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Automatic Power-Sharing Modification of P/V Droop Controllers in Low-Voltage Resistive Microgrids

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Automatic Power Sharing Modification of P/V Droop Controllers in Resistive Microgrids

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Abstract—Microgrids are receiving an increasing interest to integrate the growing share of distributed generation (DG) units in the electrical network. For the islanded operation of the microgrid, several control strategies for *the primary control* have been developed *to ensure a stable microgrid operation*. In low-voltage microgrids, active power/voltage (P/V) droop controllers are gaining attention as they take into account the resistive nature of the network lines and the lack of directly-coupled rotating inertia. However, a problem often cited with these droop controllers is that the grid voltage is not a global parameter. This can influence the power sharing between different units. In this paper, it is investigated whether this is actually a disadvantage of the control strategy. It is shown that with P/V droop control, the DG units that are located electrically far from the load centres automatically deliver a lower share of the power. This automatic power sharing modification can lead to decreased line losses, thus, an overall better efficiency compared to the methods that focus on perfect power sharing. In this paper, *the P/V and P/f droop control strategies are compared with respect to this power sharing modification and the line losses.*

Index Terms—distributed generation, droop controllers, microgrid, power system losses

I. INTRODUCTION

The increased penetration of distributed generation (DG) in the electrical power system is changing the paradigm of energy production. A potential advantage of DG over the conventional centralized generation is that the energy production takes place near the consumer, which can minimize the power losses in the distribution lines. The DG units can also provide ancillary services, such as reactive power support and reserve services, which can allow deferral of investments in the energy infrastructure [1].

Microgrids are receiving a growing interest as they can provide a coordinated approach for the integration of DG [2]. The increased amount of small-scale power sources that are not directly coupled to the electrical network has led to the development of converter-based microgrids [3]. Microgrids can operate both in grid-connected and islanded mode [4], [5]. In the grid-connected operation, the DG units are grid-following in a current-control strategy [6], [7]. Generally, the DG units' delivered power is independent of the state of the electrical network, i.e., determined by a maximum power point tracking strategy. Ancillary services provided by the DG units can include reference power modification based on the state

of the electrical network, voltage support in case of voltage dips and reactive power support [8].

In the islanded operation of the microgrid, the DG units are responsible for the voltage control and the power balancing and sharing. Hence, a grid-forming control strategy is required. As microgrids have different characteristics in comparison with traditional power systems, different operation and control methods have been developed. Communication-based control strategies, such as [9], [10], are mainly developed for uninterruptible power supplies, and can achieve good results in islanded microgrids. Especially in large microgrids, **using a communication link can be impractical and costly. Because this link can form a single point of failure, it may also reduce the reliability of the system compared to the case where no communication is required for the primary control of microgrids.** Therefore, droop controllers, that are not based on a communication link, have been developed.

This paper focusses on the primary control. An overlaying secondary controller can be used for the microgrid management, such as changing the set points of the DG units for further optimisation of the system and enabling the DG units to deliver ancillary services. Opposed to the primary controller, this secondary controller is generally based on inter-unit communication.

The active power/grid frequency (P/f) droop control strategy, which is based on the conventional grid control, is widely used [5], [11]–[15]. **In conventional networks, a significant amount of rotating inertia is directly coupled to the network, e.g., the inertia of the large synchronous generators. Hence, the P/f droop control is dependent on the kinetic energy in the inertia. In microgrids, generally, less inertia is present. In this case, the P/f droops are dependent on the natural linkage between P and the phase angle difference between the DG units, which in turn is dynamically determined by the frequency f. This linkage is valid in case of inductive networks. However, often, small-scale microgrids are low-voltage distribution networks, which lack a significant amount of inertia and are mainly resistive [16], [17]. Therefore, active power/voltage (P/V) droop controllers are increasingly being considered in these microgrids because of the natural linkage between P and V [17]–[19].**

In the P/f - Q/V droop controller combination, there is a line impedance effect on the accuracy of reactive power sharing [20], [21]. Analogously, **a problem often cited** when using P/V - Q/f droop controllers is the inherent trade-off between accuracy of active power sharing and output voltage control (deviation of voltage and frequency from the nominal values) [15], [22]–[24]. **Accurate power sharing implies that**

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the power changes of the loads are picked up by the units according to their droop, i.e., a combination of their power ratings and their ability to change their output power, analogous as the droop of the large central generators. It implies that the line impedances do not influence the power sharing ratio. In the P/V droop controllers, the parameter for active power changes is the terminal voltage. This parameter is a local quantity, opposed to the grid frequency in the P/f droops, which is equal in the whole grid. Hence, the line impedance can impact the active power sharing (the ratio of delivered power of each two units).

The power sharing modification due to the line impedance effect is often mentioned as a disadvantage for P/V droops. Therefore, this paper shows that this automatic power sharing modification often benefits the line losses in the network. It generally leads to a lower output power of the units electrically far from the load centres compared to units nearby, when compared with power sharing according to the **droops**. In conclusion, the P/V droop control strategy leads to an automatic power sharing modification that can reduce the line losses in the system and hence, increase the system efficiency. This is investigated on a theoretical basis in § II and by means of simulation examples in a basic microgrid § III. In § IV, the line losses **and power sharing modification of the P/V versus P/f droop control strategies are compared in a realistic 85-node distribution system.**

II. POWER SHARING IN RESISTIVE NETWORKS

In this paper, the power sharing modification of P/V droop controllers in a low-voltage microgrid is studied. Therefore, the basic microgrid of Fig. 1 is studied. **In this paragraph,** a purely resistive microgrid is considered as P/V droops are based on the resistive character of the lines in low-voltage microgrids. **Typical R/X values vary between 2 and 8, with a value of 7.7 according to [16], [17].** For simplicity of the theoretical analysis and its conclusions, two equally-rated DG units (rated power P_nom) are connected to a load through line impedances Z_1 and Z_2. **A discussion for different unit ratings is included in §. II-B.** The power sharing in case of P/f droop control and P/V droop control are considered. **In this theoretical analysis, o**nly the active power is taken into account, thus, only the working component of the current is calculated. **The reactive power flow in the islanded microgrid should be limited. An abstraction of Q is made in the main part of the theoretical analysis, but the influence of Q is discussed briefly at the end of this section. Reactive power is also included in the extended simulation based on an 85-node microgrid.**
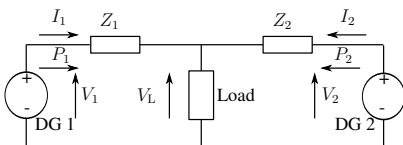

Fig. 1. Equally-rated DG units in resistive network with $R_1 < R_2$

A. Theoretical analysis

1) P/f droop control: For X-dominated lines, the P/f droop control strategy can be used (for resistive lines, P/V droop control is suited). **Both $P(f)$, which are the droops in the conventional networks, or $f(P)$ droops can be implemented. Because of the lack of inertia in the considered microgrid, the latter is chosen in this paper:**

$$f_i = f_\mathrm{nom,i} + k_\mathrm{f,i}(P_i - P_\mathrm{nom,i}), \tag{1}$$

with $i = 1,2$ and $k_\mathrm{f,i} < 0$. **The nominal power P_nom of a unit refers to the operating point of this unit. This is often the point with maximum efficiency (e.g., conventional generator) or the point determined by maximum power point tracking (e.g., renewable source). In this paper, it does not directly reflect the rating of the unit and its inverter. Therefore, like in conventional dispatchable generators, delivering more power than P_nom is possible.** The **droops are tuned such that $k_\mathrm{f,i}P_\mathrm{nom,i} = \Delta f_\mathrm{max}$,** with Δf_max dependent on the network requirements: often a 1 % limit is postulated. As grid frequency is a global parameter: $f_1 = f_2$. Hence, equally-rated units deliver the same amount of active power to the network independent on the line impedance. For the reactive power sharing, a Q/V droop controller with slope k_qv is used:

$$V_i = V_\mathrm{nom,i} + k_\mathrm{qv,i}(Q_1 - Q_\mathrm{nom,i}), \tag{2}$$

with $k_\mathrm{qv,i} < 0$. With this controller, there is a line impedance effect producing reactive power sharing mismatches [20], [21].

One remark is that P/f and P/V droop controllers intrinsically operate in different networks, namely P/f droop controllers in inductive networks and P/V droop controllers in resistive networks without rotating inertia (which is generally the case in low-voltage microgrids).

2) P/V droop control: **In resistive networks, there is a natural linkage between P and V, such that P/V droop controllers are effective.** Here, purely resistive line parameters are considered as:

- the case of a low-voltage microgrid is considered that typically has a high R/X value. E.g., a typical R/X value in low voltage lines is 7.7 according to [16], [17].
- in case of P/V droop control, resistive virtual impedance can be used in the converter, which can increase the resistive nature of the network. This virtual output impedance loop has been proposed in literature to fix the output impedance of the inverter, to increase the stability of the system and to share linear and nonlinear loads [25]–[27]. A resistive output impedance provides more damping in the system [28].

Accurate power sharing is obtained when after a load change, each DG unit changes its output power $\Delta P/P_\mathrm{nom}$ according to its ratings and specific characteristics, independent of the line impedance. In the conventional network, these ratings and characteristics are combined in the droop of the generators. Small generators and less-dispatchable units (e.g., nuclear facilities) have a lower relative ΔP after a load change compared to other units. For droop controllers in DG units, the droops are equivalent to the droops of central generators. Here, only dispatchable DG units are considered. Renewable

sources generally inject a specified amount of power, e.g., determined by maximum power point tracking, in the network, irrespective of the load changes in the network. Hence, these units do not contribute to the power sharing according to the ratings and therefore, are not droop-controlled. A method to call upon the renewables and loads in case of (extreme) load changes is using the voltage-based droop control, which is a variant of P/V droop control [18].

The P/V droop control strategy is based on:

$$V_i = V_{\text{nom,i}} - k_i(P_i - P_{\text{nom,i}}),\qquad(3)$$

where 'nom' denotes nominal values, with k the droop co-efficient ($k > 0$), which **is tuned according to**: $k_i P_{\text{nom,i}} = \Delta V_{\max}$. Generally, the P/V droop control strategy ensures that the grid voltage is close to the nominal value throughout the power system. Hence, the active power sharing is good, but not perfect if the line resistances are considered. In Fig. 1 for example, the second DG unit is located at a distance that is electrically further from the load than DG 1, i.e., $R_1 < R_2$. Accurate power sharing would involve $P_1 = P_2$. This equal power would require $V_1 = V_2$ because of the P/V droop control with equal droops and equal nominal values for the two DG units. However, this leads to a contradiction with the different voltage **drops** over the line resistances, hence, $P_1 \neq P_2$. **Therefore, in the P/V droop control, the DG units contribute to the load sharing dependent on both their ratings (droops) and the line impedances.**

It is investigated whether this modification, namely $\frac{P_1}{P_2} \neq \frac{P_{\text{nom,1}}}{P_{\text{nom,2}}} = \frac{k_2}{k_1}$, is disadvantageous. If accurate power sharing is the primary goal, this inaccuracy can be solved by means of set point changes of the droop and nominal power/voltage settings. This can be done in a slower secondary control strategy that can be communication-based. The secondary controller can also change the droops to restore the nominal values of voltage and frequency, e.g., in [24]. It can also focus on reconnecting an islanded microgrid to the main grid and minimize the fuel consumption [29].

Because the units have equal ratings:

$$V_1 - V_2 = -kP_1 + kP_2\qquad(4)$$

and in the network:

$$V_i = V_{\text{L}} + R_i I_i\qquad(5)$$

Two distinct cases can be considered:

- $I_1 < I_2$. As R_1 is lower than R_2, according to (5), this implies that $V_2 > V_1$. For the active power, $P = VI$ is valid as the active component of the current is considered. Combined, this leads to $P_2 > P_1$. However, $V_2 > V_1$ combined with (4), involves $P_1 > P_2$. This is contradiction, hence the case I_1 lower than I_2 is not possible.
- $I_1 > I_2$. Two subcases can be considered:
  - $R_2 I_2 > R_1 I_1$. Although $I_1 > I_2$, $R_2 I_2$ can be higher than $R_1 I_1$ because $R_2 > R_1$. According to (5), this leads to $V_2 > V_1$. Hence, because of the P/V droop control, $P_1 > P_2$.
  - $R_2 I_2 < R_1 I_1$. For this, a proof by contradiction is given. If $R_2 I_2 < R_1 I_1$, from (5) it follows that $V_1 > V_2$. If also $I_1 > I_2$, then $P_1 > P_2$ using $P = VI$. However, $V_1 > V_2$ combined with (4) means that

$P_2 > P_1$, which is in contradiction with the previous conclusion.

Hence:

$$\frac{R_2}{R_1} > 1 \Rightarrow \frac{P_2}{P_1} < \frac{P_{2,\text{nom}}}{P_{1,\text{nom}}}; I_1 > I_2.\qquad(6)$$

From the previous equations, it follows that the unit that is located electrically furthest from the load center will take a lower part in the power sharing. Although it seems obvious, no general conclusions about the line losses can be derived from this in the general case. However, as the electric power system is a voltage-controlled system, the voltage at each point is near its nominal value (or in strict limits), whereas the current variations can be significantly higher. Therefore, for constant power or current loads: $I_1 + I_2 \approx I_1' + I_2'$. The values with prime symbol (') refer to the case with P/f droop control. From the same assumption, voltage near its nominal value, it follows that $I_1' = I_2' = \frac{I_1 + I_2}{2}$ because of the equal power sharing. Losses comparison of

$$R_1 I_1^2 + R_2 I_2^2 \longleftrightarrow R_1 I_1'^2 + R_2 I_2'^2,\qquad(7)$$

for the P/V and P/f controlled network respectively, give:

$$\begin{aligned}
&\left[2R_1 I_1^2 + 2R_2 I_2^2 - 2R_1 I_1 I_2 - 2R_2 I_1 I_2\right] \\
&+ \left[R_1 I_1^2 + R_2 I_2^2 - R_1 I_2^2 - R_2 I_1^2\right] \longleftrightarrow 0.
\end{aligned}\qquad(8)$$

As discussed above, in the first term, $(2R_1 I_1 - 2R_2 I_2)(I_1 - I_2)$, the first factor is clearly negative and the second one positive. In the second term, $(R_1 - R_2)(I_1^2 - I_2^2)$, the first term is negative with a positive second term. Hence both terms are negative, from which it can be concluded that the losses

$$R_1 I_1^2 + R_2 I_2^2 < R_1 I_1'^2 + R_2 I_2'^2.\qquad(9)$$

Hence, the losses with P/V droops are lower than the case of P/f droops, under the aforementioned assumptions.

For units with different ratings, the droops are tuned according to $k_1 P_{\text{nom,1}} = k_2 P_{\text{nom,2}}$. For the droop control,

$$V_1 - V_2 = -k_1 P_1 + k_2 P_2\qquad(10)$$

and in the network:

$$V_1 - V_2 = R_1 I_1 - R_2 I_2\qquad(11)$$

are valid. Again, two cases can be considered, with $R_2 > R_1$:

1) $V_1 < V_2$. **Analogous to the previous case, this is advantageous for the power sharing as, then, $\frac{P_2}{P_1} < \frac{k_1}{k_2} = \frac{P_{\text{nom,2}}}{P_{\text{nom,1}}}$.**
2) $V_1 > V_2$ **is disadvantageous from the power sharing's perspective $\frac{P_2}{P_1} > \frac{k_1}{k_2} = \frac{P_{\text{nom,2}}}{P_{\text{nom,1}}}$. Together with $V_1 > V_2$, this implies that $I_1 > I_2$ and hence, $P_1 > P_2$.**
   - $k_1 > k_2$. **In this case, the furthest unit is the largest one, $P_{\text{nom,2}} > P_{\text{nom,1}}$. From above, this leads to a contradiction. Hence, if the electrically furthest unit is the largest unit, the power sharing modification is advantageous.**
   - $k_1 < k_2$. **This case does not lead to a contradiction and has a disadvantageous power sharing**

modification. As the furthest unit is the smallest one, the effect on the total line losses is however lower than in the previous case. The modification is advantageous to avoid voltage limit violation. Note also that the droop k can be shifted using a secondary controller that further optimizes the system.

In the aforementioned equations, the reactive power was not considered. For the reactive power sharing, a Q/f droop controller with slope $k_Q > 0$ is used:

$$\omega = \omega_{nom} + k_Q(Q - Q_{nom}). \tag{12}$$

As the frequency f is a global parameter, the reactive power will be properly shared. **In the previous paragraph, an abstraction was made of the reactive power. Still, Q has some influence on the power sharing.** P and Q are not fully decoupled as there is always some inductance in the lines. However, in the considered low-voltage networks, the resistance of the lines is sufficiently high such that the decoupling of P and Q is a valid assumption. Q also affects the losses of the system, but the Q flow is limited compared to P in islanded microgrids. For the P/V droops, it was shown that for equally-rated units with $R_2 > R_1$: $P_1 > P_2$. Because f is a global parameter in the related Q/f droop control: $Q_1 = Q_2$. For the P/f droops, with the same deduction: $P'_1 = P'_2$ and $Q'_1 > Q'_2$. From this, clearly, the reactive power has a tempering effect on the line losses of P/f droop controllers in the comparison of P/V - P/f droop control. As generally, the active power flow is significantly higher than the reactive power flow, the losses are still mostly advantageous for P/V droops.

B. Analytical study

In this paragraph, the same network as in the previous case is analytically studied. The P/V and P/f droop controllers are compared with respect to the power sharing modification ($\alpha = \frac{P_1/P_2}{P_{1,nom}/P_{2,nom}}$) and the system efficiency ($\eta = 1 - \frac{R_1 I_1^2 + R_2 I_2^2}{P_1 + P_2}$) as a function of the dominant parameters R_1/R_2 and $P_{1,nom}/P_{2,nom}$. The values of R_1/R_2 change from 0.2 to 20 and $P_{1,nom}/P_{2,nom}$ varies from 0.5 to 20. The sum of those parameters is kept constant, i.e., $R_1 + R_2$ and $P_{1,nom} + P_{2,nom}$.

The power sharing modification is analysed though the parameter α. A value α of one equals accurate power sharing according to the ratings, while for $\alpha > 1$, the first unit contributes more in the power sharing. Fig. 2 shows that α increases when R_1/R_2 decreases. This implies that the power sharing is dependent on the line impedances, in a manner complying with the theoretical results above, i.e., the electrically closest unit will take a larger part in the power sharing. The figure shows that the power sharing α is highly dependent on R_1/R_2, but depends on $P_{1,nom}/P_{2,nom}$ as well. For $P_{1,nom} < P_{2,nom}$:

- if $R_1 \gg R_2$, i.e., the smallest unit is the furthest one, the power sharing becomes accurate with $\alpha \approx 1$. In this case, the power sharing modification would have a low effect on reducing the line losses. This is clarified in Fig. 3, showing a highly efficient system in this case.
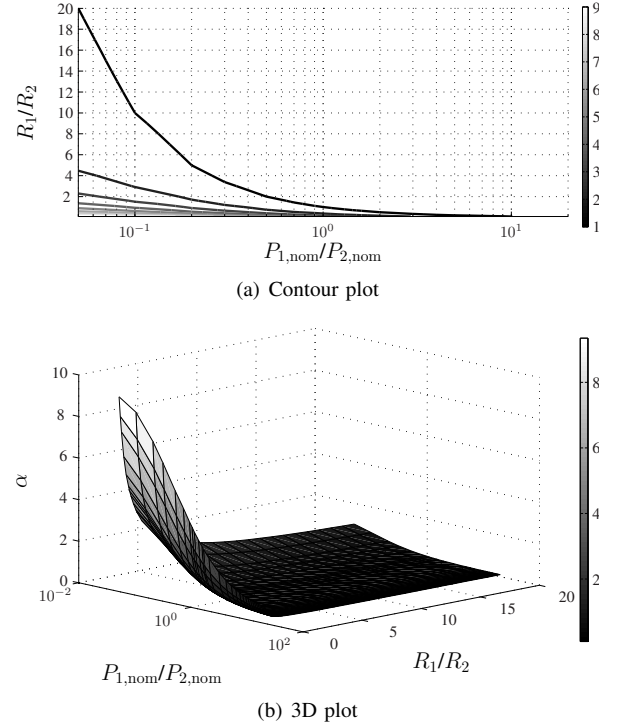

(a) Contour plot

(b) 3D plot

Fig. 2. Analytical results of the power sharing modification of P/V droops: $\alpha = \frac{P_1/P_2}{P_{1,nom}/P_{2,nom}}$

- if $R_2 > R_1$, i.e., the largest unit is the furthest one, the power sharing modification is beneficial with $\alpha > 1$.

This also complies with the theoretical analysis. Here, only values of $\alpha \geq 1$ are shown in the contour plot, for the lower values, i.e., for $P_{1,nom} > P_{2,nom}$, analogous results are obtained. For the P/f droop control, the results are not shown in a figure as a constant $\alpha = 1$ is obtained, thus, with power sharing according to the ratings and independent of the lines.

In Fig. 3, the line losses or equivalently, the system efficiency of both controllers are compared. From Fig. 3(c), it is concluded that the automatic power sharing modification leads to a higher efficiency of the P/V controllers compared to P/f control: $\eta_{PV} - \eta_{PF} > 0$.

III. BASIC EXAMPLE

The **theoretical results are verified on the basic microgrid example of Fig. 1 with $R_1 = 0.2$ Ω, $R_2 = 2$ Ω, $V_{nom} = 230$ V rms, $P_{nom} = 2.5$ kW and $P_L = 4$ kW. Here, a purely resistive network is considered (low-voltage microgrid), but in § IV, line inductance will be included as well.**

In the P/V droop control, k equals $0.0025/\sqrt{2}$ V/W (i.e., $k = \frac{\Delta V_{max}}{P_{nom}} = \frac{4.5V}{2500W}$), and a Q/f droop controller with droop 0.001 mrad/(s·VAr) is used. For the P/f - Q/V droop control (referred to as P/f), the droops are $-8 \cdot 10^{-6}$ Hz/W (i.e., $k_f = \frac{-0.125rad/s}{2500W}$) in the P/f droop control and -0.0035 V/VAr (i.e., $k_{Qv} = \frac{-8.8Vrms}{2500VAr}$) for the Q/V droop. Directly-coupled rotating inertia is lacking in the considered network, hence, the P/f controller is based on an inductive nature of the microgrid lines. As a resistive microgrid is studied in this
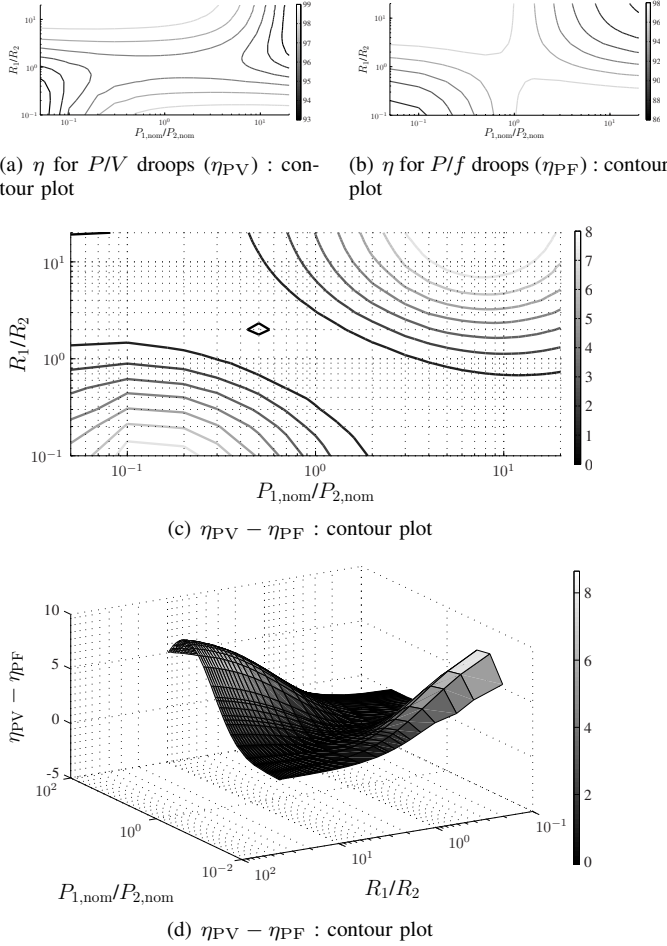
(a) η for P/V droops (η_{PV}) : contour plot

(b) η for P/f droops (η_{PF}) : contour plot

(c) $\eta_{\mathrm{PV}} - \eta_{\mathrm{PF}}$: contour plot

(d) $\eta_{\mathrm{PV}} - \eta_{\mathrm{PF}}$: contour plot

Fig. 3. Analytical results system efficiency η

example, a virtual inductive output impedance is included in the inverters, with 2 mH virtual inductance. This virtual impedance control loop has been proposed in literature to fix the output impedance of the inverter [25]–[27]:

$$v_{\mathrm{ref}} = v_{\mathrm{droop}} - x_v i_o, \qquad (13)$$

with x_v the virtual output impedance, v_{ref} the reference voltage, v_{droop} the voltage obtained by the droop controllers and i_o the output current. This control loop modifies the reference voltage v_{droop} that is obtained by the P/f and Q/V droop controllers to obtain an inductive behavior of the DG unit, i.e., $x_v = sL_v$. This allows for the P/f droop controller to obtain a stable operation.

The obtained results are summarized in Table I and comply with § II-B.

Both control strategies achieve $V \approx V_{\mathrm{nom}}$, or at least, in the voltage limits of, e.g., 10 %. In the P/V droop control, the automatic modification in the power sharing, with higher output power of the DG unit that is electrically closest to the load, leads to lower line losses. **According to the P/V droop, for example, $V_1 = 230\mathrm{V} - 0.0025/\sqrt{2}(3239 - 2500)$, such that V_1 is indeed equal to 229 V rms as shown in the table.** Note that the droop of the P/V controller is determined according to a trade-off between the power control ($\frac{P_1}{P_2}$ close to $\frac{P_{\mathrm{nom},1}}{P_{\mathrm{nom},2}}$) and voltage control ($V$ close to V_{nom}). A higher

TABLE I
EXAMPLE P/V DROOP CONTROL VERSUS P/f DROOP CONTROL

	P/V	P/f
V_1 (V)	229	222
V_2 (V)	233	238
I_1 (A)	14.16	11.08
I_2 (A)	3.55	10.35
P_1 (W)	3239	2119
P_2 (W)	827	2119
$P_{\mathrm{line,loss}}$ (W)	65	239

absolute value of this droop leads to a higher difference of the voltage from its nominal value and lower power difference. Hence, a less accurate voltage control is obtained, e.g., with droop $-0.005/\sqrt{2}$ instead of $-0.0025/\sqrt{2}$ V/W, $V_1 = 228$ V, $V_2 = 235$ V, $P_1 = 2985$ W, $P_2 = 1093$ W and the line losses equal 77 W. In this case, the voltage of both units differs more from the nominal value of 230 V, but $\frac{P_1}{P_2} = 2.73$ is closer to $\frac{P_{\mathrm{nom},1}}{P_{\mathrm{nom},2}} = 1$ compared to the equivalent value of 3.9 in Table I. Note that, here, the line resistances are chosen to be rather large to clarify the effect of power sharing modification. Practically, the line resistances will be lower leading to a lower modification of power sharing, but the same conclusions can be drawn.

As discussed above, the reactive power also has some effect on the line losses. Hence, a general comparison between P/f and P/V droops with respect to the line losses cannot be drawn, opposed to the effect on power sharing modification. P/V-Q/f droops have no circulating current, opposed to the P/f-Q/V droops. In the Q/V droop, a lower absolute value of droop k_{qv} indicates a higher reactive power difference for the same voltage difference (compared to the nominal value), hence, an increased line loss. Therefore, in the P/f droop control, circulating reactive power is obtained (1249 VAr), which is avoided in the P/V controller. **One remark concerning this circulating power, is that generally, it is practically not present. In this extreme theoretical case, pure active loads and a pure resistive network are considered, in which the P/f droop control is not the obvious approach because of the intrinsic linkage between P and V.** The reason of this circulating Q in the theoretical case is the usage of the Q/V droop controller. This is clarified by the following example. In case I_1 would be lower than I_2, $V_1 < V_2$ as $R_1 < R_2$. This would lead to $P_1 < P_2$, which is contradictory with $P_1 = P_2$ because of the P/f droop control with equal droops and equal nominal values for the two DG units. Therefore, $I_1 > I_2$ and combined with $P_1 = P_2$, this leads to $V_1 < V_2$. Because of the negative Q/V slope, this leads to a difference in reactive power, namely $Q_1 > Q_2$. As the lines are purely resistive and a pure active power load is considered, this leads to circulating power from one DG unit to the other.

IV. 85-NODE DISTRIBUTION NETWORK

The previous basic example studied a simplified low-voltage network with purely resistive lines, pure active

power loads and DG units of equal ratings. In order to verify the statement of automatic power sharing modification and reduced line losses in case of P/V droop control, in this section, a realistic distribution network is considered. In this network, inductive loads, consumption of reactive power, inductive-resistive lines and DG units of different ratings are considered. Matlab Simulink is used in order to study this network.

The line losses are calculated in a 85-node distribution network, the data of the system are given in [30], [31]. A figure of the system is shown in [30]. This paper also provides all the details of the lines and loads. The network has a nominal voltage of 11 kV and has 75 loads. **The R/X value of the network lines equals 2.4. The loads are modelled as RL loads with**

$$R = \frac{V_{\text{nom}}^2}{P_{\text{nom}}} \tag{14}$$

and $X/R = 1$. Analogous as in [30], the power factor of all loads is 0.7 lagging. **The differences between the model of [30] and the model discussed below are limited:**

- **The distribution network in [30] is a balanced three-phase radial system. Here, it is seen in its single-phase equivalent.**
- **The network of [30] has no DG units, while here, six DG units are included. Their nominal power and node of location are shown in Table II. The DG units are connected to the microgrid through a small line resistance of 0.1 Ω.**

TABLE II
DG UNITS PLACEMENT AND RESULTS FOR GRID-CONNECTED SYSTEM,
P_{nom} EQUALS DELIVERED POWER

node	P_{nom} (kW)	node	P_{nom} (kW)
6	500	54	200
22	120	76	200
47	332	82	800

The following cases are compared:

- grid-connected system with six DG units modelled as PQ generators with power factor one
- islanded system with six DG units with P/V droop control
- islanded system with six DG units with P/f droop control

A. Grid-connected system with six DG units modelled as PQ generators with power factor one

In this case, the DG units are grid-following PQ generators with a power factor of one. Grid-following units deliver their nominal power to the network, **i.e., P_{nom} determined by maximum power point or maximum efficiency, and do not change this value in case of load changes. Hence, these units are current-controlled.** In steady-state, the grid delivers 870 kW and 2404 kVAr to the microgrid and the DG units deliver their nominal power as shown in Table. II.

B. Islanded system with six DG units having P/V droop control

In this case, the 85-node network is islanded. As in the islanded mode, no main grid is available, at least one grid-forming unit is required. Note that all six DG units are considered as dispatchable DG units. Renewable sources can be included as well, but as they generally are not dispatched, they can be seen as negative loads. The renewables do not influence the power sharing ratio of the dispatchable units, and hence, are not considered in this paper. Droop control is used for the power balancing and power sharing, thus, analogous as in the conventional network, the dispatchable units are voltage-controlled. Therefore, they are modelled as ac voltage sources. This is contrary to the grid-connected DG units in the previous cases that had a grid-following control algorithm and, hence, were modelled as ac current sources. The P/V of (3) and Q/f of (12) droop controllers are used, with $k_q = 1.5e^{-7}$ Hz/VAr for each DG unit, $k = \frac{700}{P_{\text{nom}}}$ V/W, P_{nom} the nominal power of the DG unit and $Q_{\text{nom}} = 0$ VAr. **For the tuning, $\Delta V_{\max} = 700$ V has been used, reflecting a 6.5 % voltage limit.** The rms voltage $V_{\text{g,ref}}$ and frequency f determine the reference voltage $v_{\text{g,droop}}$ of the droop controller. Also, a virtual resistive output impedance of $z_v = 3\Omega$ is included in the inverters, such that the output voltage v_{g} of the DG unit equals:

$$v_{\text{g}} = v_{\text{g,droop}} - z_v i_{\text{DG}} \tag{15}$$

with i_{DG} the output current. All DG units deliver almost equal reactive power, namely 387 kVAr. The reason is the combination of equal droops, equal nominal reactive power and because f is a global parameter in the microgrid.

The simulation results for active power and terminal voltage are summarized in Table III. **In the grid-connected case, the utility network was exporting power to the microgrid. To cope with this loss of power input due to the islanding of the system, the DG units deliver more power compared to the grid-connected case, thus P is higher than the nominal value.** From the line/load data and the figure of the distribution network in [30], clearly, DG 6 lies closer to the load centres compared to DG 82 which lies near the edges of the system. Hence, the equivalent line resistance $R_6 < R_{82}$. **According to the power sharing modification studied in this paper,** it can be expected that $|\Delta P_6| > |\Delta P_{82}|$. The value ΔP of the DG units should be compared because of the different ratings of the DG units, with $\Delta P = \frac{P - P_{\text{nom}}}{P_{\text{nom}}}$. This expected power sharing modification is indeed valid as P_6 has risen with 38 % while P_{82} has risen with only 17 %. Hence, $\frac{P_6}{P_{82}} = 0.74$ instead of the nominal value of 0.63. This is compatible with the electrical distance of the DG units from the load centres and hence, benefits the line losses in the system. **This complies with the expected power sharing modification because of the usage of P/V droop controllers.** The calculated line losses equal 35.9 kW.

C. Islanded system with six DG units having P/f droop control

Analogous as the previous case, the R/X value of the network lines in the considered 85-node system is approximately

TABLE III
ISLANDED MICROGRID: DG WITH P/V DROOP CONTROL

node	P (kW)	node	P (kW)
6	690	54	290
22	180	76	319
47	427	82	933

2.4. However, in inertia-less networks, the P/f droop control strategy is highly dependent on a linkage between P and f, which is present in inductive networks, but not in resistive networks. Therefore, a virtual output inductance L_v of 50 mH is included in the inverters. The DG units are equipped with P/f droop control of (1) and Q/V droop control of (2), with $k_{Qv} = -6.5e^{-5}$ V/VAr for each DG and $k_f = \frac{-1}{2\pi P_{nom}}$ Hz/W.

The obtained results are summarized in Table IV. Perfect power sharing is obtained, e.g., $\frac{P_6}{P_{82}} = 0.63$, which equals the nominal value. Hence, P_i/P_j is constant and equal to $P_{i,nom}/P_{j,nom}$ for all P/f droop controlled DG units. This is advantageous as the units always deliver power according to their ratings, but, opposed to the P/V controllers, no automatic power sharing modification is obtained. The overall line losses equal 47.04 kW, which is higher than in the case of the P/V droop control (31 %).

An important remark is that the line losses between the P/f and P/V droop control strategies are difficult to compare **in general** as these controllers **normally** operate in networks with different characteristics. P/f **droops are generally used in inductive networks and/or networks with inertia. The** P/V **droops are fitted for inertia-less resistive networks, which is often the case in the low-voltage microgrids.**

TABLE IV
ISLANDED MICROGRID: DG WITH P/f DROOP CONTROL

node	P (kW)	node	P (kW)
6	663	54	265
22	159	76	265
47	441	82	1058

V. CONCLUSION

In this paper, the power sharing between multiple DG units is compared. Firstly, in P/V droop control, an inherent trade-off between accuracy of active power sharing and voltage regulation (voltage near the nominal value) is present. Hence, the ratio of delivered power of each two DG units can differ from the ratio of their nominal active power because of the line parameters. This paper shows that this automatic power sharing modification is in the sense that the DG units that are near the load centres, when considering the electrical distance, automatically take a larger part in the power sharing than the ones further away. **Hence, the power sharing modification of** P/V **controllers is beneficial with respect to the line losses.**
Secondly, also P/f **droop control is included in the DG units with a virtual inductive output impedance, to cope with the mainly resistive network lines in the considered low-voltage microgrids.** The P/f droop controls strategy

achieves accurate active power sharing. Hence, it does not have the automatic power sharing modification of the P/V droop control strategy.

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