor the liquid and gas flow rates. Five liquid flow rates (0.1, 0.2, 0.3, 0.4 and 0.5 L min<sup>-1</sup>) and three gas flow rates (0.1, 0.2 and 0.3 L min<sup>-1</sup>) were investigated, resulting in a total of 15 combinations. These ranges of flow rates correspond to those expected in full-scale airlift tubular membrane systems [5]. For each experimental condition, surface shear stresses are measured for a period of 10 seconds, and recorded at a frequency of 1000 Hz [6]. All experimental conditions are replicated six times.

The electrochemical probes are made from two platinum wires imbedded flush to the inside surface of the tube wall to avoid them having an effect on the flow field. A detailed description of the directional electrochemical probes is presented in [7]. Measurements from the directional electrochemical probes are measured as volt and can be converted to a mass transfer coefficient ( $k_m$ ), which can be used to calculate shear stresses using the following equation [7]:

$$\tau_w = \mu_L \frac{1.561 d_e}{D_f^2} k_m^3 \quad (2)$$

where  $d_e$  is the diameter of the probe (m),  $D_f$  is the diffusion coefficient of ferricyanide ( $7.14 \cdot 10^{-10} \text{ m}^2 \text{ s}^{-1}$  [8]) and  $\mu_L$  is the dynamic viscosity of the solution (= 0.001 Pa s). The detailed procedure to obtain Eq. (2) can be found in [5]. Eq. (2) correlates mass transfer to shear stress, which is the objective of this work.

## 2.2 Slug flow

To reduce the fouling on the membrane, air is introduced to create a two-phase flow. In vertical tubes, there are four specific flow patterns: bubbly, slug, churn and annular flow. Their respective structure depends on the superficial velocities, surface tension and densities of the fluids. It was found that the setup under study is operated in the slug flow region (Taylor bubbles). In the slug flow that typically builds up, three different zones can be distinguished (Fig. 2): 1) the falling film zone, i.e. the zone where the bubble is passing, 2) the wake zone, i.e. the zone just behind the bubble where mixing of liquid and gas takes place and 3) the liquid zone [2].

## 2.3 Mass Transfer Coefficient

Single-phase flow: During the filtration process, the separation between the sludge and the solute occurs at the membrane, giving an increase in the solute concentration near the membrane surface. This is called concentration polarization [9] which is function of the mass

transfer coefficient. The latter can be obtained by electrochemical methods as presented in section 2.1 or by using dimensionless relationships function of the  $Sh$  and depending on the flow regime (Tab. 2). However, it is important to note that the relationships are for smooth tubes only and they are not defined in the transition regime ( $2000 < Re < 4000$ ). A weighting factor approach can be used here to determine the  $Sh$  number in the transition regime [10].

Two-phase flow: In a slug flow, each zone has its own mass transfer coefficient. Fig. 3 illustrates the mass transfer coefficients for each zone. It is possible to observe that if the flow would be single phase the value of the mass transfer coefficient in the liquid slug would be lower compared to the falling film and wake zone due to higher liquid velocities. Therefore, the mass transfer coefficient increases due to the two-phase cross flow. [2] and [11] proposed equations for each zone based on hydrodynamics models and mass balances of slug flow. However, these models require extensive experimental measurements and mathematical derivation.

#### 2.4 Heat Transfer Coefficient

Single-phase flow: The heat transfer for single phase flow in a tube depends on the  $Nu$  number and the flow regime (Tab 2). The  $Nu$  number is the ratio of convective to conductive heat transfer normal to the boundary and the  $Pr$  number is the ratio of the momentum diffusivity and the thermal diffusivity. They are the thermal counterpart for the  $Sh$  and  $Sc$  the number. Comparing the heat and mass transfer analogies for single phase flow, it is possible to observe that the structure is the same but coefficients in the equations are slightly different (Tab. 2).

Two-phase flow: The  $Nu$  number for two phase flow ( $Nu_{tp}$ ) is defined by  $Nu_{tp} = h_{tp} d / k_{c,tp}$  [12].

Where  $h_{tp}$  is the heat transfer coefficient for two-phase flow and  $k_{c,tp}$  is the thermal conductivity coefficient for two-phase flow, which is defined by  $k_{c,tp} = (1-x)k_{c,L} + xk_{c,G}$ . Here,  $k_{c,L}$  and  $k_{c,G}$  are the thermal conductivity of the liquid and gas respectively and  $x$  is the vapour quality. The heat transfer coefficient for two-phase flow is defined by [13]:

$$h_{tp} = F_p h_L \left[ 1 + 0.55 \left( \frac{x}{1-x} \right)^{0.1} \left( \frac{1-F_p}{F_p} \right)^{0.4} \right] \cdot \left( \frac{Pr_G}{Pr_L} \right)^{0.25} \left( \frac{\mu_L}{\mu_G} \right)^{0.25} (I^*)^{0.25} \quad (3)$$

where  $F_p$  is the flow pattern factor (dimensionless),  $h_L$  is the heat transfer coefficient for the liquid and  $I^*$  is the inclination factor (dimensionless). The definition of these parameters can be found in [13]. The subscripts  $G$  and  $L$  are gas and liquid respectively. The heat transfer

coefficient for the liquid ( $h_L$ ) in single phase flow are given in Tab 2 as function of the  $Nu$  number. The liquid ( $Re_L$ ) and superficial gas ( $Re_{SG}$ ) Reynolds numbers are defined by:

$$Re_L = \frac{\rho_L u_{SL} d}{(1 - \alpha_p) \mu_L} \quad (4)$$

$$Re_{SG} = \frac{\rho_G u_{SG} d}{\mu_G} \quad (5)$$

This liquid Reynolds number gives a better representation for the liquid phase heat transfer ( $h_L$ ) and it works well in their two-phase heat transfer relationship for various gas-liquid combinations and flow patterns. Eq. (3) is valid for  $Re_L$  from 750 to  $1.3 \cdot 10^5$  and  $Re_{SG}$  from 14 to  $2.1 \cdot 10^5$ .

### 2.5 Heat-and-Mass Transfer Analogy

The Lewis number (Eq. (1)) can be used for both laminar and turbulent regimes. Moreover, it allows to determine either the heat or mass transfer, given one of them is known as the Lewis number can be computed independently. The exponent  $n$  is usually  $1/3$ . When there is filtration, it is possible to assume that the filtration has no effect on the hydrodynamics due to the fact that the permeate flow is less than 1 % of the cross-flow and it is assumed that it does not affect the slug flow. Therefore, the filtration process is not taken into account to develop a relationship for the mass transfer coefficient.

## 3. RESULT AND DISCUSSION

### 3.1 Shear profiles

Single-phase flow: Initial measurements were performed for single-phase flow to calibrate the wall shear stress with theoretical equations using the friction factor (Tab. 2). The friction factor is used in internal flow calculations and it expresses the linear relationship between mean flow velocity and shear stress at the wall. The mass transfer coefficient obtained from the electrochemical setup was converted into shear stress (Fig. 4a). Also, it was found that the flow was in laminar regime. Subsequently, the  $Sh$  number was computed and compared to the relationship for single phase flow (Tab. 2) to check the validity of the Leveque equation (Fig. 4b). Using SPSS v15 to estimate the coefficient, the proposed model becomes:

$$Sh = 1.495 \left( Re Sc \frac{d}{L} \right)^{\frac{1}{3}} \quad (6)$$

which is 8% lower compared to the theoretical model and can be used as a starting point for the analysis of the two-phase flow.

The mass transfer coefficient for the probe was defined in Eq. (2), from which is necessary to extrapolate to the mass transfer coefficient for the tube as follows:

$$k_m = 0.862 \frac{d_e}{d} \left( \frac{\tau_w D_f^2}{\mu d_e} \right)^{\frac{1}{3}} \quad (7)$$

Now the objective is to determine the wall shear stress from the heat transfer point of view (using the  $Nu$  number). It is possible to write the wall shear stress as function of the  $Sh$  number as follows:

$$\tau_w = \frac{1.561 \mu D_f}{d_e^2} Sh^3 \quad (8)$$

The Lewis number can be written as:

$$Sh = Nu \left( \frac{Sc}{Pr} \right)^{\frac{1}{3}} \quad (9)$$

Combining Eq. (8) and (9) yields:

$$\tau_w = \frac{1.561 \mu D_f}{d_e^2} \left( \frac{Sc}{Pr} \right) Nu^3 \quad (10)$$

This is a general equation valid for single phase flow. Nevertheless, the behaviour of two-phase flow is different and, hence, some corrections are needed.

Two-phase flow: Typical voltage results obtained using the electrochemical shear probes, and the corresponding shear stresses, are presented in [5] and will not be shown here. It is important to highlight, nevertheless, that gas slugs rising in vertical tubes were observed to periodically coalesce when trailing slugs reached the wake of the leading slugs, accelerating the tailing slugs to finally coalesce with the leading slug. For this reason, the shear stress profiles induced by successive slugs were not exactly the same. As a result, the profile of shear stresses in successive shear events, induced by rising gas slugs, varied considerably over time. Shear Stress Histograms (SSH) were used to explore the effect of the different experimental conditions investigated (Fig. 5) on the resulting shear stresses [5].



From Fig. 6, it is possible to distinguish two peaks in the SSH: one peak occurs at positive shear value and is caused by the liquid slugs and a second peak occurs at a negative shear value and is caused by the gas slugs. The magnitude of the frequency for both peaks is, however, different for the different gas-liquid flow rate combinations.

Therefore, Eq. (10) can be written for two zones, instead of 3 zones for simplicity (Fig. 2 and 3): One zone for the liquid slug ( $ls$ ) and one zone for the gas slug ( $gs$ ) (this zone will include the falling film zone and the wake zone, because in the SSH, the wake zone cannot be distinguished):

$$\tau_{w,ls} = \frac{1.561 \mu D_f}{d_e^2} \left( \frac{Sc_L}{Pr_L} \right) Nu_L^3 \quad (11)$$

$$\tau_{w,gs} = \frac{1.561 \mu D_f}{d_e^2} \left( \frac{Sc_{tp}}{Pr_{tp}} \right) Nu_{tp}^3 \quad (12)$$

From the SSH, it is possible to get the average shear stress for the liquid and the gas slug peaks to feed Eq. (11) and (12) respectively. Due to the fact that the length of the bubbles is different, caused by coalescence, the values obtained from the SSH distribution are just averages. Therefore, a correction factor needs to be added. This correction factor should consider the fact that the hydraulic diameter changes in the falling film zone. The correction factor is function of the Reynolds number as follows:

$$t = a_1 Re_i^{a_2} \quad (13)$$

This empirical factor considers several characteristics of the flow, such as: coalescence of bubbles, bubble length, hydraulic diameter and transition regime, as the transition regime is not defined. Eq. (11) and (12) become:

$$\tau_{w,ls} = \frac{1.561 \mu D}{d_e^2} \left( \frac{Sc_L}{Pr_L} \right) t_{ls}^3 Nu_L^3 \quad (14)$$

$$\tau_{w,gs} = \frac{1.561 \mu D}{d_e^2} \left( \frac{Sc_{tp}}{Pr_{tp}} \right) t_{gs}^3 Nu_{tp}^3 \quad (15)$$

It is important to highlight that the power 3 in the  $t$  coefficients of Eq. (14) and (15) is just to maintain the same exponent of the  $Nu$  number. For simplicity, the first term of the equation is grouped in a constant.

$$a_3 = \frac{1.561 \mu D_f}{d_e^2} \left( \frac{Sc_L}{Pr_L} \right) \approx \frac{1.561 \mu D_f}{d_e^2} \left( \frac{Sc_{tp}}{Pr_{tp}} \right) \quad (16)$$

Re-writing Eq. (14) and (15) and combining with (13) yields:

$$\tau_{w,ls} = a_3 \left( a_{1,ls} \text{Re}_i^{a_{2,ls}} \right)^3 Nu_L^3 \quad (17)$$

$$\tau_{w,gs} = a_3 \left( a_{1,gs} \text{Re}_i^{a_{2,gs}} \right)^3 Nu_{TP}^3 \quad (18)$$

The correction factor can now be determined from fitting Eq. (17) and (18) to experimentally gathered data. This was done through a power-law regression with the software SPSS v15 using:

$$a_{1,ls} \text{Re}_i^{a_{2,ls}} = \left( \frac{\tau_{w,ls}}{a_3 Nu_L^3} \right)^{\frac{1}{3}} = t_l \quad (19)$$

$$a_{1,gs} \text{Re}_i^{a_{2,gs}} = \left( \frac{\tau_{w,gs}}{a_3 Nu_{tp}^3} \right)^{\frac{1}{3}} = t_g \quad (20)$$

The problem now arises as to which Reynolds number to use (liquid or superficial gas Reynolds number) in the correction factor. For this purpose, the  $R^2$  can be used as goodness of fit criteria. Results are summarized in Tab. 3.

From Tab. 3, the values that are in bold provide the best fit to the experimental data. Both liquid and gas slugs were found to be more dependent of the  $\text{Re}_L$  rather than the  $\text{Re}_{SG}$ . The  $\text{Re}_L$  considers the mixture velocity and the void fraction of the gas slug which is clearly important to account for the liquid and gas slugs. On the other hand, the  $\text{Re}_{SG}$  was expected to yield a bad correlation for the liquid slug (i.e. no liquid velocity is included in the Reynolds number). However, it was expected that it would provide a good correlation for the gas slug, which is clearly not the case. The reason for that could be that it should include the combined liquid and gas velocities to account for the increase in gas velocity due to buoyancy effects. Therefore, it was chosen to use  $\text{Re}_L$  in Eq. (19) and (20). Fig. 6 shows the power-law relationships for the liquid and gas slug.

From Fig. 6, it is possible to observe that the liquid Reynolds number is adequate to fit the empirical Eq. (19) and (20) to experimental data. The recovered parameters for Eq. (17) and (18) are shown in Tab. 4.

Therefore the final expressions of Eq. (17) and (18) have the form:

$$\tau_{w,ls} = \left(48.900 \text{Re}_L^{-0.295}\right)^3 Nu_L^3 \quad (21)$$

$$\tau_{w,gs} = \left(138.741 \text{Re}_L^{-1.196}\right)^3 Nu_{tp}^3 \quad (22)$$

From Fig. 6, it is possible to observe that the results of the heat-and-mass transfer relationship are adequate to predict the shear stress for the liquid and gas slug. The above analysis indicates that relatively simple dimensionless models can be used to describe the shear stress in the slug flow. Note that since the relationships presented in Eq. (21) and (22) are empirical, care must be taken when using them for design purposes. It is worth mentioning that this kind of analogies assume Newtonian behaviour. Given that sludge only exhibits slight non-Newtonian behaviour (flow behaviour index close to unity it is assumed).

## CONCLUSIONS

To determine the mass transfer coefficient experimentally is an arduous task and requires a lot of time and experimental work. Besides, it can only be done for solutions where the mass diffusion coefficient and the chemical reactions are well known. Therefore, to apply it in an activated sludge, which is a heterogeneous mixture causes severe difficulties. To overcome this, a setup with shear probes and an electrolytic solution was used to measure the shear stress and the mass transfer coefficient. Based on that, a heat transfer relationship, which is well studied in the literature, is suggested, to determine the shear stress using the Sherwood number. A validation with experimental measurements was made and proved that this type of analogy is valid. The outcome of the mass transfer coefficient was validated and an empirical expression was developed in function of the Nusselt number.

## ACKNOWLEDGMENT

This research project has been supported by a Marie Curie Early Stage Research Training Fellowship of the European Community's Sixth Framework Programme under contract number MEST-CT-2005-021050. Funding for the infrastructure used to measure surface shear stress was provided by the Natural Science and Engineering Research Council of Canada (NSERC).

## NOMENCLATURE

$C$	Concentration, g L <sup>-1</sup>
$c_p$	Specific heat, kJ kg <sup>-1</sup> K <sup>-1</sup>
$d$	Tube diameter, m
$d_e$	Probe diameter, m
$D_f$	Diffusion coefficient, m <sup>2</sup> s <sup>-1</sup>

$F_p$	Flow pattern factor, dimensionless
$h$	heat transfer coefficient, $\text{W m}^{-2} \text{K}^{-1}$
$I^*$	Inclination factor, dimensionless
$J$	Flux, $\text{m}^2 \text{s}^{-1}$
$k_c$	Thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$
$k_m$	Mass transfer coefficient, $\text{m s}^{-1}$
$Nu$	Nusselt number, dimensionless
$Pr$	Prandtl number, dimensionless
$Re$	Reynolds number, dimensionless
$Sc$	Schmidt number, dimensionless
$Sh$	Sherwood number, dimensionless
$t$	Correction factor, dimensionless
$x$	Vapour quality, dimensionless

#### Greek symbols

$\alpha$	Void fraction, dimensionless
$\rho$	Density, $\text{kg m}^{-3}$
$\mu$	Viscosity, $\text{Pa s}$
$\tau$	Wall shear stress, $\text{Pa}$

#### Subscript

$B$	Bulk
$G$	Gas
$gs$	Gas slug
$L$	Liquid
$ls$	Liquid slug
$M$	Membrane
$SG$	Superficial gas
$tp$	Two-phase
$W$	Wall

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Figure 1. Description of the electrochemical shear measurement setup [5].

Figure 2. Zones in the slug flow [2] (left) and numerical simulation of a Taylor bubble rising through stagnant glycerine in a vertical tube [14] (right)

Figure 3. Mass transfer coefficients in different zones of the slug flow [2].

Figure 4. Reynolds number vs a) shear stress and b)  $Sh$  number comparison for both the experimental data and theoretical equations.

Figure 5. Relative frequency vs shear stress for a liquid combination of  $0.1 \text{ L min}^{-1}$  and three gas flow rates ( $0.1, 0.2$  and  $0.3 \text{ L min}^{-1}$ ) [5].

Figure 6. Liquid Reynolds number vs the correction factor of Eq. (22) and (23) for the liquid and gas slug respectively.

## Heat-And-Mass Transfer Relationship to Determine Shear Stress in Tubular Membrane Systems

Corresponding Author: Nicolas Ratkovich  
Address: Department of Civil Engineering, Aalborg University,  
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark  
Telephone: +45 9940 8572  
Fax: +45 9814 2555  
Email address: [nr@civil.aau.dk](mailto:nr@civil.aau.dk)

Dear editor

The breakthrough of membrane bioreactors (MBR) in wastewater treatment is still hampered by poor understanding of the fouling phenomena and of the air scouring used to cure it. Mathematical models combined with experimental data have proven to be a good combination to build up process knowledge and eventually optimize the system in terms of design and operation.

The objective of this study was to calculate the mass transfer coefficient in an efficient and accurate way. Indeed, for accurate determination, numerous complex experimental measurements are required. Therefore, this work proposes an alternative method that uses already existing heat transfer relationships for two phase flow and links them through a dimensionless number to the mass transfer coefficient (Sherwood number) to obtain an empirical relationship which can be used to determine the shear stress.

Sincerely,

Nicolas Ratkovich

Figure 1  
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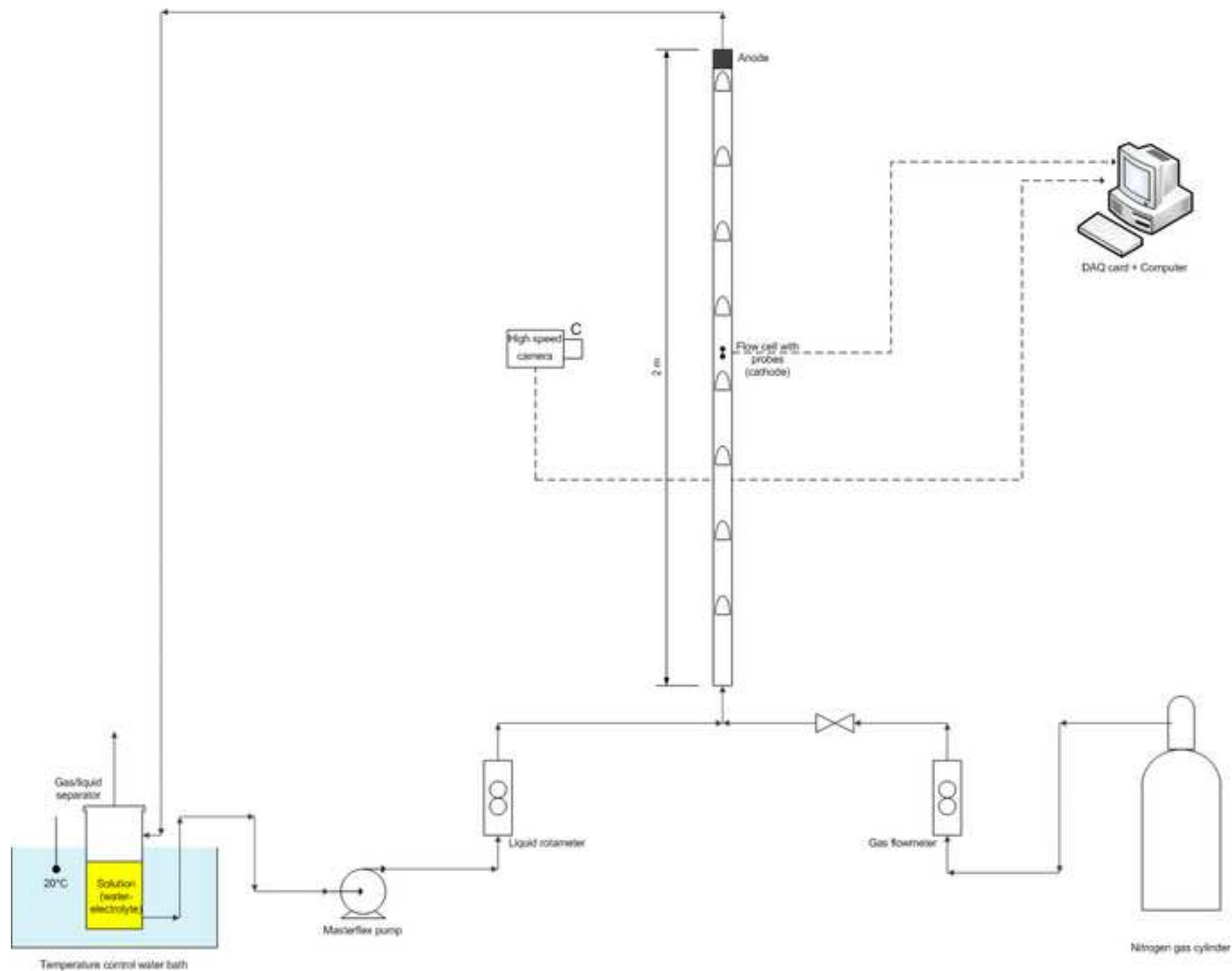




Figure 2  
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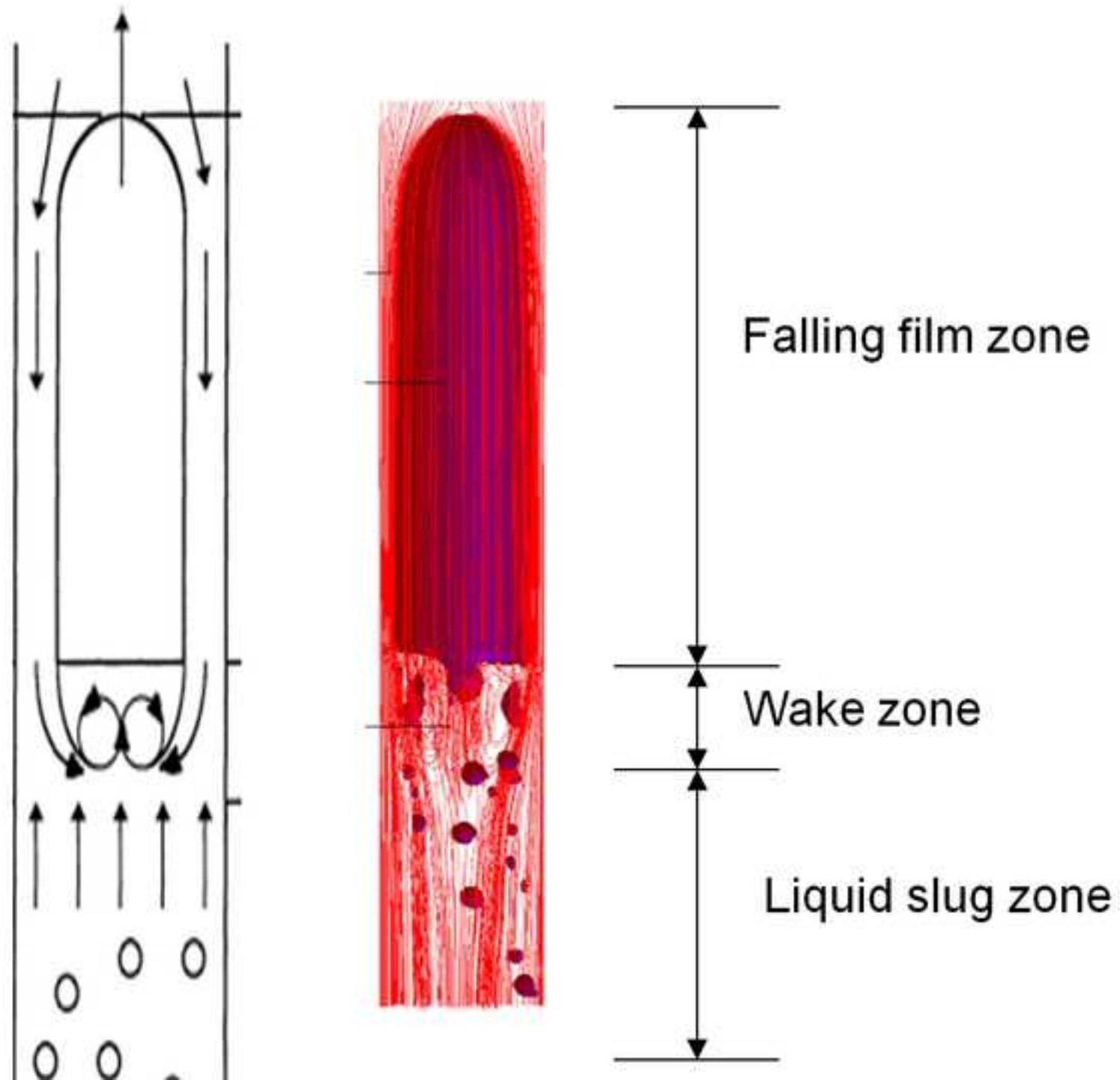


Figure 3  
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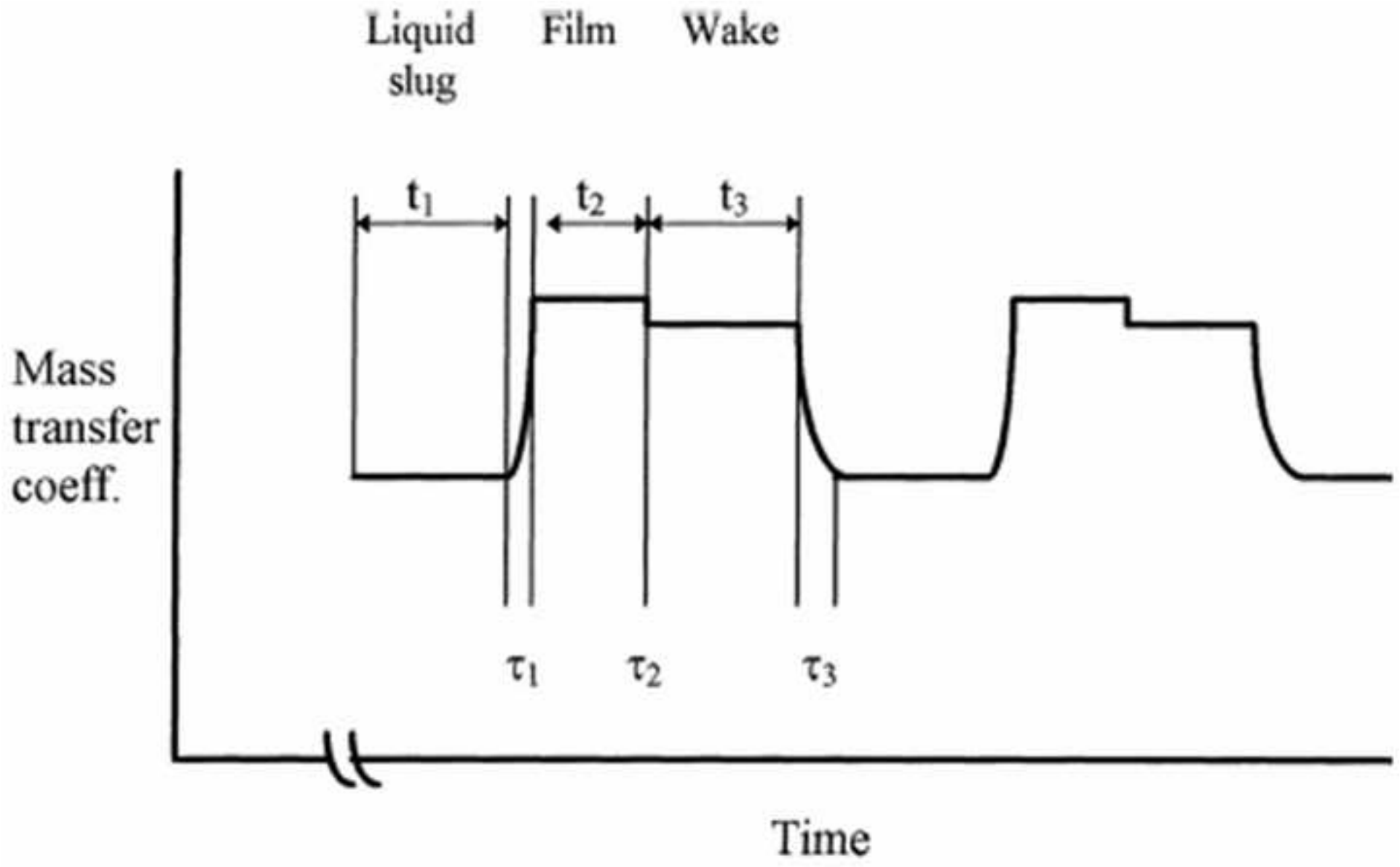


Figure 4a  
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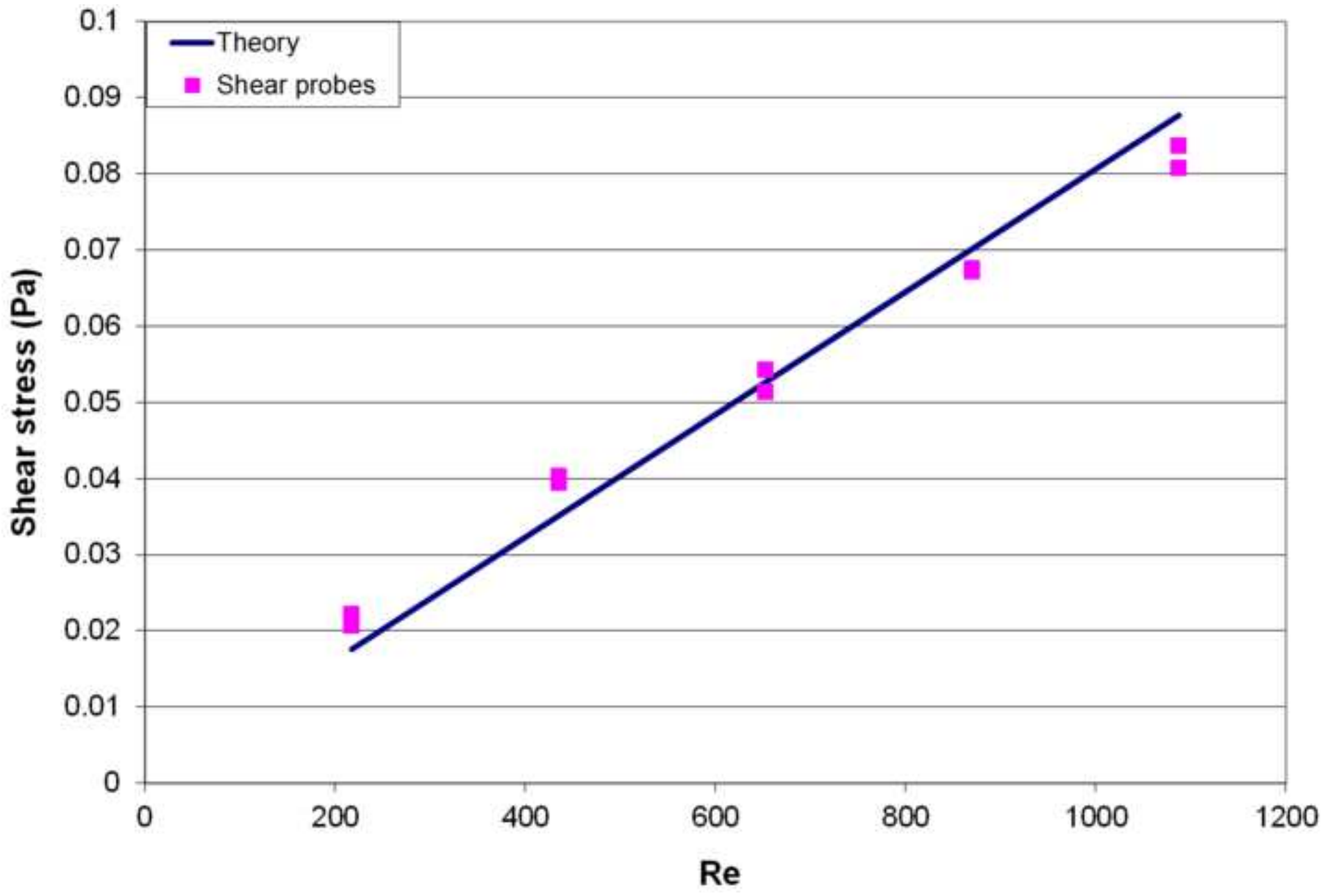


Figure 4b  
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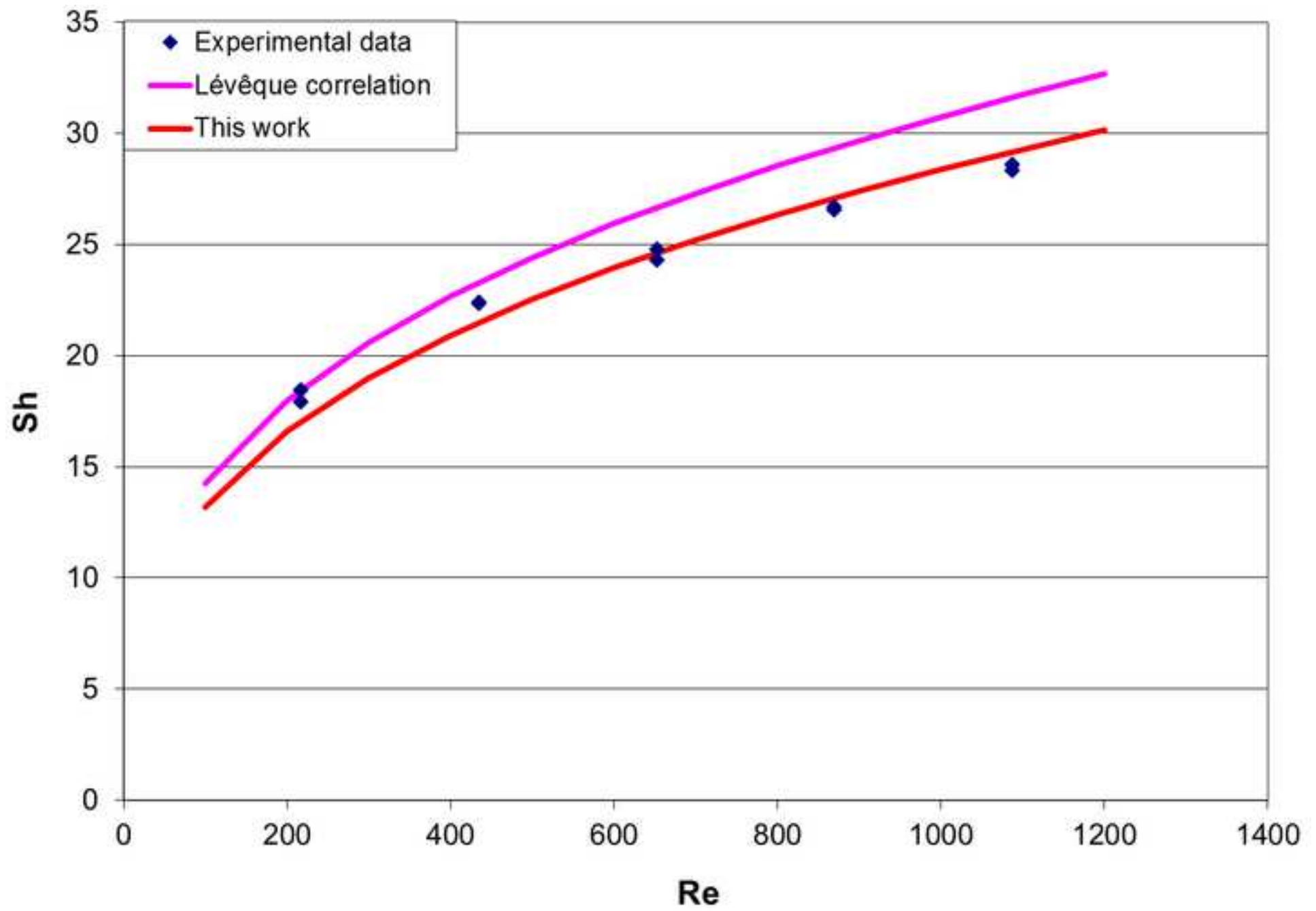


Figure 5  
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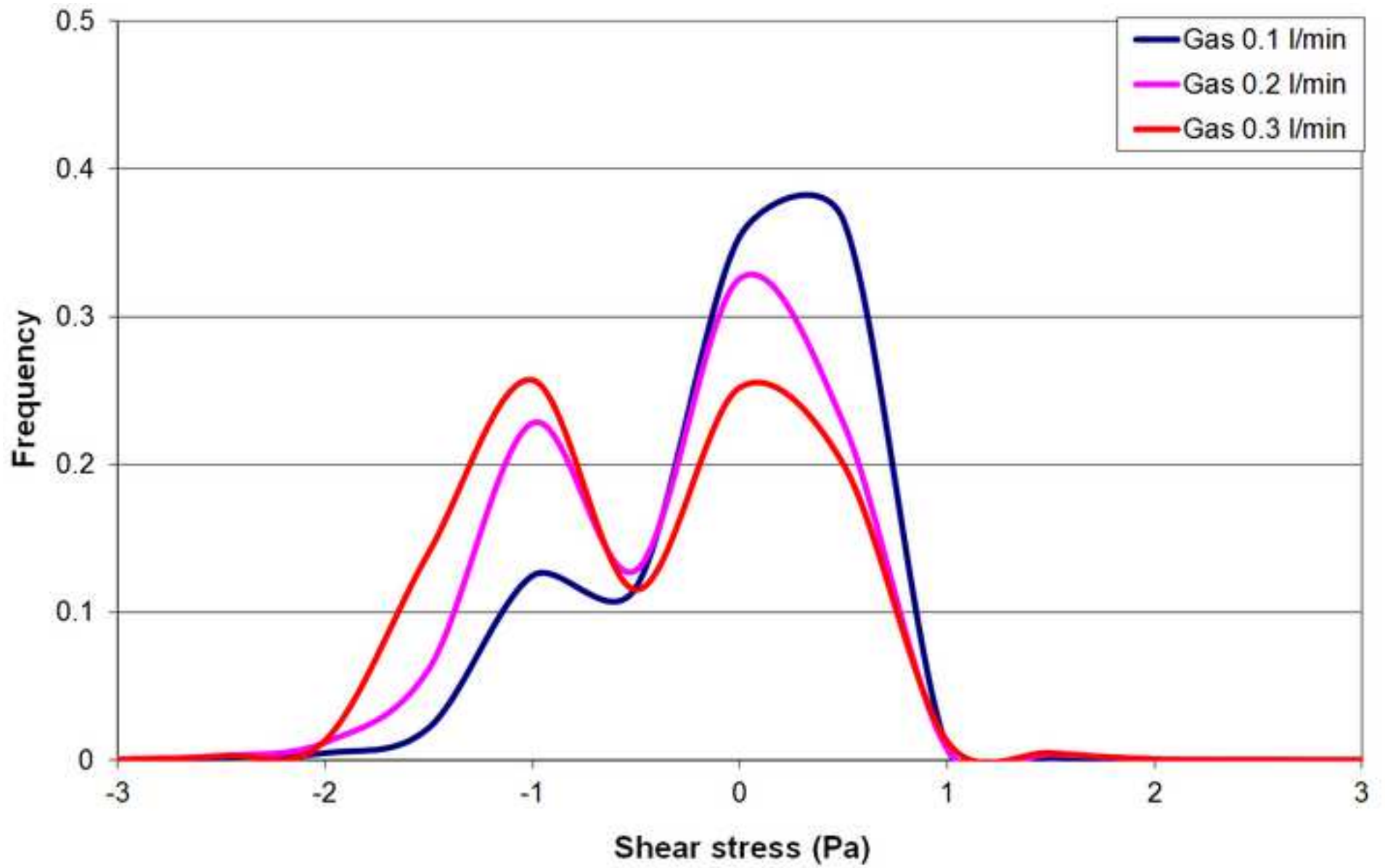
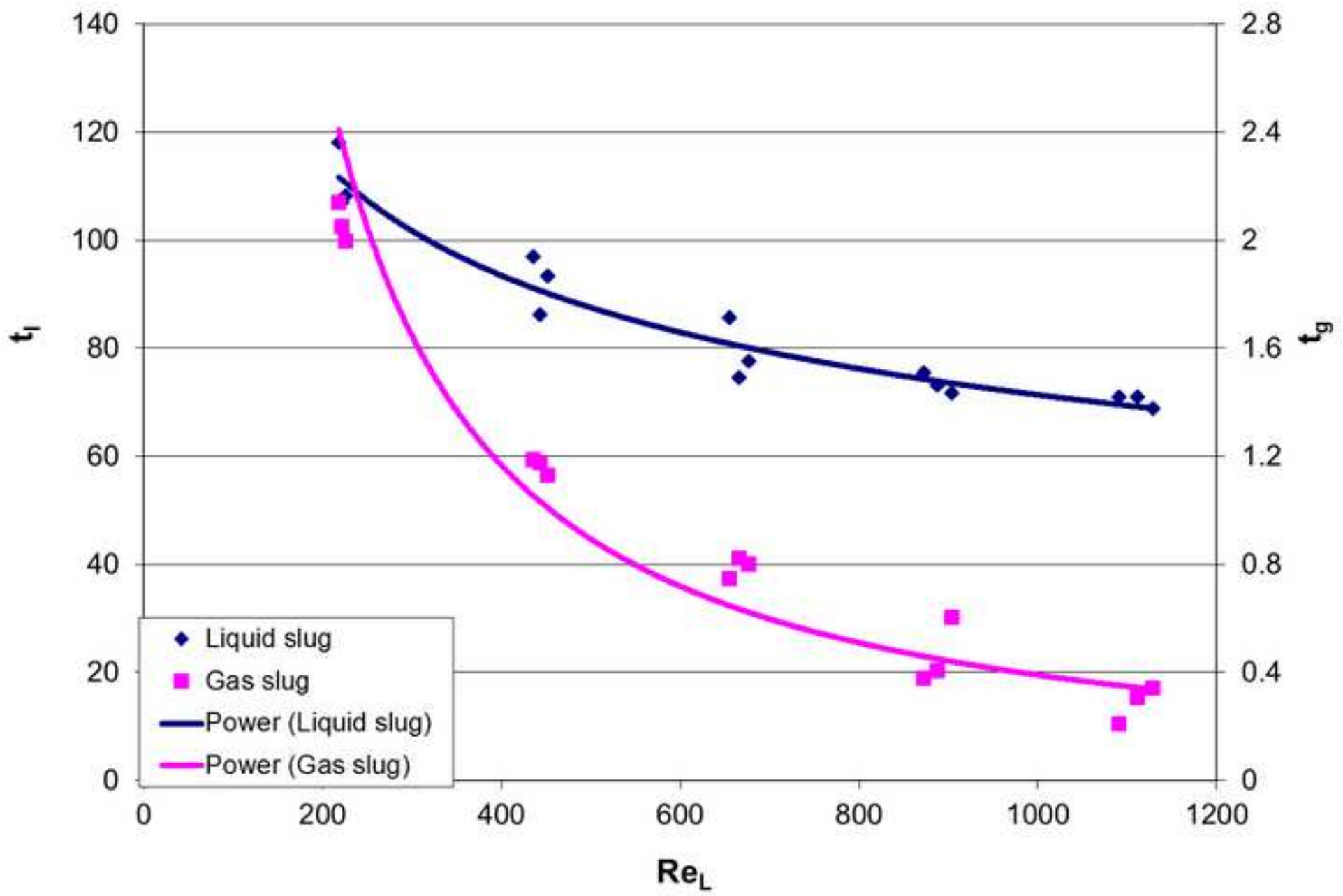


Figure 6  
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## \*Conflict of Interest Statement

There are not any actual or potential conflict of interest of this work

Table 1. Dimensionless heat and mass transfer numbers

<b>Heat transfer</b>	$Nu = \frac{hd}{k_c}$	$Pr = \frac{c_p \mu}{k_c}$
<b>Mass Transfer</b>	$Sh = \frac{k_m d}{D_f}$	$Sc = \frac{\mu}{\rho D_f}$

Table 2. Relationship among wall shear stress, mass and heat transfer

	<b>Mass transfer</b>	<b>Wall friction</b>	<b>Heat transfer</b>
	$Sh = \text{function}(d, Re, Sc)$	$f = \text{function}(d, Re)$	$Nu = \text{function}(d, Re, Pr)$
<b>Dimensionless numbers</b>	$Sh = \frac{k_m d}{D_f}$ $Sc = \frac{\mu}{\rho D_f}$	$Re = \frac{\rho u d}{\mu}$ $f = \frac{8\tau_w}{\rho u^2}$	$Nu = \frac{hd}{k_c}$ $Pr = \frac{c_p \mu}{k_c}$
<b>Laminar (Re &lt; 2000)</b>	$Sh = 1.62 \left( Re Sc \frac{d}{L} \right)^{\frac{1}{3}}$	$f = 64 Re^{-1}$	$Nu = 1.86 \left( Re Pr \frac{d}{L} \right)^{\frac{1}{3}} \left( \frac{\mu_B}{\mu_W} \right)^{0.14}$
<b>Turbulent (Re &lt; 2000)</b>	$Sh = 0.04 Re^{0.8} Sc^{\frac{1}{3}}$	$f = 0.25 \left[ \log_{10} \left( \frac{\varepsilon}{3.7d} + \frac{5.74}{Re^{0.9}} \right) \right]^{-2}$	$Nu = 0.027 Re^{0.8} Pr^{\frac{1}{3}} \left( \frac{\mu_B}{\mu_W} \right)^{0.14}$
<b>Analogy</b>	$\frac{Sh}{Nu} = \left( \frac{Sc}{Pr} \right)^{\frac{1}{3}} = Le^{\frac{1}{3}}$		

\* The subscripts  $B$  and  $W$  are for the bulk and wall respectively.



Table 3.  $R^2$  of the different Reynolds number.

	$t_l$	$t_g$
$Re_L$	<b>0.973</b>	<b>0.955</b>
$Re_{SG}$	0.163	0.102

Table 4. Parameters of Eq. (17) and (18)

	Liquid slug	Gas slug
$a_1$	545.738	1508.757
$a_2$	-0.295	-1.196
$a_3$	0.00072	0.00077