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–Paper–

CLOSED-LOOP IDENTIFICATION FOR CONTROL OF LINEAR PARAMETER VARYING SYSTEMS

Jan Bendtsen Klaus Trangbaek

ABSTRACT

This paper deals with system identification for control of linear parameter varying systems. In practical applications, it is often important to be able to identify small plant changes in an incremental manner without shutting down the system and/or disconnecting the controller; unfortunately, closed-loop system identification is more difficult than open-loop identification. In this paper we prove that the so-called Hansen Scheme, a technique known from linear time-invariant systems theory for transforming closed-loop system identification problems into open-loop-like problems, can be extended to accommodate linear parameter varying systems as well. We investigate the identified subsystem's parameter dependency and observe that, under mild assumptions, the identified subsystem is affine in the parameter vector. Various identification methods are compared in direct and Hansen Scheme setups in simulation studies, and the application of the Hansen Scheme is seen to improve the identification performance.

Key Words: Closed-loop system identification, Linear parameter varying systems, Youla-Kucera parameterisation

I. Introduction

Industrial control systems are typically in operation for extensive periods of time, amongst other things due to the fact that once a functioning system has been commissioned and brought into operation, it is very costly in terms of engineering manpower and loss of production output (and hence income) to take the system out of action in order to maintain and update it. On the other hand, most large-scale industrial systems are subject to frequent changes and modifications, which may change the dynamics of various subsystems of the overall plant. Thus, it is often the case that a control system can be improved after initial commissioning, as more actual operation data becomes available.

Assuming that a good, or at least acceptable, model for the original system already exists, however, it seems wasteful to estimate the total model from scratch in case of limited structural modifications. Motivated by this observation, we study incremental modelling for control of plants running in closed loop in this paper.

In particular, we look at the so-called *Hansen scheme* [1, 2, 3], which, given a nominal system model and controller, allows open-loop-like system identification unmodelled dynamics parameterised via a technique called *dual Youla-Kucera factorisation*—see the survey paper [4] and the references therein for further details. It is worth noting here that several rigorous studies show that models obtained with the Hansen scheme are distinctly superior to models obtained from 'direct' identification methods when it comes to subsequent controller design [5, 6].

In this paper, we show how the Hansen scheme can be reformulated to deal with *linear parameter varying* (LPV) systems [7, 8, 9, 10]. Please note that we are *not* proposing a new identification method as such; it remains necessary to employ an established LPV

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identification method for LPV identification of the dual Youla-Kucera parameter. Rather, our aim is to remove some of the specific closed-loop difficulties from the identification setting in order to facilitate subsequent control design.

There are already a number of methods for identification of LPV systems available in the literature, e.g., [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23], all of which can, in principle, be used in the setup we shall present in the following with little modification.

The main contribution of the present paper is to show that the Hansen scheme can be formulated for LPV systems in a non-conservative setting using the notions of *LPV stability* shown via *polyhedral Lyapunov functions* [24]. The work presented here is related to results presented in [25] and [26], which presented similar results in a quite general, nonlinear setting. However, by restricting the class of systems under consideration here, we are able to present an explicit methodology for the identification and control design, which is suitable for controller updating as it focuses on incremental modelling.

The outline of the rest of the paper is as follows. Section II provides some important preliminary results on the notion of LPV stability employed in the rest of the paper. Section III then presents a Youla-Kucera parametrisation of LPV systems, after which Section IV shows how the Hansen scheme is cast in this framework. Section V investigates the identified subsystem's parameter dependency, whereupon Section VI compares several open- and closed-loop identification schemes on a simple simulation example. Finally, Section VII sums up the conclusions of the work.

II. LPV Stability

In this work, we consider discrete-time linear parameter-varying (LPV) systems G_θ with a minimal state space realisation given by matrix functions $A_\theta \in \mathbb{R}^{n\times n}, B_\theta \in \mathbb{R}^{n\times m}, C_\theta \in \mathbb{R}^{p\times n}$ and $D_\theta \in \mathbb{R}^{p\times m}$, mapping an input signal vector $u \in \mathbb{R}^m$ to an output measurement signal $y \in \mathbb{R}^p$. Specifically, we deal with systems of the form

$$G_\theta: \quad x_{k+1} = A_{\theta(k)}x_k + B_{\theta(k)}u_k \qquad (1)$$
$$y_k = C_{\theta(k)}x_k + D_{\theta(k)}u_k \qquad (2)$$

where $\theta(k) \in \mathbb{R}^q$ is an external scheduling parameter, which is allowed to vary as a function of time but not as a function of the system states x. Since we only allow θ to depend on k, we will simply write θ rather than $\theta(k)$ in the following. We require that θ belongs to the bounded compact set

$$\Theta = \left\{ \theta \in \mathbb{R}^q \,\middle|\, \theta_i \geq 0, \ \sum_{i=1}^{q} \theta_i = 1 \right\}$$

and that $A_\theta, B_\theta, C_\theta$ and D_θ are continuous, bounded functions of $\theta \in \Theta$ (only).

For notational convenience, we will use the shorthand

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & D_\theta \end{array}\right]$$

for the LPV system (1)–(2) in the sequel.*

If D_θ is nonsingular, i.e., D_θ^{-1} is well defined for all θ, the LPV system G_θ has an inverse operator

$$G_\theta^{-1} = \left[\begin{array}{c|c} A_\theta + B_\theta D_\theta^{-1} C_\theta & B_\theta D_\theta^{-1} \\ \hline D_\theta^{-1} C_\theta & D_\theta^{-1} \end{array}\right]$$

in the sense that $G_\theta G_\theta^{-1} = G_\theta^{-1} G_\theta = I$, where I is the identity, for any trajectory of θ. We will ensure invertibility by construction whenever necessary in the sequel.

Next, consider the autonomous LPV system $x_{k+1} = A_\theta x_k$ along with the Lyapunov function candidate $V(x) = \|Wx\|_\infty$, where $W \in \mathbb{R}^{\mu\times n}$ is a constant matrix of rank n. $V(x)$ is a positive definite function with $V(0) = 0$, and computing the sample-to-sample difference yields

$$\begin{aligned} V(x_{k+1}) - V(x_k) &= \|Wx_{k+1}\|_\infty - \|Wx_k\|_\infty \\ &= \|WA_\theta x_k\|_\infty - \|Wx_k\|_\infty \end{aligned}$$

which is negative if A_θ is sufficiently small; this can be tested via algebraic means. If the autonomous part of an LPV system admits such a Lyapunov function for all $\theta \in \Theta$, we say that it is *LPV stable*.

In particular, it is known that a *polytopic* LPV system, i.e., a system where $A_\theta, B_\theta, C_\theta$ and D_θ are given as convex combinations of fixed matrices A_i, B_i, C_i and $D_i, i = 1, \ldots, q$, admits a polyhedral Lyapunov function if the associated matrix equalities hold for each vertex system. Furthermore, it is shown in [24] that the existence of a polyhedral Lyapunov function is in fact *equivalent* to LPV stability for polytopic LPV systems. That is, this class of Lyapunov functions is non-conservative, as opposed to e.g. quadratic Lyapunov functions in the sense that one

*Please note that this notation should not be confused with "transfer functions"; throughout the paper we strictly consider operators defined in state space, as given by (1)–(2), with $x_0 = 0$ unless otherwise noted.

may find examples of stable polytopic LPV systems that do not permit a quadratic Lyapunov function, but it is not possible to find stable polytopic LPV systems that do not permit a polyhedral Lyapunov function. We require the following technical result:

Lemma 1 *[24] $V(x) = \|Wx\|_\infty$ is a (polyhedral) Lyapunov function for the polytopic autonomous LPV system $x_{k+1} = A_\theta x_k$ if and only if there exist matrices $Q_i \in \mathbb{R}^{\mu\times\mu}$ such that $WA_i = Q_iW$ and $\|Q_i\|_\infty < 1$ for $i = 1, \ldots, q$.*

Based on Lemma 1 we can show the following simple, yet important result for connection of LPV systems.

Lemma 2 *Suppose two autonomous LPV systems $x_{1,k+1} = A_\theta^{11}x_{1,k}$ and $z_{2,k+1} = A_\theta^{22}z_{2,k}$ are LPV stable; then for any continuous and bounded A_θ^{21} of appropriate dimensions, the autonomous LPV system*

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} A_\theta^{11} & 0 \\ A_\theta^{21} & A_\theta^{22} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} \tag{3}$$

is also LPV stable.

Proof: According to Lemma 1, since the systems $x_{1,k+1} = A_\theta^{11}x_{1,k}$ and $z_{2,k+1} = A_\theta^{22}z_{2,k}$ are LPV stable, there exist matrices $W^1, W^2, Q_\theta^1, Q_\theta^2$ of appropriate dimensions with $\|Q_\theta^1\|_\infty < 1, \|Q_\theta^2\|_\infty < 1$ such that

$$\begin{bmatrix} W^1 & 0 \\ 0 & W^2 \end{bmatrix} \begin{bmatrix} A_\theta^{11} & 0 \\ 0 & A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1 & 0 \\ 0 & Q_\theta^2 \end{bmatrix} \begin{bmatrix} W^1 & 0 \\ 0 & W^2 \end{bmatrix}$$

for $\theta \in \Theta$. Also, we have

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ 0 & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1.$$

Turning to the combined system (3), if we can find a scalar $\beta > 0$ and a θ-dependent matrix Q_θ^{21} such that

$$\begin{bmatrix} W^1 & 0 \\ 0 & \beta W^2 \end{bmatrix} \begin{bmatrix} A_\theta^{11} & 0 \\ A_\theta^{21} & A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1 & 0 \\ Q_\theta^{21} & Q_\theta^2 \end{bmatrix} \begin{bmatrix} W^1 & 0 \\ 0 & \beta W^2 \end{bmatrix}$$

and

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ Q_\theta^{21} & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1$$

hold for every $\theta \in \Theta$, then we can conclude that the system is LPV stable by invoking Lemma 1. Rewriting the matrix equality above, we get

$$\begin{bmatrix} W^1A_\theta^{11} & 0 \\ \beta W^2A_\theta^{21} & \beta W^2A_\theta^{22} \end{bmatrix} = \begin{bmatrix} Q_\theta^1W^1 & 0 \\ Q_\theta^{21}W^1 & \beta Q_\theta^2W^2 \end{bmatrix}$$

which is satisfied iff $\beta W^2A_\theta^{21} = Q_\theta^{21}W^1\ \forall\theta \in \Theta$.

Since W^1 has full row rank, it has a left pseudo-inverse $W^{1\dagger}$; thus, we may choose $Q_\theta^{21} = \beta W^2A_\theta^{21}W^{1\dagger}$ with β sufficiently small to satisfy

$$\left\| \begin{bmatrix} Q_\theta^1 & 0 \\ \beta W^2A_\theta^{21}W^{1\dagger} & Q_\theta^2 \end{bmatrix} \right\|_\infty < 1 \quad \forall\theta \in \Theta$$

which is always possible since A_θ^{21} is bounded. $\triangleleft$

III. Basic Parametrisation

In the rest of the paper, we will assume that the plant and the nominal model G_θ are strictly proper, i.e.

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & 0 \end{array}\right] \tag{4}$$

and that that they are both stabilised by an observer-based LPV controller of the form

$$K_\theta = \left[\begin{array}{c|c} A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta \\ \hline F_\theta & 0 \end{array}\right] \tag{5}$$

for all $\theta \in \Theta$, where F_θ and L_θ are such that $\bar{x}_{k+1} = (A_\theta + B_\theta F_\theta)\bar{x}_k$ and $\hat{x}_{k+1} = (A_\theta + L_\theta C_\theta)\hat{x}_k$ are LPV stable.

Any G_θ that satisfies the above assumption for any trajectory of $\theta \in \Theta$, can be written as a right, respectively left, coprime factorisation of the form:

$$G_\theta = N_\theta M_\theta^{-1} = \tilde{M}_\theta^{-1}\tilde{N}_\theta \tag{6}$$

where $N_\theta, M_\theta, \tilde{M}_\theta$ and $\tilde{N}_\theta$ are LPV stable operators of a specific form given below. Correspondingly, K_θ can be factorised as

$$K_\theta = U_\theta V_\theta^{-1} = \tilde{V}_\theta^{-1}\tilde{U}_\theta \tag{7}$$

with LPV stable $U_\theta, V_\theta, \tilde{U}_\theta, \tilde{V}_\theta$. The factors are given as

$$\begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} = \left[\begin{array}{c|cc} A_\theta + B_\theta F_\theta & B_\theta & -L_\theta \\ \hline F_\theta & I & 0 \\ C_\theta & 0 & I \end{array}\right] \tag{8}$$

$$\begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} = \left[\begin{array}{c|cc} A_\theta + L_\theta C_\theta & -B_\theta & L_\theta \\ \hline F_\theta & I & 0 \\ C_\theta & 0 & I \end{array}\right] \tag{9}$$

Then, it is possible to check that

$$\begin{aligned} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} &= \begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} \begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} \\ &= \begin{bmatrix} M_\theta & U_\theta \\ N_\theta & V_\theta \end{bmatrix} \begin{bmatrix} \tilde{V}_\theta & -\tilde{U}_\theta \\ -\tilde{N}_\theta & \tilde{M}_\theta \end{bmatrix} \end{aligned} \tag{10}$$

holds; this equation is referred to as the *double Bezout identity*.

Finally, we introduce the *upper linear fractional transformation* of appropriately block-partioned systems

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix}$$

and Δ defined as

$$\mathcal{F}_u\left(G, \Delta\right) = \Pi_{22} + \Pi_{21}\Delta(I - \Pi_{11}\Delta)^{-1}\Pi_{12}$$

provided the inverse exists (see also [27, Chap. 10]). We have the following result.

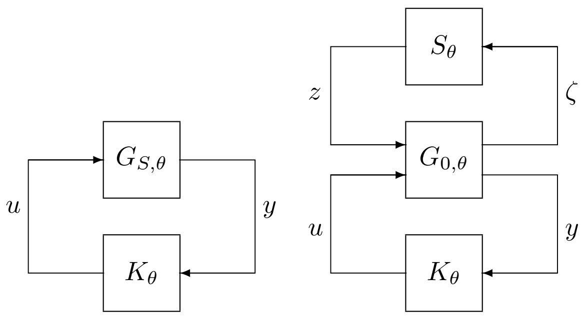

Fig. 1. *All LPV systems $G_{S,\theta}$ stabilised by the LPV controller K_θ (left) can be represented by a nominal system $G_{0,\theta}$ stabilised by K_θ and a dual Youla-Kucera parameter S_θ (right).*

Theorem 1 *Let $G_\theta = N_\theta M_\theta^{-1}$ with state space realisation (4) be LPV stabilised by a feedback controller $K_\theta = U_\theta V_\theta^{-1}$ with state space realisation (5) (see Figure 1). Let F_θ and L_θ be matrix functions such that $\bar{x}_{k+1} = (A_\theta + B_\theta F_\theta)\bar{x}_k$ and $\hat{x}_{k+1} = (A_\theta + L_\theta C_\theta)\hat{x}_k$ are LPV stable for all $\theta \in \Theta$. All plants stabilised by K_θ can be parametrised as $G_{S,\theta} = \mathcal{F}_u\left(G_{0,\theta}, S_\theta\right)$, where*

$$G_{0,\theta} = \left[\begin{array}{c|cc} A_\theta & -L_\theta & B_\theta \\ \hline -F_\theta & 0 & I \\ C_\theta & I & 0 \end{array}\right]$$

and $S_\theta = \left[\begin{array}{c|c} A_{S,\theta} & B_{S,\theta} \\ \hline C_{S,\theta} & 0 \end{array}\right]$ is any proper LPV stable system. S_θ is denoted the dual Youla-Kucera parameter.

Proof: We first show that under the given assumptions, K_θ stabilises $G_{S,\theta}$. The upper loop in the right part of Figure 1 is closed, yielding $G_{S,\theta}$ in the left part of the figure:

$$\begin{aligned} G_{S,\theta} &= \mathcal{F}_u\left(G_{0,\theta}, S_\theta\right) \\ &= \left[\begin{array}{cc|c} A_{S,\theta} & -B_{S,\theta}F_\theta & B_{S,\theta} \\ -L_\theta C_{S,\theta} & A_\theta & B_\theta \\ \hline C_{S,\theta} & C_\theta & 0 \end{array}\right] \end{aligned} \tag{11}$$

and when connecting K_θ as shown to this system, we obtain the autonomous LPV system

$$\begin{bmatrix} \xi_{k+1} \\ \eta_{k+1} \\ \chi_{k+1} \end{bmatrix} = \begin{bmatrix} A_{S,\theta} & -B_{S,\theta}F_\theta & 0 \\ 0 & A_\theta + L_\theta C_\theta & 0 \\ -L_\theta C_{S,\theta} & -L_\theta C_\theta & A_\theta + B_\theta F_\theta \end{bmatrix} \begin{bmatrix} \xi_k \\ \eta_k \\ \chi_k \end{bmatrix}$$

where ξ is the state vector of S_θ, χ is the controller state vector and $\eta = x - \chi$ is the difference between the state vector of $G_{0,\theta}$ and K_θ. Since $A_{S,\theta}$, $A_\theta + L_\theta C_\theta$ and $A_\theta + B_\theta F_\theta$ are all LPV stable, and $B_{S,\theta}F_\theta$, $L_\theta C_{S,\theta}$ and $L_\theta C_\theta$ are bounded for bounded θ, we can then conclude that the closed-loop system is LPV stable by applying Lemma 2 twice in succession.

We then show that, given $K_\theta = U_\theta V_\theta^{-1}$, a nominal $G_\theta = N_\theta M_\theta^{-1}$ stabilised by K_θ and a $G_{S,\theta}$ also stabilised by K_θ, there exists an S_θ (connected as shown in Fig. 1) such that the interconnection of $G_{0,\theta}$ and S_θ is identical to $G_{S,\theta}$.

We construct the dual Youla-Kucera parameter as $S_\theta = \mathcal{F}_u\left(\bar{G}_\theta, G_{S,\theta}\right)$, where

$$\bar{G}_\theta = \left[\begin{array}{c|cc} A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta & B_\theta \\ \hline F_\theta & 0 & I \\ -C_\theta & I & 0 \end{array}\right]$$

First, we note that the $(1,1)$-block subsystem of $\bar{G}_\theta$ is identical to K_θ (cf. (5)); thus, since $\mathcal{F}_u\left(K_\theta, G_\theta\right)$ is LPV stable, $S_\theta = \mathcal{F}_u\left(\bar{G}_\theta, G_{S,\theta}\right)$ is also LPV stable. Secondly, it is fairly easy to see that

$$\mathcal{F}_u\left(G_{0,\theta}, \bar{G}_\theta\right) = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

which is the identity of $\mathcal{F}_u\left(\cdot, \cdot\right)$. Thus,

$$\begin{aligned} \mathcal{F}_u\left(G_{0,\theta}, S_\theta\right) &= \mathcal{F}_u\left(G_{0,\theta}, S_\theta\right) \\ &= \mathcal{F}_u\left(G_{0,\theta}, \mathcal{F}_u\left(\bar{G}_\theta, G_{S,\theta}\right)\right) \\ &= \mathcal{F}_u\left(\mathcal{F}_u\left(G_{0,\theta}, \bar{G}_\theta\right), G_{S,\theta}\right) \\ &= G_{S,\theta}. \end{aligned}$$

which completes the proof. $\triangleleft$

Note that knowledge of a specific polytopic Lyapunov function is not required in the proof; we simply require the state transformations to be independent of the system states.

By Theorem 1, all LPV systems stabilized by K_θ can be written as $G_{S,\theta} = \mathcal{F}_u\left(G_{0,\theta}, S_\theta\right)$, with $G_{0,\theta}$ given in the theorem. By inspection, it is seen that

$$
\begin{aligned}
G_{0,\theta} &= \left[\begin{array}{c|cc} A_\theta & -L_\theta & B_\theta \\ \hline -F_\theta & 0 & I \\ C_\theta & I & 0 \end{array}\right] \\
&= \begin{bmatrix} -M_\theta^{-1}U_\theta & M_\theta^{-1} \\ \tilde{M}_\theta^{-1} & G_\theta \end{bmatrix} \\
&= \begin{bmatrix} -M_\theta^{-1}U_\theta & M_\theta^{-1} \\ V_\theta - N_\theta M_\theta^{-1}U_\theta & N_\theta M_\theta^{-1} \end{bmatrix}
\end{aligned}
$$

where the last equality is obtained by the Bezout identity. Then, it can be checked that

$$
\begin{aligned}
\mathcal{F}_u\left(G_{0,\theta}, S_\theta\right) &= (N_\theta + V_\theta S_\theta)(M_\theta + U_\theta S_\theta)^{-1} \\
&= \left(\tilde{M}_\theta + S_\theta \tilde{U}_\theta\right)^{-1}\left(\tilde{N}_\theta + S_\theta \tilde{V}_\theta\right)
\end{aligned} \tag{12}
$$

This setup is depicted in Figure 2 and will be used in the following.

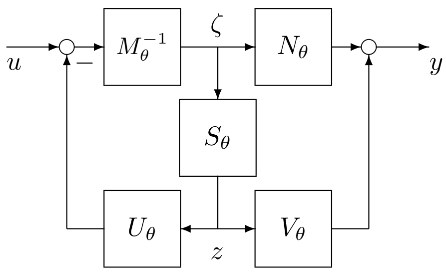

Fig. 2. *Dual Youla-Kucera parametrisation of all proper polytopic LPV plants stabilised by the LPV controller* $K_\theta = U_\theta V_\theta^{-1}$.

IV. Open-Loop-Like System Identification

Next, we consider system identification of an LPV system $G_{S,\theta}$. Output measurements are related to the input through the expression

$$y = G_\theta u + n_y$$

and a good estimate of $\bar{G}_\theta$ can be obtained if u and n_y are uncorrelated, using any of the methods mentioned in the Introduction.

Unfortunately, in a closed-loop setting u is *not* uncorrelated with n_y, since the noise is fed back through the controller, and the frequency content in u may be severely limited in closed-loop operation as well, especially in near-steady state operation. To alleviate these drawbacks, we recast the closed-loop system identification problem into an 'open-loop-like' problem.

We assume that a nominal state space LPV model of an existing system, G_θ, has been found. The system takes control signals u as input, and yields corresponding output measurements y, which are affected by additive noise $n_y \in \mathbb{R}^p$. The parameter variation θ is measurable and satisfies the assumptions in the previous sections.

Based on this model, a stabilising observer-based LPV controller K_θ of the form (5) with stable observer and state feedback dynamics has been designed, for instance using the methods in [28]. However, for some reason, e.g., monitoring of the plant during operation, it is suspected that there is additional un-modelled dynamics, which we wish to identify.

Since K_θ stabilises $G_{S,\theta}$ and (12) is a *full parametrisation* of all LPV systems stabilised by K_θ, Theorem 1 ensures that there exists an (LPV stable) parameter system S_θ such that $G_{S,\theta}$ can be written as in (12) (or, equivalently, as in (11)).

Consider now the setup shown in Figure 3, where K_θ and G_θ are shown in their factorised form as in (7) and (6), respectively. $n' = (\tilde{M}_\theta + S_\theta \tilde{U}_\theta)n_y$ is the measurement noise that would normally affect the measurements y, relocated in the block diagram to affect the output of the parameter system instead, and r_1 and r_2 are external excitation signals.

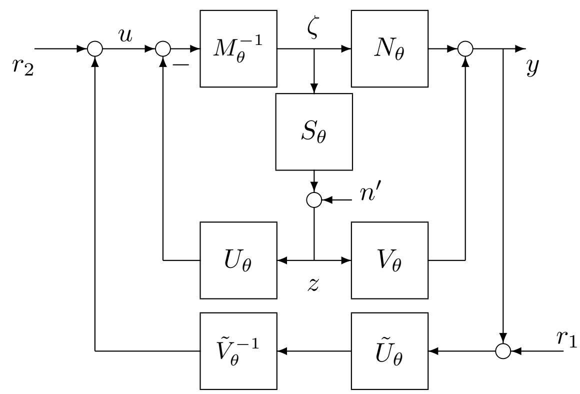

Fig. 3. *'Hansen scheme' setup for closed-loop system identification. The identification of* S_θ *based on samples of* ζ *and* z *is an open-loop identification problem.*

From the block diagram, we find the following relations:

$$(N_\theta + V_\theta S_\theta)\zeta = y - V_\theta n' \tag{13}$$

and

$$
\begin{aligned}
(M_\theta + U_\theta S_\theta)\zeta &= u - U_\theta n' \\
&= r_2 + \tilde{V}_\theta^{-1}\tilde{U}_\theta(y + r_1) - U_\theta n'
\end{aligned} \tag{14}
$$

Applying the LPV operators $\tilde{V}_\theta$ and $\tilde{U}_\theta$ to (13) and (14), respectively, subtracting (14) from (13) and using the Bezout identity then results in

$$\zeta = \tilde{U}_\theta r_1 + \tilde{V}_\theta r_2 \tag{15}$$

In a similar vein, from the block diagram, we have the relations

$$\begin{aligned} M_\theta \zeta &= u - U_\theta z \\ N_\theta \zeta &= y - V_\theta z \end{aligned}$$

Applying the LPV stable filters $\tilde{N}_\theta$ to the top expression and $\tilde{M}_\theta$ to the bottom one, subtracting one from the other and using the Bezout identity then results in

$$z = \tilde{M}_\theta y - \tilde{N}_\theta u \tag{16}$$

Thus, ζ and z can be obtained by filtering measurements through known, stable LPV filters. Furthermore, assuming n_y is independent of r_1 and r_2, then ζ is independent of n' as well.

As a consequence, although u and y are measured in closed-loop, the identification of S_θ using the signals θ, z and ζ becomes equivalent to an open-loop LPV identification problem.

V. Parameter dependency

As argued above, the Hansen Scheme allows open-loop-like identification of S_θ. However, in order to use several of the LPV identification methods mentioned in the Introduction, it is particularly convenient if the system to be identified is affine in θ, which is clearly not evident from Equation (12). Thus, in this section, we investigate what assumptions must be imposed on the overall system's dependency on θ in order to justify identification of an affine S_θ.

Theorem 2 *Suppose an LPV plant*

$$G_{S,\theta} = \left[\begin{array}{c|c} \Phi_\theta & \Gamma_\theta \\ \hline H_\theta & 0 \end{array}\right]$$

where $\Phi_\theta \in \mathbb{R}^{n\times n}$, $\Gamma_\theta \in \mathbb{R}^{n\times m}$ and $H_\theta \in \mathbb{R}^{p\times n}$ are matrix-valued functions of the parameter $\theta \in \Theta$, is known to be stabilised by an LPV controller K_θ with state space realisation (5). Let K_θ be designed based on a nominal plant model $G_\theta \neq G_{S,\theta}$ with state space realisation (4), and let G_θ and K_θ be factorised as given in (8)–(9).

Then the dual Youla-Kucera parameter S_θ in (12) has the state space realisation

$$S_\theta = \left[\begin{array}{cc|c} \Phi_\theta & \Gamma_\theta F_\theta & \Gamma_\theta \\ -L_\theta H_\theta & A_\theta + B_\theta F_\theta + L_\theta C_\theta & B_\theta \\ \hline H_\theta & -C_\theta & 0 \end{array}\right] \tag{17}$$

Proof: We isolate S_θ in (12) and use the Bezout identity to obtain

$$S_\theta = V_\theta^{-1}(G_{S,\theta}K_\theta - I)^{-1}(G_\theta - G_{S,\theta})M_\theta \tag{18}$$

Next, by inserting the expressions

$$V_\theta^{-1} = \left[\begin{array}{c|c} A_\theta + B_\theta F_\theta + L_\theta C_\theta & L_\theta \\ \hline C_\theta & I \end{array}\right]$$

$$\begin{aligned}(G_{S,\theta}K_\theta - I)^{-1} &= \left[\begin{array}{cc|c} \Phi_\theta & \Gamma_\theta F_\theta & 0 \\ 0 & A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta \\ \hline H_\theta & 0 & -I \end{array}\right]^{-1} \\ &= \left[\begin{array}{cc|c} \Phi_\theta & \Gamma_\theta F_\theta & 0 \\ -L_\theta H_\theta & A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta \\ \hline -H_\theta & 0 & -I \end{array}\right]\end{aligned}$$

$$G_\theta - G_{S,\theta} = \left[\begin{array}{cc|c} A_\theta & 0 & B_\theta \\ 0 & \Phi_\theta & \Gamma_\theta \\ \hline C_\theta & -H_\theta & 0 \end{array}\right]$$

and

$$M_\theta = \left[\begin{array}{c|c} A_\theta + B_\theta F_\theta & B_\theta \\ \hline F_\theta & I \end{array}\right]$$

in (18), we get (19) on the following page. Let $\phi_k \in \mathbb{R}^{6n}$ denote the state vector of (19). Then, by applying the state transformation $\psi_k = T\phi_k$, where

$$T = \begin{bmatrix} I & 0 & -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & I & 0 \\ 0 & 0 & -I & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & I & 0 & -I \end{bmatrix}$$

and removing two sets of unobservable and two sets of uncontrollable states, we see that (19) may be reduced to (17). $\triangleleft$

Theorem 2 shows that the dual Youla-Kucera parameter runs the risk of becoming nonlinear in θ if both of the factors in either of the products $\Gamma_\theta F_\theta$, $B_\theta F_\theta$, $L_\theta H_\theta$ or $L_\theta C_\theta$ are θ-dependent. To put it differently, suppose for instance that Φ_θ, A_θ, F_θ and L_θ are affine functions of θ, while the in- and output matrices are constants, i.e., $B_\theta = B$, $C_\theta = C$, $H_\theta = H$ and $\Gamma_\theta = \Gamma$; then each of the state space matrices in S_θ willl depend affinely on θ. We shall assume this in the following example.

$$S_\theta = \left[\begin{array}{cccccc|c} A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta H_\theta & 0 & -L_\theta C_\theta & L_\theta H_\theta & 0 & 0 \\ 0 & \Phi_\theta & \Gamma_\theta F_\theta & 0 & 0 & 0 & 0 \\ 0 & -L_\theta H_\theta & A_\theta + B_\theta F_\theta + L_\theta C_\theta & -L_\theta C_\theta & L_\theta H_\theta & 0 & 0 \\ 0 & 0 & 0 & A_\theta & 0 & B_\theta F_\theta & B_\theta \\ 0 & 0 & 0 & 0 & \Phi_\theta & \Gamma_\theta F_\theta & \Gamma_\theta \\ 0 & 0 & 0 & 0 & 0 & A_\theta + B_\theta F_\theta & B_\theta \\ \hline C_\theta & -H_\theta & 0 & -C_\theta & H_\theta & 0 & 0 \end{array}\right] \quad (19)$$

VI. Simulation Example

We consider the following unstable system with a single time varying parameter $0 \leq \theta \leq 1$:

$$\begin{aligned} x_{k+1} &= \Phi_\theta x_k + \Gamma u_k + K v_k \\ y_k &= H x_k + v_k, \\ \Phi_\theta &= \begin{bmatrix} 0.9 & 0.05 & 0.1 & -0.3 & 0.4 \\ -0.2-0.7\theta & 0.9 & 0 & 0 & 0 \\ 0 & 0.1 & 0.9 & 0.1 & -0.1 \\ 0.3+\theta & 0 & 0 & 0 & 0.3+\kappa \\ 0 & 0.3 & -0.3 & 0.3 & 0.92+0.05\theta \end{bmatrix}, \\ \Gamma &= \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \; K = \begin{bmatrix} -0.8 \\ 0.3 \\ 0 \\ 0 \\ -0.7 \end{bmatrix}, \\ H &= \begin{bmatrix} 0 & 1 & 2 & 1 & -1 \end{bmatrix}, \end{aligned}$$

with $\kappa = 0.3$ and $E\{v_k v_k^T\} = 10^{-6}$. We assume that we already have a reasonably accurate nominal model (A_θ, B, C) of the deterministic part. A_θ is equal to Φ_θ, except that the model assumes $\kappa = 0$, while the input and output matrices are correctly identified, i.e., $B = \Gamma$, $C = H$.

The system is open loop unstable and only barely detectable and stabilisable; in fact, although the model error may seem small, even a slightly larger error can in fact easily cause an unstable closed loop.

A stabilising LPV controller

$$\begin{aligned} x_{c,k+1} &= (A_\theta + BF_\theta + L_\theta C)x_{c,k} - L_\theta y_k \\ u_k &= F_\theta x_{c,k} \end{aligned}$$

with

$$\begin{aligned} F_\theta &= \begin{bmatrix} 0.11-0.27\theta & 0.42 & -0.43 & 0.12+0.05\theta & 0.7 \end{bmatrix} \\ L_\theta &= \begin{bmatrix} 0.87-0.37\theta \\ -0.26-0.77\theta \\ -0.19 \\ 0.47+0.4\theta \\ 0.87 \end{bmatrix} \end{aligned}$$

has been designed for the system. It satisfies the requirements given in Theorem 1 for all $\theta \in [0\ ;\ 1]$.

In closed loop operation, excitation in the form of white noise with variance 1 is added to the input (r_2 in Figure 3). The full output measurement sequence is shown in Figure 4 and a zoom of the signals along with the auxiliary signals used in the Hansen scheme is shown in Figure 5.

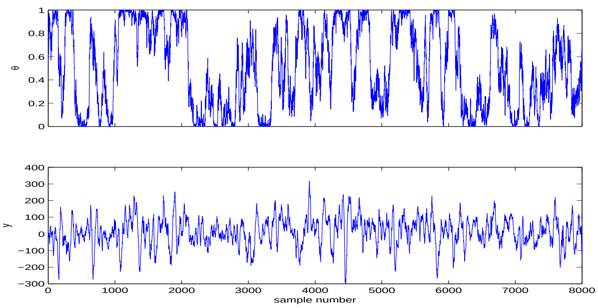

Fig. 4. *Measurement data for system identification. Top: $\theta(k)$; bottom: y_k*

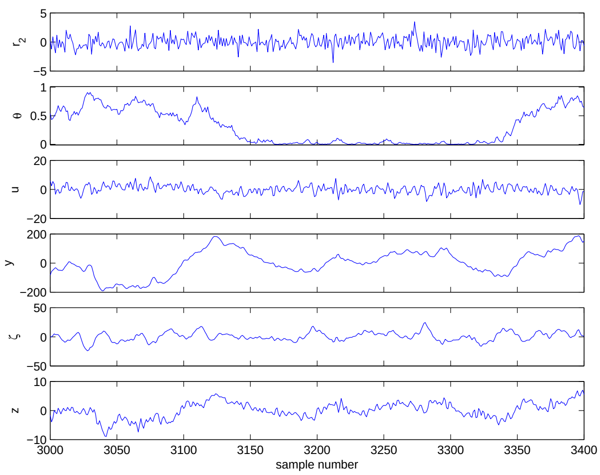

Fig. 5. *Zoom of measurement data, indluding auxiliary signals. From top to bottom: $r_{2,k}$; $\theta(k)$; u_k; y_k; ζ_k; z_k*

In all the identifications, models on the form $\hat{x}_{k+1} = \hat{A}_\theta \hat{x}_k + \hat{B}_\theta u_k$, $\hat{y}_k = \hat{C}\hat{x}_k$ are assumed, with $\hat{A}_\theta$ and $\hat{B}_\theta$ depending linearly on θ.

In order to evaluate the obtained models, the ν-gap between the model and the real system is computed.

The ν-gap is a value between 0 and 1 that expresses the difference between two transfer functions in terms of their similarity with respect to closed loop operation; that is, if the ν-gap between two plant models is small, then a good controller designed for one transfer function will also work well with the other [29]. The ν-gap is only defined for LTI systems, so the comparisons strictly speaking only hold for fixed values of θ. Here, the ν-gap is evaluated for θ frozen at 0, 0.5 and 1.

The identifications are performed using an increasing number of samples, in order to evaluate how much excitation is needed. Two identification methods, ARX and PBSIDopt, are tested, both in a direct form and using the Hansen scheme. The state space matrices are found by minimising the prediction error using least squares methods. Note that we do not assume any explicit knowledge of which entries in A_m are erroneous, so a direct grey box approach is not possible.

The first identification method examined is the LPV ARX method found in e.g. [11] and [17]. Here, the state estimate simply consists of delayed outputs and inputs. In the direct application, the method is simply fed measured input and output data, and a model with 5 delayed outputs and 5 delayed inputs is identified. We assume a zero-order polynomial dependence on θ in the identification. The dash-dot line in Figure 6 shows the ν-gap as a function of the number of samples used. For $\theta = 1$ the model is acceptable, but for $\theta = 0$ and $\theta = 0.5$, even large numbers of samples do not yield acceptable models. Making delayed values of θ available to the identification algorithm did not improve the model, either.

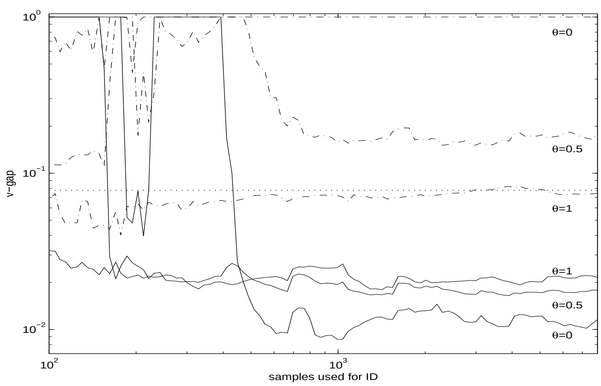

Fig. 6. *ν-gap for models identified using ARX methods, with frozen values of θ, as a function of increasing sample size. Dash-dot: direct identification; solid: Hansen scheme*

Next, the ARX method is used to identify a dual Youla parameter in a Hansen scheme. First the data is filtered as discussed in Section IV. Then the ARX method is used to identify S_θ, again with 5 delayed outputs and 5 delayed inputs, which is then combined with the nominal model as in Eqn. (11). The resulting model error is shown by the solid lines in Figure 6. The dotted lines show the ν-gap for the nominal model (which is approximately 0.08 for all frozen θ), indicating that a significant improvement is achieved with a reasonably small number of samples.

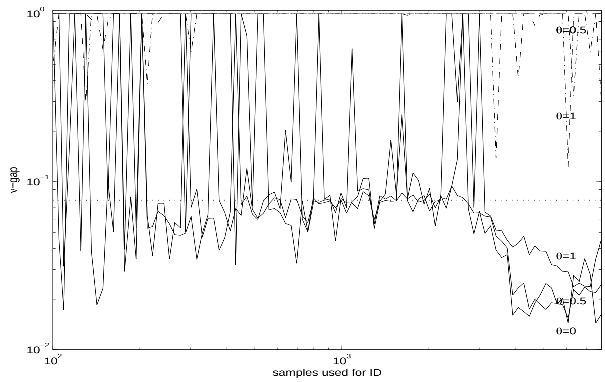

Fig. 7. *ν-gap for models identified using PBSID, with frozen values of θ, as a function of increasing sample size. Dash-dot: direct identification; solid: Hansen scheme*

The second method examined is PBSIDopt, which is presented in an LPV version in [20]. In this approach, a subspace method is used to construct the state estimates, and consequently requires a lot of computational power.

First PBSIDopt (with a window length of 9) is applied directly to the measurements to obtain a 5th order LPV model, and the result, shown by the dash-dot lines in Figure 7, is quite poor. Changing the window length did not improve the identification noticeably.

Next, PBSIDopt (again with a window length of 9) is applied to obtain a 7th order LPV model of S_θ in the Hansen scheme. The ν-gaps of the resulting model is shown with solid lines in Figure 7; as can be seen, the ν-gap drops below those of the nominal model when more then 3000 samples are used. The result is not as good as for the Hansen ARX method, but it is a definite improvement over using PBSIDopt directly.

Figure 8 shows Bode plots for all the models obtained with the maximum number of samples, with θ frozen at 0.9. The picture is similar for all other values of θ; the Hansen scheme is able to capture the spike, whereas the direct methods are not.

The reason that the Hansen scheme improves on the identification is likely different for the two different

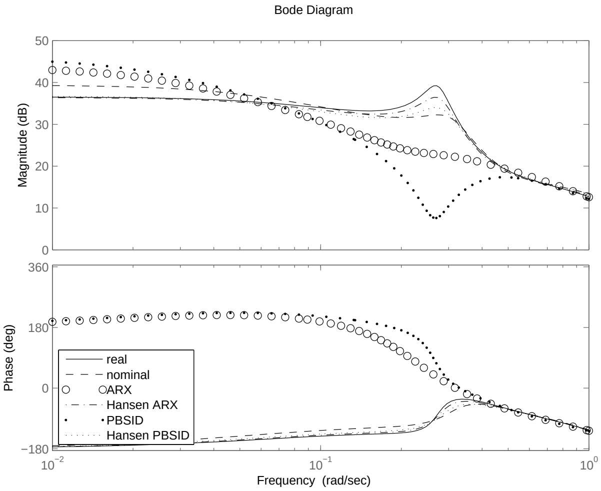

Fig. 8. *Bode plots for different models identified using PBSIDOpt, with* $\theta = 0.9$.

identification methods. For the ARX case, the closed-loop nature of the data affects the direct ARX method, and the Hansen scheme helps to decouple these effects. In PBSIDopt, the main approximation lies in assuming that the state transition is zero beyond the window length; in this example this is not the case. The Hansen scheme, on the other hand, focuses on the identification of a subsystem, where this assumption is closer to being satisfied.

VII. Discussion

In this paper we considered incremental system identification of LPV systems that are modified during online operation, for instance due to replacement and/or addition of system components (so-called *plug-and-play control*). We used the notion of polyhedral Lyapunov functions to prove the existence of a dual Youla-Kucera parameter system for proper polytopic LPV systems in a non-conservative manner. Then we showed how the Hansen scheme can be used for incremental system identification of such LPV systems in an open-loop-like setting. The method is an extension of the Hansen scheme for LTI systems. This particular approach is suited for systems where dynamic elements are changed during online operation, e.g. due to replacement or introduction of new sensors, actuators or other components; only the changed dynamics need to be identified, while nominal plant and controller information may be retained.

REFERENCES

1. F. Hansen, G. Franklin, and R. Kosut, “Closed-loop identification via the fractional representation: Experiment design,” in *Proc. of American Control Conference*, 1989.
2. T. T. Tay, J. B. Moore, and R. Horowitz, “Indirect adaptive techniques for fixed controller performance enhancement,” *International Journal of Control*, vol. 50, pp. 1941–1960, 1989.
3. P. Ansay, M. Gevers, and V. Wertz, “Identification with the Youla parameterization in identification for control,” in *Proc. of the American Control Conference*, 1999.
4. B. D. O. Anderson, “From Youla-Kucera to identification, adaptive and nonlinear control,” *Automatica*, vol. 34, pp. 1485–1506, 1998.
5. M. Gevers, L. Ljung, and P. V. D. Hof, “Asymptotic variance expressions for closed-loop identification,” *Automatica*, vol. 37, no. 5, pp. 781–786, 2001.
6. S. Douma, P. V. den Hof, and O. Bosgra, “Controller tuning freedom under plant identification uncertainty: double Youla beats gap in robust stability,” *Automatica*, vol. 39, no. 2, pp. 325–333, 2003.
7. F. Bruzelius, S. Petterson, and C. Breitholtz, “Linear parameter-varying descriptions of nonlinear systems,” in *Proc. of the 2004 American Control Conference*, 2004.
8. R. Toth, *Modeling and Identification of Linear Parameter-varying Systems*, ser. Lecture Notes in Computer Science. Springer-Verlag, 2010, vol. 403.
9. B. S. Hong, T. Y. Lin, and W. J. Su, “Lpv modeling and synthesis for dc propulsion of electric scooters,” *Asian Journal of Control*, vol. 14, no. 3, pp. 744–755, August 2011.
10. L. Song and J. Yang, “Smooth switching output tracking control for LPV systems,” *Asian Journal of Control*, 2012, *to appear*.
11. L. Giarre, D. Baruso, P. Falugi, and B. Bamieh, “LPV model identification for gain scheduling control: An application to rotating stall and surge control problem,” *Control Engineering Practice*, vol. 14, no. 4, pp. 351–361, 2006.
12. M. Sznaier and M. C. Mazzaro, “An LMI approach to control-oriented identification and model (in) validation of LPV systems,” *IEEE Transactions on Automatic Control*, vol. 48, pp. 1619–1625, 2003.
13. F. Bianchi and R. Sanchez-Pena, “Robust identification/invaldation in an LPV framework,” *International Journal of Robust and Nonlinear Control*, vol. 20, no. 3, pp. 301–312, 2010.

14. J. V. Salcedo and M. Martinez, “LPV identification of a turbocharged diesel engine,” *Applied Numerical Mathematics*, vol. 58, pp. 1553–1571, 2008.
15. K. Hsu, T. Vincent, and K. Poolla, “Nonparametric methods for the identification of linear parameter varying systems,” in *Proceedings of IEEE Intl. Conf. on Computer-Aided Control Systems*, 2008, pp. 846–851.
16. J. Caigny, J. Camino, and J. Swevers, “Interpolating model identification for siso linear parameter-varying systems,” *Mechanical Systems and Signal Processing*, vol. 23, no. 8, pp. 2395–2417, 2009.
17. M. Butcher, A. Karimi, and R. Longchamp, “On the consistency of certain identification methods for linear parameter varying systems,” in *Proceedings of the 17th IFAC World Congress*, Seoul, Korea, Jul. 2008.
18. V. Laurain, M. Gilson, R. Toth, and H. Garnier, “Refined instrumental variable methods for identification of LPV Box-Jenkins models,” *Automatica*, vol. 46, no. 6, pp. 959–967, 2010.
19. P. L. dos Santos, J. A. Ramos, and J. L. M. de Carvalho, “Identification of linear parameter varying systems using an iterative deterministicstochastic subspace approach,” in *Proc. of the European Control Conference*, 2007, pp. 4867–4873.
20. J. van Wingerden and M. Verhaegen, “Subspace identification of bilinear and LPV systems for open- and closed-loop data,” *Automatica*, vol. 45, no. 2, pp. 372–381, 2009.
21. R. Toth, C. Lyzell, M. Enqvist, P. Heuberger, and P. van den Hof, “Order and structural dependence selection of LPV-ARX models using a non-negative garrote approach,” in *Proc. of 48th IEEE Conference on Decision and Control*, Dec. 2009.
22. V. Verdult and M. Verhaegen, “Subspace identification of multivariable linear parameter varying systems,” *Automatica*, vol. 38, pp. 805–814, 2002.
23. R. Toth, P. Heuberger, and P. van den Hof, “Asymptotically optimal orthonormal basis functions for LPV system identification,” *Automatica*, vol. 45, no. 6, pp. 1359–1370, 2009.
24. F. Blanchini, S. Miani, and F. Mesquine, “A separation principle for linear switching systems and parametrization of all stabilizing controllers,” *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 279–292, 2009.
25. S. Dasgupta and B. D. O. Anderson, “A parametrization for the closed-loop identification of nonlinear time-varying systems,” *Automatica*, vol. 32, pp. 1349–1360, 1996.
26. K. Fujimoto, B. Anderson, and F. Bruyne, “A parameterization for closed-loop identification of nonlinear systems based on differentially coprime kernel representations,” *Automatica*, vol. 37, no. 6, pp. 1893–1907, 2001.
27. K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Prentice-Hall, 1996.
28. W. Heemels, J. Daafouz, and G. Millerioux, “Design of observer-based controllers for LPV systems with unknown parameters,” in *Proceedings of Conf. Decision and Control*, 2009, pp. 1836–1841.
29. G. Vinnicombe, *Uncertainty and Feedback*. Imperial College Press, 2001.
30. J. Bendtsen and K. Trangbaek, “Gain scheduling control based on closed-loop system identification,” in *Proc. of IFAC Workshop on Adaptation and Learning in Control and Signal Processing*, Yokohama, Japan, Sep. 2004.