Simple uncertainty budget and assessment with the Kragten method: Examples for building physics

Johra, Hicham

DOI (link to publication from Publisher):
10.54337/aau633631860

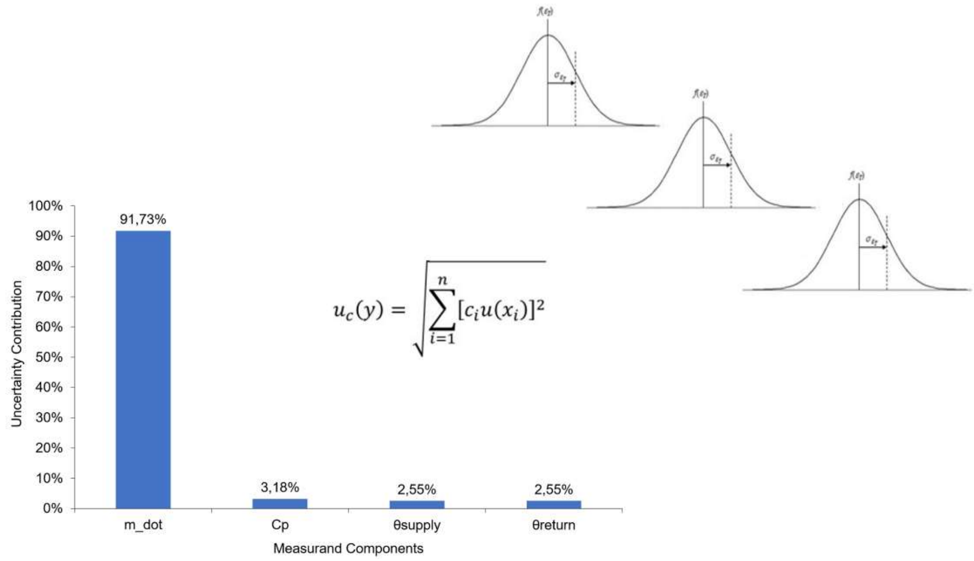
Publication date:
2024

Document Version
Publisher's PDF, also known as Version of record

DEPARTMENT OF THE BUILT ENVIRONMENT
AALBORG UNIVERSITY

Simple uncertainty budget and assessment with the Kragten method: Examples for building physics

Hicham Johra



ISSN 1901-7286
DCE Lecture Notes No. 85

Aalborg University
Department of the Built Environment
Division of Sustainability, Energy & Indoor Environment

DCE Lecture Notes No. 85

Simple uncertainty budget and assessment with the Kragten method: Examples for building physics

by

Hicham Johra

January 2024

Scientific Publications at the Department of the Built Environment

Technical Reports are published for timely dissemination of research results and scientific work carried out at the Department of the Built Environment at Aalborg University. This medium allows publication of more detailed explanations and results than typically allowed in scientific journals.

Technical Memoranda are produced to enable the preliminary dissemination of scientific work by the personnel of the Department of the Built Environment where such release is deemed to be appropriate. Documents of this kind may be incomplete or temporary versions of papers—or part of continuing work. This should be kept in mind when references are given to publications of this kind.

Contract Reports are produced to report scientific work carried out under contract. Publications of this kind contain confidential matter and are reserved for the sponsors and the Department of the Built Environment. Therefore, Contract Reports are generally not available for public circulation.

Lecture Notes contain material produced by the lecturers at the Department of the Built Environment for educational purposes. This may be scientific notes, lecture books, example problems or manuals for laboratory work, or computer programs developed at the Department of the Built Environment.

Theses are monograms or collections of papers published to report the scientific work carried out at the Department of the Built Environment to obtain a degree as either PhD or Doctor of Technology. The thesis is publicly available after the defence of the degree.

Latest News is published to enable rapid communication of information about scientific work carried out at the Department of the Built Environment. This includes the status of research projects, developments in the laboratories, information about collaborative work and recent research results.

Published 2024 by
Aalborg University
Department of the Built Environment
Thomas Manns Vej 23
DK-9220 Aalborg Ø, Denmark

Printed in Aalborg at Aalborg University

ISSN 1901-7286
DCE Lecture Notes No. 85

Table of Contents

1. Foreword

The aim of this lecture note is to present and exemplify the Kragten method [1] to calculate the combined uncertainty (uncertainty budget) of a measurand from the standard uncertainty estimates of individual inputs of that measurand, and the mathematical formulation of that measurand. If these two elements are not available, the Kragten method cannot be applied. The method also provides sensitivity (significance) assessment of the different components (inputs) in the combined uncertainty budget. The Kragten method for uncertainty calculation is very simple yet a robust and accurate alternative to the more complex GUM or Monte Carlo simulation methods. It can be performed with a simple spreadsheet tool (e.g., MS Excel) with minimum risks of mistakes. This method is adequate for the field of building physics, energy in buildings and indoor environmental engineering.

This lecture note also provides examples of uncertainty calculations (budgets) for common measurands and metrics in the field of building physics, energy in buildings and indoor environmental engineering. These examples are attached to the present lecture note document (Excel spreadsheet documents).

One should note that this lecture note does not cover the process of estimating the standard uncertainty of the individual inputs of the measurand. Those standard uncertainties should be obtained from technical documentation, models, or estimates from measurements (e.g., 1σ standard deviation of a set of repeated measurements on measurand that is assumed to remain constant over the monitoring period), and converted into standard uncertainties (1σ confidence interval assuming a normal probability distribution or the errors).

2. Introduction to the Kragten method

Stating the uncertainty of a result, simulation, calculation or measurement is crucial in all fields of science and engineering. Indeed, making a decision or drawing conclusions from results might be very different if these results have an uncertainty of ±1% or ±50% (for a given confidence interval). Unfortunatelly, uncertainty statements are not always present in scientific and technical documentation. This might be due to the relative difficulty for most people to grasp this concept and apply formal methods for uncertainty assessment and analysis.

If several methods exist to determine combined uncertainties, many can appear very complex to implement and perform, such as the GUM method (Guide to the Expression of Uncertainty in Measurement) or the Monte Carlo simulation method.

Uncertainty budget is commonly performed according to the first law of propagation of variances (see GUM) with the following equation:

$$u^2(Y) = \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{\partial Y}{\partial x_i}\frac{\partial Y}{\partial x_j}\right) cov(x_i, x_j) \qquad (1)$$

With the measurand *Y*, the squared uncertainty of this measurand $u^2(Y)$, and $u^2(x_i)$, the squared uncertainty of the different measurand's input variables x_i in the measurand's mathematical formulation (for a 1σ confidence interval and assuming a normal distribution for the uncertainty of each input variable). The measurand's uncertainty $u(Y)$ is evaluated by combining the uncertainties of each variable x_i and the covariance $cov(x_i,x_j)$ between each correlated variable. If all the input variables x_i and x_j are independent, the covariance terms are zero: $cov(x_i,x_j) = 0$.

However, this approach can present some limitations if the measurand's mathematical formulation has some strong non-linearity. Moreover, if the measurand's formulation is very complex and the variables are correlated, the calculation of the partial derivatives of *Y* with regards to x_i ($\partial Y/\partial x_i$ or sensitivity coefficient c_i) and the covariance terms $cov(x_i,x_j)$ can be very difficult and prone to calculation mistakes [2].

To tackle this issue, the Kragten numerical method modifies the propagation law of uncertainties by replacing the sensitivity coefficients c_i corresponding to the partial derivatives of the measurand *Y* by the ratio of the change in the measurand ΔY for a small change Δx_i in one of the input variables by the small change of this input variable x_i:

$$\frac{\partial Y}{\partial x_i} \approx \frac{\Delta Y}{\Delta x_i} \rightarrow u(Y, x_i) = \frac{\partial Y}{\partial x_i} u(x_i) \approx \frac{\Delta Y}{\Delta x_i} u(x_i) \rightarrow u^2(Y, x_i) \approx \left(\frac{\Delta Y}{\Delta x_i}\right)^2 u^2(x_i) \qquad (2)$$

Thus, each uncertainty contribution (component) $u(Y,x_i)$ is calculated between two values of the measurand *Y*: one corresponding to the estimated value x_i and the other one corresponding to the estimated value x_i corrected (addition or subtraction) by the uncertainty $u(x_i)$ of this variable x_i. This approach represents a gradient method similar to the that of the first order finite difference one [2].

With the above transformation, the uncertainty of the measurand can thus be calculated from the propagation of variances in Eq. 1 as a simplified equation:

$$u^2(Y) = \sum_{i=1}^{n} u^2(Y, x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\big(u(Y, x_i)\cdot u(Y, x_j)\big)cov(x_i, x_j) \qquad (3)$$

If assuming that all input variables x_i of the measurand's mathematical formulation (function) are non-correlated, the covariance terms $cov(x_i,x_j)$ are zero. Eq. 3 can thus be further simplified as follows:

$$u^2(Y) = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}\right)^2 u^2(x_i) = \sum_{i=1}^{n} u^2(Y, x_i) \approx \sum_{i=1}^{n}\left(\frac{\Delta Y}{\Delta x_i}\right)^2 u^2(x_i) \qquad (4)$$

This approximated numerical method (differentiation) presents the advantage of only requiring the mathematical formula used to compute the measurand uncertainty without complex computation of partial derivatives of covariances. Moreover, this method takes into account the interactions of the variables within the mathematical formulation and non-linearities.

Under these simplification assumptions, the Kragten method is a reliable and simpler way to combine uncertainty from multiple components of a measurand. It is thus a very convenient and accurate alternative to GUM to calculate uncertainty for most scientific and engineering applications with a lower risk of miscalculations or mistakes.

One should not that the Kragten method requires that the mathematical formulation of the measurand is known and can be computed (or approximated). It also assumes that all uncertainties (standard uncertainty of the input variables and combined uncertainty) are set according to a normal probability distribution of the errors. If there is no mathematical formulation of the measurand, one should then consider using the Monte Carlo method.

Cremona et al., (2018) have assessed the validity of the Kragten numerical method against the GUM method and the Monte Carlo simulation method. They concluded that the Kragten method gives results that are very close to those of the GUM and the Monte Carlo methods but with very little computation burden and mathematical complexities [2].

3. Further mathematical demonstration of the Kragten method

As shown above, if the the input variables x_i of the mathematical function $Y=f(x_1, x_2,\dots,x_n)$ for computing the measurand Y are assumed to be non-correlated, then the combined standard uncertainty $u_c(Y)$ of this measurand can be calculated as follows:

$$u_c{}^2(Y) = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}\right)^2 u^2(x_i) = \sum_{i=1}^{n} u^2(Y, x_i)$$

$$\rightarrow u_c(Y) = \sqrt[2]{\sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}\right)^2 u^2(x_i)} = \sqrt[2]{\sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}u(x_i)\right)^2}$$

$$\rightarrow u_c(Y) = \sqrt[2]{\left(\frac{\partial Y}{\partial x_1}u(x_1)\right)^2 + \left(\frac{\partial Y}{\partial x_2}u(x_2)\right)^2 + \cdots + \left(\frac{\partial Y}{\partial x_n}u(x_n)\right)^2}$$

$$\rightarrow u_c(Y) = \sqrt[2]{u^2(Y,x_1) + u^2(Y,x_2) + \cdots + u^2(Y,x_n)}$$

With $u(x_i)$ the standard uncertainty of the input variable x_i for the measurand's function $Y=f(x_i)$.

The uncertainty contribution's significance S [%] of the input variable x_j can thus be computed as:

$$S_j = \frac{u^2(Y,x_j)}{u_c{}^2(Y)} = \frac{\left(\frac{\partial Y}{\partial x_j}u(x_j)\right)^2}{\sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}u(x_i)\right)^2}$$

The uncertainty component (contribution) for the variable x_i is:

$$u(Y,x_i) = \frac{\partial Y}{\partial x_i}u(x_i)$$

If applying the numerical approximation (finite difference) to the partial derivative above, one can express the uncertainty component for the variable x_i as:

$$u(Y,x_i) = \frac{\partial Y}{\partial x_i}u(x_i) \approx \frac{\Delta Y}{\Delta x_i}u(x_i)$$

For the first input variable (component) x_1 of the measurand's formulation $Y=f(x_1, x_2,\dots, x_n)$, the measurand's variation ΔY can expressed as:

$$\Delta Y = f(x_1 + \Delta x_1, x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)$$

For a small variation Δx_1 of the first variable x_1.

The uncertainty component (contribution) for the first variable x_1 can thus be written as:

$$u(Y,x_1) = \frac{\partial Y}{\partial x_1}u(x_1) \approx \frac{\Delta Y}{\Delta x_1}u(x_1) = \frac{f(x_1+\Delta x_1, x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)}{\Delta x_1}u(x_1)$$

To compute this uncertainty component, one should then decide on what is the value of Δx_1, the small variation of the first variable x_1. Since this Δx_1 is ought to be very small compared to the value of x_1, it is proposed to set the former as equal to the uncertainty $u(x_1)$ (which is also supposed to be relatively small) of the variable x_1:

$$\Delta x_1 = u(x_1)$$

The uncertainty component for the first variable x_1 can thus be written as:

$$u(Y,x_1) = \frac{\partial Y}{\partial x_1}u(x_1) \approx \frac{\Delta Y}{\Delta x_1}u(x_1) = \frac{f(x_1+u(x_1), x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)}{u(x_1)}u(x_1)$$

Which can be simplified as follows:

$$u(Y,x_1) \approx \frac{f(x_1+u(x_1), x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)}{\cancel{u(x_1)}}\cancel{u(x_1)}$$

$$\rightarrow u(Y,x_1) \approx f(x_1+u(x_1), x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)$$

Proceeding accordingly for each input of the measurand's formulation, one can thus compute the combined standard uncertainty $u_c(Y)$ of the measurand as:

$$u_c(Y) = \sqrt[2]{u^2(Y,x_1) + u^2(Y,x_2) + \cdots + u^2(Y,x_n)}$$

With:

$$u(Y,x_1) = f(x_1+u(x_1), x_2, \dots, x_n) - f(x_1, x_2, \dots, x_n)$$

$$u(Y,x_2) = f(x_1, x_2+u(x_2), \dots, x_n) - f(x_1, x_2, \dots, x_n)$$

…

$$u(Y,x_n) = f\big(x_1, x_2, \dots, x_n+u(x_n)\big) - f(x_1, x_2, \dots, x_n)$$

Here again, $u(Y,x_i)$ is the uncertainty component (contribution) of the input variable x_i for the measurand Y, $f(x_1, x_2,\dots,x_n)$ is the mathematical formulation of the measurand Y with the input variables $x_1, x_2,\dots,x_n$ (with set given values), each with a set given standard uncertainty $u(x_1), u(x_2),\dots, u(x_n)$, respectively.

4. Overview of the spreadsheet-based Kragten method for uncertainty budget and analysis

This section introduces the overview of the spreadsheet used to perform the Kragten method for uncertainty budget for the measurement of the thermal conductivity in a horizontal insulation layer (see **Figure 1**), followed by a barplot presenting the distribution of the contribution (significance) of each component of the measurand (see **Figure 2**).

To conduct this uncertainty budget with the Kragten method, one only needs the mathematical formulation of the measurand, the value of each input in the formulation of the measurand, and the standard uncertainty of each input in the formulation of the measurand (standard uncertainty for 1σ: covering factor k = 1).

In the present case of the uncertainty budget for the measurement of the thermal conductivity, the components' standard uncertainties are as follows:

- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the total heat flow: 0.5 W
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the parasitic heat flow: 0.28 W
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the thickness of the insulation layer: 0.01 m
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the length of the insulation layer: 0.02 m
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the width of the insulation layer: 0.02 m
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the insulation temperature on the hot side: 0.04 °C
- Standard uncertainty (1σ confidence interval; covering factor k = 1) for the insulation temperature on the cold side: 0.18 °C

The combined expanded uncertainty of the thermal conductivity is thus 0.041 W/m.K (3σ confidence interval; coverage factor k=3) (see **Figure 1**). In addition, one can see in **Figure 2** that the uncertainty of the total heat flow measurement is the most significant contributor to the overall combined uncertainty of the thermal conductivity measurement.

The calculation of this example with the Kragten method can be found in the spreadsheets attached to that document.

The uncertainty of the different measurand's components could be obtained directly from technical documentation, empirical data (estimate of measurement uncertainty from the standard deviation of multiple consecutive measurements) or simulation data in which each component can be varied independently of all the other ones.

Quantification of combined uncertainty using the Kragten spreadsheet method

Definition of the measurand:
λ: Thermal conductivity [W/m.K]

Mathematical expression of the measurand: $\lambda = \frac{(Q_{meas} - Q_{para}) \times d}{L \times W \times (\theta_{hot} - \theta_{cold})}$

Definition of terms (measurant components):

Q_{meas}: *Measured total heat flow [W]*
Q_{para}: *Parasitic heat flow [W]*
d: *Thickness of the insulation [m]*
L: *Length of the insulation [m]*
W: *Width of the insulation [m]*
θ_{hot}: *Insulation temperature on the hot side [°C]*
θ_{cold}: *Insulation temperature on the cold side [°C]*

(green)	Inputs from the user
(beige)	Component value
(grey)	Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18

Measurand evaluation with the incremented component variables

4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

Difference between measurand value and measurand evaluated with the incremented component variables

-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

$(\text{Difference})^2$

1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)
λ	W/m.K	4,112E-02	1,51E-03	3,7%	2,30E-06

Contribution (Significance) of the different components to the squared combined standard uncertainty (u_c^2) of the measurand

Q_{meas}	Q_{para}	d	L	W	θ_{hot}	θ_{cold}	Sum
51,0%	16,0%	20,5%	1,4%	2,2%	0,4%	8,4%	100%

Measurand	Measurand unit [S.I.]	Value of the measurand	Coverage factor: k	Combined expanded uncertainty (3σ; k=3) of measurand	Relative expanded uncertainty (3σ; k=3) of measurand
λ	W/m.K	4,112E-02	3	4,54E-03	11,1%

The thermal conductivity λ is 0,0411202008397178 W/m.K ± 0,00454488311733916 W/m.K (3σ confidence interval; coverage factor k=3)

Figure 1: *Example of uncertainty budget with the spreadsheet Kragten method (thermal conductivity measurement).*

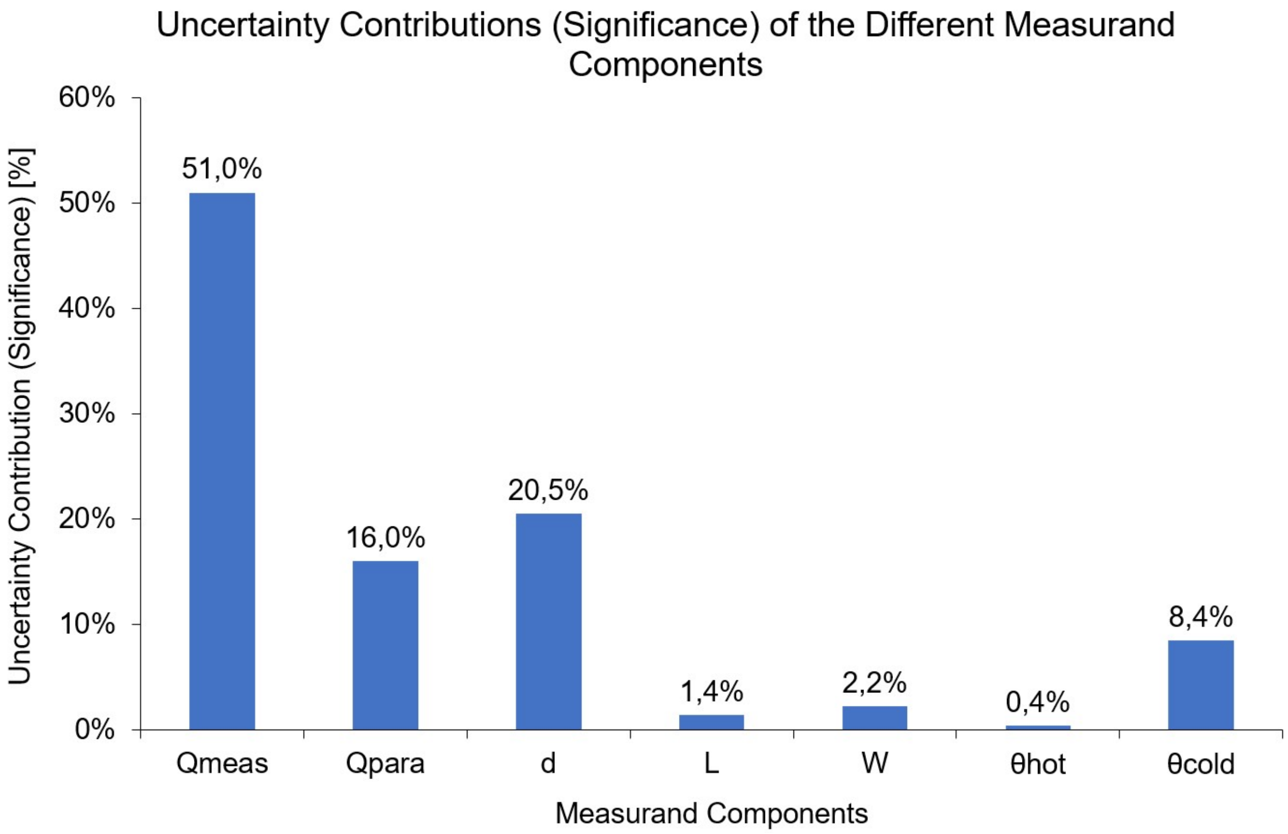


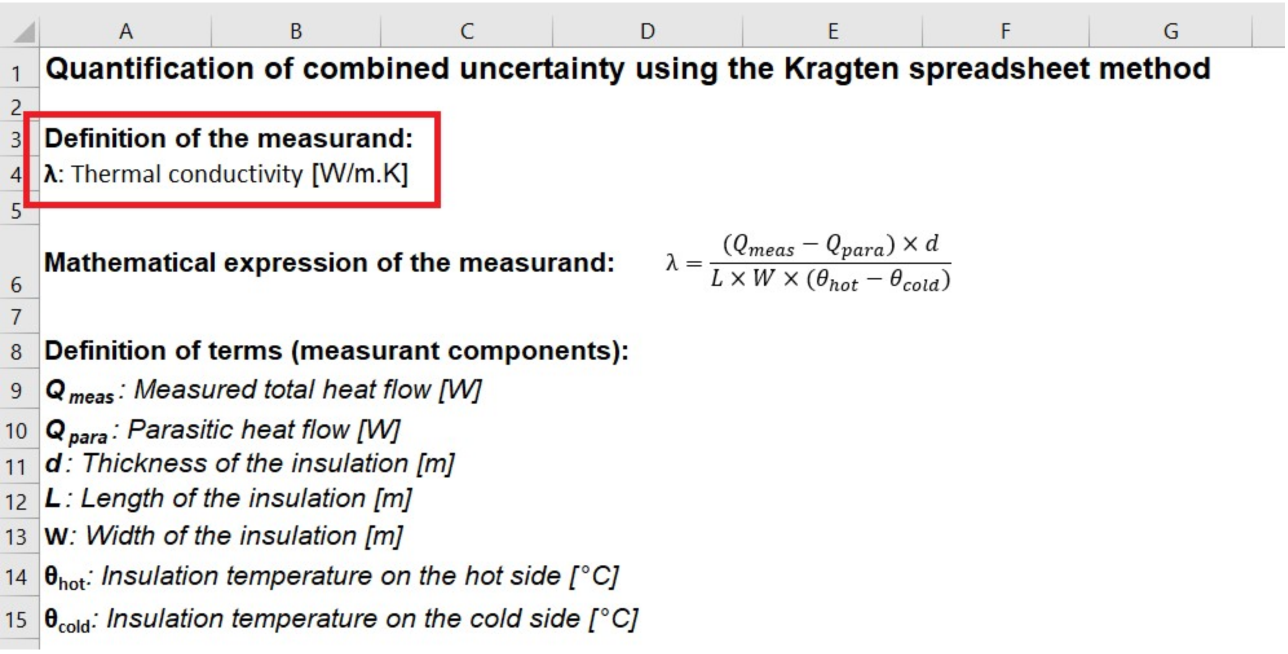
Figure 2: *Example of uncertainty contribution (significance) analysis of the different measurand components with the Kragten method (thermal conductivity measurement).*

5. Step-by-step guidelines for using the spreadsheet-based Kragten method for uncertainty budget and analysis

To perform an uncertainty budget with the Kragten method, one is encouraged to re-use and modify one of the example spreadsheets attached to the present document and adapt it accordingly to the formulation of the measurand under consideration. Follow the next steps to modify the spreadsheet and perform the uncertainty budget correctly. Here, the uncertainty budget for measuring thermal conductivity in an insulation layer is used as an illustrative example.

1/ Copy one of the Excel spreadsheet documents and rename it adequately to reflect the measurand for which the uncertainty budget is computed.

2/ Open the spreadsheet and select the first "Uncertainty budget" sheet. On the top of the sheet, indicate the definition of the measurand with its symbol, full descriptive name, and unit (S.I. unit).


Quantification of combined uncertainty using the Kragten spreadsheet method

Definition of the measurand:
λ: Thermal conductivity [W/m.K]

Mathematical expression of the measurand: $$\lambda = \frac{(Q_{meas} - Q_{para}) \times d}{L \times W \times (\theta_{hot} - \theta_{cold})}$$

Definition of terms (measurant components):
Q_{meas}: *Measured total heat flow [W]*
Q_{para}: *Parasitic heat flow [W]*
d: *Thickness of the insulation [m]*
L: *Length of the insulation [m]*
W: *Width of the insulation [m]*
θ_{hot}: *Insulation temperature on the hot side [°C]*
θ_{cold}: *Insulation temperature on the cold side [°C]*

3/ Indicate the mathematical formulation/expression of the measurand with appropriate symbols for each of its inputs.

Quantification of combined uncertainty using the Kragten spreadsheet method

Definition of the measurand:

λ: Thermal conductivity [W/m.K]

Mathematical expression of the measurand: $\lambda = \frac{(Q_{meas} - Q_{para}) \times d}{L \times W \times (\theta_{hot} - \theta_{cold})}$

Definition of terms (measurant components):

$\mathbf{Q_{meas}}$: *Measured total heat flow [W]*

$\mathbf{Q_{para}}$: *Parasitic heat flow [W]*

d: *Thickness of the insulation [m]*

L: *Length of the insulation [m]*

W: *Width of the insulation [m]*

$\boldsymbol{\theta_{hot}}$: *Insulation temperature on the hot side [°C]*

$\boldsymbol{\theta_{cold}}$: *Insulation temperature on the cold side [°C]*

4/ Indicate the corresponding symbol, definition, and unit (S.I. unit) of each input present in the mathematical formulation of the measurand. There is no need to include mathematical constants (e.g., “2” or “pi” or “e”) which do not have any uncertainty on their value (or are assumed not to have any uncertainty or the latter being negligible).

Quantification of combined uncertainty using the Kragten spreadsheet method

Definition of the measurand:

λ: Thermal conductivity [W/m.K]

Mathematical expression of the measurand: $\lambda = \frac{(Q_{meas} - Q_{para}) \times d}{L \times W \times (\theta_{hot} - \theta_{cold})}$

Definition of terms (measurant components):

$\mathbf{Q_{meas}}$: *Measured total heat flow [W]*

$\mathbf{Q_{para}}$: *Parasitic heat flow [W]*

d: *Thickness of the insulation [m]*

L: *Length of the insulation [m]*

W: *Width of the insulation [m]*

$\boldsymbol{\theta_{hot}}$: *Insulation temperature on the hot side [°C]*

$\boldsymbol{\theta_{cold}}$: *Insulation temperature on the cold side [°C]*

5/ Move on to the computation matrix below.

	Inputs from the user
	Component value
	Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
		↓	→					↓			

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

↓

Difference between measurand value and measurand evaluated with the incremented component variables						
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

↓

$(\text{Difference})^2$						
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

6/ Adjust the number of lines to accommodate all components (inputs) defined above in the measurand's mathematical formulation: remove or insert new lines.

	Inputs from the user
	Component value
	Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
		↓	→					↓			

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

↓

Difference between measurand value and measurand evaluated with the incremented component variables						
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

↓

$(\text{Difference})^2$						
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

7/ Add the corresponding component (input) symbols, unit (S.I. unit), the value of the input/component, and value of the standard uncertainty corresponding to a 1σ (coverage factor k=1) confidence interval. The relative standard uncertainty [%] is simply calculated as the ratio of the standard uncertainty of a given component variable by the value of this component.

Inputs from the user
Component value
Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

Difference between measurand value and measurand evaluated with the incremented component variables						
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

$(\text{Difference})^2$						
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

8/ Then move on to the computation matrix to the right.

Inputs from the user
Component value
Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

Difference between measurand value and measurand evaluated with the incremented component variables						
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

$(\text{Difference})^2$						
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

9/ Adjust the number of columns in the matrix for it to stay a squared matrix (same number of rows and columns.

16													
17		Inputs from the user											
18		Component value											
19		Incremented component variable: component value + standard uncertainty											
20													
21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW		$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00		21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00		2,00	2,00
24	*d*	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60		0,60	0,60
25	*L*	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53		4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62		3,60	3,60
27													
28	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00		22,04	22,00
29	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00		5,00	5,18
30			↓	→						↓			
31						Measurand evaluation with the incremented component variables							
32						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02		4,102E-02	4,156E-02
33										↓			
34						Difference between measurand value and measurand evaluated with the incremented component variables							
35						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04		9,653E-05	-4,400E-04
36										↓			
37						$(\text{Difference})^2$							
38						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08		9,32E-09	1,94E-07

10/ Copy-paste the grey cells on the matrix diagonal to complete the newly created diagonal cells. The diagonal cells should be the sum of the component value and the standard uncertainty of that component for the corresponding row. The formula of the cell conserves the column number/letter but not the row number.

SUM | =$C27+$D27

	A	B	C	D	E	F	G	H	I	J	K	L	M
16													
17		Inputs from the user											
18		Component value											
19		Incremented component variable: component value + standard uncertainty											
20													
21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW		$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00		21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00		2,00	2,00
24	*d*	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60		0,60	0,60
25	*L*	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53		4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62		3,60	3,60
27											$D27		
28	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00		22,04	22,00
29	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00		5,00	5,18
30			↓	→						↓			
31						Measurand evaluation with the incremented component variables							
32						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02		4,102E-02	4,156E-02
33										↓			
34						Difference between measurand value and measurand evaluated with the incremented component variables							
35						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04		9,653E-05	-4,400E-04
36										↓			
37						$(\text{Difference})^2$							
38						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08		9,32E-09	1,94E-07

11/ Complete all the other non-diagonal cells of the matrix as a copy of the component value for the corresponding row.

SUM =$C26

Inputs from the user
Component value
Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW		$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00		21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00		2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60		0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53		4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	=$C26	3,60	3,60
										0,00		
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00		22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00		5,00	5,18

↓ →

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02		4,102E-02	4,156E-02

↓

Difference between measurand value and measurand evaluated with the incremented component variables							
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04		9,653E-05	-4,400E-04

↓

$(Difference)^2$							
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08		9,32E-09	1,94E-07

12/ All yellow cells should thus contain a copy of the component's value. All grey cells should contain the sum of the component value and the standard uncertainty of that component for the corresponding row.

Inputs from the user
Component value
Incremented component variable: component value + standard uncertainty

Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18

↓ →

Measurand evaluation with the incremented component variables						
4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

↓

Difference between measurand value and measurand evaluated with the incremented component variables						
-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04

↓

$(Difference)^2$						
1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

13/ Indicate the correct column name on the top of the computation matrix: it corresponds to the content of the grey cell in the corresponding column and indicates that it is the sum of the corresponding component (in this example, “L” is the length) and a small variation of that component (in this example, “δL” a small variation of the length), with this small variation of the component being equal to the uncertainty of that component.

16		
17		Inputs from the user
18		Component value
19		Incremented component variable: component value + standard uncertainty
20		

21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
24	d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
25	L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
27	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
28	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
29			↓	→					↓			
30						Measurand evaluation with the incremented component variables						
31						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02
32									↓			
33						Difference between measurand value and measurand evaluated with the incremented component variables						
34						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04
35									↓			
36						$(\text{Difference})^2$						
37						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

14/ Under the computation matrix, the zone entitled “Measurand evaluation with the incremented component variables” is the implementation of the measurand’s mathematical formulation using the values of the component above it as inputs to compute the measurand value.

16		
17		Inputs from the user
18		Component value
19		Incremented component variable: component value + standard uncertainty
20		

21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
24	d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
25	L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
27	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
28	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
29			↓	→					↓			
30						Measurand evaluation with the incremented component variables						
31						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02
32									↓			
33						Difference between measurand value and measurand evaluated with the incremented component variables						
34						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04
35									↓			
36						$(\text{Difference})^2$						
37						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07

15/ Pay particular attention to the correct formulation implementation in the Excel sheet.

=(F22-F23)*F24/(F25*F26*(F27-F28))

tandard uncertainty

F	G	H	I	J	K	L
$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
21,50	21,00	21,00	21,00	21,00	21,00	21,00
2,00	2,28	2,00	2,00	2,00	2,00	2,00
0,60	0,60	0,61	0,60	0,60	0,60	0,60
4,53	4,53	4,53	4,55	4,53	4,53	4,53
3,60	3,60	3,60	3,60	3,62	3,60	3,60
22,00	22,00	22,00	22,00	22,00	22,04	22,00
5,00	5,00	5,00	5,00	5,00	5,00	5,18
Measurand evaluation with the incremented component variables						
F28))	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02

16/ Move downleft to the cell corresponding to the "Value of the measurand" and implement the measurand's mathematical formulation there using only the value of the component variables (without any addition of any uncertainty. Ensure the measurand's formulation implementation is the same as in the previous step.

SUM =(C22-C23)*C24/(C25*C26*(C27-C28))

	A	B	C	D	E	F	G	H	I	J	K	L
21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	d+δd	L+δL	W+δW	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
24	d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
25	L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
27	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
28	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
29			↓	→					↓			
30						Measurand evaluation with the incremented component variables						
31						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02
32									↓			
33						Difference between measurand value and measurand evaluated with the incremented component variables						
34						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04
35									↓			
36						$(Difference)^2$						
37						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07
38			↓						↓			
39	Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum($Difference^2$)						
40	λ	W/m.K	C28))	1,51E-03	3,7%	2,30E-06						

17/ For each column of the computation matrix, under the corresponding “Measurand evaluation with the incremented component variables” cell, one can find the “Difference between measurand value and measurand evaluated with the incremented component variables” cell. Make sure that all cells are completed.

SUM | × ✓ fx | `=$C$40-G31`

	A	B	C	D	E	F	G	H	I	J	K	L
21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
24	d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
25	L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
27	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
28	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
29			↓	→					↓			
30						Measurand evaluation with the incremented component variables						
31						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02
32									↓			
33						Difference between measurand value and measurand evaluated with the incremented component variables						
34						-1,082E-03	`=$C$40-G31`	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04
35									↓			
36						$(\text{Difference})^2$						
37						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07
38			↓						↓			
39	Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)						
40	λ	W/m.K	4,112E-02	1,51E-03	3,7%	2,30E-06						
41			↓	↓	←				↓			

18/ All differences between the measurand evaluated with the variable values and evaluated with incremented component variables are then squared and summed up. Make sure that all “(Difference)2) cells are completed and that the “Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)” encompasses all cells of the “(Difference^2)”.

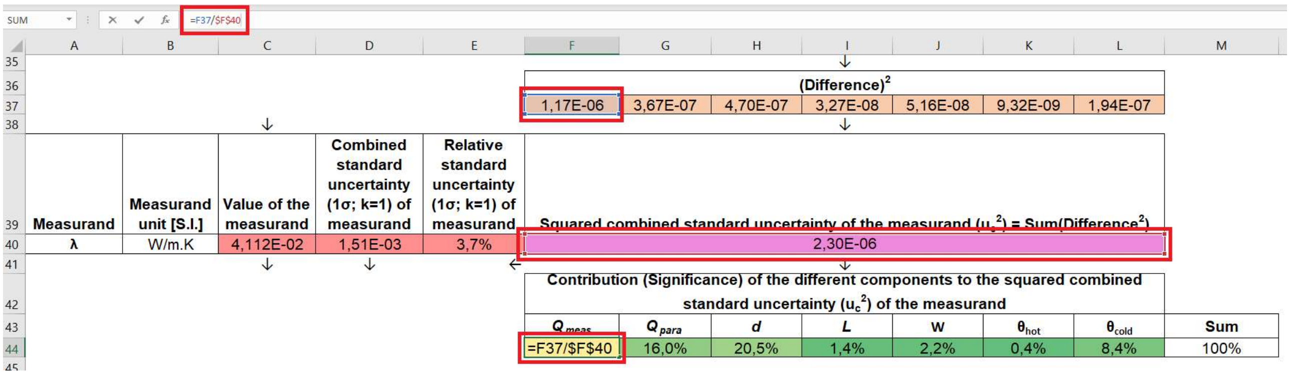
	A	B	C	D	E	F	G	H	I	J	K	L
21	Component	Component unit [S.I.]	Value of component variable	Standard uncertainty (1σ; k=1) of component variable	Relative standard uncertainty (1σ; k=1) of component	$Q_{meas}+\delta Q_{meas}$	$Q_{para}+\delta Q_{para}$	$d+\delta d$	$L+\delta L$	$W+\delta W$	$\theta_{hot}+\delta\theta_{hot}$	$\theta_{cold}+\delta\theta_{cold}$
22	Q_{meas}	W	21,00	0,50	2,4%	21,50	21,00	21,00	21,00	21,00	21,00	21,00
23	Q_{para}	W	2,00	0,28	14,0%	2,00	2,28	2,00	2,00	2,00	2,00	2,00
24	d	m	0,6	0,01	1,7%	0,60	0,60	0,61	0,60	0,60	0,60	0,60
25	L	m	4,53	0,02	0,4%	4,53	4,53	4,53	4,55	4,53	4,53	4,53
26	W	m	3,6	0,02	0,6%	3,60	3,60	3,60	3,60	3,62	3,60	3,60
27	θ_{hot}	°C	22	0,04	0,2%	22,00	22,00	22,00	22,00	22,00	22,04	22,00
28	θ_{cold}	°C	5	0,18	3,6%	5,00	5,00	5,00	5,00	5,00	5,00	5,18
29			↓	→					↓			
30						Measurand evaluation with the incremented component variables						
31						4,220E-02	4,051E-02	4,181E-02	4,094E-02	4,089E-02	4,102E-02	4,156E-02
32									↓			
33						Difference between measurand value and measurand evaluated with the incremented component variables						
34						-1,082E-03	6,060E-04	-6,853E-04	1,807E-04	2,272E-04	9,653E-05	-4,400E-04
35									↓			
36						$(\text{Difference})^2$						
37						1,17E-06	3,67E-07	4,70E-07	3,27E-08	5,16E-08	9,32E-09	1,94E-07
38			↓						↓			
39	Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)						
40	λ	W/m.K	4,112E-02	1,51E-03	3,7%	2,30E-06						
41			↓	↓	←				↓			

19/ Under the "Squared combined standard uncertainty of the measurand", one can find the zone where the contribution (significance) of the different components is calculated. Ensure each component's symbol is included in the same order as in the computation matrix.

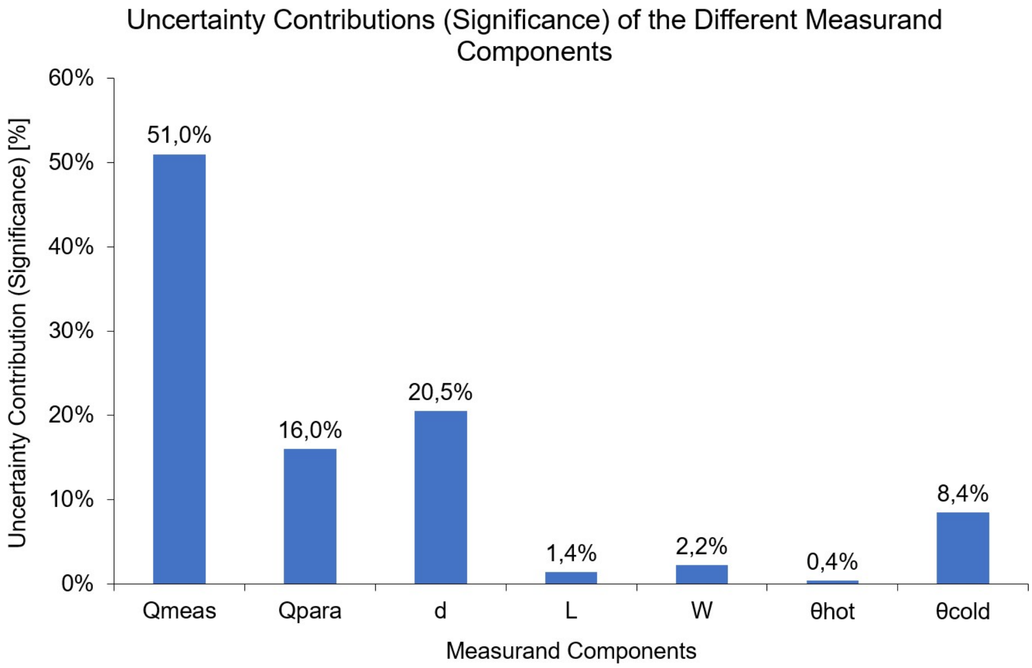
Contribution (Significance) of the different components to the squared combined standard uncertainty (u_c^2) of the measurand							
Q_{meas}	Q_{para}	d	L	W	θ_{hot}	θ_{cold}	Sum
51,0%	16,0%	20,5%	1,4%	2,2%	0,4%	8,4%	100%

20/ In the cells under, each component's contribution (significance) in the total combined uncertainty of the measurand is evaluated. Each component's contribution is calculated as the squared difference of the measurand evaluation with and without the component increment divided by the total squared combined standard uncertainty of the measurand.

Make sure that all cells are completed and that the sum of the contributions is equal to 100%.



21/ These contributions (significance) of the different components are also represented in the barplot of the second sheet in the spreadsheet.



22/ On the left of the spreadsheet, one can find the combined standard uncertainty for the measurand, which corresponds to the square root of the "Squared combined standard uncertainty of the measurand".

Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)
λ	W/m.K	4,112E-02	1,51E-03	3,7%	2,30E-06

Contribution (Significance) of the different components to the squared combined standard uncertainty (u_c^2) of the measurand

Q_{meas}	Q_{para}	d	L	w	θ_{hot}	θ_{cold}	Sum
51,0%	16,0%	20,5%	1,4%	2,2%	0,4%	8,4%	100%

23/ At the bottom of the spreadsheet, one can find the final results of the uncertainty budget: the value of the measurand, followed by the coverage factor "k" of the confidence interval for which the uncertainty is calculated, followed by the combined expanded uncertainty of the measurand (expanded to the coverage factor "k=3" or "3σ") and followed by the relative expanded uncertainty (the expanded uncertainty divided by the value of the measurand).

Measurand	Measurand unit [S.I.]	Value of the measurand	Combined standard uncertainty (1σ; k=1) of measurand	Relative standard uncertainty (1σ; k=1) of measurand	Squared combined standard uncertainty of the measurand (u_c^2) = Sum(Difference^2)
λ	W/m.K	4,112E-02	1,51E-03	3,7%	2,30E-06

Contribution (Significance) of the different components to the squared combined standard uncertainty (u_c^2) of the measurand

Q_{meas}	Q_{para}	d	L	w	θ_{hot}	θ_{cold}	Sum
51,0%	16,0%	20,5%	1,4%	2,2%	0,4%	8,4%	100%

Measurand	Measurand unit [S.I.]	Value of the measurand	Coverage factor: k	Combined expanded uncertainty (3σ; k=3) of measurand	Relative expanded uncertainty (3σ; k=3) of measurand
λ	W/m.K	4,112E-02	3	4,54E-03	11,1%

The thermal conductivity λ is 0,0411202008397178 W/m.K ± 0,00454488311733916 W/m.K (3σ confidence interval; coverage factor k=3)

24/ Finally, at the bottom of the spreadsheet, one can find the full statement of the measurand and its expanded uncertainty with mention of the confidence interval. Before using that statement in a document, adjust the number of decimal digits in the measurand's value and associated uncertainty to most the significant digit (figure) of the uncertainty (or 2 significant figure is the second significant decimal digit is a 5). E.g., in the present case, the statement with trimmed digits would be "**The thermal conductivity λ is 0.041 W/m.K ±0.0045 W/m.K (3σ confidence interval)**".

Measurand	Measurand unit [S.I.]	Value of the measurand	Coverage factor: k	Combined expanded uncertainty (3σ; k=3) of measurand	Relative expanded uncertainty (3σ; k=3) of measurand
λ	W/m.K	4,112E-02	3	4,54E-03	11,1%

The thermal conductivity λ is 0,0411202008397178 W/m.K ± 0,00454488311733916 W/m.K (3σ confidence interval; coverage factor k=3)

6. List of uncertainty analysis examples provided with this document

6 examples of Kragten uncertainty budgets are provided in 6 Excel spreadsheets attached with this lecture note. These examples are for common calculations in the field of building physics and can be reused or modified directly to fit many purposes:

- Uncertainty budget for density measurement of a large sample. File name: *Kragten_uncertainty_budget_density_large_sample.xlsx*
- Uncertainty budget for effective air permeability measurement. File name: *Kragten_uncertainty_budget_effective_air_permeability.xlsx*
- Uncertainty budget for heat losses calculation. File name: *Kragten_uncertainty_budget_heat_losses.xlsx*
- Uncertainty budget for power measurement of a hydronic heating or cooling system. File name: *Kragten_uncertainty_budget_power_hydronic_system.xlsx*
- Uncertainty budget for Rayleigh number calculation. File name: *Kragten_uncertainty_budget_Rayleigh_number.xlsx*
- Uncertainty budget for thermal conductivity measurement. File name: *Kragten_uncertainty_budget_thermal_conductivity.xlsx*

7. Further reading and resources

The Kragten method is very well introduced by Ivo Leito in the following video: https://youtu.be/qRx8cFVitgk?si=GIAUN3BSBV48ZK3h

Further videos on the topic of estimation of measurement uncertainty can be found in the following video playlist: https://www.youtube.com/@estimationofmeasurementunc1399

References

[1] J. Kragten (1994). Calculating Standard Deviations and Confidence Intervals with a Universally Applicable Spreadsheet Technique. Analyst 119(10), 2161-2165. https://doi.org/10.1039/an9941902161

[2] P. Cremona, T. Rogaume, F. Richard, B. Batiot (2018). Application of the Kragten method in order to evaluate the uncertainty of the heat release rate determination using of the cone calorimeter. Proceedings of the 3rd European Symposium on Fire Safety Science, IOP Conference Series: Journal of Physics 1107, 032019. https://doi.org/10.1088/1742-6596/1107/3/032019

Recent publications in the Lecture Note Series

Hicham Johra. Simple data pre-processing of the laser flash analysis results from the LFA 447 apparatus. DCE Lecture Notes No. 72. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Description of the laser flash analysis method for thermal diffusivity measurement with the LFA 447. DCE Lecture Notes No. 73. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Guide to manually refill the liquid nitrogen tank of the LFA 447 apparatus. DCE Lecture Notes No. 74. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Description of the Guarded Hot Plate Method for Thermal Conductivity Measurement with the EP500. DCE Lecture Notes No. 75. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Thermophysical Properties of Building Materials: Lecture Notes. DCE Lecture Notes No. 76. Department of Civil Engineering, Aalborg University, 2019.

Hicham Johra. Assembling temperature sensors: thermocouples and resistance temperature detectors RTD (Pt100). Lecture Notes No. 78. Department of Civil Engineering, Aalborg University, 2020.

Hicham Johra. Overview of the Coefficient of Performance (COP) for conventional vapour-compression heat pumps in buildings. Lecture Notes No. 79. Department of the Built Environment, Aalborg University, 2022.

Martin Veit, Hicham Johra. How to use Arduino UNO for low-cost temperature measurements with Pt 100 sensors. DCE Lecture Notes No. 80. Department of the Built Environment, Aalborg University, 2022.

Hicham Johra. What is building energy flexibility – demand response? DCE Lecture Notes No. 81. Department of the Built Environment, Aalborg University, 2023.

Hicham Johra. Air permeameter for porous building materials: Aalborg University prototype 2023. DCE Lecture Notes No. 84. Department of the Built Environment, Aalborg University, 2023.

ISSN 1901-7286
DCE Lecture Notes No. 85