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# Traffic Load on Interconnection Lines of Generalized Double Ring Network Structures

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**Abstract** — Generalized Double Ring (N2R) network structures possess a number of good properties, but being not planar they are hard to physically embed in communication networks. However, if some of the lines, the interconnection lines, are implemented by wireless technologies, the remaining structure consists of two planar rings, which are easily embedded by fiber or other wired solutions. It is shown that for large N2R structures, the interconnection lines carry notably lower loads than the other lines if shortest-path routing is used, and the effects of two other routing schemes are explored, leading to lower load on interconnection lines at the price of larger efficient average distance and diameter.

**Keywords** — Communication Networks, Network Topology, Combining Wired and Wireless Networks, Network Planning.

## 1. Introduction

Many applications such as tele robotics[1][2] and tele operations[3] are currently migrating from LAN to WAN environments. This trend is expected to continue and will put a huge pressure on Internet infrastructures at all levels, in terms of not only bandwidth but also reliability[4]. While fiber networks offer almost unlimited bandwidth, it is still necessary to develop physical network topologies, which offer sufficient levels of reliability. Most networks are today based on ring topologies, which offer two independent paths between any pair of nodes. While being easy to implement and embed, they suffer from large hop counts, and even though easy protection and restoration schemes are supported, they do not handle failures well: any single failure results in notably larger hop counts, implying a huge increase in traffic load as well as transmission delay, and in case of two failures the network is disconnected. The Generalized Double Rings (N2R) structures introduced recently[5] offer 3 independent paths between any pair of nodes and high levels of symmetry. However, like other 3-regular structures with fairly short distances such as chordal rings[6], they are not planar and thus hard to physically implement by fiber without compromising line independency.

While no other wired or wireless technology offer a bandwidth comparable to that of fiber networks, wireless technologies are developing fast, and the idea of combining wired and wireless networks to obtain network structures with good structural properties seems interesting. Despite expected tech-

nological developments, it is likely to be suitable only for structures where the wireless parts carry significantly lower traffic than the wired parts. It was indicated that using shortest-path routing in N2R structures, some lines would carry a limited amount of traffic[7]. This is investigated further in this paper, forming a base for designing networks which are fairly easy to implement and possess good structural properties. To our knowledge, load distribution has not been studied in this perspective before.

## 2. Preliminaries

A network structure  $S$  is a set of nodes and a set of bi-directional lines, where each line connects two nodes. A structure can be considered a model of a network, abstracting from specific physical conditions such as node equipment, medias and wiring. The definition of a structure is similar to that of a simple graph in graph theory. A path between two distinct nodes  $u$  and  $v$  is a sequence of nodes and lines:  $(u = u_0), e_1, u_1, e_2, u_2, \dots, u_{n-1}, e_n, (u_n = v)$ , such that every line  $e_i$  connects the nodes  $u_{i-1}$  and  $u_i$ . The length of a path corresponds to the number of lines it contains, so in the case above the path is of length  $n$ . The distance between a pair of distinct nodes  $(u, v)$  corresponds to the length of the shortest path between them and is written  $d(u, v)$ . This paper considers only 3-connected structures, i.e. between every pair of distinct nodes there exists three different paths, which share no nodes or lines. The size of a structure equals the number of nodes it contains. A structure has a planar representation if it can be drawn with no lines or nodes crossing or overlapping each other. A structure with a planar representation is said to be planar. Average distance and diameter of a structure are defined as follows. The average of  $d(u, v)$  over all pairs of distinct nodes  $u$  and  $v$  is said to be the average distance, and the maximum of  $d(u, v)$  over all pairs of distinct nodes is said to be the diameter.

N2R structures are defined as follows. Let  $p$  and  $q$  be positive integers, such that  $p \geq 3$ ,  $q < \frac{p}{2}$  and  $\gcd(p, q) = 1$ .  $p$  and  $q$  then define a  $N2R(p; q)$  structure  $S$  which consists of two rings, an outer ring and an inner ring, each containing  $p$  nodes. The nodes of the outer ring are labeled  $o_0, o_1, \dots, o_{p-1}$  and the nodes of the inner ring labeled  $i_0, i_1, \dots, i_{p-1}$ . Thus,  $S$  contains  $2p$  nodes. For each  $i$  such that  $0 \leq i \leq p-1$  there

exists a line between each of the following pairs of nodes:

- $(o_i, o_{i+1(\text{mod } p)})$  (lines of the outer ring: outer lines)
- $(i_i, i_{i+q(\text{mod } p)})$  (lines of the inner ring: inner lines)
- $(o_i, i_i)$  (interconnection lines)

$N2R(p; 1)$  is called the Double Ring (*DR*), and the diameter given by  $\lfloor \frac{p}{2} \rfloor + 1$ . Since the diameter increases linearly with the structure size, it is useful for reference purposes.

The set of lines of the inner ring is denoted  $L_i$ , the set of lines of the outer ring is denoted  $L_o$  and the set of interconnection lines is denoted  $L_{io}$ . Even though  $N2R$  structures are not in general planar, any  $N2R$  structure from which one of the sets of lines  $L_i$ ,  $L_o$  or  $L_{io}$  is removed has a planar representation. Furthermore, any  $N2R$  structure can be physically implemented in a way where only the lines of either  $L_i$ ,  $L_o$  or  $L_{io}$  are crossing each other. Figure 1 shows  $N2R(11; 3)$  drawn according to the definition and as two (planar) rings, where only interconnection lines need to cross each other, making them candidates for the wireless part of the network.

For a given  $N2R$  structure, the average-path load is defined for each of the set of lines  $L_i$ ,  $L_o$  and  $L_{io}$  as follows. Assume that paths are set up between any pair of distinct nodes, giving a total of  $p(2p-1)$  paths,  $p_1, \dots, p_{p(2p-1)}$ . Any such path  $p_j$  of length  $|p_j|$  consists of  $|p_j|_{L_i}$  lines of  $L_i$ ,  $|p_j|_{L_o}$  lines of  $L_o$  and  $|p_j|_{L_{io}}$  lines of  $L_{io}$ . Note that these values depend on how the shortest-paths are chosen.  $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_i}}{p(2p-1)}$  is the average-path load on inner lines,  $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_o}}{p(2p-1)}$  the average-path load on outer lines and  $\frac{\sum_{j=1}^{p(2p-1)} |p_j|_{L_{io}}}{p(2p-1)}$  the average-path load on interconnection lines. Adding these three values, the total average-path load is obtained, equaling the average distance if all paths are chosen to be shortest paths. Any shortest path between nodes of the same ring will use 0 or 2 interconnection lines, and any shortest path between nodes of different rings will use exactly one interconnection line[7]. This implies that the average-path load on interconnection lines is between 0.5 and 1.5, implying a limited traffic load on these. Where it does not lead to confusion, we may simply write load instead of average-path load.

Routing policies are introduced, which constrain the use of interconnection lines. In this way, one path is chosen between each pair of distinct nodes, but it does not need to be a shortest path. Taking the average over these path lengths, the efficient average distance is obtained, given that routing policy. Similarly, the efficient diameter is obtained by taking the maximum over these path lengths.

### 3. Methods

The study is carried out in three steps. In each step, different policies for structure selection and routing apply. Structure selection policies are used for choosing  $q$  given  $p$ , reflecting that

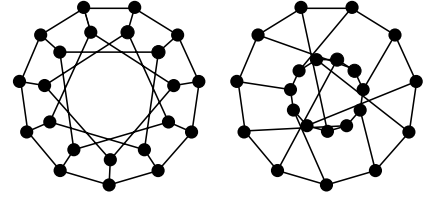


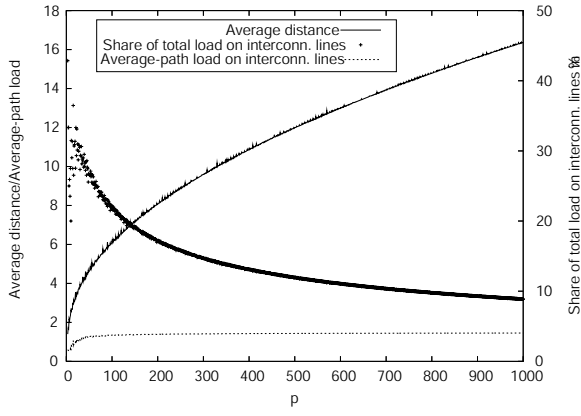
Figure 1.  $N2R(11; 3)$  drawn in two different ways.

for each value of  $p$  several different structures can exist with different characteristics.

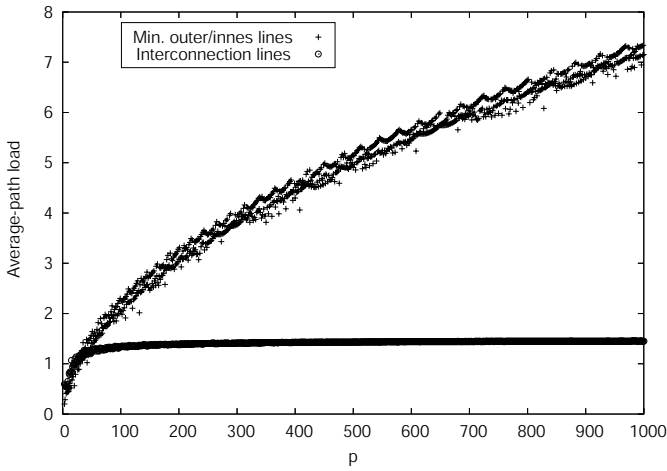
In the first step, the load on interconnection lines is compared to the load on other lines.  $q$  is for each value of  $p$  initially chosen to minimize diameter and to the largest possible extent also average distance. It was shown[8] that this leads to structures with average distance minimized or nearly minimized. This selection policy may result in several values of  $q$  being chosen. Routing, or path selection, is done using shortest paths in three variants; first, the shortest-paths are chosen to minimize the load on interconnection lines, second they are chosen to minimize the load on inner lines, and finally they are chosen to minimize the load on outer lines. When several values of  $q$  exist, further selection is done for each of the three routing schemes by choosing  $q$  to minimize the load on the lines of which the load is minimized. Thus, for each value of  $p$  the lowest possible load for each set of lines is obtained.

In the second step, two approaches to further reduce the load on interconnection lines, at the price of higher efficient average distances and diameters, are studied.  $q$  is chosen as before, but where this results in several values of  $q$ , only those resulting in the lowest possible load on interconnection lines are chosen. During this step, two routing schemes are evaluated. Both use shortest paths between nodes in different rings, but for pairs of nodes in the same ring, restrictions on the use of interconnection lines apply. This is done for each of the schemes as follows, where  $x_{diam}$  and  $x_{avg}$  must be chosen in each case. In Routing Scheme 1 (*RS1*), a path containing interconnection lines is chosen if and only if the lengths of all paths not containing interconnection lines exceed either the diameter of the structure or  $x_{diam}\%$  of the diameter of the *DR* with the same number of nodes, whichever value is largest. In routing scheme 2 (*RS2*), a path containing interconnection lines is chosen if and only if the lengths of all paths not containing interconnection lines exceed the length of a shortest path by at least  $x_{avg}\%$ .

The two schemes are evaluated separately. First,  $x_{diam}$  is varied in steps of 10, and evaluated for  $x_{diam} = 0, 10, 20, \dots, 100$ . Next,  $x_{avg}$  is also varied in steps of 10, i.e.  $x_{avg} = 10, 20, \dots, 100$ .  $x_{avg} = 0$  is not used. At the end of this step, *RS1* and *RS2* are compared. For each considered set of values of  $p$ ,  $q$  and  $x_{diam}$ , a value of  $x_{avg}$  is determined which result in a structure with the same load on interconnection lines. If no value of  $x_{avg}$  satisfies this,  $x_{avg}$  is first determined by the lowest value of  $x_{avg}$  resulting in the load on



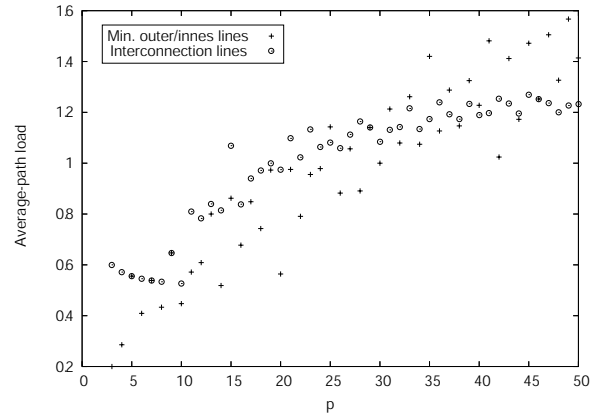
**Figure 2.** Contribution from interconnection lines to average distance, assuming shortest-path routing and avoiding interconnection lines where possible.



**Figure 3.** Average-path loads with shortest-path routing, minimizing the load on interconnection, outer or inner lines. Of the two latter, only the minimum value is shown for each value of  $p$ .

interconnection lines being lower than that of  $RS1$ . An adjustment is then made by allowing an additional number of paths to use the interconnection lines, such that the load on interconnection lines equal that of  $RS1$ . These paths are chosen to minimize efficient average distance and to the largest possible extent also efficient diameter. Now, for each considered value of  $p$ , the two ways of obtaining a certain load on interconnection lines are compared by efficient average distance and efficient diameter.

$q$  was in the previous steps chosen to minimize diameter, average distance and load on interconnection lines given shortest-path routing. If the revised routing schemes are used, this may not be optimal. In the last step, it is studied if other values of  $q$  perform better when  $RS1$  is used, varying  $x_{diam}$  from 0 to 100 in steps of 10. Given  $p$  and  $x_{diam}$ , it is determined which value of  $q$  result in the best performance. Using efficient average dis-



**Figure 4.** Average-path loads with shortest-path routing, minimizing the load on either interconnection, outer or inner lines. Of the two latter, only the minimum value is shown for each value of  $p$ .

tance, efficient diameter and average-path load on interconnection lines as performance parameters, this is done as follows. For the considered values of  $p$  and  $x_{diam}$ , all permitted values of  $q$  with diameter and average distance less than or equal to the efficient diameter and efficient average distance respectively, are evaluated. The resulting efficient average distance, efficient diameter and average-path load on interconnection lines are then compared to the values obtained in the second step.

All calculations are performed for all  $p \leq 1000$  on a standard PC, using C programs. All paths constructed are either shortest paths or paths running along the inner or outer ring, and together with the symmetries, this makes it possible to calculate all the desired values within acceptable calculation times.

## 4. Results

Figure 2 shows that for large values of  $p$ , interconnection lines carry significantly lower loads than other lines using shortest-path routing and avoiding interconnection lines if possible. For  $p$  small, the interconnection lines carry appr. 33% of the total load, a number decreasing as  $p$  increases. The distribution of the remaining load depends on the chosen routing strategy. Figures 3-4 show the potentials when reducing the load on the different sets of lines. For  $p \geq 45$  the interconnection lines allow for the lowest loads, but for  $p < 45$  the picture is more mixed.

By revising the routing scheme it is possible to reduce the load on interconnection lines significantly, but it has its costs in terms of average distance and diameter.  $RS1$  leads to distances and loads as shown in Figures 5-10. In Figures 5,7,9  $x_{diam}$  is varied in steps of 10, but only a selection of these results are shown in Figures 6,8,10 to increase readability. Among the 998 considered values of  $p$ , there are 73 cases where more than one value of  $q$  exist, and in some of these cases, the efficient average distances depend on further selection of  $q$ . Over these 73 cases and the 11 values of  $x_{diam}$  from 0 – 100, the average

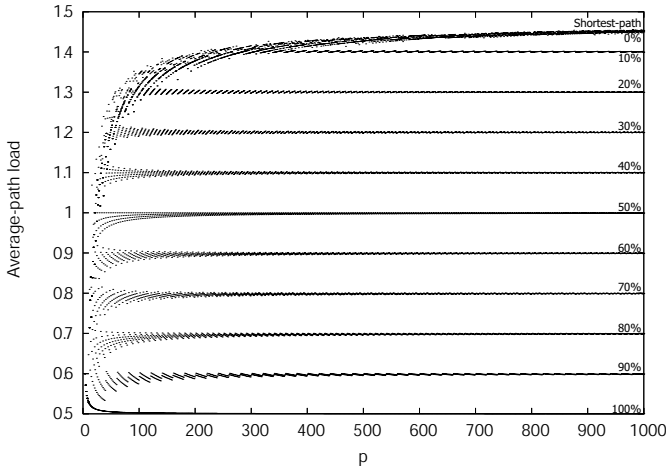


Figure 5. Interconnection line loads using RS1, varying  $x_{diam}$ .

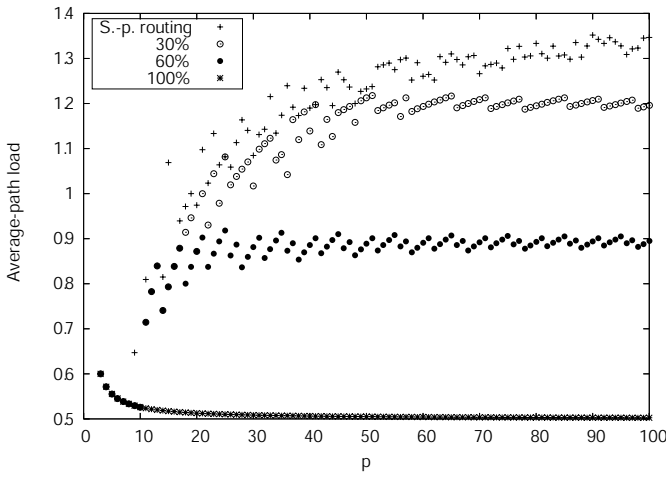


Figure 6. Interconnection line loads using RS1, varying  $x_{diam}$ .

difference between maximum and minimum efficient average distance is 0.42% of the minimum. In no case the difference exceeds 2.35%.  $q$  is chosen to minimize the efficient average distance. This choice affects no other parameters.

*RS2* leads to distances and loads as shown in Figures 11-13. The further selection of  $q$  is slightly more difficult here, because the choice of  $q$  affects the line load on interconnection lines as well as efficient average distance and diameter. Over the 73 cases with multiple values of  $q$  and the 10 values of  $x_{avg}$  (10 – 100), the differences between maximum and minimum values are on average 1.48% (load on interconnection lines), 2.30% (efficient average distance) and 4.35% (efficient diameter) of the minimum. First,  $q$  is chosen to minimize the load on interconnection lines, which reduces the number of values of  $p$  with multiple values of  $q$  to on average (over the 10 values of  $x_{avg}$ ) 33.9. From this point,  $q$  is chosen to minimize efficient diameter and where this leads to multiple candidates finally to minimize efficient average distance. Over the on average 33.9

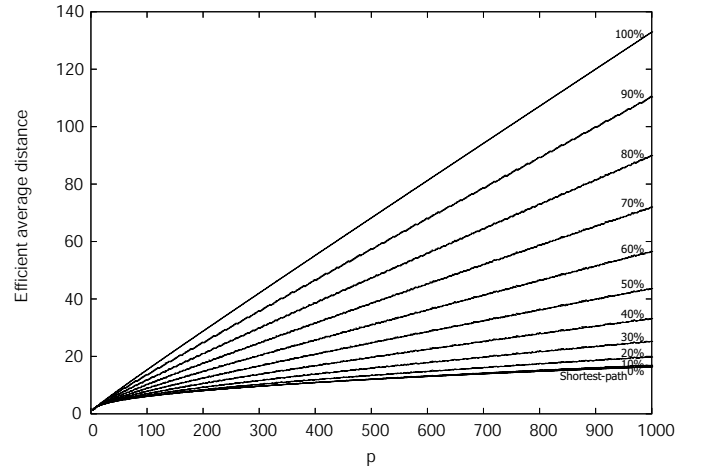


Figure 7. Efficient average distances using RS1, varying  $x_{diam}$ .

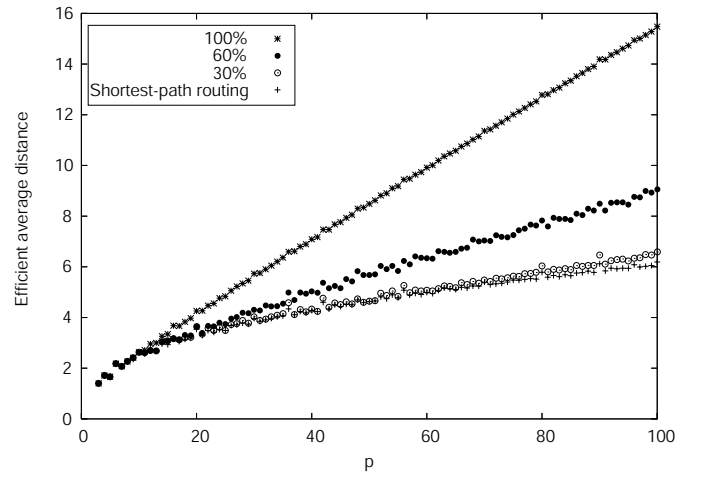


Figure 8. Efficient average distances using RS1, varying  $x_{diam}$ .

values of  $p$  with multiple values of  $q$ , this leads to efficient average distances 0.043% over the minimum obtained when minimizing the load on interconnection lines. In no case the chosen efficient average distance exceed the minimum value by more than 1.15%.

A direct comparison of the two approaches shows that in order to obtain the same load on interconnection lines, *RS2* resulted in larger or equal efficient diameters and smaller or equal efficient average distances than *RS1*. For each value of  $p$ , the differences in some cases depend on the value of  $q$ , in which case  $q$  is chosen first to maximize the relative difference in efficient average distance and second to minimize the relative difference in efficient diameter, giving an impression of the trade-offs. In general, the relative differences become smaller when  $p$  becomes large, which is illustrated for  $x_{diam} = 60$  in Figure 14. Table 1 shows for each value of  $x_{diam}$  the relative differences in efficient average distance and diameter.

For all considered values of  $p$  and  $x_{diam}$ , it was determined

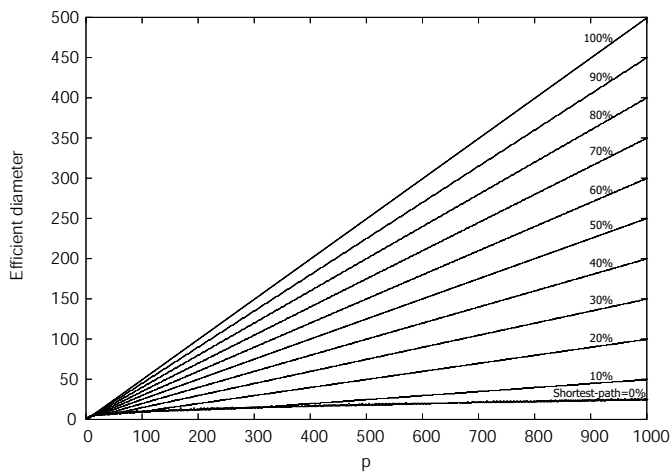


Figure 9. Efficient diameters using RS1, varying  $x_{diam}$ .

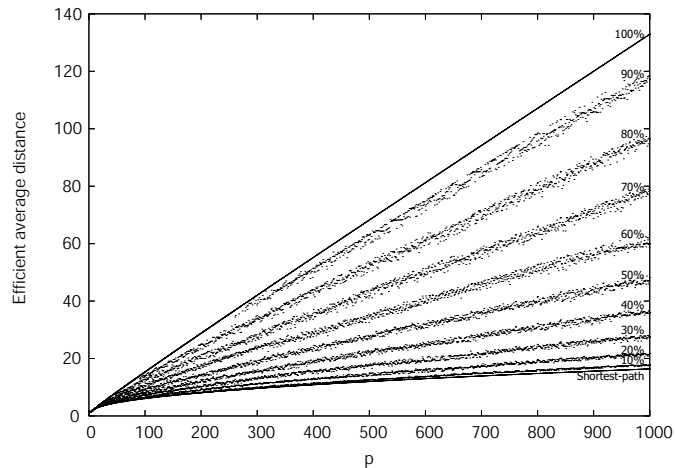


Figure 12. Efficient average distances using RS2, varying  $x_{avg}$ .

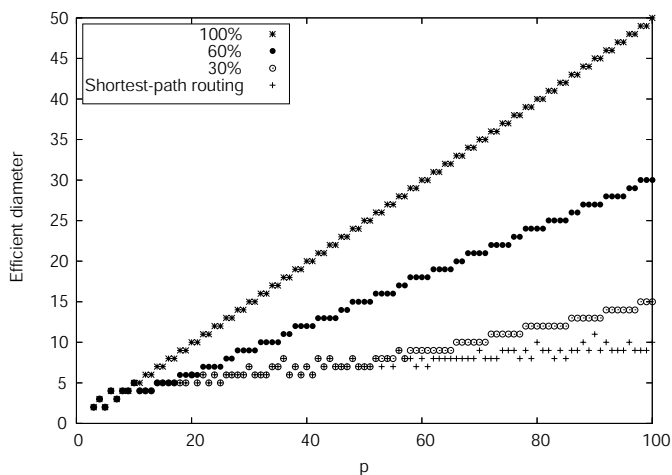


Figure 10. Efficient diameters using RS1, varying  $x_{diam}$ .

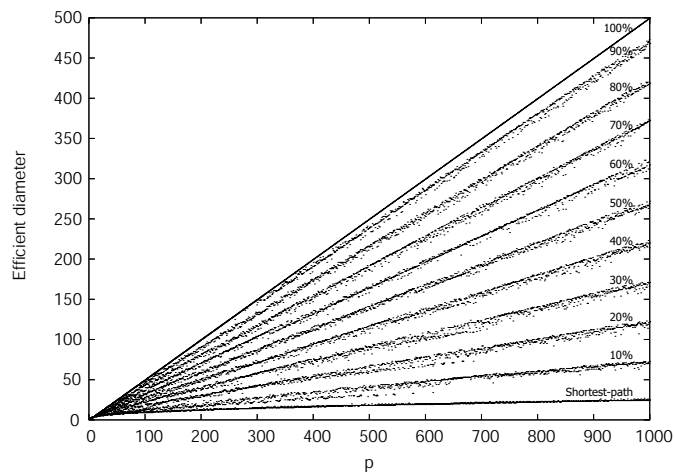


Figure 13. Efficient diameters using RS2, varying  $x_{avg}$ .

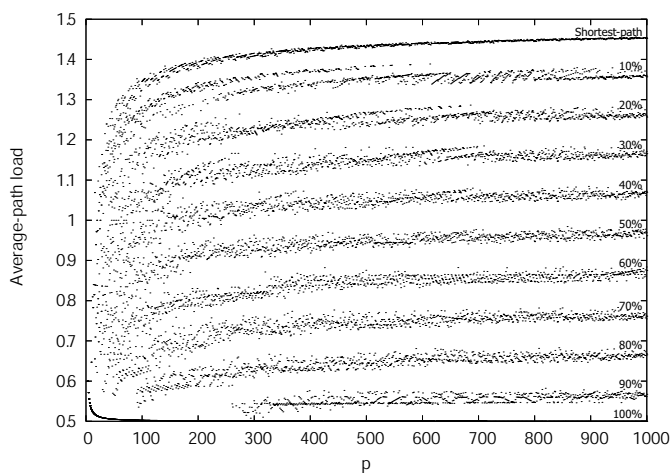


Figure 11. Interconnection line loads using RS2, varying  $x_{avg}$ .

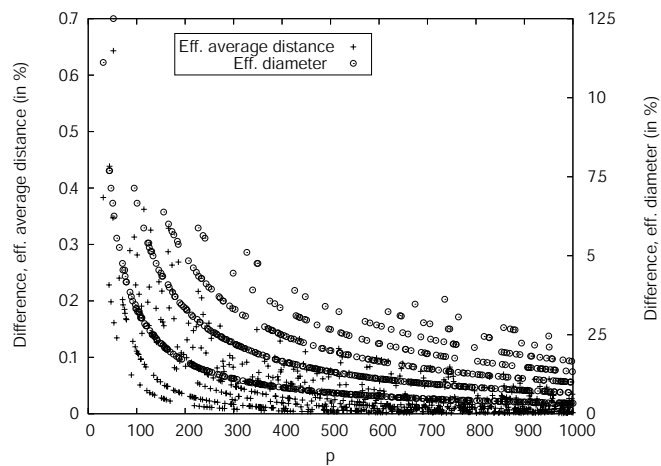


Figure 14. Reduction in eff.avg.dist./increase in eff.diam. with RS2 instead of RS1,  $x_{diam} = 60$  and same interconnection load.

if another value of  $q$  would result in a better performance than the values of  $q$  determined during the second step. It turned out that in every case, the load on interconnection lines and efficient diameter remained the same, but for some values of  $p$  it was possible to reduce the efficient average distance. The results are listed in Table 2; for each value of  $x_{diam}$  the number of values of  $p$  for which at least one better  $q$ -value exist is shown together with the potential maximum and average reductions, taken over these values of  $p$ .

## 5. Conclusion and Discussion

It was shown that using shortest-path routing, the load on interconnection lines is limited for any  $N2R$  structure, and for  $p \geq 45$  the average-path load on interconnection lines is smaller than the load on inner and outer rings. The differences increase with the size of the structures. On average, the shortest-paths use 0.5-1.5 interconnection lines, and even though this number grows fast towards 1.5 with the size of the structures, the average path length increases significantly faster. It should however be kept in mind that the absolute traffic load on interconnection lines grow faster with the size of the structures, because more nodes create more traffic. This is not reflected by the measures used in the paper.

It was also shown that the load on interconnection lines can be further reduced by changing the routing scheme, but this also imply significantly larger efficient average distance and efficient diameter; to reduce the load on interconnection lines to approximately 0.5, the efficient diameter approaches that of the  $DR$ . Two revised routing schemes were proposed. Given the decreased interconnection line load, one minimized the efficient diameter and the other the efficient average distance, but it turned out that the differences between them were in general insignificant. If networks are implemented combining fiber/wireless solutions, it may be appropriate to use such a revised routing scheme to prefer the use of fiber lines, also reflecting the fact that fiber transmissions are faster and with fewer errors than wireless transmissions; a longer path using only fiber and allowing for optical switching may be better than a shorter combined fiber/wireless path.

Structures were chosen to minimize diameter, average distance and load on interconnection lines, and even with the revised routing schemes, this seem to be a fairly good choice. The results indicate that networks with  $N2R$  topologies can be implemented physically by using wireless solutions for some or all of the interconnection lines. However, this requires more research in combining wired and wireless networks into one common network.

In access networks, a large part of the traffic is usually one-to-all traffic, going to and from a gateway to the Internet. In this case, the traffic will most likely not be distributed evenly on the interconnection lines, and it might be advantageous to implement some of these lines by fiber and some by wireless technologies.

**Table 1. Efficient average distances and diameters of RS2 compared to RS1 for values of  $p$  where they are not equal. Differences in % of RS1-values.**

$x_{diam}$	No. $p$ 's	Avg.diff, eff.avg.	Max.diff, eff.avg.	Avg.diff, eff.diam.	Max.diff, eff.diam.
0	334	0.0531%	0.784%	6.84%	25.0%
10	578	0.0631%	0.784%	7.10%	25.0%
20	658	0.0791%	0.784%	5.24%	25.0%
30	635	0.0755%	0.784%	3.61%	25.0%
40	600	0.0614%	0.637%	2.79%	20.0%
50	554	0.0446%	1.02%	2.12%	16.7%
60	692	0.0436%	0.643%	1.79%	12.5%
70	639	0.0377%	1.35%	1.48%	14.3%
80	635	0.0323%	0.877%	1.40%	14.3%
90	643	0.0236%	0.340%	1.15%	7.69%
100	0	0%	0%	0%	0%

**Table 2. Reductions in eff.avg.distance, choosing  $q$  differently.**

$x_{diam}$	Number of $p$ 's	Avg. red.	Max red.
0	43	0.317%	2.52%
10	349	0.270%	2.52%
20	511	0.629%	2.52%
30	554	0.654%	2.52%
40	587	0.547%	2.52%
50	635	0.474%	2.45%
60	604	0.364%	2.35%
70	676	0.295%	2.92%
80	719	0.240%	3.47%
90	643	0.134%	1.79%
100	49	0.0864%	1.79%

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