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Behaviour of Cohesionless Soils During Cyclic Loading

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Abstract

Offshore wind turbine foundations are typically subjected to cyclic loading from both wind and waves, which can lead to unacceptable deformations in the soil. However, no generally accepted standardised method is currently available, when accounting for cyclic loading during the design of offshore wind turbine foundations. Therefore a literature study is performed in order to investigate existing research treating the behaviour of cohesionless soils, when subjected to cyclic loading. The behaviour of a soil subjected to cyclic loading is found to be dependent on; the relative density, mean effective stresses prior to cyclic loading, cyclic and average shear stresses and the drainage conditions.

1   Introduction

Offshore wind turbine foundations are typically subjected to cyclic loading from both wind and waves. It is therefore important that not only the static load-bearing capacity is investigated, but also the cyclic load-bearing capacity. However, at present there is no generally accepted standardised method, which can be applied in order to determine the cyclic load-bearing capacity for offshore wind turbine foundations.

In order to understand the effects that cyclic loading has on cohesionless soils, literature on the topic from different authors has been gathered and a literature study is presented in this article. The purpose is to describe how the soil behaves when subjected to cyclic loading. As mentioned cyclic loading can be caused by environmental loads from wind and waves. This form of loading will have an effect on soil properties such as soil stiffness, shear strength, and void ratio.

The stresses in this article are mapped by the Cambridge method where the deviatoric stress, q, and the mean principle stress, p, are defined as

$$q = \sigma_1 - \sigma_3 \tag{1}$$

$$p = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \tag{2}$$

2   Characteristic Line

The transition from compressive to dilative behaviour is denoted as the *characteristic state*, and is illustrated for different stress paths by dots in Figure 1. The characteristic state is defined as the state where $\delta\varepsilon_v/\delta\varepsilon_1$ is equal to zero, and plotted in a p' - q diagram they construct a straight line through origio. This line is defined as the *characteristic line*, and the angle of the characteristic line is referred to as the *characteristic friction angle*, φ_{cl}. Stress states below the characteristic line leads to contraction ($\Delta\varepsilon_v > 0$) whereas if a stress state is above the characteristic line it leads to dilation ($\Delta\varepsilon_v < 0$). This means that a dense soil following a given stress path starting from below the characteristic line to a stress point above it, will first contract, then dilate when it crosses the characteristic line.

A similar transition occurs in the undrained state, where the so-called *phase transformation line* describes the change in incremental pore pressure, Δu, going from positive to negative increments. The phase transformation stress is defined as where p' has a vertical tangent, i.e. where the mean effective
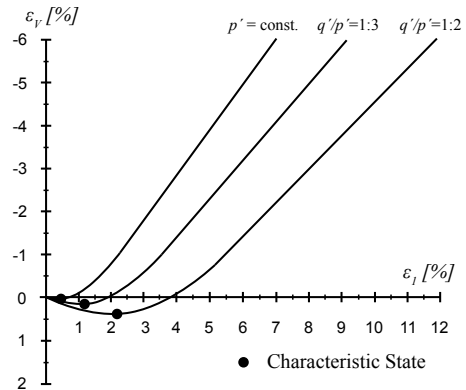

Figure 1: *Volumetric strain as a function of the axial strains during a triaxial compression test on a dense sand for a specimen with equal height and diameter.(Ibsen, 1998)*

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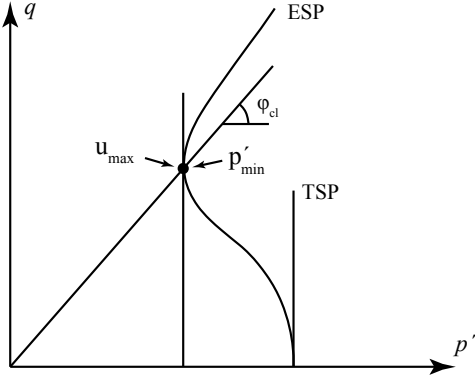[2] *Prof., Department of Civil Engineering, Aalborg University, Denmark*

Figure 2: *Phase transformation state in undrained triaxial compression tests on sand with p = constant. ESP = Effective Stress Path. TSP = Total Stress Path. (Ibsen, 1998).*

stress reaches the lowest value, p'_{min}, as shown in Figure 2.

2.1 Influence of Relative Density

Figure 3 shows the failure envelopes for sands with different relative densities, I_D, along with the characteristic line. It is seen that sands with higher relative density dilates more and therefore gains a higher ultimate shear strength. However for very loose sands and for sands with a very high confining pressure the characteristic line coincides with the failure envelope. The latter case is due to crushing of the particles.

3 Monotonic Triaxial Tests

Triaxial tests, whether they are cyclic or monotonic, can be conducted in several ways. They can be drained or undrained, consolidated or unconsolidated and furthermore the consolidation can be made isotropic or anisotropic. When performing triaxial tests, the soil specimen should reflect the site conditions. This entails in most cases that only
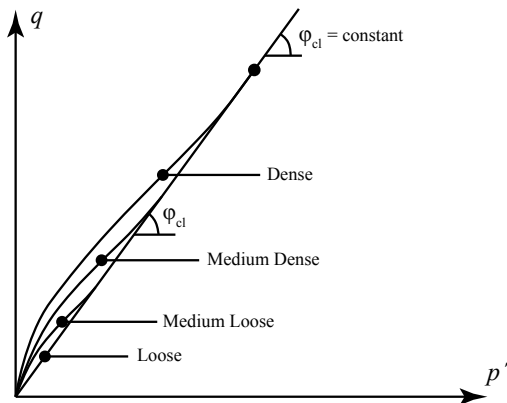

Figure 3: *Variation of drained shear strength envelope for sand with relative density (Ibsen and Lade, 1998a).*
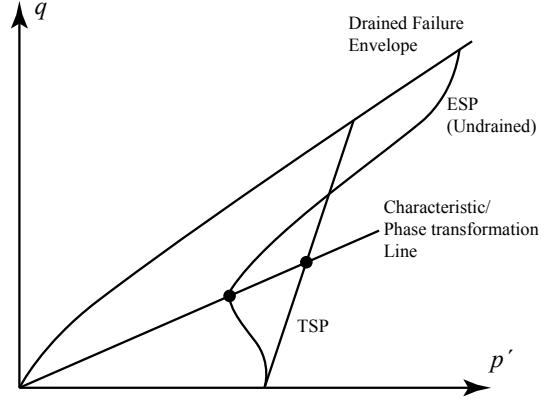

Figure 4: *Difference between a drained and an undrained test starting from the same mean effective stress.*

anisotropic consolidated tests should be used, and the drainage can be chosen so it corresponds to the site specific situation. The response of the soil will be different according to the relative density. In the following section it is only the behaviour of dense, i.e. dilative specimens, that will be treated since these are most common offshore (Lesny, 2010).

3.1 Drained vs. Undrained

In drained triaxial tests the pore pressure is allowed to dissipate and no excess pore pressure is generated. This makes the effective stresses equal to the total stresses and they will follow the total stress path (TSP) in a $p' - q$ diagram as shown in Figure 4.

In undrained triaxial tests no volume change is possible and therefore excess pore pressure is generated. The stresses will therefore follow the effective stress path (ESP) in Figure 4. Below the phase transformation line this will lead to an increase in pore pressure and thereby a drop in effective stresses. When the stress crosses the phase transformation line the soil specimen will attempt to dilate, and therefore negative pore pressure is generated. This leads to an increase in effective stresses, which is why a dilative soil sample can withstand a larger load in the undrained condition compared to the drained condition.

3.2 Undrained Shear Strength

The drained shear strength, τ_f, accounts for the friction angle, the effective mean stress and cohesion, and is given as

$$\tau_f = \frac{1}{2} \cdot \frac{6 \, \sin \varphi}{3 - \sin \varphi} \left(p' + c' \cot \varphi' \right) \qquad (3)$$

where $c' = 0$ for cohesionless soils. In the undrained case for sand, the undrained shear strength, c_u, can be used instead of τ_f according to Ibsen and Lade (1998b). Therefore, the use of the above expression

is extended to the undrained case by adding the initial pore pressure, u_0, and the pore pressure at which cavitation occurs u_{cav}, which results in equation (4). The used effective mean stress, p'_{df}, is the one which corresponds to failure in the drained case.

$$c_u = \frac{1}{2} \cdot \frac{6 \, \sin\varphi}{3 - \sin\varphi} \, \left(p'_{df} + u_0 + u_{cav}\right) \qquad (4)$$

The argument for using the above expression is that the undrained bearing capacity for a dense sand is governed by cavitation, as negative pore pressure develops during loading (Ibsen and Lade, 1998b). It is therefore important to include the pore pressure when calculating c_u in the undrained case for sand. The effect of adding the initial pore pressure, u_0, and the pore pressure at cavitation, u_{cav}, is illustrated in Figure 5. The figure illustrates the effective stress paths for two examples with the same initial effective mean stress, p'_0. The two examples end up having a different undrained shear strength, because of differences in initial pore pressure. Following the total stress path will lead to drained failure in point (a), which is the point where p'_{df} is measured. From this point the amount of initial pore pressure and the pore pressure at cavitation is added to p'_{df}. This means that a higher amount of initial pore pressure will lead to a higher value of the undrained shear strength before failure is reached, which is illustrated by point (b) and (c).

4 Cyclic loading

A definition of cyclic loading is needed in order to determine how to conduct laboratory tests with cyclic loading on cohesionless soil. In Peralta (2010) a definition of cyclic loading is given as a load frequency between 0 and 1 Hz, as shown in Table 1. Furthermore, inertia forces can be neglected due to the low frequency, and the accumulated strain is predominantly plastic.

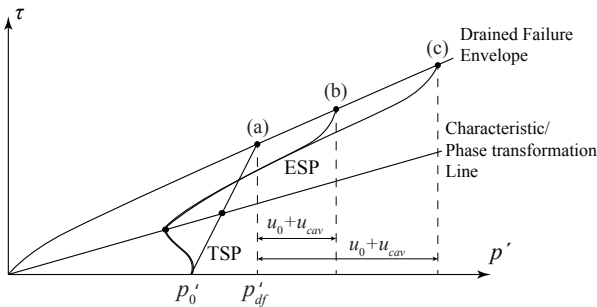Cyclic loading is defined by two components; the *average shear stress*, τ_a, and a *cyclic shear stress*,

Figure 5: *Illustration of the effect of including initial pore pressure and the pore pressure at cavitation to the drained failure criterion.*
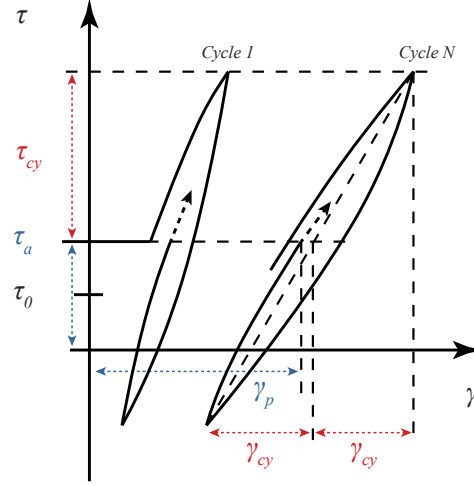
Figure 6: *Definition of stress and strain under cyclic loading (Andersen, 2009).*

τ_{cy}, which is the amplitude of a load cycle. These are depicted in Figure 6.

Failure caused by cyclic loading is defined as either 15 % of *permanent shear strain*, γ_p, or 15 % *cyclic shear strain*, γ_{cy}, according to Andersen (2009). The cyclic and permanent shear strains are also depicted in Figure 6.

4.1 Critical States During Cyclic Loading

In order to determine failure during cyclic loading the concept of *cyclic limit state*, CLS, is used. The cyclic limit state describes the upper bound for non-failure conditions of cyclic loaded soils. The cyclic limit state is a straight line in the $p' - q$ space, on which a single point is defined as the *critical level of repeated loading*, CLRL. CLRL is by Ibsen (1998) and Peralta (2010) defined as the upper bound stress level for a given soil at which strains and/or pore pressures accumulate continuously and lead to failure, and is therefore the shear stress level at the CLS-line in the $p' - q$ space.

Laboratory tests of soils under cyclic loading has shown, that soils subjected to a finite number of load cycles not necessarily reach failure, i.e. the cyclic limit state. In some cases the soil will instead reach a state of equilibrium before failure thereby producing only an elastic response, i.e no plastic strain or pore pressure accumulation with additional load cycles. This phenomenon is also known as *shakedown*.

4.2 Cyclic Stable State

A stress state where the positive and negative pore pressures generated neutralize each other is known as the *cyclic stable state*. For the undrained state it is defined as $\Sigma\Delta u = 0$ during a cycle. Ibsen (1998) performed nine undrained cyclic tests on a sand with $I_D = 0.78$ and equal height and diameter. The tests showed that if the mean deviatoric stress is lower

Table 1: *Approximate classification of repeated loading of soils. (Peralta, 2010)*

Repeated Loading of Soils	Cyclic	Cyclic-Dynamic	Dynamic
Frequency	0 to 1 Hz	1 to 10 Hz	> 10Hz
Inertia	No (negligible)	Yes (relevant)	Yes (relevant)
Strain accumulation	Predominantly plastic	Plastic and elastic	Predominantly elastic

than the cyclic stable state, positive pore pressure is generated. Opposite, a negative pore pressure is generated each time the mean deviatoric stress level becomes higher than the cyclic stable state. This is seen in Figure 7, which shows the nine cyclic tests and the generation of either positive or negative pore pressure. The cyclic loading leads the effective mean stress towards the cyclic stable state in each test. When the cyclic stable state has been reached the effective mean stress does not change and the cyclic loading will not lead to any further hardening or softening of the soil (shakedown).

4.3 Shakedown Theorem

For an elastic-perfectly plastic material subjected to cyclic loading, the shakedown theorem states that the five cases in Figure 8 can occur (Goldscheider, 1977). It could be questioned if it also can be used for soil, which is an elasto-plastic material. However, Goldscheider (1977) found by experiments, that the theorem partially can be used on cohesionless soils. One exception though, was that a pure elastic response of the soil was never observed during the performed cyclic triaxial tests.

Based on some of the cases within the shakedown theorem, different failure modes of soils due to cyclic loading are illustrated in Figure 9. Figure 9(a) shows incremental collapse, where the strain incre-
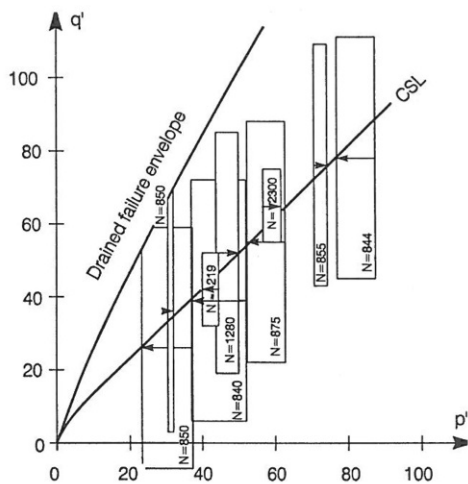

Figure 7: *The effective stress path of nine cyclic tests. The test is performed on Lund No. 0 with $I_D = 0.78$ and specimens with equal height and diameter. CSL is the Cyclic Stable Line, N is the number of cycles added to the test and the arrow describes the changes in effective mean stress. (Ibsen, 1998)*

Case 1: Elastic response
By sufficiently low cyclic load amplitudes, the response of the structure is elastic with no plastic deformations whatsoever;

Case 2: Ordinary collapse
By sufficiently high cyclic load amplitudes, the load carrying capacity of the structure becomes exhausted and failure occurs instantaneously as plastic, unconstrained deformations develop and the structure collapses – this is also known as ordinary collapse;

Case 3: Incremental collapse
By cyclic load amplitudes less than the collapse load given in (Case 2) and if the plastic strain increments are of the same sign (plastic strain increases incrementally), then the total accumulated plastic deformation of the structure increases indefinitely and becomes so large after a sufficient number of cycles so that it becomes unserviceable. This phenomenon is termed incremental collapse;

Case 4: Alternating plasticity
By cyclic load amplitudes less than the collapse load given in (Case 2) and if the plastic strain increments in each cycle changes sign, then the strain per cycle tends to cancel out the previous strain increment so that no further increase of the overall plastic deformations occurs or the total plastic deformation remains small. This case has been termed as alternating plasticity. In this case, residual forces or stresses remain in the material that do not become constant but tend to change cyclically with time. The plastic work increases indefinitely with number of cycles and at some local points of the structure, material may break due to low-cycle fatigue;

Case 5: Shakedown
In the last case, it may happen that for lower cyclic load amplitudes, an initial plastic deformation of the structure develops but, after a certain finite number of load cycles, the cyclic response of the structure eventually becomes elastic and the structure stabilizes. The stabilization of accumulated plastic deformations is termed as shakedown or adaptation. A significant feature of shakedown are residual stresses in the material that are self-equilibrating which remains constant with time (or number of cycles).

Figure 8: *General shakedown cases for an elasto-plastic material (Goldscheider, 1977).*
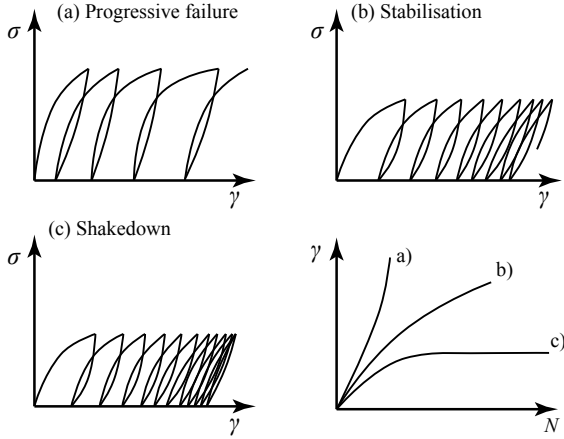
Figure 9: *Visualisation of the shakedown theorem.*

Figure 10: *Results of a consolidated anisotropical undrained simple shear test on seabed sand. Cyclic shear stress is equal to 15 kPa and effective vertical consolidation stress equal to 75 kPa. (Randolph and Gouvernec, 2011)*

ment increases for every cycle. In Figure 9(b), which is also a form of incremental collapse, the strain increment decreases for larger number of cycles without ever reaching a stable state and therefore failure will eventually occur. Figure 9(c) illustrates shakedown, where the strain increment decreases with increasing number of cycles, but never reaches failure.

4.4 Liquefaction

A special failure mode is known as *liquefaction*. This failure mode can occur when cohesionless soils are exposed to cyclic loading in the undrained state. In this case, there is a probability that the effective stresses will reach zero due to pore pressure build-up and the soil will behave as a liquid with no bearing capacity, as shown in Figure 10. The first time the effective stress reaches zero the soil will try to dilate and negative pore pressure will be generated, which leads to an increase in effective stresses. As cyclic loading continues this pattern will repeat itself and an increase in shear strains is observed as illustrated in Figure 11.

5 Response Due to Cyclic Loading

The response from cyclic loading varies from the response of monotonic loaded tests. The effects of pore pressure build-up in undrained cyclic tests are especially critical for the effective stresses. Furthermore, the response is dependent on whether the test is performed as a direct simple shear test or a triaxial test, which will be outlined in the following sections.

5.1 Cyclic Simple Shear Test

Randolph and Gouvernec (2011) conducted an undrained cyclic simple shear test on cohesionless soil influenced by two-way symmetric loading with a cyclic shear stress, τ_{cy}, equal to 15 kPa as shown in
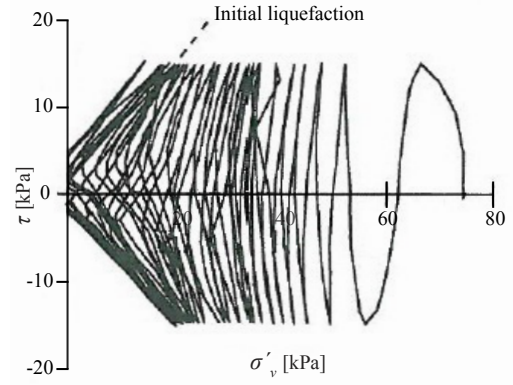
Figure 10. The specimen will try to contract leading to an increase in excess pore pressure, which results in a decrease in effective vertical stress, σ'_v. Unlike a monotonic test excess pore pressure continues to increase with repeated load cycles until the effective vertical stress becomes zero. After this point has been reached the specimen tends to dilate which causes a decrease in excess pore pressure and thereby an increase in effective vertical stress. This leads to the butterfly shaped stress paths in Figure 10.

Figure 11 shows the τ - γ diagram from the same test performed by Randolph and Gouvernec (2011). From the figure it is seen that the shear strain is very small until initial liquefaction is reached. From this point additional load cycles leads to a significant increase in shear strain.

5.2 Cyclic Triaxial Test

As stated earlier the soil response from triaxial tests is different from the response obtained by cyclic sim-
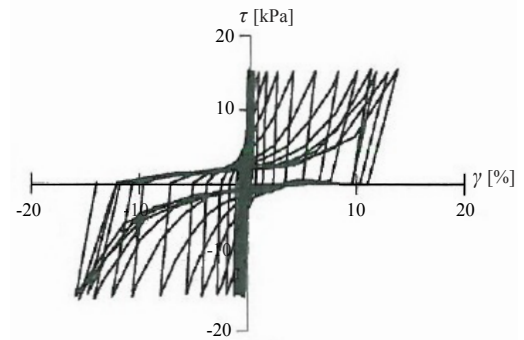

Figure 11: *Results of a consolidated anisotropical undrained simple shear test on seabed sand. Cyclic shear stress is equal to 15 kPa and effective vertical consolidation stress equal to 75 kPa. (Randolph and Gouvernec, 2011)*
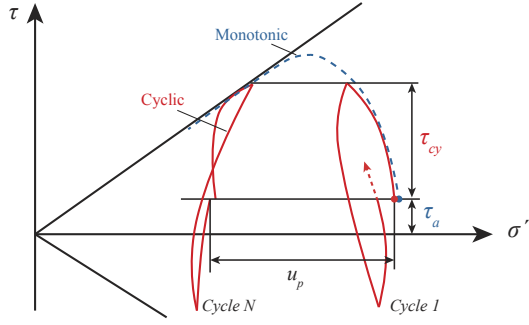
Figure 12: *Effective stress path for an undrained cyclic tri-axial test. (Andersen, 2009)*

Figure 14: *Strain contour diagram for sand. Average shear stress equal to zero. (Randolph and Gouvernec, 2011)*

ple shear tests. The difference is that during triaxial response pore pressure is reduced when unloaded compared to a cyclic simple shear test where pore pressure still builds up during unloading. However, in cyclic triaxial tests pore pressure build-up still occurs during the course of one cycle (Andersen, 2009). This can be seen when comparing the $\sigma' - \tau$ diagram for a cyclic triaxial test in Figure 12 and a cyclic simple shear test in Figure 10. Moreover it can be observed that the initial stress path is the same as a monotonic test, i.e. until the shear stress reaches its maximum value for the first time, as seen in Figure 12.

Two-way loading is defined by Andersen (2009) as if the shear stresses changes sign and one-way loading if the shear stresses always have either a positive or negative value. In cyclic simple shear tests subjected to two-way loading the soil have the same strength when developing negative and positive shear strain. In two-way loaded triaxial tests the soil will be affected of both compression and extension. In this case the soil do not have the same strength when developing negative and positive shear strain, because the extension strength is lower than the compression strength. Figure 13 shows various cyclic loading conditions for both the cyclic simple shear test and the cyclic triaxial test with the differences in response in a γ - τ diagram.

5.3 Cyclic Load Ratio

Cyclic load ratio is a normalisation of the cyclic shear stress. For cohesive soils it is normalised with respect to the undrained shear strength, and for friction materials the normalisation parameter is the vertical effective consolidation stress. In order to determine how many cycles a sample can withstand before it reaches a maximum shear strain value, Randolph and Gouvernec (2011) made a strain contour diagram as seen in Figure 14. The figure illustrates strain contours for sand from one undrained monotonic and four undrained cyclic symmetric simple shear tests with a cyclic load ratio, τ_{cyc}/s_{uss}, equal to 0.8, 0.6, 0.4 and 0.28. The number of cycles to reach a shear strain with a magnitude of 0.2,
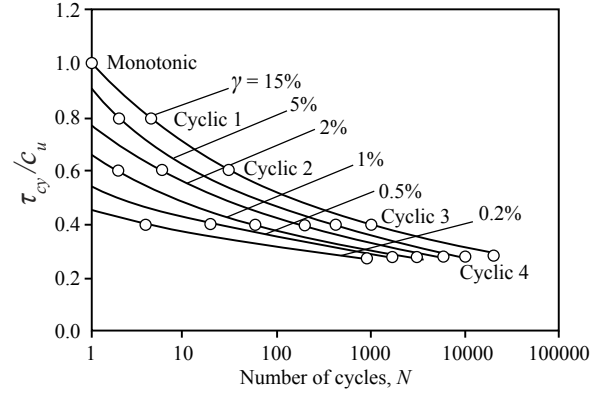
0.5, 1, 2, 5 or 15 % can be identified for any value of τ_{cyc}/s_{uss}.

Randolph and Gouvernec (2011) identified the cyclic load ratio as a very important factor for the bearing capacity of soils when subjected to cyclic loading. As an example it can be seen in Figure 14 that a cyclic load ratio of 0.28 will produce a shear strain of 0.2 % after approximately 1000 cycles. With an increase of the cyclic load ratio to 0.40 a shear strain of 0.2 % will be obtained after only 6 cycles.

5.4 Average Load Ratio

In addition to the cyclic load ratio, Andersen (2009) found that the *average load ratio* also has a large effect on the cyclic load-bearing capacity for soil. The average load ratio is defined as the average shear stress normalised in the same manner as the cyclic load ratio. Furthermore, he showed that the development of shear strain is not dependent on the maximum shear stress, but the ratio between cyclic and average shear stress. This can be seen in Figure 15, where different loadings that all have the same maximum shear stress yield very different results based on their average and cyclic shear stresses.

These effects are combined in Figure 16, together with the number of cycles to failure. This method was suggested by Andersen and Berre (1999), and has the advantage compared to the strain contour diagram, that the average load ratio is also taken into account. It should be noted that failure in Figure 16 is defined as only 3 % average or cyclic shear strain.
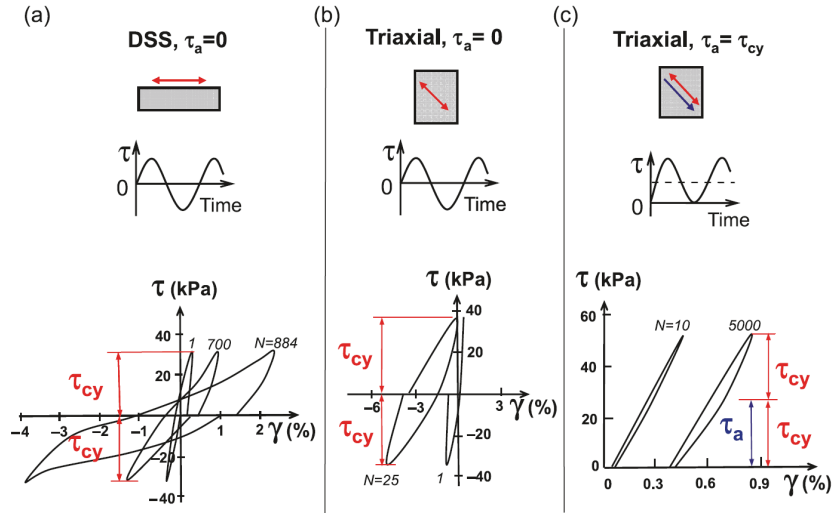
Figure 13: *Stress-strain behaviour under various cyclic loading conditions. (Andersen, 2009)*

Figure 15: *Results from cyclic triaxial tests with the same maximum stress, τ_{max}. (Andersen, 2009)*

Test	τ_{max}	τ_a	τ_{cy}	Result
A	50	0	50	Failure (γ = 15%) 10 cycles
B	50	25	25	γ_p = 0.8%, γ_{cy} = 0.3% 2500 cycles
C	50	42.5	7.5	γ_p = 0.03%, γ_{cy} = 0.02% 2500 cycles

Figure 16: *Number of cycles to failure, depending on cyclic and average load ratio. (Andersen and Berre, 1999)*

6   Conclusion

Cohesionless soils subjected to cyclic loading are influenced by several factors. Most dominating are the average and cyclic load ratios. A small increase in load ratio can mean a significant reduction in the cyclic load bearing capacity. It is also important to take both load ratios into account at the same time and not just the cyclic load ratio.

Cyclic loading also has an influence on the pore pressure in the undrained case. As cyclic loading progresses pore pressure will build up and potentially become equal to the total stresses. When this happens the effective stresses will become zero and liquefaction occurs, producing large shear strains. The opposite can also occur when the stress state is located above the cyclic stable line, thereby creating negative pore pressure, and a subsequent increase in effective stresses. Lastly, shakedown can occur resulting in no pore pressure build-up or increase in shear strains.

The initial pore pressure is found to have a significant impact on the undrained shear strength when conducting monotonic triaxial tests. An increase in the initial pore pressure will give an increase in undrained shear strength due to extra pore pressure before cavitation occurs.

Another relevant parameter is the relative density and its influence on the drained failure envelope. A dense sample will have a higher bearing capacity due to its ability to dilate.
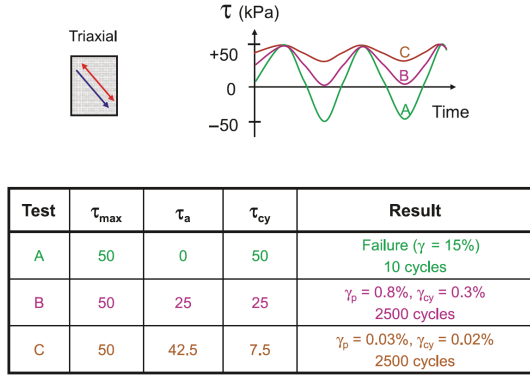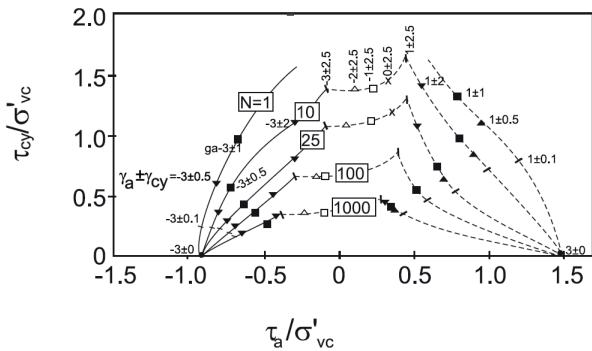
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