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Sturm, Bob L.; Daudet, Laurent

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# Recursive Nearest Neighbor Search in a Sparse and Multiscale Domain for Comparing Audio Signals

Bob L. Sturm (EURASIP # 7255)<sup>a,\*</sup>, Laurent Daudet (EURASIP # 2298)<sup>b</sup>

<sup>a</sup>*Department of Architecture and Media Technology, Aalborg University Copenhagen  
Lautrupvang 15, 2750 Ballerup, Denmark*

<sup>b</sup>*Institut Langevin (LOA), Université Paris Diderot – Paris 7,  
UMR 7587, 10, rue Vauquelin, 75231 Paris, France*

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## Abstract

We investigate recursive nearest neighbor search in a sparse domain at the scale of audio signals. Essentially, to approximate the cosine distance between the signals we make pairwise comparisons between the elements of localized sparse models built from large and redundant multiscale dictionaries of time-frequency atoms. Theoretically, error bounds on these approximations provide efficient means for quickly reducing the search space to the nearest neighborhood of a given data; but we demonstrate here that the best bound defined thus far involving a probabilistic assumption does not provide a practical approach for comparing audio signals with respect to this distance measure. Our experiments show, however, that regardless of these non-discriminative bounds, we only need to make a few atom pair comparisons to reveal, e.g., the origin of an excerpted signal, or melodies with similar time-frequency structures.

*Keywords:* Multiscale decomposition; sparse approximation; time-frequency dictionary; audio similarity

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## 1. Introduction

2 Sparse approximation is essentially the modeling of data with few terms  
3 from a large and typically overcomplete set of atoms, called a “dictionary” [24].

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\*Corresponding author

Email addresses: `boblsturm@gmail.com` (Bob L. Sturm (EURASIP # 7255) ),  
`laurent.daudet@espci.fr` (Laurent Daudet (EURASIP # 2298))

4 Consider an  $\mathbf{x} \in \mathbb{R}^K$ , and a dictionary  $\mathcal{D}$  composed of  $N$  unit-norm atoms in the  
5 same space, expressed in matrix form as  $\mathbf{D} \in \mathbb{R}^{K \times N}$ , where  $N \gg K$ . A pursuit  
6 is an algorithm that decomposes  $\mathbf{x}$  in terms of  $\mathbf{D}$  such that  $\|\mathbf{x} - \mathbf{D}\mathbf{s}\|^2 \leq \epsilon$  for  
7 some error  $\epsilon \geq 0$ . (In this paper, we work in a Hilbert space unless otherwise  
8 noted.) When  $\mathbf{D}$  is overcomplete,  $\mathbf{D}$  has full row rank and there exists an infinite  
9 number of solutions to choose from, even for  $\epsilon = 0$ . Sparse approximation aims  
10 to find a solution  $\mathbf{s}$  that is mostly zeros for  $\epsilon$  small. In that case, we say that  $\mathbf{x}$   
11 is sparse in  $\mathcal{D}$ .

12 Matching Pursuit (MP) is an iterative descent sparse approximation method  
13 based on greedy atom selection [17, 24]. We express the  $n$ th-order model of the  
14 signal  $\mathbf{x} = \mathbf{H}(n)\mathbf{a}(n) + \mathbf{r}(n)$ , where  $\mathbf{a}(n)$  is a length- $n$  vector of weights,  $\mathbf{H}(n)$   
15 are the  $n$  corresponding columns of  $\mathbf{D}$ , and  $\mathbf{r}(n)$  is the residual. MP augments  
16 the  $n$ th-order representation,  $\mathcal{X}_n = \{\mathbf{H}(n), \mathbf{a}(n), \mathbf{r}(n)\}$ , according to:

$$\mathcal{X}_{n+1} = \left\{ \begin{array}{l} \mathbf{H}(n+1) = [\mathbf{H}(n) | \mathbf{h}_n], \\ \mathbf{a}(n+1) = [\mathbf{a}^T(n), \langle \mathbf{r}(n), \mathbf{h}_n \rangle]^T, \\ \mathbf{r}(n+1) = \mathbf{x} - \mathbf{H}(n+1)\mathbf{a}(n+1) \end{array} \right\} \quad (1)$$

17 using the atom selection criterion

$$\mathbf{h}_n = \arg \min_{\mathbf{d} \in \mathcal{D}} \|\mathbf{r}(n) - \langle \mathbf{r}(n), \mathbf{d} \rangle \mathbf{d}\|^2 = \arg \max_{\mathbf{d} \in \mathcal{D}} |\langle \mathbf{r}(n), \mathbf{d} \rangle| \quad (2)$$

18 where  $\|\mathbf{d}\| = 1$  is implicit. The inner product here is defined  $\langle \mathbf{x}, \mathbf{y} \rangle \triangleq \mathbf{y}^T \mathbf{x}$ . This  
19 criterion guarantees  $\|\mathbf{r}(n+1)\|^2 \leq \|\mathbf{r}(n)\|^2$  [24]. Other sparse approximation  
20 methods include orthogonal MP [28], orthogonal least squares (OLS) [41, 33],  
21 molecular methods [9, 38, 19], cyclic MP and OLS [36], and minimizing a relaxed  
22 sparsity measure [6]. These approaches have higher computational complexities  
23 than MP, but can produce data models that are more sparse.

24 Sparse approximation is data-adaptive and can produce parametric and mul-  
25 tiscale models having features that function more like mid-level “objects” than  
26 low-level projections onto sets of vectors [9, 19, 38, 8, 22, 27, 32, 34, 43]. These  
27 aspects make sparse approximation a compelling complement to state-of-the-art  
28 approaches for and applications of comparing audio signals based upon, e.g.,

29 monoresolution cepstral and redundant time-frequency representations, such as  
30 fingerprinting [42], cover song identification [26, 10, 3, 35], content segmentation,  
31 indexing, search and retrieval [16, 4], artist or genre classification [39].

32 In the literature we find some existing approaches to working with audio sig-  
33 nals in a sparse domain. Features built from sparse approximations can provide  
34 competitive descriptors for music information retrieval tasks, such as beat track-  
35 ing, chord recognition, and genre classification [40, 32]. Sparse representation  
36 classifiers have been applied to music genre recognition [27, 5], and robust speech  
37 recognition [12]. Parameters of sparse models can be compared using histograms  
38 to find similar sounds in acoustic environment recordings [7, 8], or atoms can  
39 be learned to compare and group percussion sounds [34]. Biologically-inspired  
40 sparse codings of correlograms of sounds can be used to learn associations be-  
41 tween descriptive high-level keywords and audio features such that new sounds  
42 can be automatically categorized, and large collections of sounds can be queried  
43 in meaningful ways [22]. Outside the realm of audio signals, sparse approxi-  
44 mation has been applied to face recognition [44], object recognition [29], and  
45 landmine detection [25].

46 In this paper, we discuss the comparison of audio signals in sparse domains,  
47 but not specifically for fingerprinting or efficient audio indexing and search —  
48 two tasks that have been convincingly solved [42, 13, 16, 18]. We explore the pos-  
49 sibilities and effectiveness of comparing, atom-by-atom, audio signals modeled  
50 using sparse approximation and large overcomplete time-frequency dictionaries.  
51 Our contributions are three-fold: 1) we generalize an iterative nearest-neighbor  
52 search algorithm to comparing subsequences [14, 15]; 2) we show that though  
53 sparse models of audio signals can be compared by considering pairs of atoms,  
54 the best bound so far derived [14, 15] does not make a practical procedure;  
55 and 3) we show experimentally that the hierarchic comparison of audio signals  
56 in a sparse domain still provides intriguing and informative results. Overall,  
57 our work here shows that a sparse domain can facilitate comparisons of audio  
58 signals in “hierarchical” ways through comparing individual elements of each  
59 sparse data model organized roughly in order of importance.

60 In the next two sections, we discuss and elaborate upon a recursive method  
 61 of nearest neighbor search in a sparse domain [14, 15]. We extend this method to  
 62 comparing subsequences, and examine the practicality of probabilistic bounds  
 63 on the distances between neighbors. In the fourth section, we describe several  
 64 experiments in which we compare a variety of audio signals through comparisons  
 65 of their sparse models. We conclude with a discussion about the results and  
 66 several future directions.

## 67 2. Nearest Neighbor Search by Recursion in a Sparse Domain

68 Consider a set of signals

$$\mathcal{Y} \triangleq \{\mathbf{y}_i \in \mathbb{R}^K : \|\mathbf{y}_i\| = 1\}_{i \in \mathcal{I}} \quad (3)$$

69 where  $\mathcal{I} = \{1, 2, \dots\}$  indexes this set, and a query signal  $\mathbf{x}_q \in \mathbb{R}^K$ ,  $\|\mathbf{x}_q\| = 1$ .  
 70 Assume that we have generated sparse approximations for all of these sig-  
 71 nals  $\hat{\mathcal{Y}} \triangleq \{\{\mathbf{H}_i(n_i), \mathbf{a}_i(n_i), \mathbf{r}_i(n_i)\} : \mathbf{y}_i = \mathbf{H}_i(n_i)\mathbf{a}_i(n_i) + \mathbf{r}_i(n_i)\}_{i \in \mathcal{I}}$  using a dic-  
 72 tionary  $\mathbf{D}$  that spans the space  $\mathbb{R}^K$ , and giving the  $n_q$ -order representation  
 73  $\{\mathbf{H}_q(n_q), \mathbf{a}_q(n_q), \mathbf{r}_q(n_q)\}$  for  $\mathbf{x}_q$ . Since  $\mathcal{D}$  spans  $\mathbb{R}^K$ ,  $\mathcal{D}$  is “complete,” and any  
 74 signal in  $\mathbb{R}^K$  is “compressible” in  $\mathcal{D}$ , meaning that we can order the represen-  
 75 tation weights in  $\mathbf{a}_i(n_i)$  or  $\mathbf{a}_q(n_q)$  in terms of decreasing magnitude, i.e.,

$$0 < |[\mathbf{a}_i(n_i)]_{m+1}| \leq |[\mathbf{a}_i(n_i)]_m| \leq C m^{-\gamma}, \quad m = 1, 2, \dots, n_i - 1 \quad (4)$$

76 for  $n_i$  arbitrarily large, with  $C > 0$ , and where  $[\mathbf{a}]_m$  is the  $m$ th element of the  
 77 column vector  $\mathbf{a}$ . This can be seen in the magnitude representation weights in  
 78 Fig. 1, which are weights of sparse representations of piano notes, described  
 79 in Section 4.1. With MP and a complete dictionary, we are guaranteed  $\gamma > 0$   
 80 because  $\|\mathbf{r}(n+1)\|^2 < \|\mathbf{r}(n)\|^2$  for all  $n$  [24].

81 Consider the Euclidean distance between two signals of the same dimension,  
 82 which is the cosine distance for unit-norm signals. Thus, with respect to this  
 83 distance, the  $\mathbf{y}_i \in \mathcal{Y}$  nearest to  $\mathbf{x}_q$  is given by solving

$$\min_{i \in \mathcal{I}} \|\mathbf{y}_i - \mathbf{x}_q\| = \max_{i \in \mathcal{I}} \langle \mathbf{x}_q, \mathbf{y}_i \rangle. \quad (5)$$



Figure 1: Gray lines show decays of representation weight magnitudes as a function of approximation order  $k$  for several decompositions of unit-norm signals (4-second recordings of single piano notes described in Section 4.1). Thick black line shows a global compressibility bound with its parameters.

84 We can express this inner product in terms of sparse representations

$$\begin{aligned} \langle \mathbf{x}_q, \mathbf{y}_i \rangle &= \langle \mathbf{H}_q(n_q) \mathbf{a}_q(n_q) + \mathbf{r}_q(n_q), \mathbf{H}_i(n_i) \mathbf{a}_i(n_i) + \mathbf{r}_i(n_i) \rangle \\ &= \mathbf{a}_i^T(n_i) \mathbf{H}_i^T(n_i) \mathbf{H}_q(n_q) \mathbf{a}_q(n_q) + O[\mathbf{r}_q, \mathbf{r}_i]. \end{aligned} \quad (6)$$

85 With a complete dictionary we can make  $O[\mathbf{r}_q, \mathbf{r}_i]$  negligible by choosing  $\epsilon$  ar-  
86 bitrarily small, so we can express (5) as

$$\max_{i \in \mathcal{I}} \langle \mathbf{x}_q, \mathbf{y}_i \rangle \sim \max_{i \in \mathcal{I}} \mathbf{a}_i^T(n_i) \mathbf{G}_{iq} \mathbf{a}_q(n_q) = \max_{i \in \mathcal{I}} \sum_{m=1}^{n_i} \sum_{l=1}^{n_q} [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{ml} \quad (7)$$

87 where  $[\mathbf{B} \bullet \mathbf{C}]_{ml} = [\mathbf{B}]_{ml} [\mathbf{C}]_{ml}$  is the Hadamard, or entry wise, product,  $[\mathbf{B}]_{ml}$   
88 is the element of  $\mathbf{B}$  in the  $m$ th row of the  $l$ th column,  $\mathbf{G}_{iq} \triangleq \mathbf{H}_i^T(n_i) \mathbf{H}_q(n_q)$  is a  
89  $n_i \times n_q$  matrix with elements from the Gramian of the dictionary, i.e.,  $\mathbf{G} \triangleq \mathbf{D}^T \mathbf{D}$ ,  
90 and finally we define the outer product of the weights

$$\mathbf{A}_{iq} \triangleq \mathbf{a}_i(n_i) \mathbf{a}_q^T(n_q). \quad (8)$$

## 91 2.1. Recursive Search Limited by Bounds

92 Since we expect the decay of the magnitude of elements in  $\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}$  to be  
93 fastest in diagonal directions by (4), we define a recursive sum along the  $M$

94 anti-diagonals starting at the top left:

$$S_{iq}(M) \triangleq S_{iq}(M-1) + \sum_{m=1}^M [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{m(M-m+1)} \quad (9)$$

95 for  $M = 2, 3, \dots, \min(n_i, n_q)$ , and setting  $S_{iq}(1) = [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{11}$ . With this we  
 96 can express the argument of (7) as

$$\langle \mathbf{x}_q, \mathbf{y}_i \rangle \approx \sum_{m=1}^{n_i} \sum_{l=1}^{n_q} [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{ml} = S_{iq}(M) + R(M) \quad (10)$$

97 where at step  $M$ , we are comparing  $M$  additional pairs of atoms to those con-  
 98 sidered in the previous steps.  $R(M)$  is a remainder that we will bound. The  
 99 total number of atom pairs contributing to  $S_{iq}(M)$  (9) is

$$P(M) \triangleq \sum_{m=1}^M m = M(M+1)/2. \quad (11)$$

100 The approach taken by Jost et al. [14, 15] to find the nearest neighbors of  
 101  $\mathbf{x}_q$  in  $\mathcal{Y}$  bounds the remainder  $R(M)$  by compressibility (4). Assuming we have  
 102 a positive upper bound on the remainder, i.e.,  $R(M) \leq \tilde{R}(M)$ , we know lower  
 103 and upper bounds on the cosine distance  $L_{iq}(M) \leq \langle \mathbf{x}_q, \mathbf{y}_i \rangle \leq U_{iq}(M)$ , where

$$L_{iq}(M) \triangleq S_{iq}(M) - \tilde{R}(M) \quad (12)$$

$$U_{iq}(M) \triangleq S_{iq}(M) + \tilde{R}(M). \quad (13)$$

104 Finding elements of  $\mathcal{Y}$  close to  $\mathbf{x}_q$  with respect to (5) can be done recursively  
 105 over the approximation order  $M$ . For a given  $M$ , we find  $\{S_{iq}(M)\}_{i \in \mathcal{I}}$ , com-  
 106 pute the remainder  $\tilde{R}(M)$ , and eliminate signals that are not sufficiently close  
 107 to  $\mathbf{x}_q$  with respect to their cosine distance by comparing the bounds. This  
 108 approach is similar to hierarchical ones, e.g., [21], where the features become  
 109 more discriminable as the search process runs. (Also note that compressibility  
 110 is similar to the argument made in justifying the truncation of Fourier series in  
 111 early work on similarity search [1, 11, 30], i.e., that power spectral densities of  
 112 many time-series decay like  $\mathcal{O}(|f|^{-b})$  with  $b > 1$ .)

113 Starting with  $M = 1$ , we compute the sets  $\{L_{iq}(1)\}_{i \in \mathcal{I}}$  and  $\{U_{iq}(1)\}_{i \in \mathcal{I}}$ ,  
 114 that is, the first-order upper and lower bounds of the set of distances of  $\mathbf{x}_q$



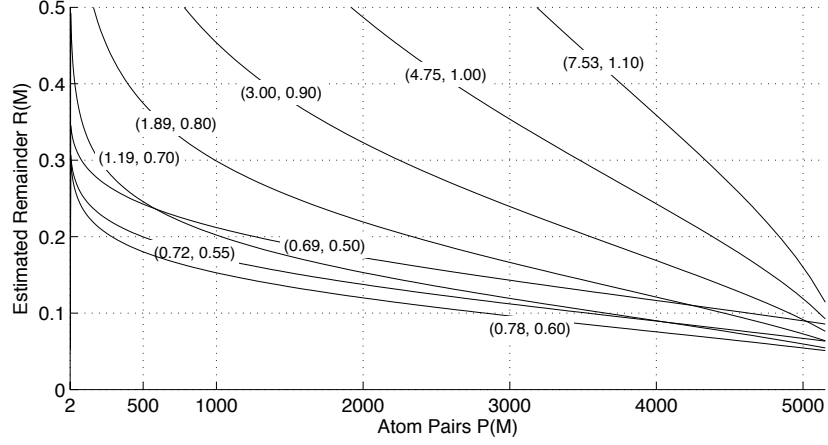


Figure 2: Estimated remainder (assuming unit-norm signals) using bound in (16) with  $p = 0.2$  (probability that remainder does not exceed bound) and  $n = 100$  (number of elements in each sparse model) as a function of the number of atom pairs already considered for several pairs of compressibility parameters  $(C, \gamma)$  estimated from the dataset used to produce Fig. 1.

115 from all signals in  $\mathcal{Y}$ . Then we find the index of the largest lower bound  
116  $i_{\max} = \arg \max_{i \in \mathcal{I}} L_{iq}(1)$ , and reduce the search space to  $\mathcal{I}_1 \triangleq \{i \in \mathcal{I} : U_{iq}(1) \geq$   
117  $L_{i_{\max}q}(1)\}$ , since all other data have a least upper bound on their inner prod-  
118 uct with  $\mathbf{x}_q$  than the greatest lower bound in the set. For the next step, we  
119 compute the sets  $\{L_{iq}(2)\}_{i \in \mathcal{I}_1}$  and  $\{U_{iq}(2)\}_{i \in \mathcal{I}_1}$ , find the index of the maxi-  
120 mum  $i_{\max} = \arg \max_{i \in \mathcal{I}_1} L_{iq}(2)$ , and construct the reduced set  $\mathcal{I}_2 \triangleq \{i \in \mathcal{I}_1 :$   
121  $U_{iq}(2) \geq L_{i_{\max}q}(2)\}$ . Continuing in this way, we find the elements of  $\mathcal{Y}$  closest  
122 to  $\mathbf{x}_q$  at each  $M$  with respect to the cosine distance by recursing into the sparse  
123 approximations of the signals.

## 124 2.2. Bounding the Remainder

125 To reduce the search space quickly we desire that (12) and (13) converge  
126 quickly to the neighborhood of  $\langle \mathbf{x}_q, \mathbf{y}_i \rangle$ , or in other words, that the bounds on  
127 the remainder quickly become discriminative. Jost et al. [14, 15] derive three  
128 different bounds on  $R(M)$ . From the weakest to the strongest, these are:

129 1.  $[\mathbf{G}_{iq}]_{ml} = 1$  (worst case scenario, and impossible for  $n > 1$ )

$$R(M) \leq C^2 (\|\mathbf{c}_M^\gamma\|_1 + \|\mathbf{d}^\gamma\|_1) \quad (14)$$

130 2.  $[\mathbf{G}_{iq}]_{ml} \sim \text{iid Bernoulli}(0.5)$ ,  $\Omega = \{-1, 1\}$  (impossible for  $n > 1$ )

$$R(M) \leq C^2 \sqrt{\ln 4} (\|\mathbf{c}_M^\gamma\|_2^2 + \|\mathbf{d}^\gamma\|_2^2)^{1/2} \quad (15)$$

131 3.  $[\mathbf{G}_{iq}]_{ml} \sim \text{iid Uniform}$ ,  $\Omega = [-1, 1]$ ,

$$R(M) \leq C^2 \sqrt{2/3} \text{Erf}^{-1}(p) (\|\mathbf{c}_M^\gamma\|_2^2 + \|\mathbf{d}^\gamma\|_2^2)^{1/2} \quad (16)$$

132 with probability  $0 \leq p \leq 1$

133 where we define the following vectors for  $n \triangleq \min(n_i, n_q)$  and  $M = 2, \dots, n$

$$\mathbf{c}_M^\gamma \triangleq \{[l(m-l+1)]^{-\gamma} : m = M+1, \dots, n; l = 1, \dots, m\} \quad (17)$$

$$\mathbf{d}^\gamma \triangleq \{[l(n-m+1)]^{-\gamma} : m = 1, \dots, n-1; l = m+1, \dots, n\}. \quad (18)$$

134 Appendix A gives derivations of these bounds, as well as the efficient computa-  
 135 tion of (16) for the special case of  $\gamma = 0.5$ . The parameters  $(C, \gamma)$  describe the  
 136 compressibility of the signals in the dictionary (4). The bounds of (15) and (16)  
 137 are much more discriminative than (14) because they involve an  $\ell_2$ -norm at the  
 138 price of uncertainty in the bound. The bound in (16) is attractive because we  
 139 can tune it with the parameter  $p$ , which is the probability that the remainder  
 140 will not exceed the bound. Figure 2 shows bounds based on (16) for several  
 141 pairs of compressibility parameters for the dataset used to produce Fig. 1.

### 142 2.3. Estimating the Compressibility Parameters

143 The bounds (14)–(16), and consequently the number of atom pairs we must  
 144 consider before the bounds become discriminable, depend on the compressibility  
 145 parameters,  $(C, \gamma)$  — which themselves depend in a complex way on the signal,  
 146 the dictionary, and the method of sparse approximation. Figure 3 shows the  
 147 error surface, feasible set, and the optimal parameters for the dataset used to  
 148 produce Fig. 1. We describe our parameter estimation procedure in Appendix  
 149 B. The resulting bound is shown in black in Fig. 1. These compressibility  
 150 parameters also agree with those seen in Fig. 2.

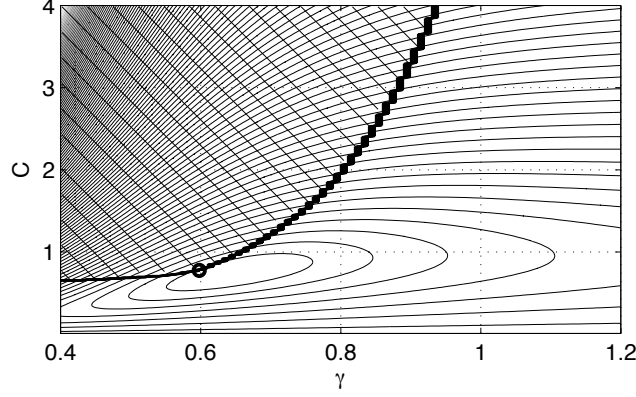


Figure 3: Error surface as a function of the compressibility parameters for the dataset used to produce Fig. 1, with the feasible set shaded at top left, and optimal parameters marked by a circle.

### 151 3. Subsequence Nearest Neighbor Search in a Localized Sparse Do- 152 main

153 The recursive nearest neighbor search so far described has the obvious limi-  
154 tation that it cannot be applied to comparing subsequences of large data vectors,  
155 as is natural for comparing audio signals. Thus, we must adapt its structure to  
156 work for comparing subsequences in a set of data

$$\mathcal{Y} \triangleq \{\mathbf{y}_i \in \mathbb{R}^{N_i} : N_i \geq K\}_{i \in \mathcal{I}} \quad (19)$$

157 (note that now we do not restrict the norms of these signals). We can create  
158 from the elements of  $\mathcal{Y}$  a new set of all subsequences having the same length as  
159 a  $K$ -dimensional query  $\mathbf{x}_q$  ( $K < N_i$ ):

$$\mathcal{Y}_K \triangleq \left\{ \mathbf{P}_t \mathbf{y}_i / \|\mathbf{P}_t \mathbf{y}_i\| : t \in \mathcal{T}_i = \{1, 2, \dots, N_i - K + 1\}, \mathbf{y}_i \in \mathcal{Y} \right\} \quad (20)$$

160 where  $\mathbf{P}_t$  extracts a  $K$ -length subsequence in  $\mathbf{y}_i$  starting a time-index  $t$  (it is an  
161 identity matrix of size  $K$  starting a column  $t$  in a  $K \times N_i$  matrix of zeros). The set  
162  $\mathcal{T}_i$  are times at which we create length- $K$  subsequences from  $\mathbf{y}_i$ . If we decompose  
163 each of these by sparse approximation, then we can use the framework in the  
164 previous section. However, sparse approximation is an expensive operation that

we want to do only once for the entire signal, and independent of the length of  $\mathbf{x}_q$ .

To address this problem, we instead approximate each element in  $\mathcal{Y}_K$  by building local sparse representations from the global sparse approximations of each  $\mathbf{y}_i$ , and then calculating their distance to  $\mathbf{x}_q$  using the framework in the previous section. From here on we consider only the  $K$ -length subsequences of a single element  $\mathbf{y}_i \in \mathcal{Y}$  without loss of generality (i.e., all other elements of  $\mathcal{Y}$  can be included as subsequences). Toward this end, consider that we have decomposed the  $N_i$ -length signal  $\mathbf{y}_i$  using a complete dictionary to produce the representation  $\{\mathbf{H}_i(n_i), \mathbf{a}_i(n_i), \mathbf{r}_i(n_i)\}$ . From this we construct the local sparse representations of  $\mathbf{y}_i$ :

$$\hat{\mathcal{Y}}_K \triangleq \left\{ \{\mathbf{P}_t \mathbf{H}_i(n_i), \xi_t \mathbf{a}_i(n_i), \mathbf{P}_t \mathbf{r}_i(n_i)\} : t \in \mathcal{T}_i \right\} \quad (21)$$

where the time partition  $\mathcal{T}_i$  is the set of all times at which we extract a  $K$ -length subsequence from  $\mathbf{y}_i$ , and  $\xi_t$  is set such that  $\|\xi_t \mathbf{P}_t \mathbf{y}_i\| = 1$ , i.e., each length- $K$  subsequence is unit-norm. For each  $K$ -dimensional subsequence, (7) now becomes

$$\begin{aligned} \max_{t \in \mathcal{T}_i} \langle \mathbf{x}_q, \mathbf{P}_t \mathbf{y}_i \rangle &= \max_{t \in \mathcal{T}_i} \left[ \langle \mathbf{H}_q(n_q) \mathbf{a}_q(n_q), \xi_t \mathbf{P}_t \mathbf{H}_i(n_i) \mathbf{a}_i(n_i) \rangle + O[\mathbf{r}_q, \mathbf{r}_i] \right] \\ &\approx \max_{t \in \mathcal{T}_i} \xi_t \mathbf{a}_i^T(n_i) \mathbf{H}_i^T(n_i) \mathbf{P}_t^T \mathbf{H}_q(n_q) \mathbf{a}_q(n_q) \\ &= \max_{t \in \mathcal{T}_i} \xi_t \sum_{m=1}^{n_i} \sum_{l=1}^{n_q} [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}(t)]_{ml} \end{aligned} \quad (22)$$

where  $\mathbf{A}_{iq}$  is defined in (8), we define the time-localized Gramian

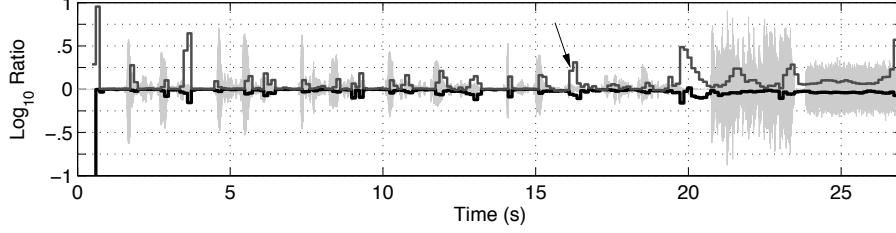
$$\mathbf{G}_{iq}(t) \triangleq \mathbf{H}_i^T(n_i) \mathbf{P}_t^T \mathbf{H}_q(n_q) \quad (23)$$

and we have excluded the terms involving the residuals because we can make them arbitrarily small.

### 3.1. Estimating the Localized Energy

The only thing left to do is find an expression for  $\xi_t$  so that each subsequence is comparable with the others with respect to the cosine distance. We

(a) Six Speech Signals (0-20 s), Music Excerpt (21-23 s), Realization of GWN (24-27 s)



(b) Music: Orchestra

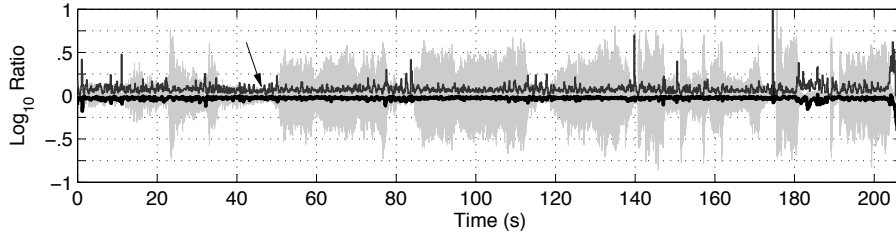


Figure 4: Short-term energy ratios,  $\log_{10}(\sum_{j=1}^{n_t} w_j a_j^2 / \|\mathbf{P}_t \mathbf{y}_i\|^2)$ , over 1 s windows (hopped 125 ms) for MP decompositions using 8xMDCT [31] to a global residual energy 30 dB below the initial signal energy. Arrow points to line (top, gray) using weighting  $w_j = 1$ . The other line (bottom, black) uses (25). Data in (a) are described in Section 4.2; data in (b) are described in Section 4.3.

186 assume that the localized energy can be approximated from the local sparse  
 187 representation in the following way assuming  $\|\mathbf{P}_t \mathbf{y}_i\| > 0$

$$\xi_t = \|\mathbf{P}_t \mathbf{y}_i\| \approx \sqrt{\mathbf{a}_i^T(n_i) \mathbf{H}_i^T(n_i) \mathbf{P}_t^T \mathbf{P}_t \mathbf{H}_i(n_i) \mathbf{a}_i(n_i)} \approx \sqrt{\sum_{j=1}^{n_t} w_j a_j^2} \quad (24)$$

188 where the  $n_t$  weights  $a_j \in \{\mathbf{a}_i(n_i)\}_m : [\mathbf{H}_i^T(n_i) \mathbf{P}_t^T \mathbf{P}_t \mathbf{H}_i(n_i)]_{ml} \neq 0, 1 \leq m, l \leq$   
 189  $n_i\}$  are those associated with atoms having support in  $[t, t + K)$ , and  $w_j$  we  
 190 define to weigh the contribution of  $a_j^2$  to the localized energy estimate. We set  
 191  $\xi_t = 0$  if  $\sum_{j=1}^{n_t} a_j^2 = 0$ .

192 If all atoms contributing to the subsequence have their entire support in  
 193  $[t, t + K)$ , and are orthonormal, then we can set each  $w_j = 1$ . This does not  
 194 hold for subsequences of a signal decomposed using an overcomplete dictionary,  
 195 as shown by Fig. 4. For much of the time we see  $\sum_{j=1}^{n_t} a_j^2 \geq \|\mathbf{P}_t \mathbf{y}_i\|^2$ , which

means our localized estimate of the segment energy is greater than its real value. This will make  $\xi_t$  and consequently (22) smaller.

Instead, we make a more reasonable estimate of  $\|\mathbf{P}_t \mathbf{y}_i\|$  by accounting for the fact that atoms can have support outside  $[t, t + K)$ . For instance, if an atom has some fraction of support in the subsequence we multiply its weight by that fraction. We thus weigh the contribution of the  $j$ th atom to the subsequence norm using

$$w_j = \begin{cases} 1, & u_j \geq t, u_j + s_j \leq t + K \\ (K/s_j)^2, & u_j < t, u_j + s_j \geq t + K \\ (u_j + s_j - t)^2/s_j^2, & u_j < t, t < u_j + s_j \leq t + K \\ (t + K - u_j)^2/s_j^2, & t \leq u_j < t + K, u_j + s_j > t + K \end{cases} \quad (25)$$

where  $u_j$  and  $s_j$  are the position and scale, respectively, of the atom associated with the weight  $a_j$ . In other words, if an atom is completely in  $[t, t + K)$ , it contributes all of its energy to the approximation; otherwise, it contributes only a fraction based on how its support intersects  $[t, t + K)$ . With this we are now slightly underestimating the localized energies, as seen in Fig. 4. In both of these cases for  $\{w_j\}$ , however, we can assume by the energy conservation of MP [24] that as the subsequence length becomes larger our error in estimating the subsequence energy goes to zero, i.e.,

$$\lim_{K \rightarrow N_i} \|\mathbf{P}_t \mathbf{y}_i\|^2 - \sum_{j=1}^{n_t} w_j a_j^2 = \|\mathbf{P}_t \mathbf{r}_i(n_i)\|^2. \quad (26)$$

With a complete dictionary, the right hand side can be made zero. Significant departures from the energy estimate of subsequences can be due to the interactions between atoms [37].

### 3.2. Recursive Subsequence Search Limited by Bounds

Now, similar to (9) and (10), we can say,

$$\langle \mathbf{x}_q, \mathbf{P}_t \mathbf{y}_i \rangle \approx \xi_t \sum_{m=1}^{n_t} \sum_{l=1}^{n_q} [\mathbf{A}_{iq}(t) \bullet \mathbf{G}_{iq}(t)]_{ml} = S_{iq}(t, M) + R(t, M) \quad (27)$$

216 where for  $M = 2, 3, \dots, \min(n_i, n_q)$ , and with  $S_{iq}(t, 1) = \xi_t[\mathbf{A}_{iq}(t) \bullet \mathbf{G}_{iq}(t)]_{11}$ ,

$$S_{iq}(t, M) \triangleq S_{iq}(t, M - 1) + \xi_t \sum_{m=1}^M [\mathbf{A}_{iq}(t) \bullet \mathbf{G}_{iq}(t)]_{m(M-m+1)}. \quad (28)$$

217 The problem of finding the subsequence closest to  $\mathbf{x}_q$  with respect to the co-  
 218 sine distance can now be done iteratively over  $M$  by bounding each remainder  
 219  $R(t, M)$  using (14), (15), or (16), and the method presented in in Section 2.1.  
 220 Furthermore, we can compare only a subset of all possible subsequences using  
 221 a coarser time partition  $\mathcal{T}_i$ .

### 222 3.3. Practicality of the Bounds for Audio Signals

223 The experiments by Jost et al. [14, 15] use small images (128 square) and  
 224 orthogonal wavelet decompositions, which do not translate to audio signals de-  
 225 composed over redundant time-frequency dictionaries. Jost et al. [14, 15] do  
 226 not state the compressibility parameters they use, but for the high-dimensional  
 227 audio signals with which we work in this paper it is not unusual to have  $\gamma \approx 0.5$   
 228 when using MP and highly overcomplete dictionaries. We find that decom-  
 229 posing 4 s segments of music signals (single channel, 44.1 kHz sample rate,  
 230 representation weights shown in Fig. 1) using the dictionary in Table 1 requires  
 231 on average 2,375 atoms to reduce the residual energy 20 dB below the initial  
 232 signal energy. Thus, for the bound (16) using  $n = 2,375$  atoms, and with the  
 233 parameters  $(C, \gamma) = (0.4785, 0.5)$  (in the feasible set), Fig. 5 clearly shows that  
 234 in order to have any discriminable bound (say  $\pm 0.2$  for unit-norm signals) we  
 235 must either select a low value for  $p$  — in which case we are assuming the first  
 236 atom comparison is approximately the cosine distance — or we must make over  
 237 a million pairwise comparisons.

238 This is not practical for signals of large dimension, and dictionaries contain-  
 239 ing billions of time-frequency atoms. There is no possibility of tabulating the  
 240 dictionary Gramian for quick lookup of atom pair correlations; and the cost of  
 241 looking up atoms in million-atom decompositions is expensive as well. It is clear  
 242 then that the tightest bound given in (16) is not practical for efficiently discrim-

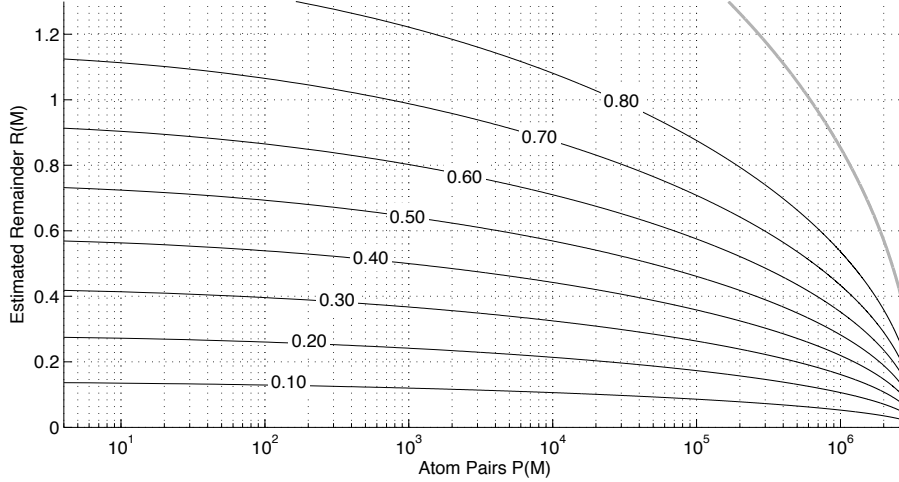


Figure 5: Estimated remainder (assuming unit-norm signals) as a function of the number of atom pairs already considered for dataset used to produce Fig. 1. Gray: bound in (15). Black, numbered: bound in (16) for several labeled  $p$  (probability that remainder does not exceed bound) with  $n = 2,375$  (number of elements in each sparse model), and  $(C, \gamma) = (0.4785, 0.5)$ .

inating distances between audio signals with respect to their cosine distance (5)  
decomposed by MP and time-frequency dictionaries.

#### 4. Experiments in Comparing Audio Signals in a Sparse Domain

Though approximate nearest neighbor subsequence search of sparsely approximated audio signals with the bound (16) is impractical, we have found that approximating the cosine distance in a sparse domain has some intriguing behaviors. We now present several experiments where we compare different types of audio data in a sparse domain under a variety of conditions. All signals are single channel, and have a sampling rate of 44.1 kHz. We decompose each by MP [17] using either the dictionary in Table 1, or the 8xMDCT dictionary [31].

##### 4.1. Experiment 1: Comparing Piano Notes

In this experiment, we look at how well low-order sparse approximations of sampled piano notes embody their harmonic characteristics by comparing



$s$ (samples/ms)	$\Delta_u$ (samples/ms)	$\Delta_f$ (Hz)
128/3	32/0.7	43.1
256/6	64/2	43.1
512/12	128/3	43.1
1024/23	256/6	43.1
2048/46	512/12	21.5
4096/93	1024/23	10.8
8192/186	2048/46	5.4
16384/372	4096/93	2.7
32768/743	8192/186	1.3

Table 1: Time-frequency dictionary parameters (44.1 kHz sampling rate): atom scale  $s$ , time resolution  $\Delta_u$ , and frequency resolution  $\Delta_f$ . Finer frequency resolution for small-scale atoms is achieved with interpolation by zero-padding.

257 them using the methods presented in Section 2. The data in set ‘A’ are 68 notes  
258 (chromatically spanning A0 to G#6) on a real and somewhat in-tune piano; and  
259 in set the data ‘B’ are 39 notes (roughly a C major scale C0 to D6) on a real  
260 and very out-of-tune piano with very poor recording conditions. We truncate  
261 all signals to have a dimension of 176,400 (4 seconds), and decompose each  
262 by MP [17] over a redundant dictionary of time-frequency Gabor atoms, the  
263 parameters of which are summarized in Table 1. We stop each decomposition  
264 once its residual energy drops 40 dB below the initial energy. We normalize the  
265 weights of each model by the square root energy of the respective signal. We do  
266 not align the time-domain signals such that the note onsets occur at the same  
267 time. Figure 1 shows the ordered decays of the weights in the sparse models of  
268 data set ‘A’.

269 Figure 6(a) shows the magnitude correlations between all pairs of signals in  
270 set ‘A’ evaluated in the time-domain. The overtone series is clear as diagonal  
271 lines offset at 12, 19, 24, and 28, semitones from the main diagonal. Figure 6(b)  
272 shows the approximated magnitude correlations (9) using only  $M = 10$  atoms  
273 from each signal approximation (thus  $P(10) = 55$  atom pairs). Even though the

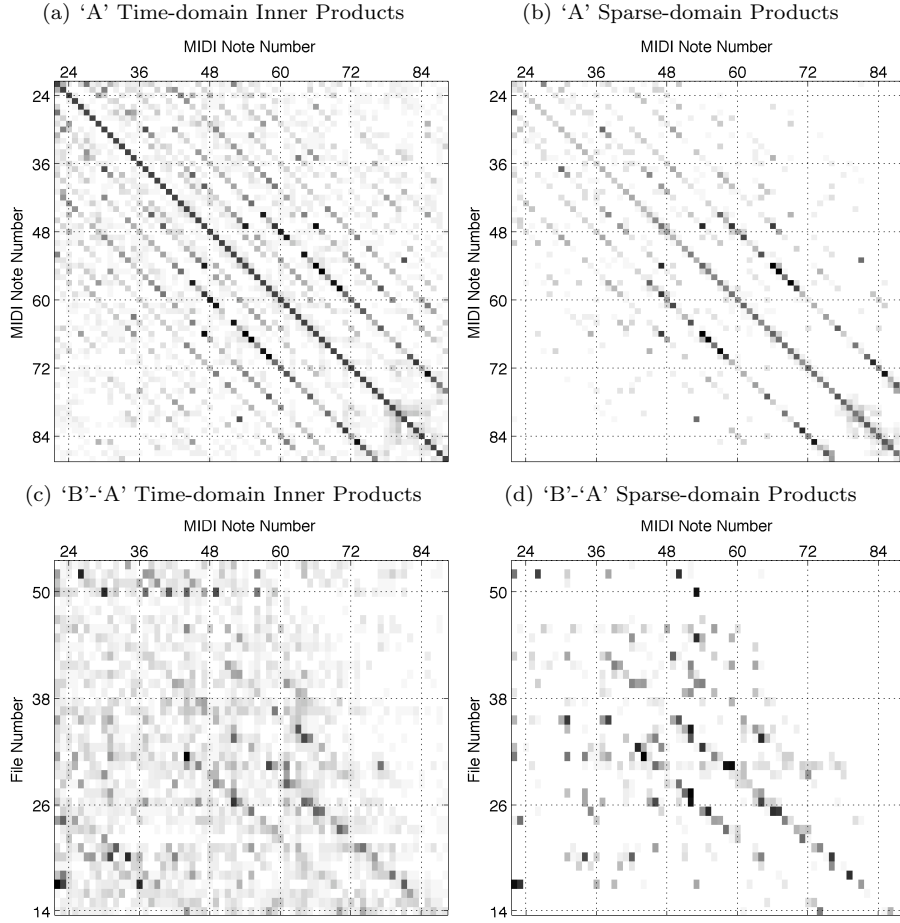


Figure 6:  $|S_{iq}(10)|$  (9) for two sets of recorded piano notes. (a) and (b): Set ‘A’ compared with itself in time and sparse domains ( $M = 10$ ). (c) and (d): Set ‘B’ (rows) compared with set ‘A’ (columns) in time and sparse domains ( $M = 10$ ). Elements on main diagonals in (a) and (b) are scaled by 0.25 to increase contrast of other elements.

mean number of atoms in this set of models is about 7000 we still see portions of the overtone series. The diagonal in Fig. 6(b) does not have a uniform color because low notes have longer sustain times than high notes, and the sparse models thus have more time-frequency atoms with greater energies spread over the support of the signal. Figure 6(c) show the magnitude correlations between sets ‘B’ and ‘A’ evaluated in the time-domain; and Fig. 6(d) shows the

280 magnitude correlations (9) using only  $M = 10$  atoms from each model. In a  
 281 sparse domain, we can more clearly see the relationships between the two sets  
 282 because the first 10 terms of each model are most likely related to the stable  
 283 harmonics of the notes, and not to the noise. We can see a diatonic scale starting  
 284 around MIDI number 36, as well as the fact that the pitch scale in data set ‘B’  
 285 lies somewhere in-between the semitones in data set ‘A’.

286 Figure 7(a) shows the approximate magnitude correlations  $|S_{iq}(M)|$  (9), as  
 287 well as the upper and lower bounds on the remainder using the tightest bound  
 288 (16) with  $p = 0.2$ , and  $n = 100$ , for the signal A3 from set ‘A’ and the rest of the  
 289 set. Here we can see that the lower bound for the largest magnitude correlation  
 290 exceeds the upper bound of all the rest after comparing only  $M = 19$  atoms  
 291 from each decomposition. All but five of the signals can be excluded from the  
 292 search after  $M = 6$ . The four other signals having the largest approximate  
 293 magnitude correlation are labeled, and are harmonically related to the signal  
 294 through its overtone series. With a signal selected from set ‘B’ and compared to  
 295 set ‘A’, Fig. 7(b) shows that we must compare many more atoms between the  
 296 models until the bounds have any discriminability. After  $P(M) = 1500$  atom  
 297 comparisons we can see that the largest magnitudes  $|S_{iq}(M)|$  (9) are roughly  
 298 harmonically related to the detuned D5 from set ‘B’.

299 As a final part of this experiment, we look at the effects of comparing atoms  
 300 with parameters within some subset. As done in Fig. 6(d), we compare the  
 301 sparse approximations of two different sets of piano notes, but here we only  
 302 consider those atoms that have scales greater than 186 ms. This in effect means  
 303 that we look for signals that share the same “long-term” time-frequency be-  
 304 haviors. The resulting  $|S_{iq}(10)|$  (9) is shown in Fig. 8. We see correlations  
 305 between the notes much more clearly compared with Fig. 6(d). Removing the  
 306 short-term phenomena improves “tonal”-level comparisons between the signals  
 307 because non-overlapping yet energetic short atoms are replaced by atoms rep-  
 308 resentative of the note harmonics.

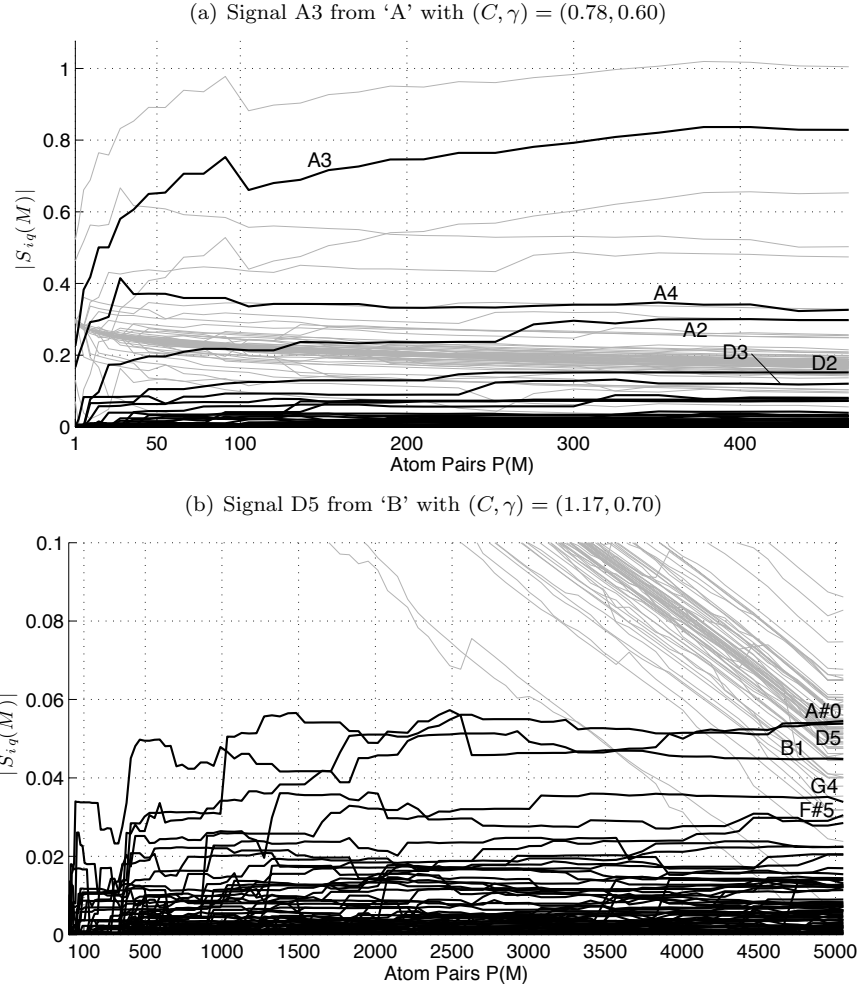


Figure 7: Black:  $|S_{iq}(M)|$  (9) as a function of the number of atom pairs considered for the set of piano notes in 'A' with a signal from either (a) 'A' (note A3) or (b) 'B' (note D5 approximately). Gray: for each  $S_{iq}(M)$ , magnitudes of  $L_{iq}(M)$  (12) and  $U_{iq}(M)$  (13) using bound in (16) with  $p = 0.2$  (probability that remainder does not exceed bound), and  $n = 100$  (number of elements in each sparse model). Largest magnitude correlations are labeled. Note differences in axes.

#### 4.2. Experiment 2: Comparing Speech Signals

In this experiment, we test how efficiently using (28) we can find in a speech signal the time from which we extract some  $\mathbf{x}_q$ . We also test how distortion in the query affects these results. We make a signal by combining six segments of

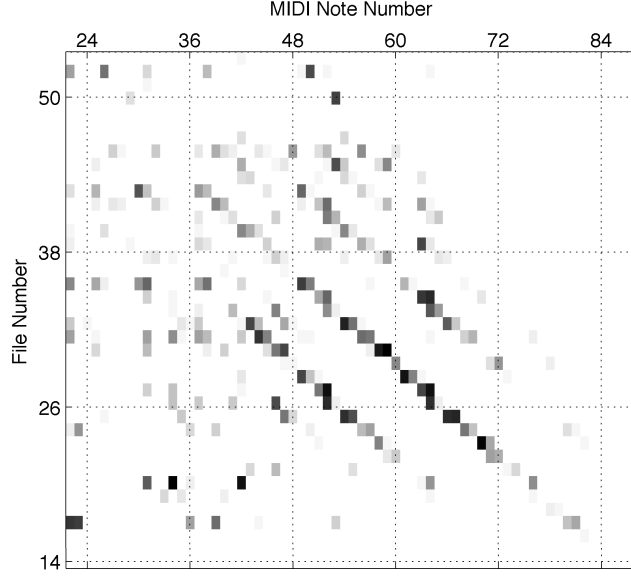


Figure 8:  $|S_{iq}(10)|$  (9) for two sets of recorded piano notes in a sparse domain using only the atoms with duration at least 186 ms. Compare with Fig. 6(d).

313 speech, a short music segment, and white noise, shown in Fig. 4(a). The six  
 314 speech segments are the same phrase spoken by three females and three males:  
 315 “Cottage cheese with chives is delicious.” We extract from one of these speech  
 316 signals the word “cheese,” to create  $\mathbf{x}_q$  with duration of 603 ms, shown at top in  
 317 Fig. 9. We decompose this signal using MP and the 8xMDCT dictionary [31].

318 We distort the query in two ways: with additive WGN (AWGN), and with  
 319 an interfering sound having a high correlation with the dictionary. In the first  
 320 case, shown in the middle in Fig. 9, the signal  $\mathbf{x}'_q = (\alpha\mathbf{x}_q + \mathbf{n})/||\alpha\mathbf{x}_q + \mathbf{n}||$  is  
 321 the original  $\mathbf{x}_q$  distorted by a unit-norm AWGN signal  $\mathbf{n}$ . We set  $\alpha = 0.3162$   
 322 such that  $10\log_{10}(|\alpha\mathbf{x}_q|^2/||\mathbf{n}||^2) = 20\log_{10}(|\alpha|) = -10$  dB. For this signal,  
 323 we find the following statistics from 10,000 realizations of the AWGN signal:  
 324  $E[|\langle \mathbf{x}_q, \mathbf{n} \rangle|] \approx 1 \times 10^{-5}$ ,  $\text{Var}[|\langle \mathbf{x}_q, \mathbf{n} \rangle|] \approx 4 \times 10^{-6}$ . We also find the fol-  
 325 lowing statistics for the 8xMDCT dictionary:  $E[\max_{\mathbf{d} \in \mathcal{D}} |\langle \mathbf{n}, \mathbf{d} \rangle|] \approx 5 \times 10^{-4}$ ,  
 326  $\text{Var}[\max_{\mathbf{d} \in \mathcal{D}} |\langle \mathbf{n}, \mathbf{d} \rangle|] \approx 2 \times 10^{-5}$ . Thus, the noise signal is not well-correlated  
 327 either with the original signal or the 8xMDCT dictionary. In the second case,

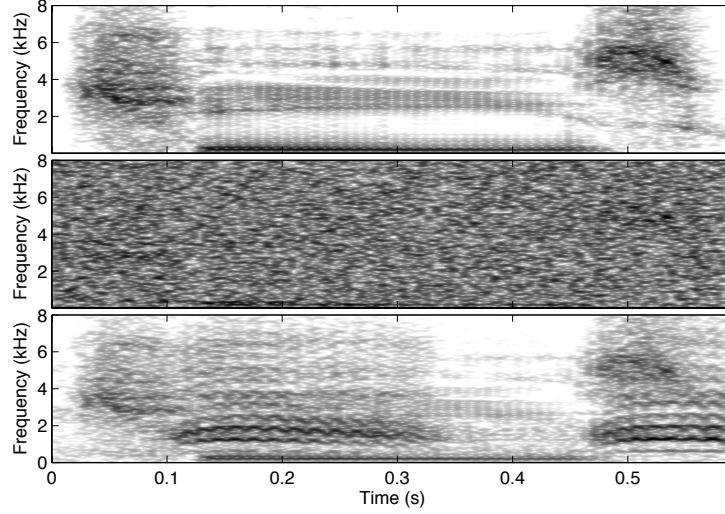


Figure 9: Log spectrograms of the query signals with which we search. Top: query of male saying “cheese.” Middle: query distorted with additive white Gaussian noise (AWGN) with  $\text{SNR} = -10$  dB. Bottom: query distorted with interfering crow sound with  $\text{SNR} = -5$  dB.

328 shown at the bottom of Fig. 9, we distort the signal by adding the sound of  
 329 a crow  $\mathbf{c}$  so that  $\mathbf{x}'_q = (\alpha\mathbf{x}_q + \mathbf{c})/||\alpha\mathbf{x}_q + \mathbf{c}||_2$  with  $||\mathbf{c}|| = 1$ . Here, we set  
 330  $\alpha = 0.5623$  given by  $20\log_{10}(|\alpha|) = -5$  dB. For this interfering signal, we find  
 331 that  $|\langle \mathbf{x}_q, \mathbf{c} \rangle| \approx 2 \times 10^{-3}$ , but  $\max_{\mathbf{d} \in \mathcal{D}} |\langle \mathbf{c}, \mathbf{d} \rangle| \approx 0.21$ , which is higher than  
 332  $\max_{\mathbf{d} \in \mathcal{D}} |\langle \mathbf{x}_q, \mathbf{d} \rangle| \approx 0.17$ . In this case, unlike for the AWGN interference, it is  
 333 likely that the sparse approximation of the signal with the crow interference will  
 334 have atoms in its low-order model due to the crow and not the speech. We do  
 335 not expect the AWGN interference to be a part of the signal model created by  
 336 MP until much later iterations.

337 Fig. 10 shows  $|S_{iq}(t, M)|$  (28) aligned with the original signal for four values  
 338 of  $M$  using the sparse approximations of the clean and distorted signals. We plot  
 339 at the rear of these figures the localized magnitude time-domain correlation of  
 340 the windowed and normalized signal with the query  $\mathbf{x}_q$ . In Fig. 10(a), using the  
 341 clean  $\mathbf{x}_q$ , we clearly see its position even when using a single atom pair for each  
 342 100 ms partition of the time-domain. We see the same behavior in Fig. 10(b)–

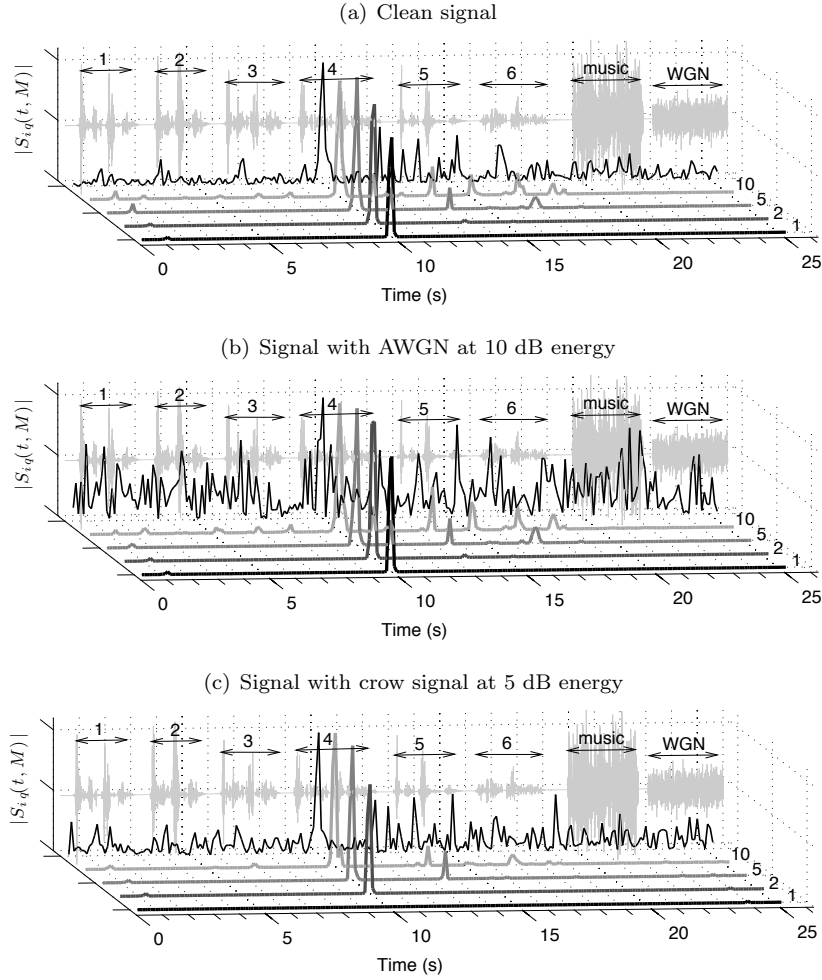


Figure 10:  $|S_{iq}(M, t)|$  (28) as a function of time and the number of atoms  $M$  (labeled at right) considered from each representation for each localized sparse approximation. Localized magnitude correlation of each signal with query is shown by the thin black line in front of the gray time-domain signal at rear.

343 (c) for the two distorted signals, but in the case where the crow sound interferes  
 344 we find the query for  $M \geq 2$ , or with at least three atom pair comparisons. The  
 345 first atom of the decomposed query with the crow is modeling the crow and not  
 346 the content of interest, and so we must increase the order of the model to find  
 347 the location of  $\mathbf{x}_q$ . As we increase the number of pairs considered we also find

#	Clean Signal			Signal + WGN			Signal + Crow		
	$t$ (s)	$ S_{iq} $	content	$t$ (s)	$ S_{iq} $	content	$t$ (s)	$ S_{iq} $	content
1	<b>10.0</b>	<b>0.798</b>	<b>“cheese”</b>	<b>10.0</b>	<b>0.236</b>	<b>“cheese”</b>	<b>10.0</b>	<b>0.409</b>	<b>“cheese”</b>
2	13.6	0.199	“cheese”	13.6	0.080	“cheese”	13.6	0.060	“cheese”
3	11.3	0.153	“-ives is-”	15.1	0.051	“delicious”	16.9	0.030	“cheese”
4	16.9	0.149	“cheese”	11.3	0.045	“-ives is-”	6.9	0.012	“cheese”
5	15.1	0.141	“delicious”	16.9	0.042	“cheese”	1.3	0.011	“cheese”
6	18.3	0.076	“delicious”	18.3	0.028	“delicious”	18.3	0.010	“delicious”
7	1.3	0.057	“cheese”	8.1	0.014	“delicious”	13.2	0.010	“cottage”
8	8.1	0.035	“delicious”	12.0	0.012	“-licious”	15.1	0.009	“delicious”
9	2.4	0.026	“delicious”	5.2	0.011	“delicious”	16.0	0.004	“cott-”
10	6.9	0.024	“cheese”	6.8	0.010	“cheese”	22.8	0.003	WGN

Table 2: Times, values and signal content for first 10 peaks in  $|S_{iq}(t, 10)|$  ( $P(10) = 55$ ) in Figs. 10(a)–(c). Highest-rated distances in each (bold) points to the origin of signal.

other segments that point in the same direction as  $\mathbf{x}_q$ . Table 2 gives the times and content of the ten largest values in  $|S_{iq}(t, 10)|$ . For the clean and AWGN distorted  $\mathbf{x}_q$ , “cheese” appears five of the six times it exists in the original signal. Curiously, these same five instances are the five smallest distances when  $\mathbf{x}_q$  has the crow interference.

We perform the same test as above but using a much longer speech signal (about 21 minutes in length) excerpted from a book-on-CD, “The Old Man and the Sea” by Ernest Hemingway, read aloud by a single person. From this signal we create several queries  $\mathbf{x}_q$ , from words to sentences to an entire paragraph of duration 35 s. We decompose each signal over the dictionary in Table 1 until the global residual energy is 20 dB below the initial energy. The approximation of the entire 21 m signal has 1,004,001 atoms selected from a dictionary containing 2,194,730,297 atoms.

One  $\mathbf{x}_q$  we extract from the signal is the spoken phrase, “the old man said” (861 ms in length). This phrase appears 26 times in the long excerpt. We evaluate  $|S_{iq}(t, M)|$  (28) every 116 ms, and find the time at which  $\mathbf{x}_q$  originally appears using only  $M = 1$  atom pair comparisons for each time partition. The



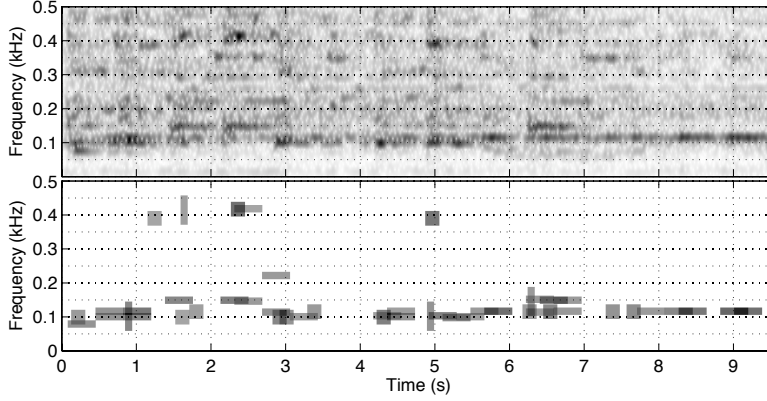


Figure 11: Polyphonic orchestral query: sonogram (top) and time-frequency tiles (bottom) of 50-order sparse approximation.

365 next highest ranked positions have values of 75% and 67% that of the largest  
 366  $|S_{iq}(t, 1)|$ . When  $M = 50$ , the values of the second and third largest values  
 367  $|S_{iq}(t, 50)|$  drop to 62% and 61% that of the largest value. In the top 30 ranked  
 368 subsequences for  $M = 5$  we find only one of the other 25 appearances of “the old  
 369 man said” (rank 26); but we also find “the old man agreed” (rank 11), and “the  
 370 old man carried” (rank 16). All other results have minimal content similarity to  
 371 the signal, but have time-frequency overlap in parts of the atoms of each model.

372 We perform the same test with a sentence extracted from the signal, “They  
 373 were as old as erosions in a fishless desert” (2.87 s), which only appears once. No  
 374 matter the  $M = [1, 50]$  we use, the origin of the excerpt remains at a rank of 6  
 375 with a value  $|S_{iq}(t, 50)|$  at 67.5% that of the highest rank subsequence. We find  
 376 that if we shift the time partition forward by 11.6 ms its ranking jumps to first,  
 377 with the second ranked subsequence at 73%. We observe a similar effect for a  
 378 query consisting of an entire paragraph (35 s). We find its origin by comparing  
 379  $M = 2$  or more atoms from each model using a time partition of 116 ms. This  
 380 result, however, disappears when we evaluate  $|S_{iq}(t, M)|$  using a coarser time  
 381 partition of 250 ms.

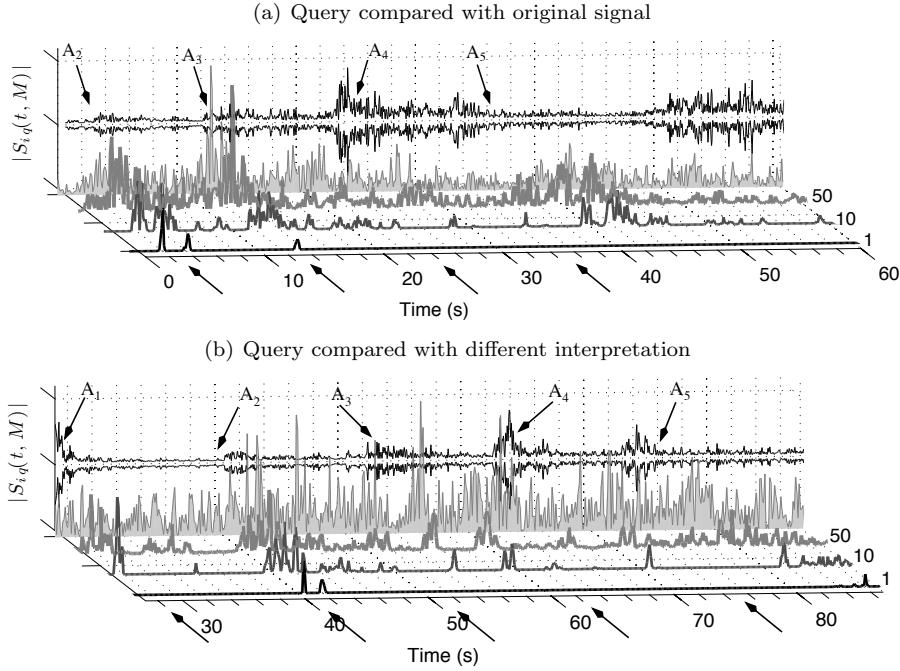


Figure 12:  $|S_{iq}(t, M)|$  (28) for three values of  $M$  for the query and two different signals. Arrows mark the appearances of the ‘A’ theme, and their appearance number. Magnitude correlation of query with localized and normalized signal is shown by the solid gray area in front of the black time-domain signal at rear.

#### 4.3. Experiment 3: Comparing Music Signals

While the previous experiment deals with single-channel speech signals, in this experiment we make comparisons between polyphonic musical signals excerpted from a commercial recording of the fourth movement of *Symphonie Fantastique* by H. Berlioz. For the query, we use a 10.31 s segment of the third appearance of the ‘A’ theme of the movement (bars 33 – 39, located around 13 – 22 s in Fig. 4(b)). Figure 11 shows the sonogram and time-frequency tiles of the model of  $\mathbf{x}_q$  using the 50 atoms with the largest magnitude weights selected from the 8xMDCT dictionary [31]. We add no interfering signals as we do in the previous experiment.

Fig. 12(a) shows  $|S_{iq}(t, M)|$  (28) over the first minute of the original signal, for three values of  $M$ , including  $M = 50$ , the time-frequency representation of

394 which is shown at bottom of Fig. 11. For  $|S_{iq}(t, 50)|$  we can see a strong spike  
 395 located around 13 s corresponding with the query, but we also see spikes at  
 396 about 2 s and around 43 s. The former set of spikes correspond with the second  
 397 appearance of the ‘A’ theme, when only low bowed strings are playing the theme  
 398 in G-minor. This is quite similar to the instrumentation of the query: low bowed  
 399 strings and a legato bassoon in counterpoint in Eb-major. The latter set of spikes  
 400 is around the end of the fifth appearance of the theme, which is played in G-  
 401 minor on low pizzicato strings with a staccato bassoon. For  $M = 10$ , we see a  
 402 conspicuous spike at the time of the fifth appearance around 34 s, as well as of  
 403 the fourth appearance around 24 s, where the theme is played in Eb-major like  
 404 the query. Finally, we test how the sparse approximation of this query compares  
 405 with subsequences from a different recording of this movement, which is also in  
 406 a different tempo. Figure 12(b) shows  $|S_{iq}(t, M)|$  (28) for three different values  
 407 of  $M$ . We see high similarity with the first and second appearances of the main  
 408 theme, but not the third, which is what the query contains.

#### 409 4.4. Discussion

410 There is no reason to believe that a robust and accurate speech or melody  
 411 recognition system can be created by comparing only the first few elements  
 412 of greedy decompositions in time-frequency dictionaries. What appears to be  
 413 occurring for the short signals, both the “cheese” and “the old man said,” is  
 414 that the first few elements of their sparse and atomic decomposition create a  
 415 prosodic representation that is comparable to others at the atomic level. For  
 416 the longer signals, such as sentences, paragraphs, and orchestral theme, a few  
 417 atoms cannot adequately embody the prosody, but we still see that by only  
 418 making a few comparisons we are able to locate the excerpted signal — as long  
 419 as the time partition is fine enough. This is due to the atoms of the models  
 420 acting in some sense as a time-frequency fingerprint, an example of which is in  
 421 Fig. 11. Through the cosine distance, the relative time and frequency locations  
 422 of the atoms in the query and subsequence are being compared. Subsequences  
 423 that share atoms in similar configurations will be gauged closer to  $\mathbf{x}_q$  than those

424 that do not.

425 By using the cosine distance it is not unexpected that (28) will be extremely  
426 sensitive to a partitioning of the time-domain. This comes directly from the  
427 definition of the time-localized Gramian (23), as well as the use of a dictionary  
428 that is not translation invariant. There is no need to partition the time axis  
429 when using a parameterized dictionary if we assume that some of the atoms  
430 in the model of  $\mathbf{x}_q$  will have parameters that are nearly the same as some of  
431 those in the relevant localized sparse representations. In such a scenario, we can  
432 search a sparse representation for the times at which atoms exist that are similar  
433 in scale and modulation frequency to those modeling  $\mathbf{x}_q$ . Then we can limit our  
434 search to those particular times without considering any uniform and arbitrary  
435 partition of the time-domain. With non-linear greedy decomposition methods  
436 such as MP and time-variant dictionaries, however, such an assumption cannot  
437 be guaranteed; but its limits are not yet well-known.

## 438 5. Conclusion

439 In this paper, we have extended and investigated the applicability of a  
440 method of recursive nearest neighbor search [14, 15] for comparing audio signals  
441 using pairwise comparisons of model elements in a sparse domain. The multi-  
442 scale descriptions offered by sparse approximation over time-frequency dictio-  
443 naries are especially attractive for such tasks because they provide flexibility in  
444 making comparisons between data, not to mention a capacity to deal with noisy  
445 signals. After extending this method to the task of comparing subsequences  
446 of audio signals, we find that the strongest bound known for the remainder is  
447 too weak to quickly and efficiently reduce the search space. Our experiments  
448 show, however, that by comparing elements of sparse models we can judge  
449 with relatively few comparisons whether signals share the same time-frequency  
450 structures, and to what degrees, although this can be quite sensitive to the  
451 time-domain partitioning. We also see that we can approach such comparisons  
452 hierarchically, starting from the most energetic content to the least, or starting

453 from the longest scale phenomenon to the shortest.

454 We are continuing this research in multiple directions. First, since we know  
 455 that the inner product matrix  $\mathbf{G}_{iq}(t)$  (23) will be very sparse for all  $t$  in time-  
 456 frequency dictionaries, this motivates designing a tighter bound based on a  
 457 Laplacian distribution of elements in  $\mathbf{G}_{iq}(t)$  with a large probability mass ex-  
 458 actly at zero. This bound would be much more realistic than that provided by  
 459 assuming the elements of the Gramian are distributed uniform (16). Another  
 460 part of the problem is of course that the sums in (9) and (28) are not such that  
 461 at step  $M$  the  $P(M)$  largest magnitude values of  $\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}$  are actually being  
 462 summed. By our assumption in (4), we know that the decay of the magnitudes  
 463 of the elements in  $\mathbf{A}_{iq}$  will be quickest in diagonal directions, but dependent  
 464 upon the element position in the matrix. These diagonal directions are simply  
 465 given by

$$\begin{bmatrix} \partial/\partial\gamma_i \\ \partial/\partial\gamma_q \end{bmatrix} m^{-\gamma_i} l^{-\gamma_q} = -m^{-\gamma_i} l^{-\gamma_q} \begin{bmatrix} \gamma_i/m \\ \gamma_q/l \end{bmatrix} \quad (29)$$

466 where we now recognize that the weights of two different representations can  
 467 decay at different rates. With this, we can make an ordered set of index pairs  
 468 by

$$\Lambda = \{(m, l)_\lambda : |[\mathbf{A}_{iq}]_\lambda| \geq |[\mathbf{A}_{iq}]_{\lambda+1}|\}_{\lambda=1,2,\dots,n_i n_q} \quad (30)$$

469 and define a recursive sum for  $1 < m \leq n_i n_q$

$$S_{iq}(m) \triangleq S_{iq}(m-1) + [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{\Lambda_m} \quad (31)$$

470 setting  $S_{iq}(1) = [\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}]_{11}$ . We do not yet know the extent to which this  
 471 approach can ameliorate the problems with the non-discriminating bound (16),  
 472 as we have yet to design an efficient way to generate a satisfactory  $\Lambda$ , and  
 473 estimate the bounds of the corresponding remainder — whether it is like that  
 474 in (16), or another that uses the fact that  $\mathbf{G}_{iq}(t)$  will be very sparse, even when  
 475  $\mathbf{x}_q = \mathbf{y}_i$ . We think that using a stronger bound and this indexing order will  
 476 significantly reduce the number of pairwise comparisons that must be made

before determining a subsequence is not close enough with respect to the cosine distance. Furthermore, we can make the elements of  $\mathbf{A}_{iq}$  decay faster, and thus increase  $\gamma$ , by using other sparse approximation approaches, such as OMP [28, 23] or CMP [36]. And we cannot forget the implications of choosing a particular dictionary. In this work we have used two different parametric dictionaries, one of which is designed for audio signal coding [31]. Another interesting research direction is to use dictionaries better suited for content description than coding, such as content-adapted dictionaries [20, 2, 19].

Finally, and specifically with regards to the specific problem of similarity search in audio signals, the cosine distance between time-domain samples makes little sense because it is too sensitive to signal waveforms whereas human perception is not. Instead, many other possibilities exist for comparing sparse approximation, such as comparing low-level histograms of atom parameters [7, 34]; comparing mid-level structures such as harmonics [9, 38, 8]; and comparing high-level patterns of short atoms representing rhythm [32]. There also exists the Matching Pursuit Dissimilarity Measure [25], where the atoms of one sparse model are used to decompose another signal, and vice versa to see how well they model each other. We are exploring these various possibilities with regards to gauging more generally similarity in audio signals at multiple levels of specificity within a sparse domain.

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## 506 Appendix A. Proof of Remainder Bounds

507 To show (14), we can bound  $R(M)$  loosely by assuming the worst case sce-  
 508 nario of  $[\mathbf{G}_{iq}]_{ml} = 1$  for all its elements. Knowing that  $R(M)$  is the sum of the  
 509 elements of the matrix  $\mathbf{A}_{iq} \bullet \mathbf{G}_{iq}$  except for the first  $P(M)$  values, and assuming  
 510 (4), we can say

$$\begin{aligned} C^{-2}R(M) &\leq \sum_{m=M+1}^n \sum_{l=1}^m [l(m-l+1)]^{-\gamma} + \sum_{m=1}^{n-1} \sum_{l=m+1}^n [l(n-m+1)]^{-\gamma} \\ &= \|\mathbf{c}_M^\gamma\|_1 + \|\mathbf{d}^\gamma\|_1. \end{aligned} \quad (\text{A.1})$$

511 where  $\mathbf{c}_M^\gamma$  and  $\mathbf{d}^\gamma$  are defined in (17) and (18). This worst case scenario is not  
 512 possible using MP because of its update rule (1).

513 We can find the tighter bound in (15) by assuming the distribution of  
 514 signs of the elements of  $\mathbf{G}_{iq}$  is Bernoulli equiprobable, i.e.,  $P\{[\mathbf{G}_{iq}]_{ml} = 1\} =$   
 515  $P\{[\mathbf{G}_{iq}]_{ml} = -1\} = 0.5$ . Thus, defining a random variable  $b_i : \mathbb{R} \mapsto \{-1, 1\}$ ,  
 516 and its probability mass function  $f_B(b_i) = 0.5\delta(b_i + 1) + 0.5\delta(b_i - 1)$  using the  
 517 Dirac function,  $\delta(x)$ , we create a random vector  $\mathbf{b}$  with  $n^2 - P(M)$  elements  
 518 independently drawn from this distribution. Placing this into the double sums  
 519 of (A.1) provides the bound

$$C^{-2}R(M) \leq \left\| \mathbf{b}^T \begin{bmatrix} \mathbf{c}_M^\gamma \\ \mathbf{d}^\gamma \end{bmatrix} \right\| \leq \|\mathbf{c}_M^\gamma\|_1 + \|\mathbf{d}^\gamma\|_1. \quad (\text{A.2})$$

520 This weighted Rademacher sequence has the property that [14]

$$P\{|\mathbf{b}^T \mathbf{s}| > R\} \leq 2 \exp(-R^2/2\|\mathbf{s}\|_2^2), R > 0 \quad (\text{A.3})$$

521 which becomes  $P\{|\mathbf{b}^T \mathbf{s}| \leq R\} \geq \max\{0, 1 - 2 \exp(-R^2/2\|\mathbf{s}\|_2^2)\}$  by the ax-  
 522 ioms of probability. With this we can find an  $R$  such that  $P\{|\mathbf{b}^T \mathbf{s}| \leq R\}$  will  
 523 be greater than or equal to some probability  $0 \leq p \leq 1$ , i.e.,

$$R(p) = (\|\mathbf{c}_M^\gamma\|_2^2 + \|\mathbf{d}^\gamma\|_2^2)^{1/2} \left[ 2 \ln \frac{2}{1-p} \right]^{1/2}. \quad (\text{A.4})$$

524 This value can be minimized by choosing  $p = 0$ , for which we arrive at the  
 525 residual upper bound in (15). Note that even though we have set  $p = 0$ , we still

526 have an unrealistically loose bound by the impossibility of MP of choosing two  
 527 sets of atoms for which all entries of their Gramian  $\mathbf{G}_{i_q}$  are in  $\{-1, 1\}$ .

528 Finally, to show (16), we can model the elements of the Gramian as random  
 529 variables,  $u_i : \mathbb{R} \mapsto [-1, 1]$ , independently and identically distributed uniformly

$$f_U(u_i) = \begin{cases} 0.5, & -1 \leq u_i \leq 1 \\ 0, & \text{else.} \end{cases} \quad (\text{A.5})$$

530 Substituting this into (14) gives a weighted sum of random variables satisfying

$$C^{-2}R(M) \leq \left| \mathbf{u}^T \begin{bmatrix} \mathbf{c}_M^\gamma \\ \mathbf{d}^\gamma \end{bmatrix} \right| \leq \|\mathbf{c}_M^\gamma\|_1 + \|\mathbf{d}^\gamma\|_1. \quad (\text{A.6})$$

531 where  $\mathbf{u}$  is the random vector. For large  $M$ , this sum has the asymptotic  
 532 property [14, 15]:

$$P\{|\mathbf{u}^T \mathbf{s}| < R\} = \text{Erf} \sqrt{\frac{3R^2}{2\|\mathbf{s}\|_2^2}}. \quad (\text{A.7})$$

533 Setting this equal to  $0 \leq p \leq 1$  and solving for  $R$  produces the upper bound  
 534 (16). We can reach the upper bound (15) if we set  $p = 0.9586$ , but note that  
 535 (16) can be made zero. This bound can still be extremely loose because the  
 536 Gramian of two models in time-frequency dictionaries will be highly sparse.

537 Computing the  $\ell_2$ -norm in these expressions, however, leads to evaluating  
 538 the double sums

$$\|\mathbf{c}_M^\gamma\|^2 = \sum_{m=M+1}^n \sum_{l=1}^m \frac{1}{[l(m-l+1)]^{2\gamma}} \quad (\text{A.8})$$

$$\|\mathbf{d}^\gamma\|^2 = \sum_{m=1}^{n-1} \sum_{l=m+1}^n \frac{1}{[l(n-m+1)]^{2\gamma}} \quad (\text{A.9})$$

539 which can be prohibitive for large  $n$ . The dimensionality of  $\mathbf{c}_M^\gamma$  is  $n(n+1)/2 -$   
 540  $P(M)$ , and of  $\mathbf{d}^\gamma$  is  $n(n-1)/2$ . We approximate these values in the following  
 541 way for  $\gamma = 0.5$ , using the partial sum of the harmonic series

$$\sum_{m=1}^n \frac{1}{m} = \ln n + \gamma_E + \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} + \mathcal{O}(n^{-6}) \quad (\text{A.10})$$



542 where  $\gamma_E \approx 0.5772$  is the Euler-Mascheroni constant. To find  $\|\mathbf{d}^{0.5}\|^2$

$$\begin{aligned}
\|\mathbf{d}^{0.5}\|^2 &= \sum_{m=1}^{n-1} \sum_{l=m+1}^n \frac{1}{l(n-m+1)} \\
&= \sum_{m=1}^{n-1} \frac{1}{n-m+1} \left[ \sum_{l=1}^n \frac{1}{l} - \sum_{l=1}^m \frac{1}{l} \right] \\
&\approx \sum_{m=1}^{n-1} \frac{1}{n-m+1} \left[ \ln n/m - \frac{n-m}{2nm} + \frac{n^2-m^2}{12n^2m^2} \right]. \tag{A.11}
\end{aligned}$$

543 To find  $\|\mathbf{c}_M^{0.5}\|^2$  we first use partial fractions and then the partial sum of the  
544 harmonic series:

$$\begin{aligned}
\|\mathbf{c}_M^{0.5}\|^2 &= \sum_{m=M+1}^n \sum_{l=1}^m \frac{1}{l(m-l+1)} \\
&= \sum_{m=M+1}^n \frac{1}{m+1} \sum_{l=1}^m \frac{1}{l} + \frac{1}{m-l+1} \\
&\approx \sum_{m=M+1}^n \frac{1}{m+1} \left[ \ln m + \gamma_E + \frac{1}{2m} - \frac{1}{12m^2} + \sum_{l=1}^m \frac{1}{l} \right] \\
&\approx 2 \sum_{m=M+1}^n \frac{1}{m+1} \left( \ln m + \gamma_E + \frac{1}{2m} - \frac{1}{12m^2} \right). \tag{A.12}
\end{aligned}$$

545 With these expressions we can avoid double sums in calculating the bounds.

## 546 Appendix B. Estimating the Compressibility Parameters

547 The compressibility parameters  $(C, \gamma)$  must be estimated for the set of  
548 weights in  $\hat{\mathcal{Y}}$  (??), as well as those of  $\mathbf{x}_q$ . Since by (4) the parameters  $(C, \gamma)$   
549 bound from above the decay of all the ordered weights, only the largest mag-  
550 nitude weights matter for their estimation. Thus, we define a vector,  $\mathbf{a}$ , of the  
551 largest  $n$  magnitude weights from each row in the set  $\{\{\mathbf{a}_i(n_i)\}_{i \in \mathcal{I}}, \mathbf{a}_q(n_q)\}$ ,  
552 which is equivalent to taking the largest weights at each approximation order.  
553 Good compressibility parameters can be given by

$$\min_{C, \gamma} \|\mathbf{C}\mathbf{z}^\gamma - \mathbf{a}\|^2 + \lambda C \text{ subject to } \mathbf{C}\mathbf{z}^\gamma \succeq \mathbf{a} \tag{B.1}$$

554 where we define  $\mathbf{z}^\gamma \triangleq [1, 1/2^\gamma, \dots, 1/n^\gamma]^T$ , and add a multiple of  $C$  in order to  
555 keep it from getting too large since the bounds (14)–(16) are all proportional to

556 it. The constraint is added to ensure all elements of the difference  $C\mathbf{z}^\gamma - \mathbf{a}_i$  are  
 557 positive such that (4) is true.

558 To remove the  $\gamma$  component from the exponent, and since all of the elements  
 559 of  $\mathbf{z}$  and  $\mathbf{a}$  are positive and non-zero, we can instead solve the problem

$$\begin{aligned} \min_{C, \gamma} & \|\ln C\mathbf{1} + \gamma \ln \mathbf{z} - \ln \mathbf{a}\|^2 + \lambda \ln C \\ &= \min_{C, \gamma} \left[ (\ln C)^2 n + \gamma^2 \|\ln \mathbf{z}\|^2 + \|\ln \mathbf{a}\|^2 + \lambda \ln C \right. \\ & \quad \left. + 2\gamma (\ln \mathbf{z})^T (\ln C\mathbf{1} - \ln \mathbf{a}) - 2 \ln C (\ln \mathbf{a})^T \mathbf{1} \right] \quad (\text{B.2}) \end{aligned}$$

560 subject to the constraint  $C\mathbf{z}^\gamma \succeq \mathbf{a}$ . Taking the partial derivative of this with  
 561 respect to  $\gamma$  and  $C$ , we find

$$\gamma_o = \frac{(\ln \mathbf{z})^T (\ln \mathbf{a} - \ln C\mathbf{1})}{\|\ln \mathbf{z}\|^2} \quad (\text{B.3})$$

$$C_o = \exp \left[ \lambda + \frac{1}{n} \sum_{i=1}^n [\ln \mathbf{a} - \gamma \ln \mathbf{z}]_i \right]. \quad (\text{B.4})$$

562 Starting with some initial value of  $C$  then, we use the following iterative method

- 563 1. solve for  $\gamma$  given a  $C$  in (B.3);
- 564 2. find the new  $C$  in (B.4) using this  $\gamma$ ;
- 565 3. set  $C' = \exp [\max(\ln \mathbf{a} - \gamma_o \ln \mathbf{z})]$  and evaluate the error  $\|C'\mathbf{z}^\gamma - \mathbf{a}\|^2$ ;
- 566 4. repeat until the error begins to increase.

567 The factor  $\lambda$  in effect controls the step size for convergence. A typical value  
 568 we use is  $\lambda = \pm 0.03$  based on experiments (the sign of which depends on if the  
 569 objective function decreases with decreasing  $C$ ).

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