MC-Parameter Calibration of Baskarp Sand No. 15

L. B. Ibsen
M. Hanson
T. Hjort
M. Thaarup
Scientific Publications at the Department of Civil Engineering

**Technical Reports** are published for timely dissemination of research results and scientific work carried out at the Department of Civil Engineering (DCE) at Aalborg University. This medium allows publication of more detailed explanations and results than typically allowed in scientific journals.

**Technical Memoranda** are produced to enable the preliminary dissemination of scientific work by the personnel of the DCE where such release is deemed to be appropriate. Documents of this kind may be incomplete or temporary versions of papers—or part of continuing work. This should be kept in mind when references are given to publications of this kind.

**Contract Reports** are produced to report scientific work carried out under contract. Publications of this kind contain confidential matter and are reserved for the sponsors and the DCE. Therefore, Contract Reports are generally not available for public circulation.

**Lecture Notes** contain material produced by the lecturers at the DCE for educational purposes. This may be scientific notes, lecture books, example problems or manuals for laboratory work, or computer programs developed at the DCE.

**Theses** are monograms or collections of papers published to report the scientific work carried out at the DCE to obtain a degree as either PhD or Doctor of Technology. The thesis is publicly available after the defence of the degree.

**Latest News** is published to enable rapid communication of information about scientific work carried out at the DCE. This includes the status of research projects, developments in the laboratories, information about collaborative work and recent research results.
MC-Parameter Calibration for Baskarp Sand No. 15

Parameters for the Mohr Coulomb constitutive model is determined for Baskarp Sand No. 15 (Aalborg University Sand No. 1). The parameters have been determined based on a combination of cone penetration tests with a mini-CPT cone and drained triaxial tests.

1 Introduction

The soil parameters are dependent on both confining pressure and density index. The density index of Baskarp Sand can be determined by conducting cone penetration tests with the mini-CPT cone used in the Geotechnical Engineering Laboratory at Aalborg University:

\[ I_D = 5.14 \left( \frac{\sigma'_{e0}}{(q_c)^{0.75}} \right)^{-0.42} \]  \hspace{1cm} (1)

where

- \( \sigma'_{e0} \) is the vertical effective stress [MPa]
- \( q_c \) is the cone resistance [MPa]

The unit weight of Baskarp Sand No. 15 is estimated to 20 [kN/m^3] and used to calculate the vertical effective stress through the soil. The variation of the density index is then determined from the measured cone resistance of the mini-CPT cone used in the Geotechnical Engineering Laboratory. All the expressions are derived from several conventional drained triaxial tests at two different density indices performed on Baskarp Sand No. 15 (Ibsen & Bødker, 1994). The used triaxial tests are listed in Table 1 and Table 2.
Table 1  Triaxial test data for Baskarp Sand No. 15, $e = 0.70$, ID $\approx 51\%$

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$\epsilon$ [-]</th>
<th>$I_D$ [%]</th>
<th>$\sigma_3$ [kPa]</th>
<th>$p'$ [kPa]</th>
<th>$q$ [kPa]</th>
<th>$p'$ [kPa]</th>
<th>$q$ [kPa]</th>
<th>$\varphi_a$ [$^\circ$]</th>
<th>$\psi$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9301_26</td>
<td>0.704</td>
<td>50</td>
<td>5</td>
<td>14.22</td>
<td>19.96</td>
<td>7.53</td>
<td>7.28</td>
<td>46.6</td>
<td>12.2</td>
</tr>
<tr>
<td>9301_24</td>
<td>0.696</td>
<td>52</td>
<td>10.1</td>
<td>26.92</td>
<td>31.59</td>
<td>13.41</td>
<td>10.23</td>
<td>45.6</td>
<td>12.4</td>
</tr>
<tr>
<td>9301_25</td>
<td>0.696</td>
<td>52</td>
<td>20</td>
<td>45.85</td>
<td>54.07</td>
<td>29.79</td>
<td>29.37</td>
<td>41.4</td>
<td>11.3</td>
</tr>
<tr>
<td>9301_22</td>
<td>0.703</td>
<td>50</td>
<td>40.1</td>
<td>93.38</td>
<td>102.82</td>
<td>67.89</td>
<td>83.38</td>
<td>42.0</td>
<td>9.6</td>
</tr>
<tr>
<td>9301_20</td>
<td>0.705</td>
<td>50</td>
<td>80.1</td>
<td>164.37</td>
<td>237.39</td>
<td>132.87</td>
<td>158.32</td>
<td>37.7</td>
<td>10.3</td>
</tr>
<tr>
<td>9301_21</td>
<td>0.695</td>
<td>53</td>
<td>160</td>
<td>324.94</td>
<td>371.24</td>
<td>267.45</td>
<td>322.35</td>
<td>37.4</td>
<td>9.0</td>
</tr>
<tr>
<td>9301_27</td>
<td>0.698</td>
<td>52</td>
<td>320</td>
<td>644.62</td>
<td>726.05</td>
<td>529.62</td>
<td>628.85</td>
<td>37.1</td>
<td>9.8</td>
</tr>
<tr>
<td>9301_28</td>
<td>0.698</td>
<td>52</td>
<td>640.1</td>
<td>1243.70</td>
<td>1390.40</td>
<td>1062.2</td>
<td>1266.7</td>
<td>35.9</td>
<td>8.5</td>
</tr>
<tr>
<td>9301_29</td>
<td>0.698</td>
<td>52</td>
<td>800.2</td>
<td>1529.40</td>
<td>1704.80</td>
<td>1339.2</td>
<td>1617.8</td>
<td>35.3</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Table 2  Triaxial test data for Baskarp Sand No. 15, $e = 0.61$, ID $\approx 80\%$

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$\epsilon$ [-]</th>
<th>$I_D$ [%]</th>
<th>$\sigma_3$ [kPa]</th>
<th>$p'$ [kPa]</th>
<th>$q$ [kPa]</th>
<th>$p'$ [kPa]</th>
<th>$q$ [kPa]</th>
<th>$\varphi_a$ [$^\circ$]</th>
<th>$\psi$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9301_12</td>
<td>0.616</td>
<td>78</td>
<td>5</td>
<td>19.96</td>
<td>44.88</td>
<td>9.6</td>
<td>13.81</td>
<td>54.9</td>
<td>17.7</td>
</tr>
<tr>
<td>9301_11</td>
<td>0.608</td>
<td>81</td>
<td>10.1</td>
<td>31.59</td>
<td>64.47</td>
<td>16.87</td>
<td>20.62</td>
<td>49.6</td>
<td>18.1</td>
</tr>
<tr>
<td>9301_10</td>
<td>0.609</td>
<td>81</td>
<td>20.1</td>
<td>54.07</td>
<td>101.90</td>
<td>33.83</td>
<td>41.49</td>
<td>45.8</td>
<td>17.4</td>
</tr>
<tr>
<td>9301_04</td>
<td>0.608</td>
<td>81</td>
<td>39.9</td>
<td>102.82</td>
<td>188.76</td>
<td>71.76</td>
<td>95.59</td>
<td>44.7</td>
<td>16.7</td>
</tr>
<tr>
<td>9301_02</td>
<td>0.607</td>
<td>81</td>
<td>100.2</td>
<td>237.39</td>
<td>411.56</td>
<td>202.9</td>
<td>308.1</td>
<td>42.3</td>
<td>10.5</td>
</tr>
<tr>
<td>9301_03</td>
<td>0.612</td>
<td>80</td>
<td>160.7</td>
<td>371.24</td>
<td>631.62</td>
<td>277.87</td>
<td>352.42</td>
<td>41.5</td>
<td>14.9</td>
</tr>
<tr>
<td>9301_07</td>
<td>0.615</td>
<td>79</td>
<td>320.1</td>
<td>726.05</td>
<td>1217.90</td>
<td>547.18</td>
<td>681.54</td>
<td>41.0</td>
<td>14.4</td>
</tr>
<tr>
<td>9301_08</td>
<td>0.617</td>
<td>78</td>
<td>640.2</td>
<td>1390.4</td>
<td>2250.60</td>
<td>1078.8</td>
<td>1316.0</td>
<td>39.6</td>
<td>12.9</td>
</tr>
<tr>
<td>9301_32</td>
<td>0.614</td>
<td>79</td>
<td>800.2</td>
<td>1704.8</td>
<td>2713.80</td>
<td>1370.8</td>
<td>1712.1</td>
<td>39.0</td>
<td>12.2</td>
</tr>
</tbody>
</table>

2  Strength Parameters

In the following, the strength parameters for Baskarp Sand No. 15 are determined.

2.1  Friction Angle and Cohesion

The friction angle and cohesion for Baskarp Sand No. 15 are determined by the following three expressions:

- Curved Coulomb Criterion
- Modified Schmertmann
- Linear Coulomb Criterion

Curved Coulomb Criterion

The expression is derived by fitting the Curved Coulomb Criterion to the triaxial test data and then expressing a linear relation between the asymptotic parameters $\varphi_a$ and $c_a$ and the density index. The Curved Coulomb Criterion is fitted to the failure values from the triaxial tests in a $(\sigma_3 - q)$-plot, see Figure 1.
The expression is modified to be density index dependent by expressing a linear relation between the asymptotic parameters, $\varphi_a$ and $c_a$ and the density index:

\[ \varphi_a = 0.091 \cdot I_D + 30.6 \quad \text{[°]} \]  
(2)

\[ c_a = 0.43 \cdot I_D + 7.5 \quad \text{[kPa]} \]  
(3)

The friction angle and the cohesion can now be determined by (4) and (5).

\[ \varphi = \sin^{-1} \left( \frac{\delta\varphi'}{\delta\sigma'_3} \right) \quad \text{[°]} \]  
(4)

\[ c = \frac{q_0}{\frac{2}{\sin\varphi} \cdot \cot \varphi} \quad \text{[kPa]} \]  
(5)

**Schmertmann**

After fitting The Modified Schmertmann expression to the series of triaxial tests the following variation of the secant friction angle with density index and confining pressure is found:

\[ \varphi_s = 0.152 \cdot I_D + 27.4 \cdot (\sigma'_3)^{-0.28} + 23.2 \]  
(6)

where

- $I_D$ is the density index, determined by (1) [%]
- $\sigma'_3$ is the confining pressure [kPa]

The expression is not valid for very loose sands and it goes towards infinity for very small confining pressures. The expression is in Figure 2 plotted against the results of the triaxial tests. The Modified Schmertmann expression yields a secant friction angle, hence the cohesion equals zero.
Figure 2  Secant friction angles from triaxial tests and the Modified Schmertmann expression at density indices of 51% and 80%.

Linear Coulomb Criterion
When using the Linear Coulomb Criterion, the tangent friction angle and the cohesion are taken as pressure independent. Thus, the Linear Coulomb Criterion is only density index dependent. The expression assumes a linear relation between the strength parameters and the density index.

\[
\begin{align*}
\varphi_l &= 0.11 \cdot I_D + 32.3 \quad [\degree] \\
c &= 0.032 \cdot I_D + 3.52 \quad [\text{kPa}]
\end{align*}
\]

The tangent friction angle and cohesion are fitted for triaxial tests with a confining pressure below 100 kPa, cf. Figure 3.

Figure 3  Confining pressure and deviatoric stress at failure for the triaxial tests and the Linear Coulomb Criterion at density indices of 51% and 80%.

2.2 Dilatancy Angle
The dilatancy angle also depends on the friction angle and the density index. A linear relation between the dilatancy angle and the density index is found to fit the results from the triaxial tests. The stress dependency is expressed by a power function, see (9).
where

\[ \psi = 0.195 \cdot I_D + 14.9 \cdot (\sigma_3^{'})^{0.0976} - 9.95 \]  

(9)

is the dilatancy angle [\(^\circ\)]

\( I_D \) is the density index, determined by (1) [%]

\( \sigma_3 \) is the confining pressure [kPa]

The expression is not valid for very loose sands and it goes towards infinity for very small confining pressures. The expression is fitted from the dilatancy angles determined from triaxial tests, which are plotted in Figure 4 together with the derived expression. The variation of the dilatancy angle is plotted for density indices of 51% and 80%, respectively.

![Figure 4 Dilatancy angles from the triaxial tests and the Modified Schmertmann expression at density indices of 51% and 80%.](image)

### 3 Elastic Parameters

As elastic parameters Young’s modulus and Poisson’s ratio are used.

#### 3.1 Young’s Modulus

Young’s modulus is chosen as the secant modulus at 50% strength and determined based on the triaxial tests on Baskarp Sand No. 15.

\[
E_{50} = \frac{E^{ref}}{E_{50}} \left( \frac{c \cdot \cos(\varphi_t) + \sigma_3^{'}}{c \cdot \cos(\varphi_t) + \sigma_3^{\text{ref}} \cdot \sin(\varphi_t)} \right)^n
\]  

(10)

where
\( \varphi_e \) is the tangent friction angle \([-\)\]
\( \sigma_3 \) is the confining pressure \([kPa]\)
\( \sigma_3^{ref} \) is the reference confining pressure \([kPa]\)
\( m \) is the amount of stress dependency \([-]\)
\( E_5^{ref} \) is the reference secant modulus corresponding to \( p_{ref} \) \([kPa]\)

In Figure 5, \( E_5 \) determined by means of (10) is plotted against the triaxial tests.

For a reference confining pressure of 100 kPa, the reference secant modulus, \( E_5^{ref} \) and the amount of stress dependency, \( m \), are determined. The parameter \( m \) accounting for the stress dependency is determined to 0.58. The reference secant modulus can be determined by:

\[
E_5^{ref} = 0.6322 \cdot I_D^{2.507} + 10920 \ [kPa]
\]  \( (11) \)

3.2 Poisson’s Ratio

The elastic parameter Poisson’s ratio, \( \nu \), is determined in (Andersen et al. 1998) where it is found to, \( \nu = 0.25 \), with a standard deviation of 0.06.

4 Verification of MC-parameters

The derived expressions for the MC-parameters are verified by simulating triaxial tests in ABAQUS and comparing it with results from the performed triaxial tests. The estimates of the secant modulus, \( E_5 \), Poisson’s ratio and the dilatancy angle are the same for all plots, see Table 3, but the friction angle and the cohesion are estimated as described above in three different ways, see Table 4.
Table 3  $E_{50}$, $\nu$, and $\psi$ input parameters for simulation of triaxial tests at a density index of 80%.

<table>
<thead>
<tr>
<th>$\sigma_3'$ [kPa]</th>
<th>$E_{50}$ [10$^3$ kPa]</th>
<th>$\nu$ [-]</th>
<th>$\psi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>13.0</td>
<td>0.25</td>
<td>18.0</td>
</tr>
<tr>
<td>10</td>
<td>17.1</td>
<td>0.25</td>
<td>17.7</td>
</tr>
<tr>
<td>20</td>
<td>22.1</td>
<td>0.25</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 4  Friction angle and cohesion determined by the Curved Coulomb Criterion, the Modified Schmertmann, and the linear Coulomb Criterion, $I_p$=80%.

<table>
<thead>
<tr>
<th>$\sigma_3'$ [kPa]</th>
<th>Modified Schmertmann $\varphi$ [°]</th>
<th>$c$ [kPa]</th>
<th>Curved Coulomb Criterion $\varphi$ [°]</th>
<th>$c$ [kPa]</th>
<th>Linear Coulomb Criterion $\varphi_l$ [°]</th>
<th>$c$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>52.5</td>
<td>0.1</td>
<td>48.9</td>
<td>1.9</td>
<td>40.8</td>
<td>6.0</td>
</tr>
<tr>
<td>10</td>
<td>49.8</td>
<td>0.1</td>
<td>46.5</td>
<td>3.4</td>
<td>40.8</td>
<td>6.0</td>
</tr>
<tr>
<td>20</td>
<td>47.3</td>
<td>0.1</td>
<td>44.0</td>
<td>5.8</td>
<td>40.8</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The triaxial tests performed at 5, 10 and 20 kPa with a density index of 80% are shown in Figure 6.
Figure 6 Stress-strain relations and normal- and volume strain relations for triaxial tests.
5 Conclusion
The very high friction angles at low stress levels determined from the Modified Schmertmann expression cause problems for the numerical calculations in ABAQUS, which are shown by the scatter in Figure 6(a) and Figure 6(c). In spite of that, the parameters fit the triaxial tests good. Both the Linear Coulomb Criterion and the Curved Coulomb Criterion fits the triaxial tests closely.

6 References


Recent publications in the DCE Technical Report Series