Noise upon the Sinusoids

Jensen, Karl Kristoffer

Published in:
IEEE Workshop on Applications of Signal Processing to Audio and Acoustics

DOI (link to publication from Publisher):
10.1109/ASPAA.2005.1540226

Publication date:
2005

Document Version
Early version, also known as pre-print

NOISE UPON THE SINUSOIDS

Kristoffer Jensen

Department of Medialogy
University of Aalborg Esbjerg
Niels Bohrsvej 8, 6700 Esbjerg, Denmark
krist@cs.aaue.dk

ABSTRACT

Sinusoids are used for making harmonic and other sounds. In order to having life in the sounds and adding a wide variety of noises, irregularities are inserted in the frequency and amplitudes. A simple and intuitive noise model is presented, consisting of a low-pass filtered noise, and having control for strength and bandwidth. The noise is added on the frequency and amplitudes of the sinusoids, and the resulting irregularity's (jitter and shimmer) bandwidth is derived. This, together with an overview of investigation methods of the jitter and shimmer results in an analysis of the necessary samplerate of the shimmer and jitter. A harmonic model introduces individual and common irregularity, and adds a correlation control. The model has been implemented in max/msp and used in contemporary music compositions.

1. INTRODUCTION

Noise is inherent in all musical sounds, including the human voice. Without noise and random fluctuations, most sounds are dull, lifeless, and synthetic, and noise can indeed add a wide variety of *tone* qualities to a harmonic sound.

In sound synthesis, irregularity and noise is getting more and more interest. The additive (sinusoidal) model of sound synthesis has proven its interest. The addition of (parametric) noise on the amplitudes (shimmer) and frequencies (jitter) of the sinusoids can give the sound a noisy quality, or irregularity and other effects, depending on the characteristics of the noise. Other researchers have used similar synthesis methods before. Risset and Wessel [1] used spectral line broadening for musical purposes, Fitz and Haken [2] dubbed their method bandwidth enhanced sinusoidal model, while Freed made phase modulation in the transform domain [3]. Jensen [4] investigated the perception of noise.

The noise parameters in this work are consisting of bandwidth, strength and correlation for the jitter and shimmer, respectively. The bandwidth is a measure of how *fast* the noise is changing, and the correlation is a measure of how much *alike* the noise is in one overtone to the noise of the fundamental. There is a dramatic difference in the timbre of the resulting sound in the different positions of this space [4], the shimmer is *rumbling*, *windy*, *crackling*, the jitter is *rough*, *weird*, *walking*.

A short overview of the methods to investigate shimmer and jitter is given, including Taylor series expansion, FM bandwidth, power spectrum, instantaneous amplitude and frequency, and relative peak amplitude.

A harmonic synthesis model with appropriate mapping strategies is detailed. This model has been used for musical proposes.

2. AM (shimmer) and FM (jitter)

The amplitude modulation (AM) noise (shimmer) and frequency modulation (FM) noise (jitter) is well-known from the speech community. Here, a parametric noise model is introduced that controls the characteristics of the noise with a few parameters.

2.1. Noise Model

A simple, yet intuitive noise model is used in this work. As the noise is used to modulate the sinusoids, the distribution is of importance; as is the strength of the noise, and the spectral shape. While different distributions could indeed alter the resulting sound, only the normal (Gaussian) distribution is used. The strength is defined in connection with the modulation, and not further discussed here. The white noise is filtered by a one-tap recursive filter,

$$s(t) = s(t-1) + fc \cdot rnd \tag{1}$$

where *s(t)* is the current sample, *s(t-1)* the previous sample, *fc* the filter coefficient, and *rnd* a new random value. If $fc<0$, the resulting filter shape is low-pass. In practice, the resulting noise samples needs to be normalized in amplitude,

$$\hat{s}(t) = s(t) \cdot \sqrt{1 - fc^2} \tag{2}$$

The filter coefficient *fc* can be found from the 3dB bandwidth value Δ through the following formula,

$$fc = -2 + \cos(\frac{\Delta\pi}{2sr}) + \sqrt{-1 + \left(2 - \cos(\frac{\Delta\pi}{2sr})\right)^2} \tag{3}$$

where *sr* is the sample rate. The constants are a function of the (arbitrary) –3dB cut-off frequency, Δ, that is a more intuitive parameter than the filter coefficient, which doesn't have a physical or perceptual signification.

2.2. Shimmer

The shimmer is amplitude modulation with a random signal. Ordinary AM renders a signal with two sidebands. A sinusoid modulation with another sinusoid,

$$s(t) = (1 + I\sin(\omega_m t))\sin(\omega_c t) \quad (4)$$

has two sidebands at $\omega_c \pm \omega_m$ with amplitude $I/2$, in addition to the sinusoid with frequency f_c which amplitude doesn't change with I. Replacing the modulation signal with a random signal changes the behavior of the sidebands. Instead of two fixed bands, a distribution of bands is found. This signifies that the resulting sound is essentially consisting of the carrier sinusoid with the frequency shifted noise, situated at the carried frequency.

2.3. Jitter

Jitter is frequency modulation with a random signal. The frequency modulation is known from communication theory and sound synthesis [5], in which the carrier signal is modulated by a modulation sinusoid,

$$s(t) = \sin\{\omega_c t + (1 + I\sin\{\omega_m t\})\}. \quad (5)$$

While the AM has an easy expression of two sidebands, the FM signal has an infinite number of sidebands, whose frequencies are $f_c \pm n\, f_m$ {n=0,…,∞}, and whose amplitude is the *n-th* order Bessel function of the first kind ($J_n(I)$). Thus, a sinusoid modulating another sinusoids frequency renders a spectrum with an infinity number of sidebands. If the modulating signal (phase modulation) is a random signal, each sideband frequency is changed into a random process, whose frequency is modeled as a probability density function.

The FM spectrum can be estimated by different means. Fitz and Haken [2] investigated FM with a low-intensity narrow-band random modulation signal by expressing it in the Taylor series expansion. Rocchesso [5] has given the -40dB bandwidth of a FM signal, modulated with a sinusoid, approximated to be,

$$\Delta_{FM} \approx 2I\omega_m \quad (6)$$

If the modulating signal is a low-pass random signal with bandwidth Δ, the resulting FM bandwidth is approximately 2IΔ.

The jitter bandwidth thus increases with the modulation strength. Mazo and Salz [7] investigated the power spectra of random FM signals through analysis of the ensemble average (expectation), and found the power spectrum of a sinusoid, modulated with a zero mean Gaussian random variable with variance σ^2 to be,

$$G(\omega) = 4\frac{\sigma^2}{\omega^2 + \sigma^4}. \quad (7)$$

This can be moved to the location of the modulation frequency by standard Fourier transform properties. The numerical and theoretical spectrums (using eq. 7) for a sinusoid, modulated with white Gaussian noise with four different variances, are shown in figure 1.

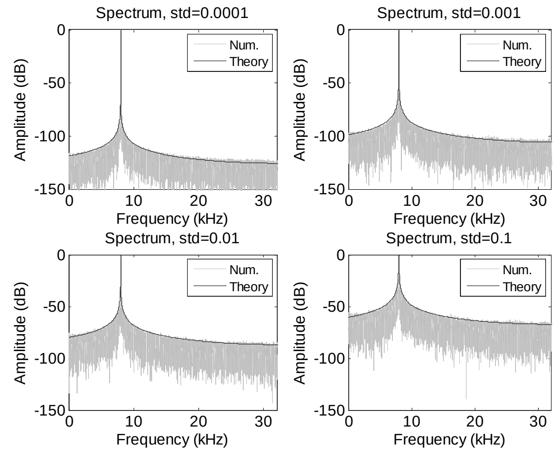

Figure 1. Numerical and theoretical spectrum for a FM signal with Gaussian noise modulation. The spectral peak is only infinite in the case of theoretic infinite duration.

The eq. 7 is a promising method to investigate the jitter behavior, but it does not currently allow the analysis of jitter with low-pass filtered noise.

2.4. Signal investigation

Here is investigated the bandwidth and amplitude envelope standard deviation of a single sinusoid modulated with low-pass filtered noise. This is done through the use of the effective bandwidth, defined as [8],

$$\Delta f = \frac{1}{N}\sqrt{\sum 2\pi(f_i - f_0)^2} \quad (8)$$

where f_i is the instantaneous frequency and f_0 is the carrier frequency. N is the length of the sound. Here, the shimmer and jitter is compared by investigating and comparing the instantaneous frequency f_i and amplitude envelope a_i. The effective bandwidth and amplitude standard deviation for shimmer and jitter are shown in figure 2 for the modulation index $I=\{0..1\}$. The shimmer and jitter behaves quite the same, with the shimmer having a slightly larger bandwidth and standard deviation. Even so, the jitter has a different sound, more band-passed, darker, while the shimmer definitely has an additive noise quality.

Another interesting parameter is the relative amplitudes of the spectral peak for the shimmer and jitter. As expected, this is always one for the shimmer, regardless of the modulation index, while it is decreasing approximately inversely proportional to I for the jitter, asymptotically approaching zero for large I.

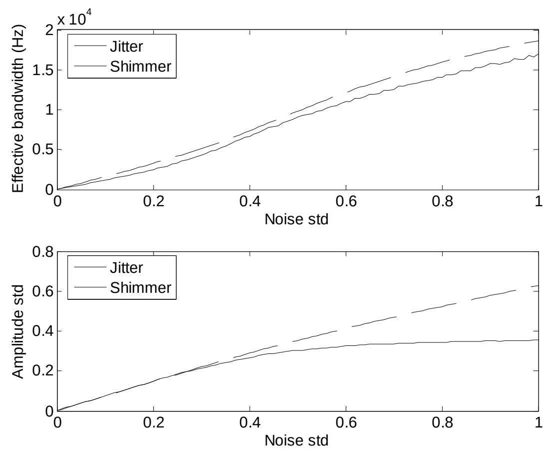

Figure 2. Jitter and Shimmer effective bandwidth (top) and amplitude envelope standard deviation (bottom).

2.5. Examples

One sinusoid with low-pass filtered shimmer and jitter has two parameters for each, noise bandwidth and modulation index.

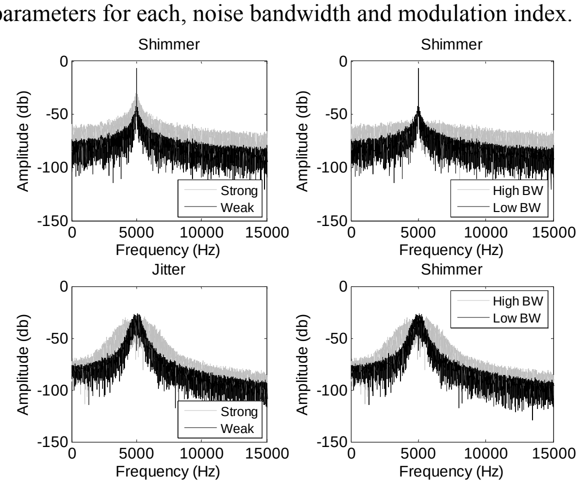

Figure 3. Four examples of shimmer (with modulation Index change, top left, with bandwidth change, top right) and jitter (with Index change, bottom left, with bandwidth change, bottom right).

For the shimmer, the modulation index changes the signal to noise ratio and the bandwidth changes the bandwidth of the signal. For the jitter, the signal bandwidth increases both with the modulation index and the noise bandwidth, the difference being that the spectral peak is diminished more with the index, while the noise bandwidth affects more of the outlier frequencies.

Four examples of shimmer and jitter are shown in figure 3. Shimmer with two different strengths (top left) and bandwidth (top right). It is clear that the strength does not affect the spectral shape, while a larger bandwidth smears the noise out on a larger frequency range. Jitter is shown bottom left with two different strength and bottom right with two different bandwidths. For the jitter, the strength definitely increases the resulting jitter frequency range, as does the bandwidth, although not with the same spectral shape.

The jitter is rendering *low-frequency random pitch variations*, when the bandwidth is low, and adding a *rough* quality to the sinusoidal, when the bandwidth is high. In contrast, the shimmer renders a *rumbling* quality to the sound for low bandwidths, and adds an *additive noise*, when the bandwidth is high.

2.5. Modulator sampling Rate

The sampling rate of the modulator influences the maximum bandwidth of the modulating noise. A large sampling rate enables the noise to cover the full spectrum, while a low modulating sampling rate only renders a noise at the vicinity of the carrier frequency. Assuming the modulator sampling rate sr_m, then the maximum shimmer and jitter bandwidths are

$$\Delta_{AM} \leq \frac{sr_m}{2}, \Delta_{FM} \leq sr_m I \tag{9}$$

3. HARMONIC SOUNDS

In the case of harmonic sounds, a (independent) shimmer and jitter random component can be added to the amplitudes and frequencies. In this case, the resulting bandwidth does not have to be larger than the fundamental frequency, giving a modulating sample rate of $2f_0$, for the shimmer and f_0/I for the jitter, if the spectrum is to be filled out between the harmonic components. A common value of I is below 0.25, which necessitates a modulating sample rate of 4 modulation samples for each period. If only shimmer is taken into account, then 2 modulation samples are enough for each period.

3.1. Shimmer and Jitter

The complete synthesis model for the shimmer and jitter is

$$s(t) = \sum_{p=1}^{P} a_p \left(1 + \sigma_p^s s_p^s\right) \sin\left(2\pi \int_0^t p f_0 \left(1 + \sigma_p^j s_p^j\right) \delta\tau\right) \tag{10}$$

where P is the number of partials, a_p is the partial amplitude, f_0 is the fundamental frequency, σ^s and σ^j are the shimmer and jitter strength (the modulation index), and s^s and s^j are the shimmer and jitter noise component.

The strength affects the amount of noise, or irregularities on the overtones. Jensen [4] proposed a wide variety of verbal attributes for the resulting sounds, depending on the parameter values, including *clean* or *calm*, *agitated* and *dirty, schizophrenic*, *windy*, *dark, splashing*, *rough*, and *screaming*.

3.2. Correlation

For the noises on the overtones, an independent and a common noise source is added to each overtone. This enables the synthesis of random fluctuating, as well as coherent sounds. The common and independent noises are filtered with the same filter, and the relative ratio is controlled by a correlation coefficient c,

$$s_p = (1-c)r_0 + cr_p \tag{11}$$

where r_0 is the common and r_p the independent noise component.
The correlation is a rather interesting parameter in the harmonic model, as it renders a large variety of different *tones*, together with the bandwidth and standard deviation. According to [4], setting the correlation coefficient c low and the bandwidth high renders a rather dark additive noise quality to the shimmer, whereas the jitter renders a more *nasal* quality. As the correlation is increased, still with high bandwidth, the shimmer gets a different *tone*, almost *splashing*, whereas the jitter becomes somehow more *insisting*, almost *screaming*. As the bandwidth is decreased, still with high correlation, the shimmer goes towards a *crackling* quality, ending in a peaceful random fluctuation. The jitter goes through several categories of *rough*, *full* qualities, ending in *random pitch*. Finally, as the correlation is now decreased, the shimmer goes from the *fluctuating* sound to a beautiful *windy* quality, and the jitter gives a more sch*izophrenic* sound. The correlation values in musical sounds are generally found between 0.25 and 0.75 [7].

4. APPLICATIONS

The shimmer and jitter has been used previously in several applications, including synthesis [1], [2], [3] and [10], analysis/synthesis [7], classification [11] and understanding of instrument performance behavior [7]. Here is detailed the reasons and ways of mapping the synthesis parameters to a control interface and the implementation in max/msp is detailed.
The parameters available in the synthesis model are the partial amplitudes (a), the fundamental frequency (f_0), the shimmer and jitter strength (σ^s and σ^j), bandwidth (Δ^s and Δ^j), and correlation (c^s and c^j), each one, except the fundamental frequency, available for each partial. In order to simplify the control task, each parameter will have an identical value for all partials, except the amplitude, which will have exponentially decreasing amplitude,

$$a_p = a_0 B^{-(p-1)} \tag{12}$$

B is found [7] from the spectral centroid (which is correlated with the perceptual brightness) by

$$B = \frac{SC}{SC-1} \tag{13}$$

Brightness is infinity when B is *1* and decreasing when B is increasing. A_0 is the loudness, and it is mapped to the decibel scale. The jitter and shimmer strength are also controlled in the *dB* scale, the jitter between *-60* to *-20 dB* and shimmer between *-40* to *0 dB*. The bandwidth is controlling the filter coefficient using eq. 3 between zero and ¼ of the sample rate, while the correlation is controlled linearly between zero (only common noise) and one (only independent noise). Harmonic sounds are assumed.

The model has been implemented in max/msp. This implementation is made to be used in connection with a multiple gestures interface [12] in music cooperative systems.

5. CONCLUSIONS

The random AM (shimmer) and FM (jitter) modulations can give rise to a large variety of sensations, when modeled in a harmonic model with modulation strength, bandwidth and correlation as only parameters. The shimmer and jitter are shown to differ mainly in spectral peak amplitude; the shimmer does not affect the spectral peak, while the spectral peak in the jitter asymptotically goes to zero with increasing modulation strength. Different jitter analysis methods with good potential are reviewed. It is shown that the noise sampling rate must be higher than 4 samples per period for normal jitter strengths. A musical piece is currently made through the theories presented in this paper.

6. REFERENCES

[1] Risset J. C., D. Wessel D., Exploration of Timbre by Analysis and Synthesis, in *The Psychology of Music*, D. Deutsch, Ed.: Academic Press. 1982.

[2] K. Fitz and L. Haken, Bandwidth Enhanced Sinusoidal Modeling in Lemur, Proc. International Computer Music Conference, Banff, 1995.

[3] Freed A., Spectral Line Broadening with Transform Domain Additive Synthesis, proceedings of the International Computer Music Conference, China. 1999.

[4] Jensen, K., Irregularities, Noise and Random Fluctuations in Musical Sounds, Journal of Music and Meaning, 2004.

[5] Chowning, J. M., The Synthesis of complex audio spectra by means of frequency modulation. J. Acoust. Soc. Am. 21(7) September, pp. 526-34. 1973.

[6] Rocchesso, D., *Introduction to Sound Processing*. Edizioni di Mondo Estremo, Firenze, 2003.

[7] Mazo, J.E., Salz J., Spectra of Frequency Modulation with Random Waveforms Information and Control 9(4): pp. 414-422. 1966.

[8] Gabor, D. , Theory of Communication, *The Journal of the Institution of Electrical Engineers*, 93(III):429-457, 1946.

[9] Jensen, K., *Timbre Models of Musical Sounds*, Ph.D. Dissertation, DIKU Report 99/7, 1999.

[10] Marentakis, G., K. Jensen, Hybrid Synthesizer: Progress Report, Workshop on current research directions in computer music, Barcelona, Spain, 2001.

[11] Jensen, K., Arnspang, J., Binary Tree Classification of Musical Instruments, Proceedings of the ICMC, Beijing, China, 1999.

[12] Jensen, K., Multiple Musical Gestures in a 2D interface. The 6th International Gesture Workshop, Université de Bretagne Sud, France. May 18-20 2005.