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Probabilistic Modelling of Information Propagation in Wireless Mobile Ad-Hoc Networks

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Abstract—**In this paper the dynamics of broadcasting wireless ad-hoc networks is studied through probabilistic modelling. A randomized transmission discipline is assumed in accordance with existing MAC definitions such as WLAN with *Decentralized Coordination* or IEEE-802.15.4. Message reception is assumed to be governed by node power-down policies and is equivalently assumed to be randomized. Altogether randomization facilitates a probabilistic model in the shape of an integro-differential equation governing the propagation of information, where brownian node mobility may be accounted for by including an extra *diffusion term*. The established model is analyzed for transient behaviour and a travelling wave solution facilitates expressions for propagation speed as well as parametrized analysis of network reliability and node power consumption. Applications of the developed models for node localization and network dimensioning are discussed at the end of the paper.**

Keywords: ad-hoc networks, broadcasting, probabilistic modelling, integro differential equations.

1 Introduction

Wireless ad-hoc networks have received much attention over the past decade; a number of technologies such as WLAN, Bluetooth, and latest ZigBee (based on the IEEE802.15.4 standard) have emerged facilitating their practical use. Various principles for routing and scheduling [1] have been studied and real life applications have been suggested within fields like personal communication, cooperative robot systems [3] and sensor networks [2].

Within routing a taxonomi of principles have been cemented in the literature ranging from static routing over hybrid mechanism to highly dynamic almost stateless protocols like flooding (see [1] for an overview).

Information propagation in ad-hoc networks resembles rumour and epidemic spreadings in complex sociologic networks. Such phenomena have traditionally been modelled by partial differential equations on random graphs or regular lattices (see e.g. [4]) or on planar continuum (see e.g. [5]).

In this paper a model for a spatial continuum is developed for communication in wireless mobile ad-hoc networks. The derived model accounts for radio signal fading, local message discard and station mobility. The developed model is accompanied by examples of applications of this type of models along with perspectives on the possibilities for enhancing the model within the presented framework.

2 Model assumptions

The following section specifies details of model assumptions, such as the distribution of nodes in the network, information propagation by radio transmission, discard of messages from memory and node mobility. All of the above model aspects are assembled into a compound model at the end of the section.

2.1 Node distribution

In wireless ad-hoc networks nodes may be distributed randomly over some domain D or they may be deployed according to some predefined plan. Additionally nodes may be stationary or mobile, e.g. a subset of nodes may belong the a fixed infrastructure, whereas a majority of nodes would be mobile as for example in an animal household system with radio based health monitoring or in populations of mobile robots.

A model of node distribution should account for both deterministic deployment and random distribution. We assign to each node i the time dependent probability measure L_i of location, i.e. $L_i(A,t)$ expresses the probability that node i is located within the subset A at time t.

Adding up for the entire set of nodes yields the additive positive measure L, i.e.

$$L(A,t) = \sum_i L_i(A,t) \tag{1}$$

where $L(A,t)$ expresses the expected number of nodes within A at time t.

2.2 Radio transmission

In the sequel we develop our model by tracing the lifespan of a particular piece of information/message M from its original birth at $t = x = 0$. The message M is initally generated and subsequently broadcast according to a randomized transmission discipline following a Poisson process. Nodes within range receive M, store it in memory and start broadcasting it, as if it were generated locally. To save battery power, nodes adopt a randomized power down schedule. Upone wake up each node determines whether it is receiving or transmitting. In reception mode nodes keep awake for some time after sleep mode is resumed and maintained for a random period of time. In transmission mode a number of messages kept in memory are broadcast after which sleep mode is resumed. Alltogether we model the transmission of M between two nodes i at x and j at y to happen with probability $\lambda\, K(x,y)\, dt$ within a time interval $[t, t+dt]$, when it is assumed that i holds M at time t and j has not received M previous to t. The intensity λ may depend on a number of factors including; power down schedule, number of messages in memory, collision probability etc., whereas the continuous function $K(x,y)$ accounts for environmental/distance effects on radio signal quality, i.e. it captures the probability that a message transmitted from x is received at y.

2.3 Message discard

At wake up time a number of messages in memory are selected randomly for broadcast. Since a large number of stored messages reduces the broadcast selection probability of each message and since memory is limited, a mechanism for discarding messages is called for. Generally we suggest a stage model, where the state of a message M evolves through a sequence of stages during its stay in each node. A final stage represents discard from memory. In this stage only a message sequence number, associated to the originating node, is kept to prohibit duplication of messages.
Transition between stages takes place according to a stage dependent Poisson process. In this manner any message life time distribution may be approximated. However in the sequel we assume only two stages; an active stage and a discarded stage, i.e. the life time of any message is exponentially distributed with the discard rate α as intensity parameter. Thus a message in the active stage at time t is dicarded with probability $\alpha\ dt$ within $[t, t+dt]$.

2.4 Network model

We denote by $f(x,t)$ the conditional probability that a generic node located at x holds M at time t. Therefore $h(A,t) = \int_A f(\cdot,t)\, dL$ expresses the expected number of nodes in A holding M at time t, whereas $\int_A 1 - f(\cdot,t) dL$ gives the expected number of nodes in A not holding M at time t. Similarly let $g(x,t)$ denote the conditional probability that a generic node located at x, discarded M previous to t. The expected number of nodes receptive to M within A at time t is therefore $\int_A 1 - f(\cdot,t) - g(\cdot,t)\, dL$. Consider two neighbourhoods A and B, where $x \in A \subset D$ and $y \in B \subset D$ then the expected number of successfull transmissions of M between A and B within $[t, t+dt]$ is given by $\int_B 1 - f(\cdot,t) - g(\cdot,t)\, dL \cdot \lambda K(x,y) h(A,t)\, dt$. Overall we consider a partition $\{A_i\}$ of D including atoms $\{p_j\}$ of positive location measure $L(p_j)$, where $x_i \in A_i$.
Including message discard, we have for any small neighbourhood A of some point $x \in D$

$$\begin{aligned} h(A,t+dt) &= h(A,t) - \alpha h(A,t)\, dt + \\ \int_A 1 - f(\cdot,t) &- g(\cdot,t)\, dL \cdot \lambda \sum_j K(x,x_j) h(A_j,t)\, dt \end{aligned} \tag{2}$$

so that

$$\begin{aligned} f(x,t+dt) &= f(x,t) - \alpha f(x,t)\, dt + \\ (1 - f(x,t) &- g(x,t)) \cdot \lambda \sum_j K(x,x_j) h(A_j,t)\, dt \end{aligned} \tag{3}$$

almost everywhere w.r.t. the location measure L. Refining the partition $\{A_i\}$ and letting dt approach zero, gives the following integro differential equation for almost every x (w.r.t. L)

$$\begin{aligned} \dot{f}(x,t) &= -\alpha f(x,t) + \\ (1 - f(x,t) &- g(x,t)) \cdot \lambda \int_D K(x,\eta) f(\eta,t)\, dL(\eta) \end{aligned} \tag{4}$$

Following an equivalent approach we may state for g

$$\dot{g}(x,t) = \alpha\, f(x,t) \tag{5}$$

2.5 Node mobility

Mobility affects information propagation in two ways; it changes node locations over time and it moves information carried by mobile nodes where messages are active. The former effect is considered above whereas the latter is treated subsequently in two stages; a model is created

where information is carried solely by physical movement and secondly this model is included as an additional term in equation 4.
Several models accounting for node mobility exist including determinstic as well as random movement. In this work we consider the case of brownian motion, i.e. where positional differences remain independent and gaussian with zero mean and variance vt, where v is the normalized speed and t is the elapsed time.
For an initial location measure $L_i(\cdot, 0)$, brownian motion generally transforms the location measure as follows

$$L_i(A,t) = \int_A \int_D \mathcal{N}(y,x,vt)\ L_i(dy,0)\ dx \qquad (6)$$

where $\mathcal{N}(\cdot, x, vt)$ is the normal density with mean x and variance vt.
Consider some subset $A \subset D$, then without radio transmission the expected number of copies in A at time $t+dt$ is given by

$$\int_A f(x,t+dt)\ L(dx,t+dt) = \qquad (7)$$
$$\int_A \int_D f(y,t)\, \mathcal{N}(y,x,v\ dt)\ L(dy,t)\ dx$$

A standard argument for stochastic differential equations [6] shows that, for uniform L, f is a solution to the diffusion equation

$$\dot{f}(x,t) = v\ \mathbf{tr} f_{XX}(x,t) \qquad (8)$$

where $\mathbf{tr} f_{XX}(x,t)$ denotes the *trace* of the Hessian $f_{XX}(x,t)$. Moreover for any initial location measure $\lim_{t\to\infty} L_i(A,t) = |A|/|D|$. Generally for an absolute continuous location measure $L(\cdot, t)$ with smooth density $f_L(\cdot, t)$, equation 7 yields a bounded time derivative $\dot{f}(x,t)$.

2.6 Compound model

We construct our compound model for a particular set of scenarios; namely where exactly one station S is known to be at x_0 at time $t = 0$, and at that particular time a message M is generated in S. All $N - 1 >> 1$ other stations are assumed to be uniformly distributed on D, so that $L(A,0) = |A|\rho + (x_0 \in A)$. After an arbitrarily small delay Δ radio broadcast happens instantaneously and all stations (including S) broadcast and discard M according to the Poisson processes discussed above.
According to 6 $L(\cdot, \Delta)$ is absolute continuous, w.r.t. Lebesque measure and its associated density $f_L(\cdot, \Delta)$ is smooth. Immediately after the first broadcast f is given by

$$f(x,\Delta) = \int_D K(x,y)\, \mathcal{N}(y,x,v\ dt)\ dy \qquad (9)$$

producing a smooth $f(\cdot, \Delta)$.
From $t = \Delta$ both the radio transmission dynamics 6 and the mobility dynamics 7 yield bounded time derivaties of f. The total expected number $h(A, t+dt)$ of stations holding M within some subset $A \in D$ at time $t + dt$ is therefore the expected number $h(A,t)$ at time t added to the number of copies entering A by radio transmission as well as the copies entering by movement and finally subtracted the number discarded. Altogether a compound model would be

$$\dot{f}(x,t) = -\alpha f(x,t) + (1 - f(x,t) - g(x,t)) \cdot$$
$$\lambda \int_D K(x,\eta) f(\eta,t)\ L(d\eta) + M(x,t) \qquad (10)$$

where M denotes the time derivative of f contributed the mobility dynamics 7.

3 Travelling wave solution

In the previous section a compound model including effects from radio transmission and movement on information propagation. From an initial moment $t = \Delta > 0$ arbitrarily close to the time $t = 0$, where a message M is generated in a station S located at x_0, information propagation dynamics 10, 5 and mobility dynamics 6 govern the evolution of the active state probability f and location measure L. Well defined initial conditions are given in terms of $f(x,\Delta) = K(x_0, x)$ and $L(\cdot, \Delta)$.
After some time location becomes close to uniform, i.e. $\lim_{t\to\infty} f_L(x,t) = \rho$ and information dynamics 10 becomes

$$\dot{f}(x,t) = -\alpha f(x,t) + (1 - f(x,t) - g(x,t)) \cdot$$
$$\lambda\rho \int_D K(x,\eta) f(\eta,t)\ d\eta + v\ \mathbf{tr} f_{XX}(x,t) \qquad (11)$$
$$\dot{g}(x,t) = \alpha\ f(x,t)$$

For $(K(x,y) = K(|x-y|)$, and large $|x|$, we shall anticipate the existence of a travelling wave solution to 11, i.e. $\frac{1}{\alpha}\ g(x,t) = w(ct - |x|)$, where c is the speed of the travelling wave and $|\cdot|$ denotes euclidian distance. Rewriting equation 11 for the *wave shape function* w gives

$$c\ w''(y) = \alpha\ w'(y) - (1 + c\ w'(y) - \alpha\ w(y)) \cdot$$
$$\lambda\rho \int_D K(x,\eta)\ w'(ct - |\eta|)\ d\eta + \qquad (12)$$
$$v\ (w'''(y) + w''(y)\frac{\mathcal{D} - 1}{|x|})$$

where $y = ct - |x|$ and $\mathcal{D}$ is the spatial dimension, which in practical situations is 2 or 3. The last term in 12 contradicts the assumed travelling wave solution. However since this term vanishes for large $|x|$, such a solution it still anticipated for large $|x|$, i.e. far from the point x_0 where M was originally generated. Also we utilize the following approximation valid for large $|x|$

$$\int_D K(x,\eta)\, w'(ct-|\eta|)\, d\eta \approx \int_{-\infty}^{\infty} H(|x|-|\eta|)\, w'(ct-|\eta|)\, d|\eta| \tag{13}$$

where the kernel H in this paper is only conjectured to exist.

The travelling wave speed which may be interpreted as the speed by which a message is carried over long distances.

3.1 Wave speed

We approximate the leading edge of the travelling wave linearly around its value for large negative y, i.e. early and far away where $f \approx 0$ and $g \approx 0$ Thus

$$\begin{aligned} & c\, w''(y) = \qquad (14)\\ & \alpha\, w'(y) + \lambda\rho \int_{-\infty}^{\infty} H(z-y)\, w'(z)\, dz + v\, w'''(y) \end{aligned}$$

which is a second order ODE in w', with two linearly independent exponential solutions. Following the so called linear conjecture [5], the speed of the leading edge defines the speed of the entire wave. The leading edge is dominated by the one exponential solution $l(y) = \exp(\beta\, y)$ with the smallest decay coefficient β. Thus

$$c\,\beta = \alpha - \lambda\rho \int_{-\infty}^{\infty} H(\tau)\ \exp(\beta\tau)\, d\tau + v\,\beta^2 \tag{15}$$

or

$$\begin{aligned} c &= \mathcal{C}(\alpha,\lambda,\rho,H,\beta) \qquad (16)\\ &= \frac{\alpha - \lambda\rho \int_{-\infty}^{\infty} H(\tau)\ \exp(\beta\tau)\, d\tau}{\beta} + v\,\beta \end{aligned}$$

The decay depends highly on initial conditions, why wave speed practically remains undetermined. However a minimum speed may be found from $c_{min}(\alpha,\lambda,\rho,H) = \min_\beta \mathcal{C}(\alpha,\lambda,\rho,H,\beta)$.

Applying the leading edge approximation to 11 and integrating over D yields for $v = 0$

$$\dot{I}(t) = I(t)\,(-\alpha + \lambda\rho \int_D K(x,\eta)\, d\eta\,) \tag{17}$$

where $I(t) = \int_D f(x,t)\, dx$. Since 17 is an approximation from above it states that for $\alpha > \lambda\rho \int_D K(x,\eta)\, d\eta$ a travelling wave solution cannot exist. Conversely we may anitcipate a travelling wave with a minimum speed $c_{min}(\alpha,\lambda,\rho,H)$ in the opposite case.

3.2 Wave shape

By definition $f(x,t)$ denotes the conditional probability that a node located at x at time t holds the message M. Likewise g denotes the probability that the node previously discarded M. Thus $f + g$ is the probability that M was previously received. However since $\lim_{t\to\infty} f(x,t) = 0$ the limit value $P = \lim_{t\to\infty} g(x,t) = \lim_{t\to\infty} \alpha w(y)$. As for the leading edge we use a linear approximation for the trailing edge around P/α, i.e. $w(y) \approx P/\alpha\,(1 - \exp(\gamma\, y))$ for large positive y. The following relation is therefore approximately fulfilled for large y.

$$\begin{aligned} & -c\gamma^2\,\phi = \alpha\gamma\,\phi + \gamma\lambda\rho\,(1 - c\gamma\, P/\alpha\,\phi - P(1-\phi)) \cdot \\ & \int_{-\infty}^{\infty} H(z) \exp(\gamma\, z)\, dz + v\,\gamma^3\phi \qquad (18) \end{aligned}$$

where $\phi = \exp(\gamma\, y)$. For large y, $\phi \approx 0$, so

$$\begin{aligned} P &= \mathcal{P}(v,\alpha,\lambda\rho,c,\gamma) \qquad (19)\\ &= 1 - \frac{v\,\gamma^2 - c\,\gamma + \alpha}{\lambda\,\rho \int_{-\infty}^{\infty} H(z) \exp(\gamma\, z)\, dz} \end{aligned}$$

Since c and γ are determined by initial conditions and therefore in practice undetermined, we settle for a lower bound $P_{min} = \min_{c,\gamma} \mathcal{P}(v,\alpha,\lambda\rho,c,\gamma)$ for P. The smallest upper bound obtainable from 19 is $P = 1$ since the second term of $\mathcal{P}$ vanishes for large γ.

4 Model applications

A compound model for information propagation is developed above, including radio transmission and node mobility.

4.1 Location estimation

A transient compound model is given in terms of the integro-differential equation 10 and the functional equation 6 tracking the time dependent distribution of mobile nodes. Together with the initial conditions put forth in sections 2.5 and 2.6, a particular conditional solution $f_{x_0}(x,t)$ to the propagation dynamics, given message generation at a particular point x_0, may be obtained numerically. Evaluating f_{x_0} at some point y_i gives the conditional reception time density $r_{x_0,i}(t)$ of messages generated at x_0 and received at y_i. From $r_{x_0,i}$ we get reception probabilities $R_{x_0,i}(\tau) = \alpha \int_0^\tau r_{x_0,i}(t)\, dt$. Reception time densities and reception probabilities may serve as the basis for the estimation of x_0, i.e. localization of the position where M was generated as well as for estimating the message generation time τ.

One applicable strategy would record reception times $t_1, .., t_R$ of M from the time t^* where M was first received at a station among $y_1, .., y_R$ until $t^* + T$. If some generation-time density p is available a Bayesian estimate is given by

$$\begin{aligned} &(\tau^*, x^*) = \qquad (20) \\ &arg\max_{\tau,x} p(\tau) \quad \Pi_{i\in R_T} R_{x,i}(t^* + T - \tau) r_{x,i}(t_i - \tau) \\ &\qquad \cdot \Pi_{i\notin R_T}(1 - R_{x,i}(t^* + T - \tau)) \end{aligned}$$

where $R_T \subseteq \{1, .., R\}$ denotes the set of reception nodes where M is received within $[t^*, t^* + T]$. For the estimator 20 it is assumed reception times $\{t_i\}$ are assumed conditionally independent given the message generation location x_0 and generation time τ.

4.2 Network dimensioning

As argued in section 3 a stationary solution for some network parameter settings, i.e. λ, α, ρ and $K(e)$ is anticipated in the shape of a travelling wave, where e denotes electrical energy consumed by each message transmission. The speed c of the wave depends on initial conditions, but a lower bound c_{min} is obtained depending solely on network settings c_{min} is likely to increase with λ, ρ and e and decrease with α. A similar argument leads to a lower bound P for the reception probability at long distances, depending only on network settings. Average energy E spent on relaying some message is given by $E = e\ P\lambda/\alpha$ at long distances. During dimensioning of the ad-hoc network demands on speed, reliability and lifetime are likely to be provided in the shape of upper delay bounds and lower reception probability bounds as well as average battery power. If messages are generated with an average rate μ in a network with N nodes the average power spent for transmission at each station is $\mu(e(1 + \lambda/\alpha) + E)$ taking into account the first and subsequent transmissions at the message generating station. All together the above dependencies reveals, in coherence with intuition, that network dimensioning is a trade off between speed/reliability and battery lifetime.

5 Conclusion

In this paper a probabilistic model is constructed for information propagation in randomized wireless ad-hoc broadcasting networks. The constructed model accounts for radio propagation by radio transmission as well as brownian motion of nodes. A well defined initial value problem is formulated, which may serve as the basis for numerical particular solutions. Also a stationary, travelling wave solution is anticipated, yielding parametric expressions for propagation speed and network reliability. Applications are suggested including a Bayesian approach for location estimation and guidelines for network dimensioning assuming a stationary solution.
Directions for future research include treatment of a variety of mobility models as well as verification of the obtained models through simulation and experimental work.

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