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Robust Nonlinear Control Design with Application to a Marine Cooling System

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Abstract: In this paper we consider design of control laws for a marine cooling system with flow dependent delays by use of principles from feedback linearization. To deal with model uncertainties and delay mismatches, a robust linear H_∞ controller is designed for the feedback linearized system. In this context, we apply a bilinear transformation to obtain a well-posed H_∞ problem. Robustness of performance for the resulting robust nonlinear control design is evaluated through a simulation example where a comparison is made to a linear control design.

Keywords: Delay compensation, feedback linearization, robust control, cooling systems, bilinear transformations

1. INTRODUCTION

Control of nonlinear systems is often achieved by use of linear controllers designed for a linear approximation of the nonlinear system at some chosen operating point. A shortcoming of this approach is that validity of the linear approximation is very often limited to a small region around the chosen operating point. As a consequence the performance of the linear controller is likely to deteriorate as the system moves away from the operating point chosen in the design [Franco et al., 2006]. One way of dealing with this is by feedback linearization where the objective is to find a state feedback control law and possibly a change of variables to transform the nonlinear system into a linear equivalent [Khalil, 1996]. However, to achieve successful cancellation of nonlinearities in the system, the plant model has to be exact. If the system is subject to uncertainties, which is the case for most real processes, the performance of the feedback linearization is degraded, sometimes even to the point where instability occurs.

In this paper we consider application of feedback linearization together with robust control design to a marine cooling system with flow dependent delays. This system was introduced in [Hansen et al., 2011a]. The use of linear robust control design in combination with feedback linearization has been investigated on several occasions, see for instance [Franco et al., 2006] and [Chang et al., 1998]. In this paper we take a heuristic approach to include time-varying state delays in the feedback linearization to achieve a linear and delay free equivalent system. However, as the marine cooling system is not exempt from neither uncertainties nor measurement noise, the cancellation of nonlinearities and delays through feedback linearization cannot be exact. Since the cooling system in question plays a vital role in the operation of a marine vessel, robustness of performance is an important aspect of the controlled cooling system. Hence, it should be ensured that inexact cancellation by the feedback linearization does not result in significant deterioration of the closed loop system performance, or even worse, causes instability. To this end, a linear H_∞ controller is designed for the feedback linearized system to deal with model uncertainties and delay estimation errors. Robustness of performance is illustrated through a simulation example, and is compared to a linear base-line control design from [Hansen et al., 2011b]. The results shows a clear improvement in both performance and robustness of performance for the design approach applied in this paper, compared to the linear reference design.

The structure of the paper is as follows: In Section 2 we introduce the model of the marine cooling system considered in this paper. Section 3 deals with the transformation of the nonlinear system into a linear equivalent using feedback linearization. Section 4 describes robust control design for the feedback linearized system, and performance of the overall control design is evaluated through a simulation example in Section 5. Finally, concluding remarks are given in Section 6.

We make use of the following notation: $\mathbb{R}$ denotes the set of real numbers while $\mathbb{R}_+$ denotes the set of non-negative real numbers. $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices and $C(\mathcal{M}, \mathcal{N})$ is the set of continuous functions mapping from $\mathcal{M}$ to $\mathcal{N}$. Vectors are written in bold, and I_n denotes the $n \times n$ identity matrix, while $\mathbf{0}_{n,m}$ denotes the $n \times m$ zero matrix.

2. MARINE COOLING SYSTEM MODEL

We consider the problem of designing robust nonlinear control laws for the marine cooling system introduced in [Hansen et al., 2011a]. A simplified diagram of the cooling system is illustrated in Fig. 1. The circuit on the left is denoted seawater (SW) circuit and has the single purpose of pumping seawater through the primary (cold) side of the heat exchanger. The objective is to remove heat from the coolant circulating through the machinery in the low temperature (LT) circuit on the right, i.e., on the secondary (hot) side of the heat exchanger. The LT circuit contains various types of auxiliary machinery in a parallel configuration, i.e. the consumers in Fig. 1 ranges from diesel generators to air condition condensers. As a consequence, the individual consumer provides very different heat loads to the cooling system and have various flow requirements. To keep the model complexity to a minimum, the same first order ODE model is applied to all consumers. It is assumed that heat exchange only takes place in the consumers of the cooling system, and in the heat exchanger.

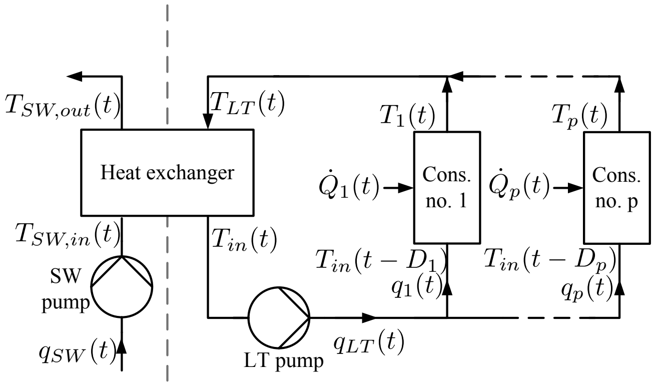

Fig. 1. Simplified layout of the cooling system considered in this work.

Using notation from Fig. 1 the dynamics of the system is governed by:

$$\dot{T}_i(t) = \frac{1}{V_i}\Big[q_i(t)(T_{in}(t-D_i(\boldsymbol{q})) - T_i(t)) + \frac{\dot{Q}_i(t)}{\rho c_p}\Big] \quad (1)$$

$$\dot{T}_{in}(t) = \frac{1}{V_{CC}}\Big[\sum_{i=1}^{p} q_i(t)(T_{LT}(t) - T_{in}(t)) + q_{SW}(t)\frac{\rho_{sw}c_{p,sw}}{\rho c_p}\Delta T_{SW}(t))\Big] , \quad (2)$$

for $i = 1, 2, \ldots, p$, where q denotes volumetric flow rate, V is internal volume, T is temperature and $\dot{Q}$ denotes heat transfer. Furthermore, ρ and c_p are respectively the density and specific heat of the coolant, $\Delta T_{SW} = T_{SW,in} - T_{SW,out}$ and $\boldsymbol{q} = [q_1, q_2, \ldots, q_p]^T$. Due to the size and layout of the cooling system, there is a transport delay in the coolant temperature from the heat exchanger to each of the consumers. Delays are modeled by:

$$D_i(\boldsymbol{q}) = \sum_{j=1}^{i} a_{m,j}\left(\sum_{k=j}^{p} q_k\right)^{-1} + \frac{a_{c,i}}{q_i} , \quad (3)$$

where $a_{m,j}$ and $a_{c,i}$ are positive, system specific constants.

In this setting, we define states, controllable inputs, and exogenous inputs as:

$$\boldsymbol{x} = \begin{bmatrix} T_1(t) \\ \vdots \\ T_p(t) \\ T_{in}(t) \end{bmatrix} \quad \boldsymbol{u} = \begin{bmatrix} q_1(t) \\ \vdots \\ q_p(t) \\ q_{SW}(t) \end{bmatrix} \quad \boldsymbol{w}_1 = \begin{bmatrix} \dot{Q}_1(t) \\ \vdots \\ \dot{Q}_p(t) \end{bmatrix} . \quad (4)$$

With the definitions in (4) the state equations can be represented as:

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{m} f_i(\boldsymbol{x}(t), \boldsymbol{x}(t - D_i(\boldsymbol{u})))u_i(t) + B_w\boldsymbol{w}_1(t) , \quad (5)$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{u} \in \mathbb{R}^m$, $\boldsymbol{w}_1 \in \mathbb{R}^d$, $B_w \in \mathbb{R}^{n\times d}$ and $f_i(\cdot)$ are smooth vector fields defined on a subset of $\mathbb{R}^n$.

We assume that the parameter varying delays are bounded, i.e. $D_i(\boldsymbol{q})$ for $i = 1, \ldots, m$ belongs to the set:

$$\begin{aligned}\mathcal{D} :=\{&D \in C(\mathbb{R},\mathbb{R}); 0 \le D \le \overline{D} < \infty \\ &\forall \boldsymbol{q} \in \mathbb{R}^p_+\} .\end{aligned} \quad (6)$$

To ensured delays are bounded entails according to (3) that inputs must be strictly positive. In this context we only consider situations where there is a positive heat load and as a consequence the temperature difference of the coolant at the in- and outlet of a consumer is always positive, i.e. $0 < (T_i(t) - T_{in}(t - D_i(\boldsymbol{q})))$ $,\forall t \in \mathbb{R}_+$. The bilinear nature of the systems means that flow rates must be strictly positive for the system to be at an equilibrium, and it is not unreasonable to bound the flows, and thereby the inputs such that: $0 < \underline{\boldsymbol{U}} \le \boldsymbol{u} \le \overline{\boldsymbol{U}}$.

Similar, disturbances are assumed to be bounded but unknown, i.e. they belong to the set:

$$\begin{aligned}\mathcal{W} :=\{&w \in C(\mathbb{R},\mathbb{R}); 0 < \underline{W} \le w \le \overline{W} < \infty \\ &\forall t \in \mathbb{R}_+\} .\end{aligned} \quad (7)$$

Initial conditions for the system in (5) are governed by:

$$\boldsymbol{x}(0) = \boldsymbol{x}_0 , \quad (8)$$

$$\boldsymbol{x}(\theta) = \phi(\theta) \quad , \quad \theta \in \left[-\overline{D}, 0\right] , \quad (9)$$

and we define that:

$$\boldsymbol{x}_t(\theta) = x(t+\theta) . \quad (10)$$

It is assumed that $\boldsymbol{x}_t(\theta)$ is available to the controller.

3. FEEDBACK LINEARIZATION DESIGN

From general feedback linearization theory it is well known that the nonlinear state equations can be linearized through state feedback of the form:

$$\boldsymbol{u} = \alpha(\boldsymbol{x}) + \gamma^{-1}(\boldsymbol{x})\boldsymbol{v} , \quad (11)$$

if the state equations follows the structure of:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\gamma(\boldsymbol{x})[\boldsymbol{u} - \alpha(\boldsymbol{x})] , \quad (12)$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\alpha : \mathbb{R}^n \mapsto \mathbb{R}^m$, $\gamma : \mathbb{R}^n \mapsto \mathbb{R}^{m\times m}$, $\gamma(x)$ is nonsingular in the domain of interest, and the pair (A, B) is controllable [Khalil, 1996].

By writing γ as a function of both states and delayed states using (3) to estimate the delays, i.e. $\gamma(\boldsymbol{x}, \boldsymbol{x}(t - \hat{D}_1), \ldots, \boldsymbol{x}(t - \hat{D}_n))$ the system in (5) is written as:

$$\dot{\boldsymbol{x}}(t) = B_v\gamma(\cdot)\boldsymbol{u}(t) + B_w\boldsymbol{w}_1(t) . \quad (13)$$

Under invertibility assumptions on γ the system is linearized through the feedback law:

$$\boldsymbol{u}(t) = \gamma(\cdot)^{-1}\boldsymbol{v}(t) , \quad (14)$$

where $\boldsymbol{v}(t)$ is a linear control input.

Since delays in the case of the marine cooling system depends on the input, a practical remark is in order. The use of an input dependent delay estimate in the feedback linearization has the immediate consequence that the control law will depend on the current input. To avoid this algebraic constraint, we approximate the delays by:

$$\hat{D}_i(\boldsymbol{q}) \approx \sum_{j=1}^{i} a_{m,j} \left(\sum_{k=j}^{p} q_k(t-\tau) \right)^{-1} + \frac{a_{c,i}}{q_i(t-\tau)} \ , \quad (15)$$

where τ is a small positive constant, i.e. we use previous input values to estimate the current delays.

To ease the notation in the following, we define:

$$\Lambda = \frac{\rho_{sw} c_{p,sw}}{\rho c_p} \Delta T_{SW}(t) \ , \quad (16)$$

$$\Phi = T_{LT}(t) - T_{in}(t) \ , \quad (17)$$

$$\Psi_i = (T_{in}(t - D_i) - T_i(t)) \ . \quad (18)$$

Bringing (1) and (2) to the form of (12) results in:

$$\dot{\boldsymbol{x}} = \overbrace{\begin{bmatrix} \frac{1}{V_1} & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \frac{1}{V_p} & 0 \\ 0 & \dots & 0 & \frac{1}{V_{cc}} \end{bmatrix}}^{B_v} \overbrace{\begin{bmatrix} \Psi_1 & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \Psi_p & 0 \\ \Phi & \dots & \Phi & \Lambda \end{bmatrix}}^{\gamma} \overbrace{\begin{bmatrix} q_1(t) \\ \vdots \\ q_p(t) \\ q_{SW}(t) \end{bmatrix}}^{\boldsymbol{u}}$$

$$+ \overbrace{\begin{bmatrix} \frac{1}{\rho c_p V_1} & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & & \frac{1}{\rho c_p V_p} \\ 0 & \dots & \dots & 0 \end{bmatrix}}^{B_w} \overbrace{\begin{bmatrix} \dot{Q}_1(t) \\ \vdots \\ \dot{Q}_p(t) \end{bmatrix}}^{\boldsymbol{w}_1} \quad (19)$$

We need to ensure that γ is non-singular, and in this case it is sufficient to look at the product of the diagonal and check if this is nonzero in the domain of interest:

$$\Lambda \prod_{i=1}^{p} \Psi_i \neq 0 \ , \quad \forall t \in \mathbb{R}_+ \ . \quad (20)$$

We have already argued that $\Psi_i < 0$, and a similar argument can be applied for Λ: Since we only consider the system during operation, the heat transfer from the LT side of the system will ensure that ΔT_{SW} will be strictly negative, and we have that $\Lambda < 0, \ \forall t \in \mathbb{R}_+$.

From the state feedback law (11), and the fact that $\alpha(x) = 0$, we get that:

$$\boldsymbol{u} = \gamma^{-1}(\cdot)\boldsymbol{v} = \begin{bmatrix} \frac{1}{\Psi_1} & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \frac{1}{\Psi_p} & 0 \\ \frac{\Phi}{\Psi_1 \Lambda} & \dots & \frac{\Phi}{\Psi_p \Lambda} & \frac{1}{\Lambda} \end{bmatrix} \boldsymbol{v} \ . \quad (21)$$

Because the feedback linearization relies on exact cancellation of nonlinear terms, any model uncertainties or mismatch between the estimated delay and actual delay will degrade the performance of the feedback linearization. This can be represented as:

$$\dot{\boldsymbol{x}} = B_v \gamma \hat{\gamma}^{-1} \boldsymbol{v} + B_w \boldsymbol{w} \quad (22)$$

$$\Downarrow$$

$$\dot{\boldsymbol{x}} = B_v (I + \Delta) \boldsymbol{v} + B_w \boldsymbol{w} \ , \quad (23)$$

where $\hat{\gamma}$ estimates the system nonlinearities and Δ represents the mismatch between the estimated and actual system nonlinearities due to uncertainties. However, rather than using the representation in (23) we accommodate for this uncertainty by adding an output disturbance term, $\boldsymbol{w}_2 \in \mathbb{R}^n$. This can be interpreted as a way of describing additive norm bounded modeling uncertainties for output feedback H_∞ control [Su et al., 2002].

By this approach the feedback linearized system can be represented as the stabilizable system:

$$\begin{aligned} \dot{\boldsymbol{x}} &= B_v \boldsymbol{v} + B_w \boldsymbol{w}_1 \\ \boldsymbol{y} &= C_0 \boldsymbol{x} + D_0 \boldsymbol{w}_2 \ , \end{aligned} \quad (24)$$

where $C_0, D_0 \in \mathcal{R}^{n \times n}$ for this system are identity matrices.

This setup is illustrated in Fig. 2 an forms the basis for the H_∞ control design in the following.

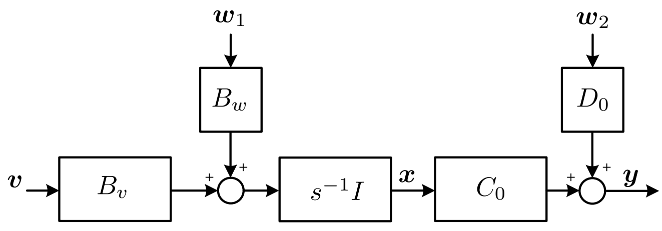

Fig. 2. Linear equivalent of feedback linearized marine cooling system with an additive uncertainty term, $\boldsymbol{w}_2$.

4. ROBUST CONTROL DESIGN

Design of the linear control input $\boldsymbol{v}(t)$ for the feedback linearized system in (24) is done by use of robust control theory. We use the standard 2×2 block formulation as illustrated in Fig. 3. This means that:

$$G(s) = C(sI - A)^{-1}B + D := \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \ , \quad (25)$$

for the system given by the state space representation A, B, C and D. Combining exogenous inputs, $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$, into a single vector and introducing an error vector, $\boldsymbol{z}$, penalizing the states and inputs, yields:

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{w}_1 \\ \boldsymbol{w}_2 \end{bmatrix} \quad (26)$$

$$\boldsymbol{z} = \boldsymbol{x} + \rho_0 \boldsymbol{v} \quad (27)$$

where $\rho_0 > 0$ is a scaling factor for the control penalty.

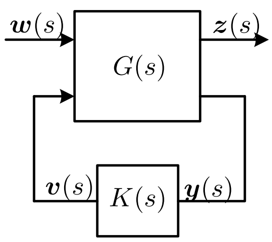

Fig. 3. The H_∞ control problem in a 2×2 block formulation.

We apply the partitioning from [Doyle et al., 1989] to (25) to achieve:

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \quad (28)$$

where B, C and D have been partitioned according to $\boldsymbol{z}$, $\boldsymbol{y}$, $\boldsymbol{w}$ and $\boldsymbol{v}$, respectively.

With the definitions of $\dot{\boldsymbol{x}}$, $\boldsymbol{y}$, $\boldsymbol{w}$ and $\boldsymbol{z}$ for the marine cooling system in question, we can write the matrices in (28) as:

$$\begin{aligned}
B_1 &= [\, B_w \;\; \mathbf{0}_{n,n} \,] && (29) & D_{11} &= \mathbf{0}_{n,(p+n)} && (33)\\
B_2 &= B_v && (30) & D_{12} &= \rho_0 I_n && (34)\\
C_1 &= I_n && (31) & D_{21} &= [\mathbf{0}_{n,p} \;\; I_n] && (35)\\
C_2 &= I_n && (32) & D_{22} &= \mathbf{0}_{n,n} && (36)
\end{aligned}$$

We now have a 2×2 block formulation of the robust control problem. However, it is evident from Fig. 2 and (24) that the system has poles on the imaginary axis. This means we are dealing with a nonstandard problem which cannot be solved with standard H_∞ theory directly. To this end we apply a bilinear transformation to shift the poles of the feedback linearized system from the origin and transform the system model into a close approximation which allows standard H_∞ control design. Once the controller is designed for the approximate model, the inverse bilinear transformation is applied to get the final controller for the original plant model [Chiang and Safonov, 1991].

We apply the $j\omega$-axis pole shifting transformation described in details in [Chiang and Safonov, 1992] given by:

$$s = \frac{\tilde{s} + p_1}{(\frac{\tilde{s}}{p_2}) + 1} \;, \quad (37)$$

where $p_1, p_2 < 0$ are the endpoints of a circle being mapped by (37) from the left s-plane into the $j\tilde{\omega}$-axis of the $\tilde{s}$-plane. Correspondingly, the inverse transformation is given by:

$$\tilde{s} = \frac{-s + p_1}{(\frac{s}{p_2}) - 1} \;. \quad (38)$$

An important aspect of the bilinear transformation in (37) is the choice of p_1 and p_2. A property of the weighted mixed sensitivity problem formulation is that any unstable plant pole within the specified control bandwidth is approximately shifted to its $j\omega$-axis mirror image once the loop is closed with an H_∞ controller. Since the bilinear transformation maps the poles from the $j\omega$-axis in the s-plane to a circle centered at $-(p_1 + p_2)/2$ in the $\tilde{s}$-plane, the parameter p_1 in (37) and (38) plays an essential role when placing dominant closed loop poles in the s-plane. Contrary to p_1, the choice of p_2 is of little importance to the design, and can be chosen such that: $p_2 \gg$ control bandwidth [Chiang and Safonov, 1991].

Having mapped the system to the $\tilde{s}$-plane using appropriate values for p_1 and p_2, we find a H_∞ optimal controller, $K(\tilde{s})$, by solving the two standard 2-Riccati equations from [Doyle et al., 1989]. The final controller, $K(s)$, is then obtained by applying the inverse bilinear transformation from (38) to $K(\tilde{s})$.

5. SIMULATION EXAMPLE

To clarify the design methodology in its entirety as well as to evaluate both performance and robustness of performance we consider a simulation example where $p = 2$, i.e. we have that:

$$\begin{aligned}
\dot{T}_1(t) &= \frac{1}{V_1}\left[q_1(t)(T_{in}(t - D_1(\boldsymbol{q})) - T_1(t)) + \frac{\dot{Q}_1(t)}{\rho c_p}\right]\\
\dot{T}_2(t) &= \frac{1}{V_2}\left[q_2(t)(T_{in}(t - D_2(\boldsymbol{q})) - T_2(t)) + \frac{\dot{Q}_2(t)}{\rho c_p}\right]\\
\dot{T}_{in}(t) &= \frac{1}{V_{CC}}\Big[\sum_{i=1}^{2} q_i(t)(T_{LT}(t) - T_{in}(t))\\
&\quad + q_{SW}(t)\frac{\rho_{sw} c_{p,sw}}{\rho c_p}\Delta T_{SW}(t))\Big] \;,
\end{aligned}$$

where the thermodynamic parameters for the cooling system is illustrated in Table 1.

Table 1. Thermodynamic parameters for cooling system.

c_p	ρ	$c_{p,sw}$	ρ_{sw}	V_{CC}	V_1	V_2
4181	1000	3993	1025	20	13.5	13.5

The corresponding delays are given by:

$$D_1(\boldsymbol{q}) = \frac{a_{m,1}}{q_1 + q_2} + \frac{a_{c,1}}{q_1} \quad (39)$$

$$D_2(\boldsymbol{q}) = \frac{a_{m,1}}{q_1 + q_2} + \frac{a_{m,2} + a_{c,2}}{q_2} \quad (40)$$

From (21) we find γ^{-1} as:

$$\gamma^{-1} = \begin{bmatrix} \frac{1}{\Psi_1} & 0 & 0 \\ 0 & \frac{1}{\Psi_2} & 0 \\ \frac{\Phi}{\Psi_1 \Lambda} & \frac{\Phi}{\Psi_2 \Lambda} & \frac{1}{\Lambda} \end{bmatrix} \quad (41)$$

We then have that:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \frac{1}{V_1} & 0 & 0 \\ 0 & \frac{1}{V_2} & 0 \\ 0 & 0 & \frac{1}{V_{CC}} \end{bmatrix} \boldsymbol{v} + \begin{bmatrix} \frac{1}{\rho c_p V_1} & 0 \\ 0 & \frac{1}{\rho c_p V_2} \\ 0 & 0 \end{bmatrix} \boldsymbol{w}_1 \quad (42)$$

$$\boldsymbol{y} = I_3 \boldsymbol{x} + I_3 \boldsymbol{w}_2 \;. \quad (43)$$

We choose $\rho_0 = 1.2$ and partition the system matrices according to (29)-(36). For the bilinear transformation we choose $p_1 = -10 \times 10^{-3}$, $p_2 = -100$ and obtain a H_∞ controller using `hinfsyn()` in Matlab.

To evaluate performance we apply the proposed controller to a nonlinear simulation model of the marine cooling system and compare the response with the base-line (PI) control design presented in [Hansen et al., 2011b]. In the first simulation scenario we consider the nominal case, where the system is subjected to step-wise disturbances while at the operating point used in the design for the base-line controller. To evaluate and compare robustness of performance for the two designs we also consider parameter perturbations of ± 50% for V_1, V_2, V_{CC}, $a_{m,1}$ and $a_{m,2}$ resulting in 32 combinations of extreme values. These combinations are all tested, and responses are plotted along the response for the nominal system for both controller design Parameters for the first simulation scenario are presented in Table 2 and disturbances are plotted in Fig. 4.

Table 2. Parameters for 1st simulation run.

$T_{1,ref}$	$T_{2,ref}$	$T_{in,ref}$	$T_{sw,in}$	$T_{sw,out}$	V_1
65	70	36	24	40	13.5±50%
$a_{m,1}$	$a_{m,2}$	$a_{c,1}$	$a_{c,2}$	V_{CC}	V_2
30±50%	40±50%	20	10	20±50%	13.5±50%

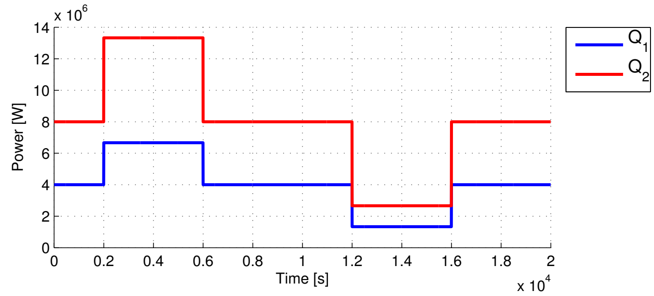

Fig. 4. Disturbances $Q_1(t)$ and $Q_2(t)$.

Responses for the base line control is shown in Fig. 5 while the response for the control design presented in this paper is illustrated in Fig. 6.

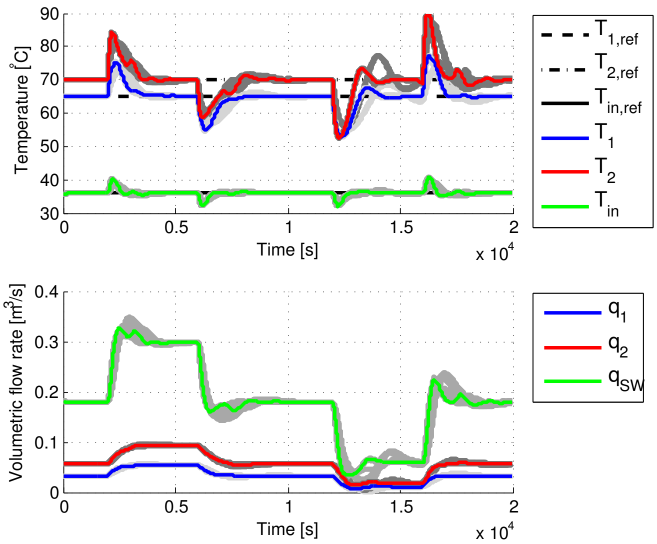

Fig. 5. Temperature response and corresponding input signals for base-line controller

Comparing Fig. 5 and Fig. 6 it is evident that the control design presented in this paper does not suffer from the same transient peaks during disturbance steps as the base-line control. However, due to the lack of integral action in the H_∞ controller, the temperature response for this control design is subject to a small steady state error. This could be avoided by introducing an integrator to the H_∞ controller as in [Su et al., 2002], which however, is outside the scope of this paper. The results also shows that even under considerable variations of system parameters the closed-loop system controlled by the control design proposed in this paper performs almost identical to the nominal one. For the case of the base-line control, the parameter variations does influence the closed-loop response, but never to the point where instability occurs.

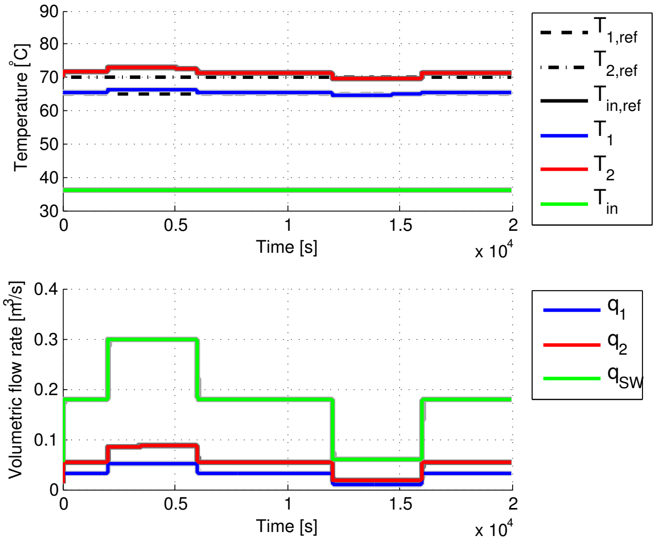

Fig. 6. Temperature response and corresponding input signals for H_∞ controller applied to feedback linearized system.

Next, we consider a scenario where the temperature references are changed, such that the base-line control is no longer at the operating point used in the design. We then subject the system to the same disturbances as in the previous case. Parameters for the second simulation run are shown in Table 3.

Table 3. Parameters for 2nd simulation run.

$T_{1,ref}$	$T_{2,ref}$	$T_{in,ref}$	$T_{sw,in}$	$T_{sw,out}$	V_1
55	60	40	24	40	13.5±50%
$a_{m,1}$	$a_{m,2}$	$a_{c,1}$	$a_{c,2}$	V_{CC}	V_2
30±50%	40±50%	20	10	20±50%	13.5±50%

Responses for the base-line control are plotted in Fig. 7 while the responses for the proposed design are plotted in Fig 8.

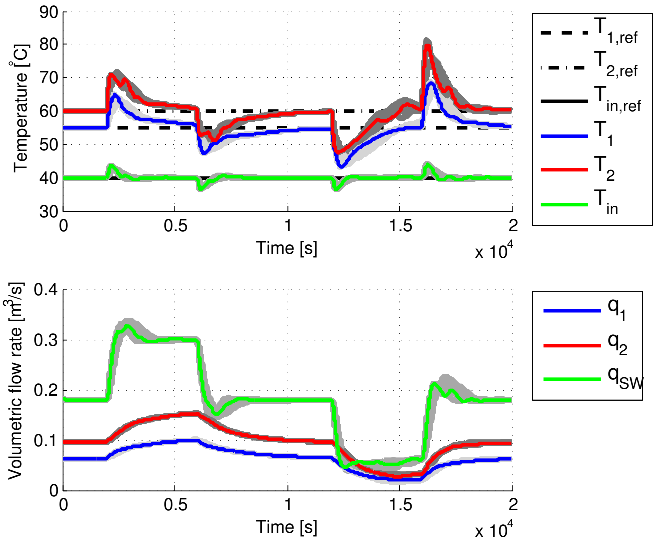

Fig. 7. Temperature response and corresponding input signals for base-line controller

As before, parameter variations does influence the closed loop response for the base-line design, while they are

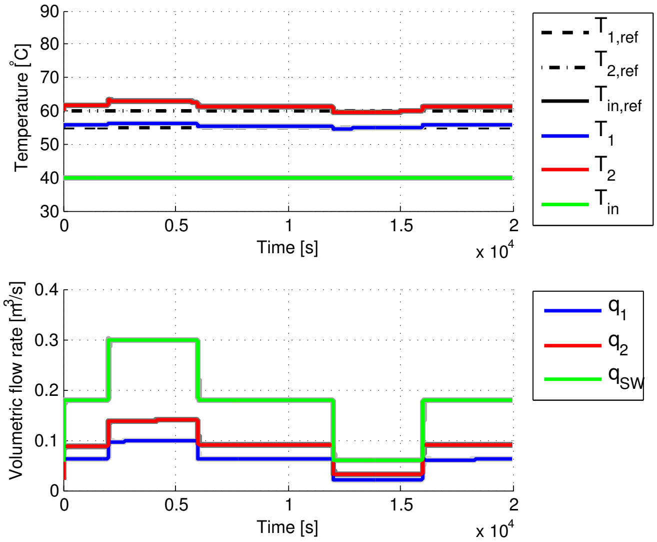

Fig. 8. Temperature response and corresponding input signals for H_∞ controller applied to feedback linearized system.

barely visible in the closed loop response for the control design presented in this paper. The change in operating conditions affects the disturbance rejection performance of the base-line control resulting in increased transient peaks when the disturbances are stepped. Contrary, the change in operating conditions causes no visible change in disturbance attenuation performance for the proposed control design.

6. CONCLUDING REMARKS

We have presented a heuristic but systematic approach to design a robust nonlinear controller for a marine cooling system with flow dependent delays. The design methodology was comprised by principles from feedback linearization to deal with delays and nonlinearities, while a H_∞ control design was used to ensure robustness towards model uncertainties and disturbances. Robustness of performance for the composite control design was illustrated through a simulation example. Results from the simulation showed that robustness towards both parameter variations and changes in operating conditions for the proposed design was significantly improved compared to a baseline PI control design. From this it is concluded that the design approach proposed in this paper yields a simple, yet effective way of compensating delays and nonlinearities while maintaining robustness of performance. However, the control does require some storage of data as the control law relies on previous values of one of the states. Also, lack of integral action in the proposed design resulted in a small steady state error, which will be addressed in future works. Future works also includes verification of the proposed design through implementation on a full scale cooling system aboard a container vessel in service.

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