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Juelsgaard, Morten; Bendtsen, Jan Dimon; Wisniewski, Rafal

Published in:
7th IFAC Symposium on Robust Control Design

DOI (link to publication from Publisher):
10.3182/20120620-3-DK-2025.00041

Publication date:
2012

Document Version
Early version, also known as pre-print

Robust Utilization of Wind Turbine Flexibility for Grid Stabilization

M. Juelsgaard, J. Bendtsen and R. Wisniewski

Dept. of Automation and Control, University of Aalborg, Denmark,
e-mail: {*mju, dimon, raf*}*@es.aau.dk*

Abstract: This work considers the use of wind turbines for stabilizing an electrical grid, by employing temporary overproduction with respect to available power. We present a simple model describing a turbine, and show how the possible period of overproduction, can be maximized through a series of convex problems, where the load is distributed among several turbines in a farm. We then present an optimization scheme that guarantees a lower limit for the overproduction period and subsequently propose an adaptive implementation that is robust against parameter uncertainties.

Keywords: Smart power applications; Wind turbine control; Robust control; Convex optimization; Adaptive control;

1. INTRODUCTION

For a number of years, the Danish use of wind turbines for electrical power generation has increased, and is further expected to increase in the future. However, incorporating significant penetration of wind power, poses several difficulties in order to maintain a stable grid and ensure security of supply. On this basis, much research is currently focused on how to incorporate wind power and other volatile energy sources into electrical grids on a far greater scale than previously, while maintaining the stability of the grid (Dansk Energi (2010), Tarnowski et al. (2009b), Margaris et al. (2010), *etc.*).

One measure of grid stability, is the grid frequency. If this deviates from the nominal value, there is an imbalance between the produced and consumed power. If the grid frequency decreases, more power needs to be produced, and vice versa. Usually this increase or decrease of production is handled by thermal plants, but recent research has also revolved around utilizing wind generated power for grid stabilization. Down regulation of the grid, *i.e.*, lowering production, can be aided by wind turbines, simply by lowering their production also. However, below rated power, wind turbines usually produce whatever power is available in the wind, so the question is whether wind turbines are also capable of participating in up regulating the grid, *i.e.*, by increasing production above available power.

This is explored by Tarnowski et al. (2009a) and Tarnowski et al. (2009b). Here it is illustrated that turbines are temporarily capable of increasing production above the available power, by extracting energy from the rotating mass in the rotor plane. Consequently, the rotor will slow down, so the overproduction can only be maintained for a limited period of time.

In this paper, we investigate some of the limitations by utilizing this overproduction. We introduce the optimization problem of exploiting the rotational energy stored in a wind farm, such that the overproduction period is maximized. This is desirable since an increased period of overproduction, would entail that turbines on a larger scale could be used for grid stabilization, as an alternative to thermal plants. We show that assuming perfect knowledge about model parameters, this problem can be solved as a series of convex problems. We then present an alternative approach, robust against uncertainties in model parameters. This alternative approach initially provides a lower bound for the overproduction period, given a certain definition of the uncertainties. Subsequently, we apply an adaptive optimization scheme for obtaining this period, while subjected to the same uncertainties.

We will initially elaborate on the background for the flexibility introduced by employing overproduction from wind turbines. This is presented in Section 2, along with the model a wind turbine. The presented model is capable of mimicking the results of Tarnowski et al. (2009a). Section 3 provides a formal formulation of the problem of maximizing the overproduction period, whereafter Section 4 considers parametric uncertainties in formulating a robust lower bound for the overproduction period, and also presents our adaptive implementation. This is followed by a numerical example in Section 5. Section 6 presents the conclusions and suggestions for further work.

2. BACKGROUND AND MODELING

This work is motivated by Tarnowski et al. (2009a), where the possible overproduction from a turbine is explored. The basic idea is that when the turbine operates at normal conditions, below rated power, and produces the available power of the wind, the rotating mass of the rotor can be seen as an energy storage. Extracting energy from this storage, entails that the power production can be increased above available power, by increasing the generator torque. However, this slows down the rotor. The rotor must not be slowed below some lower limit, so at this point, the generator torque must be reduced, allowing the rotor to accelerate back to optimal operating conditions. This is illustrated in Fig. 1(Tarnowski et al. (2009a)).

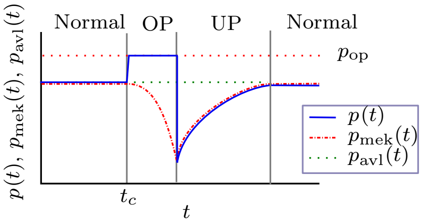

Fig. 1. The electrical, mechanical and available power of the turbine, during normal, OP (over production) and UP (under production) periods.

Fig. 1 illustrates the power production from a turbine, divided into the three different periods denoted 'Normal', 'Over production (OP)' and 'Under production (UP)' - operation. In normal operation, the power $p(t) \in \mathbf{R}_+$, produced by the turbine, equals the available power in the wind $p_{\rm avl}(t) \in \mathbf{R}_+$, *i.e.*, $p(t) = p_{\rm avl}(t)$. Here $t \in [0;\ \infty)$ is the time and $\mathbf{R}_+$ refers to the non-negative numbers. The power $p(t)$ is limited above by the rated power, $p_{\rm max}$, of the turbine, but we will disregard this concern in this work.

At some time t_c, the turbine power reference increases, for instance as a result of a frequency imbalance in the grid. The turbine is now required to produce $p(t) = p_{\rm op} > p_{\rm avl}(t)$, which is an overproduction. The electrical power can increase, however as illustrated in Fig. 1, the mechanical power in the rotor will consequently decrease, because the rotor slows down. When the lower limit is reached, the electrical power will have to be reduced, to that of the mechanical power, which now has to be returned to the vantage point, by accelerating the rotor of the wind turbine. This causes a period of underproduction, with respect to the available power.

2.1 Turbine modeling

We can model the described behavior by approximating the turbine as a flywheel, as illustrated in Fig. 2. The

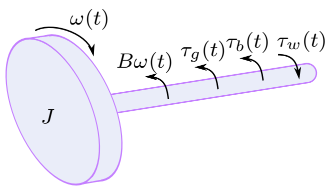

Fig. 2. Flywheel model of a wind turbine.

flywheel consist of an inertia $J \in \mathbf{R}_+$, with a rotational velocity of $\omega(t) \in \mathbf{R}_+$ and a viscous friction $B \in \mathbf{R}_+$. The rotating motion is driven by a torque $\tau_w(t) \in \mathbf{R}_+$ produced by the wind. The turbine generator affects the rotation by a torque $\tau_g(t) \in \mathbf{R}_+$, opposite the direction of rotation. Likewise, the turbine is equipped with a brake, that produces a torque $\tau_b(t) \in \mathbf{R}_+$, also opposite the rotation.

The power $p(t)$, is expressed as $p(t) = \tau_g(t)\omega(t)$, which is a non-linear expression. In the sequel, we will use a linear approximation of this expression. Even though this is not completely accurate, we will later show that the error obtained by the approximation, is only minor, when introducing on-line updates.

A model describing the turbine, can be arranged as

$$\begin{aligned} J\dot{\omega}(t) &= \text{-}B\omega(t) - \tau_g(t) - \tau_b(t) + \tau_w \\ p(t) &= \overline{\omega}\tau_g(t) + \overline{\tau}_g\omega(t) - \overline{\omega}\overline{\tau}_g, \end{aligned} \tag{1}$$

where $(\overline{\omega},\overline{\tau}_g)$ is the operating point for the linear approximation of $p(t)$. The '=' in the second line is an abuse of notation, as it is only an approximation and not an equality.

This model has two controllable inputs, $\tau_g(t)$ and $\tau_b(t)$, which both have to be non-negative. The use of both actuators counteracts the incoming wind $\tau_w(t)$, and affects the rotational velocity $\omega(t)$. Employing the generator $\tau_g(t)$ directly affects the output $p(t)$, whereas $\tau_b(t)$ has no effect on the output. As we will illustrate later, this model contains the dynamics illustrated by Fig. 1.

2.2 Farm modeling

We expand the model of a single turbine to cover an entire farm, simply by aggregation. We leave as future work, any inter turbine effects, such as changes in wind speed or turbulence throughout the farm. Models of this have been studied extensively by for instance Soleimanzadeh and Wisniewski (2010) and Knudsen et al. (2011). Including inter turbine effects in this work, could be completed by including the different phenomena in the modeling of the torque $t_{w,i}(t)$, provided by the wind, where the subscript refers to turbine i in the farm.

3. GENERAL PROBLEM DESCRIPTION

In the following, we address the general problem of maximizing the overproduction period for a farm, when presented with a demand exceeding the accumulated available power. We start by providing a formal definition of the overproduction period.

Consider a wind farm consisting of n turbines. The farm is subjected to a demand $p_{\rm dem}(t)$, given by

$$p_{\rm dem}(t) = \begin{cases} \displaystyle\sum_{i=1}^{n} p_{{\rm avl},i}(t), & t < t_c \\ (1+\gamma)\displaystyle\sum_{i=1}^{n} p_{{\rm avl},i}(t), & t \geq t_c \end{cases}, \tag{2}$$

where $\gamma > 0$.

For $t < t_c$, the demand does not exceed the accumulated available power, and can thereby be tracked closely. For $t \geq t_c$, the farm is required to overproduce. As described, the farm can only overproduce in a limited time period. We define this overproduction period by

$$T_{\rm op}(\tau_{g,i},\tau_{b,i}) = \inf\left\{ t - t_c \;\middle|\; t > t_c,\ p_{\rm dem}(t) \neq \sum_{i=1}^{n} p_i(t) \right\},$$

with $p_i(t)$ defined as in (1), and demand as defined in (2).

As the demand can be met for all $t < t_c$, $T_{\rm op}$ is the time between t_c and the first following time instance, where the demand is no longer obeyed. As $p_i(t)$ depends on $\tau_{b,i}(t)$ and $\tau_{g,i}(t)$, so does $T_{\rm op}$.

Given the model (1), the task is to chose $\tau_{g,i}(t)$ and $\tau_{b,i}(t)$, $i = 1, \dots, n$, such as to maximize the overproduction period. This can be formulated as

$$\begin{aligned}
&\underset{\tau_{g,i},\tau_{b,i}}{\text{maximize}} && T_{\text{op}}(\tau_{g,i}, \tau_{b,i}) \\
&\text{subject to} && \dot{\omega}_i(t) = \frac{-B_i}{J_i}\omega_i(t) + \frac{1}{J_i}(-\tau_{g,i}(t) + \tau_w - \tau_{b,i}(t)) \\
& && p_i(t) = \overline{\omega}_i \tau_{g,i}(t) + \overline{\tau}_{g,i}\omega_i(t) - \overline{\omega}_i\overline{\tau}_{g,i} \\
& && \omega_i(t) \geq \omega_{\min}, \quad \tau_{g,i}(t) \geq 0, \quad \tau_{b,i}(t) \geq 0 \\
& && t \geq t_c,
\end{aligned} \tag{3}$$

with initial conditions $\omega_i(t_c) = \overline{\omega}_i$ and $\tau_{g,i}(t_c) = \overline{\tau}_{g,i}$, for $i \in \{1, \dots, n\}$. The first two lines of the constraints, describes the approximated dynamics of the model. The last line encompasses the mentioned practical constraints, where ω_{min} is the allowed lower limit for the rotational velocity. It should further be stressed that the cost function in Problem (3) implies the equality constraint

$$p_{\text{dem}}(t) = \sum_{i=1}^{n} p_i(t), \quad t_c \leq t < t_c + T_{\text{op}}.$$

We can solve this problem by reformulating it as a feasibility problem. This can be done by defining

$$\tilde{T}_{\text{op}} = T_{\text{op}}(\tau_{g,i}, \tau_{b,i}),$$

and afterwards, solve the feasibility problem

$$\begin{aligned}
&\text{find} && \tau_{g,i}(t), \tau_{b,i}(t) \\
&\text{subject to} && \dot{\omega}_i(t) = \frac{-B_i}{J_i}\omega_i(t) + \frac{1}{J_i}(-\tau_{g,i}(t) + \tau_w - \tau_{b,i}(t)) \\
& && p_{\text{dem}}(t) = \sum_{i=1}^{n}(\overline{\omega}_i \tau_{g,i}(t) + \overline{\tau}_{g,i}\omega_i(t) - \overline{\omega}_i\overline{\tau}_{g,i}) \\
& && \omega_i(t) \geq \omega_{\min}, \quad \tau_{g,i}(t) \geq 0, \quad \tau_{b,i}(t) \geq 0, \\
& && 0 < t < t_c + \tilde{T}_{\text{op}},
\end{aligned} \tag{4}$$

with $i = 1, \dots, n$.

By iteratively increasing $\tilde{T}_{\text{op}}$ until (4) becomes infeasible, we find the solution to (3). An abstraction of this is presented in Fig. 3.

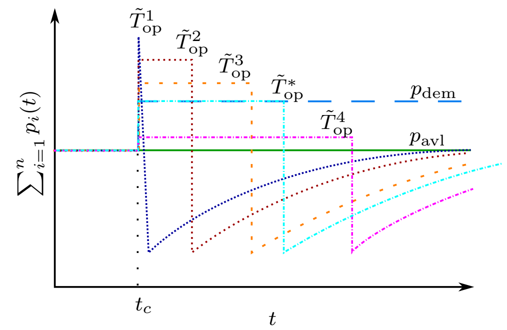

Fig. 3. Illustration of the feasibility problem, for four values of $\tilde{T}_{\text{op}}$ (short dashed), the demand p_{dem} (long dash), and the available power p_{avl} (solid).

Fig. 3 illustrates how large a power production a farm is able to maintain, if it is only required to maintain it for $\tilde{T}^1_{\text{op}}$, $\tilde{T}^2_{\text{op}}$, $\tilde{T}^3_{\text{op}}$ and $\tilde{T}^4_{\text{op}}$ seconds. For $\tilde{T}^1_{\text{op}}$, which is a very small period, the farm is capable of producing a very large power before the energy stored in the rotor is depleted. This power spike is far larger than the demand. For $\tilde{T}^2_{\text{op}} > \tilde{T}^1_{\text{op}}$, the farm can produce less power, but still more than the demand. Similarly for $\tilde{T}^3_{\text{op}}$. However, $\tilde{T}^4_{\text{op}}$ is too large, in that the farm is not able to obey the demand for the required overproduction period. By iterating, we can eventually find the value $\tilde{T}^*_{\text{op}}$, where $\tilde{T}^3_{\text{op}} < \tilde{T}^*_{\text{op}} < \tilde{T}^4_{\text{op}}$, such that the demand is exactly met, throughout the period.

For any power reference, not exceeding the rated power of the turbine, there exists a feasible solution to (4). In fact, when disregard the concern that the production from each turbine is limited above by a maximum rated power, it can be shown that for all $\gamma > 0$ in (2), there exists a $\tilde{T}_{\text{op}} > 0$, such that (4) is feasible. It will thereby always be possible to find a feasible overproduction period. For any value of $\tilde{T}_{\text{op}}$, (4) is convex, and can thereby be solved efficiently (Boyd and Vandenberghe (2004)).

In both (3) and (4), we have assumed that τ_w is constant over time, and equal for all turbines. We maintain this assumption throughout the work. Even though this to some extend reduces the practical accuracy of our results, we maintain this assumption in order to easier illustrate the implications of our results. Refer to Section 6 for arguments on improving this assumption.

The approach outlined above is completely general, in that it makes no assumptions on how large part any turbine should play in the overall overproduction, or when any individual turbine should start to overproduce, with respect to t_c. Instead it simply finds the torques that should be applied to each turbine over time, in order for the farm as a whole to obtain the longest overproduction period.

4. ROBUST OVERPRODUCTION STRATEGY

The general problem formulation in Section 3 suffers from the disadvantage of requiring information about the parameter values for all turbines in our wind farm model. In the following, we will present an alternative formulation, that is less general, however, it is developed to be robust against parametric uncertainties.

We shall employ a two step approach for arranging a robust overproduction strategy:

1. Find a lower bound on the overproduction period, under parametric uncertainties.
2. Arrange adaptive optimization scheme, for obtaining this lower bound, given the same uncertainties.

In Item 1, we find a guaranteed overproduction period, even under parametric uncertainties. It can be translated to a worst case overproduction period. In a practical setting, this worst case period would give a farm operator a guarantee for the period in which a given farm can participate in the grid stabilization, even when the operator is uncertain about specific parameter values.

In Item 2 we adaptively find the torques that should be applied each turbine, in order for the farm to overproduce for a time corresponding to the lower bound on overproduction period. In this implementation, we would never attempt to obtain an overproduction period exceeding the lower bound, in that we cannot guarantee that it is possible.

4.1 Overproduction Period Bound

Dispatch Strategy

Instead of the completely general problem outlined in (4), we can decide on the production of each turbine, based on a dispatch strategy. A dispatch strategy calculates power references $p_{\text{ref},i}(t) \in \mathbf{R}_+, i = 1, \dots, n$, to each turbine. We assume that the electrical dynamics of the turbine generator, are much faster than the mechanical dynamics of the rotor, so we can assume that $p_i(t) = p_{\text{ref},i}(t)$, if $p_{\text{ref},i}(t) < p_{\text{avl},i}(t)$.

In the following, we use the dispatch strategy

$$p_{\text{ref},i}(t) = p_{\text{avl},i}(t) + \frac{p_{\text{dem}}(t) - \sum_{i=1}^{n} p_{\text{avl},i}(t)}{n}, \tag{5}$$

If $p_{\text{dem}}(t) \leq \sum_{i=1}^{n} p_{\text{avl},i}(t)$, the references dispatched by (5) will all be less than the available power for the individual turbines, *i.e.* $p_{\text{ref},i}(t) \leq p_{\text{avl},i}(t)$, and the production will meet the demand. If however $p_{\text{dem}}(t) > \sum_{i=1}^{n} p_{\text{avl},i}(t)$, then $p_{\text{ref},i}(t) > p_{\text{avl},i}(t)$, and the turbine will only be able to follow this reference for a limited time, as the rotor will slow down.

If we employ (5), for the demand in (2), we see that the production reference to all turbines can be expressed as

$$p_{\text{ref},i}(t) = \begin{cases} p_{\text{avl},i}, & t < t_c \\ p_{\text{avl},i} + \gamma \dfrac{\sum_{j=1}^{n} p_{\text{avl},j}}{n}, & t \geq t_c \end{cases},$$

where we have omitted the time dependency on $p_{\text{avl}}(t)$, since the assumption on constant wind, entails constant available power.

Since the demand is constant for $t > t_c$, this entails that the references will be constant for $t > t_c$, *i.e.*

$$p_{\text{ref},i} = p_{\text{avl},i} + \gamma \frac{\sum_{j=1}^{n} p_{\text{avl},j}}{n}, \quad t > t_c$$

for $i = 1, \dots, n$, where $p_{\text{ref},i} > p_{\text{avl},i}$. When tracking this power reference, the model in (1) is expressed by

$$\begin{aligned} J_i \dot{\omega}_i(t) &= \text{-} B_i \omega_i(t) - \tau_{g,i}(t) + \tau_w \\ p_{\text{ref},i} &= \overline{\omega}_i \tau_{g,i}(t) + \overline{\tau}_{g,i} \omega_i(t) - \overline{\omega}_i \overline{\tau}_{g,i}, \end{aligned}$$

for $t > t_c$.

We can solve these differential equations explicitly for $\tau_{g,i}(t)$ and $\omega_i(t)$, for all i when $t > t_c$. We have disregarded the brake $\tau_{b,i}(t)$ as it should not be used when the power references are above available power. Solving the differential equations reveals

$$\tau_{g,i}(t) = \overline{d}_i + \overline{c}_i e^{\frac{\overline{\tau}_{g,i} - B_i \overline{\omega}_i}{J_i \overline{\omega}_i}(t - t_c)}, \quad \omega_i(t) = \overline{a}_i + \overline{b}_i e^{\frac{\overline{\tau}_{g,i} - B_i \overline{\omega}_i}{J_i \overline{\omega}_i}(t - t_c)}, \tag{6}$$

for $t > t_c$, where

$$\begin{aligned} \overline{a}_i &= \frac{B_i p_{\text{ref},i} + B_i \overline{\omega}_i \overline{\tau}_{g,i} - \overline{\tau}_{g,i} \tau_w}{B_i \overline{\tau}_{g,i} - B_i^2 \overline{\omega}_i} + \frac{\tau_w}{B_i} \\ \overline{b}_i &= \frac{\tau_w \overline{\tau}_{g,i} \overline{\omega}_i - B_i p_{\text{ref},i} \overline{\omega}_i - B_i \overline{\omega}_i^2 \overline{\tau}_{g,i}}{\overline{\tau}_{g,i}^2 - B_i \overline{\omega}_i \overline{\tau}_{g,i}} - \frac{p_{\text{ref},i}}{\overline{\tau}_{g,i}} \\ \overline{c}_i &= \frac{p_{\text{ref},i}}{\overline{\omega}_i} - \frac{\overline{\tau}_{g,i} \tau_w - B_i p_{\text{ref},i} - B_i \overline{\omega}_i \overline{\tau}_{g,i}}{\overline{\tau}_{g,i} - B_i \overline{\omega}_i} \\ \overline{d}_i &= \frac{\overline{\tau}_{g,i} \tau_w - B_i p_{\text{ref},i} - B_i \overline{\omega}_i \overline{\tau}_{g,i}}{\overline{\tau}_{g,i} - B_i \overline{\omega}_i} \end{aligned}$$

are all constants, with $\overline{a}_i > 0$, $\overline{b}_i < 0$, $\overline{c}_i > 0$, $\overline{d}_i > 0$, and

$$\frac{\overline{\tau}_{g,i} - B_i \overline{\omega}_i}{J_i \overline{\omega}_i} > 0.$$

From this, we can derive an expression for the time interval $T_{\text{op},i}$, where $p_i(t) = p_{\text{ref},i}$, before $\omega_i(t)$ reaches the lower bound $\omega_{\min}$. We obtain

$$T_{\text{op},i} = \frac{\overline{\omega}_i J_i}{\overline{\tau}_{g,i} - B_i \overline{\omega}_i} \ln \left(\frac{\omega_{\min} - \overline{a}_i}{\overline{b}_i} \right) \tag{7}$$

Employing the dispatch strategy (5) to obtain (7) can obviously only be suboptimal, compared to solving the general problem in (4). However, as the task is merely to provide a robust lower bound on the overproduction period, this approach is more suitable, as we shall illustrate shortly. We have further remarks concerning this in Section 6.

Equation (7) represents a mapping between the possible overproduction period, and the number of turbines in the farm. We will illustrate this with an example shortly.

Parametric Uncertainty

As seen from (7), both B_i and J_i enters explicitly in the expression for $T_{\text{op},i}$. When analyzing the gradient of (7) with respect to these model parameters, it is clear that

$$\frac{\partial T_{\text{op},i}(J_i, B_i)}{\partial J_i} > 0, \quad \forall J_i, B_i > 0,$$

and it can further be shown that

$$\frac{\partial T_{\text{op},i}(J_i, B_i)}{\partial B_i} > 0, \quad \forall J_i, B_i > 0.$$

This implies that over all possible parametrizations of B_i and J_i, the smallest possible value for $T_{\text{op},i}$ is obtained for $B_i = B_{\min}$ and $J_i = J_{\min}$. This entails, that even when the specific parameter values for the parameters across our farm model are unknown, a lower bound on the over production period can still be obtained, by analyzing a single vertice of our uncertainty set, when given as ranges on model parameters.

We can illustrate the implications of this, by the following simple example.

Example

We consider a farm consisting of n identical turbines, meaning that

$$J_1 = \cdots = J_n = J, \quad B_1 = \cdots = B_n = B,$$

which entails that all turbines have the same available power, $p_{\text{avl},i} = p_{\text{avl}}$, $i = 1, \dots, n$. The power demand during the overproduction period is given by

$$p_{\text{dem}} = \left(n + \frac{1}{2} \right) p_{\text{avl}},$$

corresponding to $\gamma = 1/(2n)$ in (2). This demand corresponds to the farm overall should produce the available power for all turbines, plus half that of one additional turbine. We define the available power as

$$p_{\text{avl}} = \overline{\tau}_g \overline{\omega},$$

where $\overline{\tau}_g = \tau_w - B\overline{\omega}$. The available power, and thereby the demand, depends on the turbine parameters

From the dispatch strategy (5), the production from each turbine during overproduction is

$$p_{\text{ref},i} = \left(1 + \frac{1}{2n} \right) p_{\text{avl}}, \quad i = 1, \dots, n. \tag{8}$$

The overproduction with respect to the available power for each individual turbine, is thereby inversely proportional to the number of turbines in the farm.

Using (7), Fig. 4 illustrates the mapping between n and T_{op}, for $n = 1, \ldots, 1000$. The figure presents four curves, corresponding to all four combinations of a maximum and minimum value of B and J. This corresponds to examining the four vertices of a parametric uncertainty region, where we do not know the specific value of B and J, but only their ranges, *i.e.*

$$B \in \mathcal{B} = [B_{\min};\ B_{\max}], \quad J \in \mathcal{J} = [J_{\min};\ J_{\max}]. \tag{9}$$

In Fig. 4, we have used $\mathcal{B} = [1; 4]$ kg $\cdot$ m^2/s, and $\mathcal{J} = [80; 120]$ kg $\cdot$ m^2. We have further used $\overline{\omega} = 4\pi$ rad/s, $\tau_w = 201$ Nm and $\omega_{\min} = 0.7\,\overline{\omega}$ rad/s.

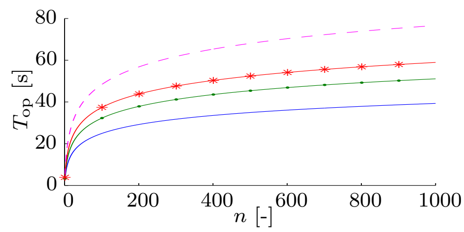

Fig. 4. The overproduction period, as a function of farm size. All turbines have the same inertia and friction, with the four combinations: $\{B_{\min}; J_{\min}\}$(Solid, Plain), $\{B_{\max}; J_{\min}\}$(Solid, Dots), $\{B_{\min}; J_{\max}\}$(Solid, Asterisk), $\{B_{\max}; J_{\max}\}$(Dashed).

Fig. 4 reveals several interesting results. First, the gain in T_{op}, obtained by increasing the number of turbines n, decreases as the farm size increases. This happens even though the overproduction performed by each individual turbine decreases, as evident by (8). This is on account of the negative exponential growth in (6).

Secondly, the specific value of the model parameters have significant impact on the overproduction period. However, as argued above, the lowest value of T_{op} is obtained for $B_i = B_{\min}$ and $J_i = J_{\min}$, $i = 1, \ldots, n$, for all n.

4.2 Robust Employment

The vertice analysis described above, gives a lower bound on the overproduction time. We now arrange a robust control strategy, capable of obtaining this lower bound, without knowing the parameters of the individual turbines. That is we only know $\mathcal{B}$ and $\mathcal{J}$ as given by (9). Further, we assume to know the available power for the farm, albeit not necessarily for the individual turbines. The demand is expressed similar to earlier, by (2).

Our approach can be expressed by the following list, which we elaborate below,

1. Assume that all turbine parameters are equal, and $B_i = B_{\min}$ and $J_i = J_{\min}$, $\forall\, i$.
2. Estimate available power for all turbines.
3. Find optimal control input to the estimated farm model.
4. Apply control signal, obtain measurements.
5. Reestimate parameters for all turbine models.
6. Continue from 2.

The first step in the list above is to assume that all turbines in the farm, are located in the same corner of the uncertainty set. This gives us a point of origin, which we can update during runtime.

The available power is estimated as

$$p^{\text{est}}_{\text{avl},i} = \overline{\tau}_{g,i}\overline{\omega}_i, \tag{10}$$

where $\overline{\tau}_{g,i}$ can be calculated as

$$\overline{\tau}_{g,i} = \tau_w - B^{\text{est}}_i\overline{\omega}_i, \tag{11}$$

B^{est}_i being the estimated friction coefficient, and $\overline{\omega}_i$ is the desired stationary rotational velocity of the rotor. B^{est}_i is initially estimated as $B_{\min}$, as given by Step 1. Afterwards, it is estimated based on measurements as described below.

Given our current estimate of the model parameters B^{est}_i and J^{est}_i, we want to find the optimal control input for our turbines to follow their references closest possible. The references are arranged using the dispatch strategy (5), using the estimated available powers.

We solve the optimization in a discretized fashion, employing a model predictive strategy (Maciejowski (2000)), for a horizon of H steps. At discrete time k, we solve the problem

$$\begin{array}{ll}\underset{\tau_{g,i},\tau_{b,i}}{\text{minimize}} & \displaystyle\sum_{j=0}^{H-1}\sum_{i=1}^{n}\Big(\lambda(p_{\text{ref},i}(k+j) - p_i(k+j))^2 \\ & \qquad +\tau_{g,i}(k+j)^2 + \tau_{b,i}(k+j)^2\Big) \\ \text{subject to} & \omega_i(k+1) = \phi^{\text{est}}_i(k)\omega_i(k) + \\ & \qquad \psi^{\text{est}}_i(k)(\text{-}\tau_{g,i}(k) - \tau_{b,i}(k) + \tau_w) \\ & p_i(k) = \overline{\omega}_i\tau_{g,i}(k) + \overline{\tau}_{g,i}\omega_i(k) - \overline{\omega}_i\overline{\tau}_{g,i} \\ & \omega_i(k) > \omega_{\min} \\ & \tau_{g,i}(k) > 0,\end{array} \tag{12}$$

for $i = 1, \ldots, n$ and $j = 0, \ldots, H-1$, where $\lambda \in \mathbf{R}_+$ is a trade-off parameter, and $\phi^{\text{est}}_i(k)$ and $\psi^{\text{est}}_i(k)$ are the discretized, current estimate of the coefficients in the dynamic equations:

$$\phi^{\text{est}}_i = 1 - \frac{B^{\text{est}}_i}{J^{\text{est}}_i}T_s, \quad \psi^{\text{est}}_i = \frac{1}{J^{\text{est}}_i}T_s.$$

Here we have used zero-order-hold for discretization, with a sample time of T_s.

The cost function employed above minimizes both the deviation from the reference, and the use of the two actuators.

We use a receding horizon strategy, and only apply the first sample of $\tau_{b,i}(k)$ and $\tau_{g,i}(k)$. From this we obtain a measurement of $\omega_i(k+1)$. We can use this to reestimate the model parameters via the least squares problem

$$\begin{array}{ll}\underset{x}{\text{minimize}} & \|F(k)x(k) - g(k)\|_2 \\ \text{subject to} & \phi^{\text{est}}_i(k) \in \Phi \\ & \psi^{\text{est}}_i(k) \in \Psi,\end{array} \tag{13}$$

with variable $x(k) = [\psi^{\text{est}}_i(k)\ \phi^{\text{est}}_i(k)]$, and

$$F(k) = \begin{bmatrix} \omega_i(k) & \tau_{g,i}(k) + \tau_{b,i}(k) - \tau_w \\ \omega_i(k-1) & \tau_{g,i}(k-1) + \tau_{b,i}(k-1) - \tau_w \\ \vdots & \vdots \\ \omega_i(1) & \tau_{g,i}(1) + \tau_{b,i}(1) - \tau_w \end{bmatrix},$$

$$g(k) = \begin{bmatrix} \omega_i(k+1) \\ \omega_i(k) \\ \vdots \\ \omega_i(2) \end{bmatrix}.$$

Above, Ψ and Φ represents the uncertainty bounds on the discretized system matrices,

$$\Phi = \left[\left(1 - \frac{B_{\max}}{J_{\min}}T_s\right); \left(1 - \frac{B_{\min}}{J_{\max}}T_s\right)\right], \quad \Psi = \left[\frac{T_s}{J_{\max}}; \frac{T_s}{J_{\min}}\right]$$

After updating the model, we can again solve (12), apply first control sample, obtain measurement, reestimate model *etc.*

It should be noted, that during runtime, we alter the optimization horizon H. In normal operation where $p_{\text{ref},i}(k) = p_{\text{avl},i}(k)$, we use a quite large value for H. However, in an overproduction situation, where $p_{\text{ref},i}(k) > p_{\text{avl},i}(k)$, we set $H = 1$, *i.e.*, we effectively remove the prediction, in our optimization. If we had continued with a long horizon, our optimization algorithm would have completely omitted to track the increased demand, in order to avoid the underproduction period illustrated in Fig. 1. Our interest is instead to track the overproduction for as long a period as possible.

The following section presents a numerical example on both obtaining the lower bound, as well as arranging the robust employment.

5. NUMERICAL EXAMPLE

Consider a farm of $n = 10$ turbines, whose parameter values are distributed uniformly between a lower and upper value. We do not know their specific parameter values, however we do know their ranges:

$$B_i \in [1;\ 4], \quad J_i \in [80;\ 120], \quad i = 1, \ldots, n.$$

We initially want to find a lower bound on the overproduction period, and afterwards use our adaptive optimization scheme to obtain it.

5.1 Overproduction Bound

As explained in Section 4.1, the worst case over production period, for a demand in the form (2) is obtained by assuming

$$B_i = B_{\min}, \quad J_i = J_{\min}, \quad i = 1, \ldots, n.$$

Employing (7), we can depict the lower bound on the overproduction for our farm, as a function of γ. This is presented in Fig. 5

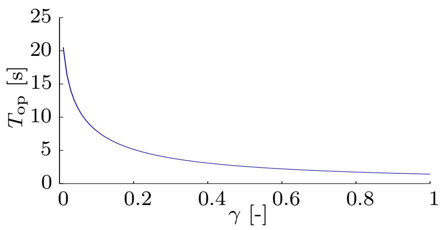

Fig. 5. The lower bound, overproduction period as a function of overproduction.

5.2 Adaptive Optimization

We implement our adaptive optimization for a case where we know the available power of the farm, albeit not of the individual turbines. Denote this available power for the farm, P_{avl}. The power demand for the farm is given by

$$p_{\text{dem}}(k) = \begin{cases} P_{\text{avl}}, & k < k_c \\ P_{\text{avl}} + \dfrac{1}{2n} P_{\text{avl}}, & k_c \leq k < k_c + T_{\text{op}} \\ P_{\text{avl}}, & k \geq k_c + T_{\text{op}} \end{cases}$$

meaning that $\gamma = 1/(2n) = 0.05$. From Fig. 5, this gives a lower bound of $T_{\text{op}} = 11$ s on the overproduction period, which we should be able to obtain. We therefore apply a power demand, with an 11 s overproduction period, and expect that this can be obeyed. We initiate the overproduction at time $k_c = 50$. After the overproduction period, we want all turbines to recover to their initial conditions, entailing a period of under production.

Following the procedure in Section 4.2, we obtain the results presented in Fig. 6 through Fig. 10. In this example we have used $\overline{\omega} = 4\pi$ rad/s, $\tau_w = 201$ Nm and $\omega_{\min} = 0.7\overline{\omega}$ rad/s, similar to the example in Section 4.1.

As each turbine model only contains two parameters, the model estimation process in Section 4.2 could in principle obtain a perfect model estimation after only two samples. To make the situation more realistic, we add noise to all measurements of the rotational velocity, such that at time k, we obtain

$$\omega_i(k+1) = \phi_i^{\text{est}} \omega_i(k) + \psi_i^{\text{est}} \left(-\tau_{g,i}(k) - \tau_{b,i}(k) + \tau_w \right) + \nu_i(k),$$

where $\nu_i(k)$ is zero-mean, normally distributed, random noise, with std. dev. σ. In the following we have chosen σ to be 1 % of the allowed range for $\omega(k)$.

In Fig. 6, the production from each turbine, as well as their references are shown, where the references are calculated by the dispatch strategy (5). Similarly, the demand tracking for the entire farm, is presented in Fig. 7

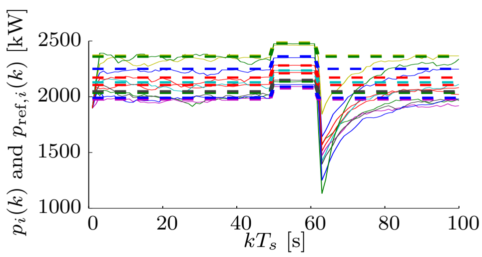

Fig. 6. The reference to each turbine (Dashed), and the actual production (Solid).

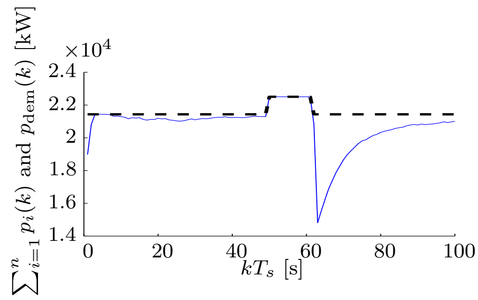

Fig. 7. The power demand (Dashed), and the farm production (Solid).

We see that the demand tracking is quite accurate in the overproduction period, despite the addition of noise. This suggests that the noise does not effect the model estimation, but rather effects the expected outcome throughout the horizon. Remember that in the overproduction period, we reduce the optimization horizon to $H = 1$, and by this, it appears that a small horizon limits the effect of the noise.

The rotational velocity is plotted in Fig. 8. As it appears, after the 11 s overproduction period, only 1 turbine is close to the lower limit $\omega_{\min} = 8.78$ rad/s. All the remaining turbines could in principle have continued to overproduce, which illustrates that the result from Section 5.1 is indeed only a lower bound.

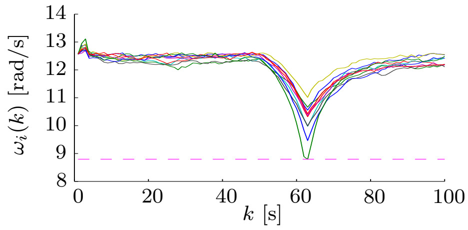

Fig. 8. The obtained rotational velocity of the turbines in the farm (Solid) and lower limit (Dashed). Only one turbine has depleted its energy storage after the overproduction period.

The applied torques are presented in Fig. 9. We see that there is an initial process of finding a steady operating point, which is on account of estimating the correct model.

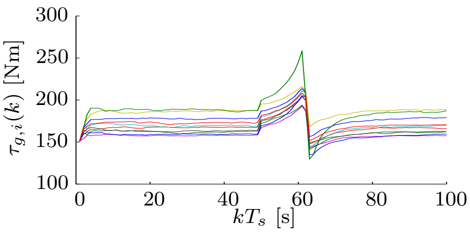

Fig. 9. The torque applied to each turbine in the farm.

In Fig. 10, we have plotted both the on-line, linear estimation of the produced power, as well as the true nonlinear production. We see that on account of continuously updating the operating point, the two are practically indistinguishable.

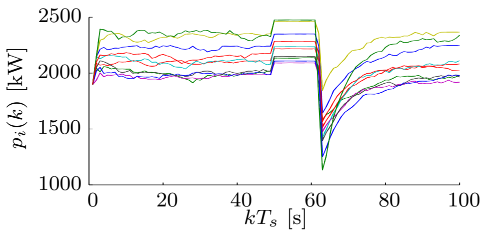

Fig. 10. Comparison between the linear approximation of the turbine power (Solid), and the actual produced power (Dashed).

6. CONCLUSION AND FURTHER WORK

In this work, we have presented an optimization-based strategy for employing the energy stored in the rotor of a wind turbine, for grid stabilization, as a period of overproduction. We have illustrated how the time an overproduction can be achieved, can be related to the size of a wind farm, and further how the maximum overproduction period can be found by solving a sequence of convex feasibility problems. Further we have demonstrated how a robust utilization of the stored energy can be arranged, when subjected to parametric uncertainties.

Throughout the design of the robust employment on the stored energy in the turbine rotors, we have employed the dispatching strategy presented in (5). Even though numerical results suggests it, we have at no point argued the optimality of this strategy, entailing that we have not argued that the lower bound we find for T_{op}, is the best lower bound. Instead we have illustrated how one might come about parametric uncertainties, in a way that ensures a worst case performance. Further work should be dedicated to either certifying optimality of the approach employed here, or arguing the optimality of a different strategy.

In the examples we have presented in this work, there is a drop in power production, immediately following the overproduction. This drop is quite significant compared to the original production level, and could cause further stability problems for the grid. The idea is here that if the overproduction period could be maintained for a sufficiently large period of time, there would be room for ramping up one or more thermal plants to cover the underproduction period. The benefit would be that wind turbines could be the primary reserve for handling grid instability, and thermal plants would then only be used secondarily, for covering the underproduction period. This would be an improvement to the current situation, where all reserves are maintained by thermal plants. This does however require some knowledge related to a guaranteed period of overproduction, which substantiates the relevance of this work.

This work have assumed constant and equal wind fields for all turbines, which will not be the case in practical situations. We have made this assumptions in order to better illustrate our approach and results. A remedy for this approach would be to include more accurate estimates of the wind fields in Equations (3), (4), and (12). We shall leave this for future work.

ACKNOWLEDGEMENTS

This work is supported by the Southern Denmark Growth Forum and the European Regional Development Fund, under the project "Smart & Cool".

The authors would like to thank Philip C. Kjær, Vestas, for constructive discussions.

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