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An Improved Model Predictive Control for DC-DC Boost Converter

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***Abstract*—DC-DC boost converter acts as one of the common interfaces in renewable energy systems. Considering its non-linear characteristics, several nonlinear control methods have been adopted. Among them, the model predictive control (MPC) is widely used. However, the common finite-set (FCS)-MPC algorithm yields a variable switching frequency. Besides, a long prediction horizon MPC is needed for the boost converter to alleviate the non-minimum phase characteristics' influence, which leads to a high computational burden. To address these issues, this paper proposes an improved MPC algorithm to guarantee stable operation. The proposed algorithm transforms the original control variable which is the switching signal into the duty cycle to generate a fixed switching frequency. Besides, by changing the prediction model, the proposed MPC algorithm performs guaranteed stability with one prediction horizon. Moreover, a Jacobian matrix is utilized to assess the stability of the proposed algorithm by determining whether its eigenvalues are in the unit cycle. Simulations are provided to prove the effectiveness of the controller.**

Keywords—DC-DC boost converter, model predictive control, fixed switching frequency, Jacobian matrix)

I. Introduction

DC-DC boost converter acts as one of the most common interfaces between the PV array and the DC link [1]-[4]. The traditional control method for boost converter is a PI controller and it is modulated with pulse width modulation (PWM) [5]. However, designing the control loop for power electronic converters should feed with their average model, which often brings offset error and is incompatible with linear controllers [6]. In recent years, based on the advancements in control theory and the maturation of digital signal processing technology, the model predictive control (MPC) attracts more attention in its application in power converters [7]-[10]. In summary, the MPC algorithm collects the system's states to predict the next states and repeats the whole process in every calculation cycle.

Compared to the conventional PID control, MPC obtains the optimal control sequence by minimizing the cost function [11]. However, the conventional finite control set (FCS) MPC algorithm yields a variable switching frequency which is undesirable as it will cause high acoustic noises and harmonics. Hence, the FCS-MPC with a fixed switching frequency has been proposed, which utilizes the external PWM. As illustrated in [6], a computationally effective MPC strategy for boost converters is proposed. With the combination of the slope of the inductor current and its corresponding operating time, the inductor current in the next sampling time can be predicted. Then, the cost function is minimized to calculate the optimal variable, and finally via the modulation of a sawtooth wave, the switching signal with a fixed switching frequency is obtained.

Despite a variable switching frequency issue being solved, the non-minimum phase characteristic of the boost converter should also be considered with the MPC algorithm. Due to the stability issue with the one prediction horizon in MPC, many previous works utilize several prediction steps [12]-[14], which require a high demand for sampling process and complex computation. In [15], a one-prediction-horizon MPC algorithm is proposed for the boost converter to solve the non-minimum phase issue where an input-state linearization is supplemented.

This paper proposes a control strategy for boost converters based on MPC with one prediction horizon. With the proposed control strategy, the system will operate with a fixed switching frequency as well as operate in a stable state. Moreover, the Jacobian matrix is utilized to prove the effectiveness and to compare the stable characteristics with the conventional MPC strategy.

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II. DC-DC Boost Converter Description

A. Common MPC algorithm for boost converter

Fig. 1 shows a circuit diagram of the DC-DC boost converter. Here, S represents the switch, D represents the diode, i_L is the inductor and v_c is the output capacitor. v_g is the input voltage and v_o is the output voltage, neglecting the series resistance, v_c equals to v_o.

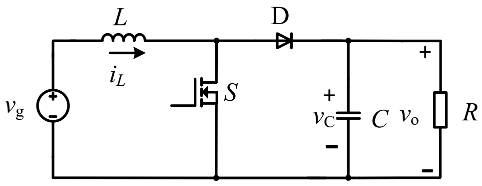

Fig. 1. Schematic of a DC-DC boost converter.

Based on the operating principle, the following state-space equation of the converter can be obtained:

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{s-1}{L} \\ \frac{1-s}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_g \tag{1}$$

where, s is the switching signal. When the switching is on, s equals to 1 otherwise it remains 0. Assuming that the sampling frequency is relatively high, the state variables in (1) can be transformed into a discrete-time pattern with the classical forward Euler approximation method, which is expressed as follows.

$$\begin{cases} \frac{di_L}{dt} = \frac{i_L(k+1) - i_L(k)}{T_s} \\ \frac{dv_c}{dt} = \frac{v_c(k+1) - v_c(k)}{T_s} \end{cases} \tag{2}$$

where T_s is the switching cycle. Combining the equation (2), the predicted inductor current and the predicted capacitor voltage at the next sampling time can be expressed as follows.

$$\begin{cases} i_L(k+1) = i_L(k) + \frac{1}{L}[v_g - (1-s)v_c(k)]T_s \\ v_c(k+1) = v_c(k) + \frac{1}{C}[i_L(k)(1-s) - \frac{v_c(k)}{R}]T_s \end{cases} \tag{3}$$

The next step is to establish the cost function and the constraints for minimizing the cost function which are expressed as the following equation. And this paper utilizes the quadratic error for the cost function:

$$\begin{cases} \min J_{ct} = \sum_{l=1}^{N} (i_L(k+l) - i_L^*)^2 + (v_c(k+1) - v_o^*)^2 \\ i_L(k+1) = i_L(k) + \frac{1}{L}[v_g - (1-s)v_c(k)]T_s \\ v_c(k+1) = v_c(k) + \frac{1}{C}[i_L(k)(1-s) - \frac{v_c(k)}{R}]T_s \end{cases} \tag{4}$$

where N is the prediction horizon and i_L^* and v_o^* are the references. Finally, a control signal s is selected and applied to the converter by minimizing the cost function in (4). However, the common principle of MPC yields a variable frequency which will cause high acoustic noises and complicates the design for passive components [3]. Besides, by using this MPC algorithm directly, the converter may undergo instability because of the non-minimum phase characteristics within one prediction horizon (k=1).

B. Common MPC algorithm for boost converter

Different from the above MPC algorithm for DC-DC boost converter, this paper proposes a simple and effective algorithm, especially in the prediction model and the control variable, which avoids inaccuracy and instability during operation. Besides, the proposed algorithm directly provides the duty cycle signal and via the modulation to generate a fixed switching frequency.

Considering the conventional control method for the boost converter, it can generate a switching signal with a constant turn-on and turn-off time with modulation. Similarly, once the MPC algorithm provides the value of the duty cycle as its output, the fixed switching signal can be also obtained via the modulation. To this end, the key issue is how to introduce the duty cycle to the cost function and obtain the optimal value.

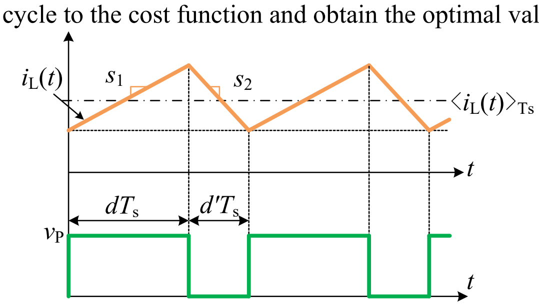

Fig. 2. Inductor current and duty cycle.

As illustrated in Fig. 2, it presents the duty cycle and the inductor current. According to the operating principle, the current in the next sampling time can be obtained where $s_1 = v_g/L$, $s_2 = (v_g - v_c)/L$:

$$i_L(k+1) = i_L(k) + s_1 dT_s + s_2(1-d)T_s \tag{5}$$

Then:

$$i_L(k+1) = i_L(k) - \frac{(1-d)}{L} v_c(k)T_s + \frac{1}{L} v_g(k)T_s \tag{6}$$

Similarly, the output voltage in the next sampling time can also be obtained as:

$$v_c(k+1) = v_c(k) + \frac{(1-d)}{C} i_L(k)T_s - \frac{1}{RC} v_c(k)T_s \tag{7}$$

It should be noticed that the initial inductor current $i_L(k)$ and capacitor voltage $v_c(k)$ in each sampling period should be the average value because the reference value is set as the static state value of the inductor current and the capacitor voltage. Otherwise, there will be an error between the sampling value and referenced value which results in an inaccuracy in the output of the boost converter.

Comparing (6) and (7) with (3), the duty cycle d is introduced as the variable to be optimized instead of the switching signal s.

The other problem which remains to be discussed is the non-minimum phase characteristics existing in the boost converter.

The following formula shows the duty cycle to output voltage transfer function of the boost converter:

$$G_{vd}(s)=\frac{V_{\mathrm{g}}}{1-D}\frac{(1-\frac{L}{(1-D)^2R}s)}{(\frac{LC}{(1-D)^2}s^2+\frac{L}{(1-D)^2R}s+1)} \tag{8}$$

Here, D is the duty cycle. Because the zero pole which equals $(1\text{-}D)^2R/L$ is in the right half plane, so the boost converter is a non-minimum phase system. Considering this characteristic, the design of the prediction model should avoid the sampling of output voltage to some extent but also not sacrifice the dynamic performance when it has load variation.

To this end, this paper proposes a compromise prediction model which predicts the inductor current only utilizing the sampling inductor current and remains the prediction model for the capacitor voltage as follows:

$$\begin{aligned}
&i_L(k+1)=i_L(k)-\frac{(1-d)}{L}\sqrt{i_L(k)v_g(k)R}T_s+\frac{1}{L}v_g(k)T_s\\
&v_c(k+1)=v_c(k)+\frac{(1-d)}{C}i_L(k)T_s-\frac{1}{RC}v_c(k)T_s
\end{aligned} \tag{9}$$

Based on the prediction model, the cost function J_{ct} is established as follows:

$$\begin{aligned}
J_{ct}&=(i_L(k+1)-i_L^*)^2+(v_c(k+1)-v_o^*)^2\\
&=(i_L(k)-\frac{(1-d)}{L}\sqrt{i_L(k)v_g(k)R}T_s+\frac{1}{L}v_g(k)T_s-i_L^*)^2\\
&+(v_c(k)+\frac{(1-d)}{C}i_L(k)T_s-\frac{1}{RC}v_c(k)T_s-v_o^*)^2
\end{aligned} \tag{10}$$

III. Stability Analysis

As seen in (4) and (10), the control variable has been changed. In this section, the stability analysis method is proposed to illustrate the advantage of the proposed algorithm.

The cost function in (10) is optimized by minimizing its value to provide an optimal variable d, that is:

$$\frac{\partial J_{ct}}{\partial d}=\frac{\partial[(i_L(k+1)-i_L^*)^2+(v_c(k+1)-v_o^*)^2]}{\partial d}=0 \tag{11}$$

Combine (10) with (11):

$$\begin{aligned}
&2(i_L(k+1)-i_L^*)\frac{\partial i_L(k+1)}{\partial d}+2(v_c(k+1)-v_o^*)\frac{\partial v_c(k+1)}{\partial d}=0\\
&=(i_L(k)-\frac{(1-d)}{L}\sqrt{i_L(k)v_gR}T_s+\frac{1}{L}v_gT_s-i_L^*)\frac{d}{L}\sqrt{i_L(k)v_gR}T_s\\
&-(v_c(k)+\frac{(1-d)}{C}i_L(k)T_s-\frac{1}{RC}v_c(k)T_s-v_o^*)\frac{d}{C}i_L(k)T_s
\end{aligned} \tag{12}$$

Then, d can be derived from (12) as:

$$\begin{aligned}
d=&\frac{(i_L(k)-\frac{1}{L}\sqrt{i_L(k)v_gR}T_s+\frac{1}{L}v_gT_s-i_L^*)\frac{1}{L}\sqrt{i_L(k)v_gR}T_s}{-\frac{1}{C^2}i_L(k)^2T_s^2-\frac{1}{L^2}i_L(k)v_gRT_s^2}\\
&-\frac{(v_c(k)+\frac{1}{C}i_L(k)T_s-\frac{1}{RC}v_c(k)T_s-v_o^*)\frac{1}{C}i_L(k)T_s}{-\frac{1}{C^2}i_L(k)^2T_s^2-\frac{1}{L^2}i_L(k)v_gRT_s^2}
\end{aligned} \tag{13}$$

To discuss the stability of the proposed algorithm, the Jacobian matrix can be derived through the linearizing of the inductor current $i_{\mathrm{L}}(k+1)$ and capacitor voltage $v_{\mathrm{c}}(k+1)$ as follows:

$$J=\begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}=\begin{bmatrix}\frac{\partial i_L(k+1)}{\partial i_L(k)} & \frac{\partial i_L(k+1)}{\partial v_c(k)}\\ \frac{\partial v_c(k+1)}{\partial i_L(k)} & \frac{\partial v_c(k+1)}{\partial v_c(k)}\end{bmatrix} \tag{14}$$

$$\begin{cases}
\partial i_L(k+1)/\partial i_L(k)=1-\frac{T_s\sqrt{v_gR}}{2L\sqrt{i_L(k)}}+\frac{\partial d}{\partial i_L(k)}\sqrt{i_L(k)v_gR}T_s+\frac{dT_s\sqrt{v_gR}}{2L\sqrt{i_L(k)}}\\
\partial i_L(k+1)/\partial v_c(k)=\frac{\partial d}{\partial v_c(k)}\sqrt{i_L(k)v_gR}T_s\\
\partial v_c(k+1)/\partial i_L(k)=\frac{1}{C}T_{\mathrm{s}}-\frac{i_L(k)}{C}T_{\mathrm{s}}\;\frac{\partial d}{\partial i_L(k)}-\frac{d}{C}T_{\mathrm{s}}\\
\partial v_c(k+1)/\partial v_c(k)=1-\frac{i_L(k)}{C}\frac{\partial d}{\partial v_c(k)}T_{\mathrm{s}}-\frac{1}{RC}T_{\mathrm{s}}
\end{cases} \tag{15}$$

Combing (13) with (15) and replacing the sampling value with stable state values I_{L} and V_{c}, the four parameters J_{11}, J_{12}, J_{21} and J_{22} can be obtained. When the eigenvalues of the Jacobian matrix are located inside the unit circle, then it proves that the MPC controlled boost converter operates in a stable state. Otherwise, it is unstable. Using the parameters in Table I, Fig. 3, and Fig. 4 show the comparison between the eigenvalues of the Jacobian matrix with a common MPC algorithm-controlled boost converter and the proposed algorithm when the load changes from 50Ω to 100Ω.

TABLE I. SYSTEM PARAMETERS

Parameters	Symbols	Values
Input voltage	V_{in}	50V
Output voltage	V_o	100V
Inductance	L	200μH
Capacitor	C	470μF
Switching frequency	f_s	10kHz
Load	R	50Ω

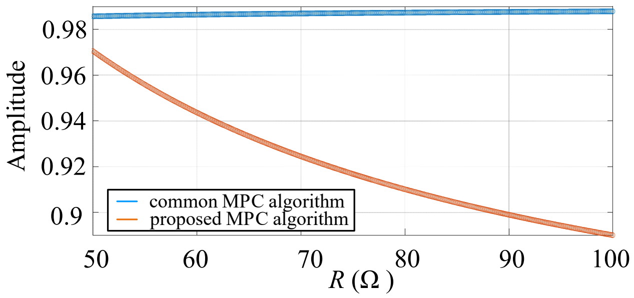

Fig. 3. Amplitude of eigenvalue λ_1 of the Jacobian matrix with load variation.

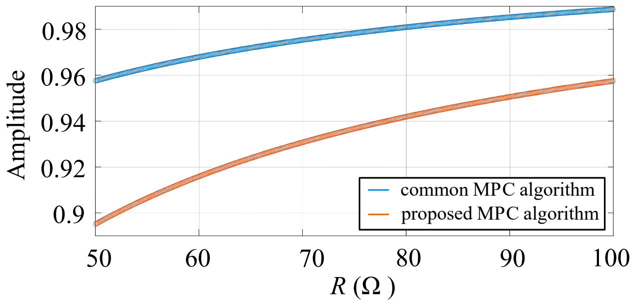

Fig. 4. Amplitude of the eigenvalue λ_2 of the Jacobian matrix with load variation.

Using the parameters in table one, Fig. 3 and Fig. 4 show the compare between the eigenvalues of the Jacobian matrix with common MPC algorithm-controlled boost converter and the proposed algorithm when the load changing from 50Ω to 100Ω.

It can be seen clearly that with the proposed algorithm, the eigenvalues of the Jacobian matrix have been changed and are located more remote from the unit value compared with the common FCS-MPC algorithm for the boost converter. With the proposed algorithm, the boost converter performs more stable with regarding to its non-minimum phase characteristics.

IV. SIMULATIONS

To verify the effectiveness of the proposed MPC algorithm-controlled DC- DC boost converter, simulations are established in three parts. Firstly, the control performance of the common FCS-MPC algorithm and the proposed FCS-MPC algorithm are compared. Fig. 5 –Fig. 6 show the output voltages and duty cycles of MPC controlled boost converter and the proposed MPC algorithm-controlled boost converter.

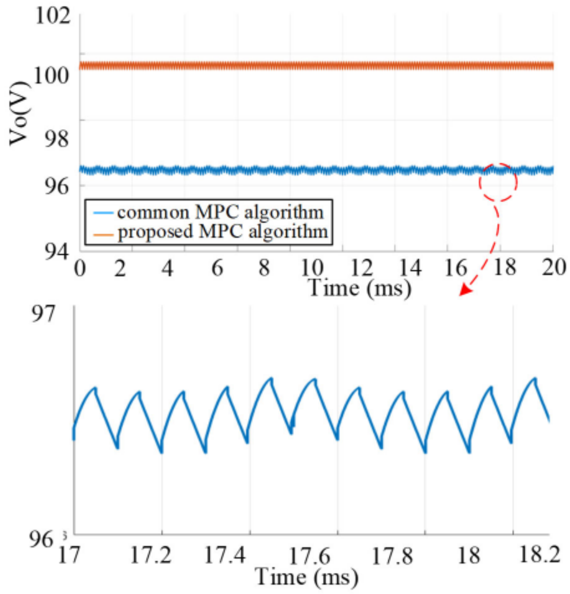

Fig. 5. Output voltage V_o with different MPC algorithms.

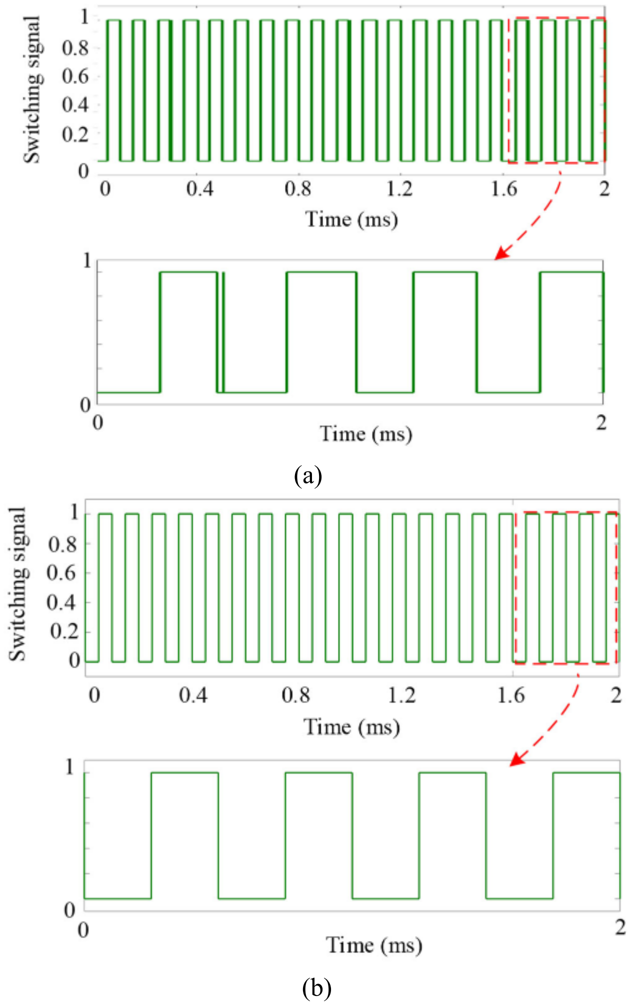

Fig. 6. Switching signal with different MPC algorithms. (a) Common MPC algorithm. (b) Proposed MPC algorithm.

With the comparison between the two algorithms, when using the proposed method, the boost converter operates more stable and tracks the reference more accurately without instability as well as ensures a fixed switching frequency. Secondly, the load transient performance is presented.

Fig. 7 –Fig. 8 show the output voltages of the proposed MPC algorithm-controlled boost converter with load steps from 50Ω to 30Ω and 30Ω to 50Ω respectively. As seen, the proposed MPC controlled boost converter can adjust to its stable state when the disturbance occurs. Besides, the system can buffer the disturbance with a slight overshoot which is about 0.5V to 1.5V, and this characteristic will prevent the system from large overvoltage and overcurrent in the dynamic process. Besides, the converter can adjust to stable state in a few switching cycles

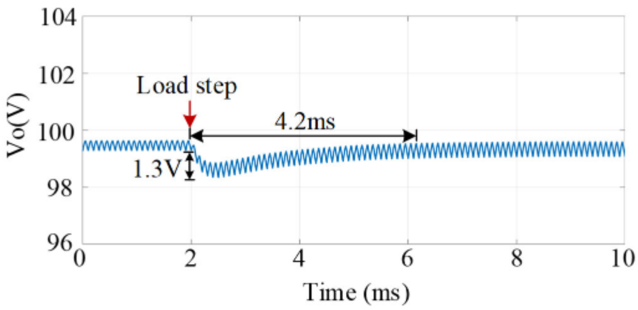

Fig. 7. Output voltage with load step from 75Ω to 50Ω with proposed MPC algorithm.

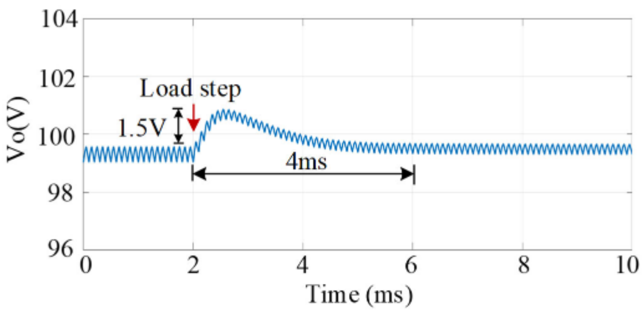

Fig. 8. Output voltage with load step from 50Ω to 75Ω with proposed MPC algorithm.

Finally, the input voltage transient performance is provided. Fig. 9-Fig. 10 show the output voltage when input voltage changes from 50V to 60 V and 60V to 50V respectively.

As seen, the proposed MPC controlled boost converter can adjust to its stable state when the input voltage changes. Besides, the output voltage can adjust into a stable state with a few switching cycles. And the overshoot which is about 0.5V to 1V, and it proves that the system can prevent large overvoltage in the dynamic process.

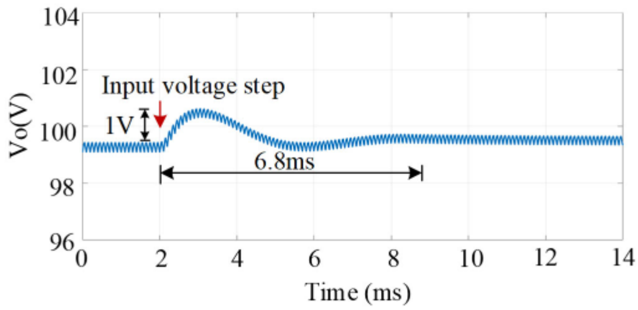

Fig. 9. Output voltage with input voltage step from 50V to 40V with the proposed MPC algorithm.

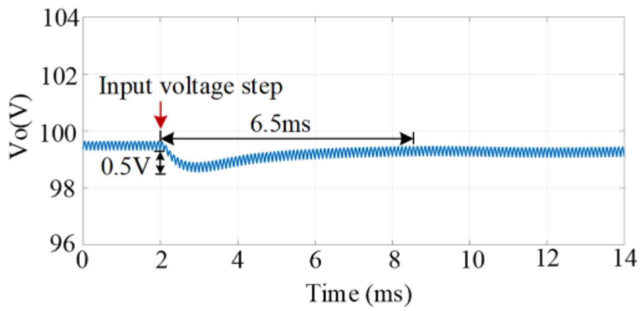

Fig. 10. Output voltage with input voltage step from 40V to 50V with the proposed MPC algorithm.

V. Conclusion

This paper proposes an improved MPC algorithm for the DC-DC boost converter. The key novelties are solving the optimization within one prediction horizon with a fixed switching frequency and providing a stability analysis method. The fixed switching frequency is realized by utilizing the duty cycle as a control variable. Besides, the proposed algorithm only costs one prediction horizon and has less complexity via changing the control objective. Finally, the Jacobian matrix is carried out for the stability assessment. In the end, the results show the proposed algorithm owns better stability. Simulations verify the effectiveness of the above theory. Based on this, the proposed MPC algorithm is proved to be well applied with a DC-DC boost converter.

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