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Fan Sun, Elisabeth de Carvalho, Chan Dai Truyen Thai, and Petar Popovski

Department of Electronic Systems, Faculty of Engineering and Science

Aalborg University, DK-9220 Aalborg, Denmark

Email: {fs,edc,ttc,petarp}@es.aau.dk

Abstract—**Joint processing of multiple communication flows in wireless systems has given rise to a number of novel transmission techniques, notably the two-way relaying based on wireless network coding. Recently, a related set of techniques has emerged, termed coordinated direct and relay (CDR) transmissions, where the constellation of traffic flows is more general than the two-way. Regardless of the actual traffic flows, in a CDR scheme the relay has a central role in managing the interference. In this paper we investigate the novel transmission modes, based on amplify-and-forward, that arise when the relay is equipped with multiple antennas and can use beamforming. We focus on one representative traffic type, with two downlink users and consider three different beamforming criteria: egoistic, altruistic, and sum-rate maximization. The sum-rate criterion leads to a non-convex problem and we introduce an iterative solution, as well as derive an upper performance bound. The numerical results demonstrate a clear benefit from usage of multiple antennas at the relay node.**

Index Terms—**Cooperative transmission, amplify-and-forward, beamforming, a priori information.**

I. INTRODUCTION

Recently there have been extensive studies on cooperative, relay-based transmission schemes for extending cellular coverage or increasing diversity. Several basic relaying transmission techniques have been introduced, such as amplify-and-forward (AF) [1], decode-and-forward (DF) [2] and compress-and-forward (CF) [3].

These transmission techniques have been applied in one-, two- or multi-way relaying scenarios. There has been a particularly high interest in two-way relaying scenarios [4], [5], [6], where throughput gains have been demonstrated by utilizing the ideas of wireless network coding. The underlying principles behind wireless network coding are (1) to aggregate and jointly process multiple communication flows and (2) intentionally allow interference and simultaneous usage of the shared wireless medium, leaving to the receivers to remove the adverse impact of interference by using any side information. Leveraging on these principles, we have proposed schemes with non-regenerative AF relaying in [7], [8] that feature more general traffic patterns compared to the two-way relaying. These schemes are termed *coordinated direct/relay* (CDR) transmissions. The CDR transmission considers scenarios where direct and relayed users (UEs) are served in uplink/downlink. The relayed UE is assumed to have no direct link to the base station (BS) due to large path loss and relies only on the amplified/forwarded signal from the relay in order to decode the signal from the BS. Transmission schemes that

are related to some of the schemes have appeared before in [9], [10], [11].

In the works that deal with CDR transmission, the relay has a central role in managing the interference. Therefore, in this work we investigate the qualitative changes and the performance improvements that arise when the relay node in the CDR schemes is equipped with multiple antennas.

We focus on one representative traffic type, with two downlink UEs. The principle can be extended to other three traffic configurations: one direct uplink UE and one relayed downlink UE; one direct downlink UE and one relayed uplink UE; two uplink UEs. In the scheme on Fig. 1, assume for example that the BS has one symbol to send to a relayed UE, while a direct user wants to receive another symbol from the BS. In a conventional cellular system, these symbols are sent over two separate downlink phases. Instead in the CDR system, the BS may first send the symbol which is received at the relay in phase 1. The relay transmits to the relayed UE and simultaneously the BS sends another symbol to the direct UE in phase 2. Enabling such simultaneous transmissions improves the *spectral efficiency* compared to the conventional method. The key point is that the direct UE can use the overheard information: the direct UE knows the interference *a priori* in phase 1, which helps to decode the desired symbol in phase 2.

Differently from the previous works, the usage of multiple antennas at the relay permits to manage the interference through beamforming. This is a significant conceptual change from the original CDR schemes, while the usage of multiple antennas at the BS and the terminals is clearly identified as a future extension. We consider AF operation at the relay, assuming that the relay and the other nodes have a perfect channel state information (CSI). We propose three options for coordinated relay beamforming aiming at maximizing the rate of the relayed UE (egoism), the rate of the direct UE (altruism), and the sum-rate:

- The egoistic relay beamforming creates severe interference to the direct UE.
- The altruistic relay beamforming focuses the transmission towards the relayed user while no interference is generated to the direct user. This creates two orthogonal downlink transmissions for individual UEs, which is an important advance compared to the CDR scheme with single-antenna relay.

- In sum-rate relay beamforming, we can improve the overall system performance by allowing the relay to create the interference to the direct UE but later on utilize the fact that the direct UE has overheard the information intended for the other UE. Beamforming here balances between egoism and altruism targets maximizing the overall sum-rate. The sum-rate maximization is achieved by a low-complexity iterative algorithm and an upper bound of the sum-rate is characterized. Simulation results confirm that the iterative design gives close performance to the upper bound.

Notation: We use uppercase and lowercase boldface letters to represent matrices and vectors, respectively. $\otimes$ refers to the Kronecker product and $||\cdot||_F^2$ denotes the Frobenius norm of a matrix. $\mathbb{I}$ is the identity matrix.

II. System Overview

The basic setup is a downlink scenario with one BS, one relay, and two UEs, see Fig. 1. The relay is equipped with M antennas. The BS and the UEs are equipped with one antenna only. The transmission from the BS to the relay has the same duration with the transmission from the relay to the direct UE.

We consider the multi-antenna relay beamforming design in the CDR transmission in Fig. 1 where the BS delivers x_1 to the relayed UE (UE 1) and x_2 to the direct UE (UE 2) in two phases. In the first slot, the BS transmits x_1 to the relay; the relay delivers it to UE 1 and the BS transmits x_2 to UE 2 simultaneously in the second slot. The elements of each channel are assumed to be independent complex Gaussian random variables with zero mean and unit variance. All links are assumed to be static within the two slots and use the same frequency band.
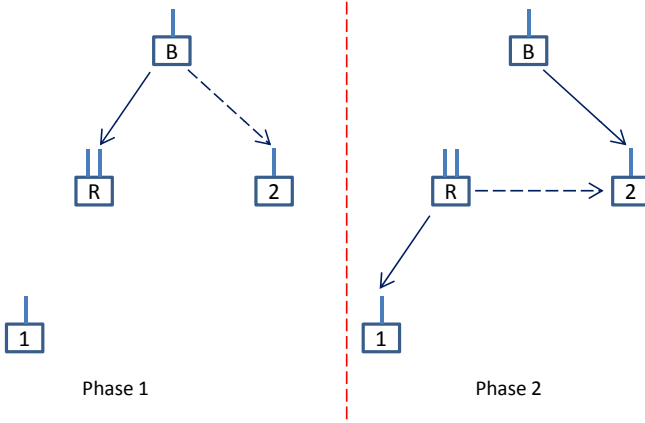

Figure 1. CDR MIMO Downlink System Model.

The received signals at the relay and UE 2 in the first slot are

$$
\begin{aligned}
\mathbf{y}_R[1] &= \mathbf{h}_{RB}x_1 + \mathbf{n}_R \\
y_2[1] &= h_{2B}x_1 + n_2[1]
\end{aligned}
\tag{1}
$$

where $\mathbf{n}_R$ is the complex white Gaussian noise vector at the relay with the covariance matrix $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^{\mathrm{H}}] = \mathbb{I}$ and $n_2[1]$ is the complex white Gaussian noise at UE 2 in the first slot with unit variance[1]. The received signals at UE 1 and UE 2 in the second slot are

$$
\begin{aligned}
y_1[2] &= \mathbf{h}_{1R}\mathbf{x}_R + n_1[2] \\
y_2[2] &= h_{2B}x_2 + \mathbf{h}_{2R}\mathbf{x}_R + n_2[2]
\end{aligned}
\tag{2}
$$

where the signal vectors transmitted from the relay is in the form $\mathbf{x}_R = \mathbf{W}\mathbf{y}_R[1]$ with $\mathbf{W}$ being the $M \times M$ relay beamforming matrix. $n_1[2]$ and $n_2[2]$ are the complex white Gaussian noise variables with unit variance each at UE 1 and UE 2 respectively. Assume P to be the transmit power of the BS, then $\mathbb{E}[|x_1|^2] = \mathbb{E}[|x_2|^2] = P$. The relay transmit power[2] is

$$
\begin{aligned}
\mathbb{E}[\mathbf{x}_R^{\mathrm{H}}\mathbf{x}_R] &= \mathrm{Tr}(P\mathbf{W}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}} + \mathbf{W}\mathbf{W}^{\mathrm{H}}) \\
&= P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{RB} + ||\mathbf{W}||_F^2 = P_R.
\end{aligned}
$$

III. Optimization Criteria for Coordinated Relay Beamforming Design

In this system, the relay is deployed to help the relayed UE which has no direct link to the BS. We propose three options for coordinated relay beamforming:

- The egoistic relay beamforming maximizes the rate of the relayed UE.
- The altruistic relay beamforming aims at rate maximization of the direct UE.
- The relay can balance between altruism and egoism by sum-rate maximization of both UEs.

We first take a look at the SNR and signal to interference plus noise ratio (SINR) expression for both UEs and then form the optimization problems for the three options, respectively.

For UE 1, starting from Eqn. (2), we will have $y_1[2] = \mathbf{h}_{1R}\mathbf{W}\mathbf{h}_{RB}x_1 + \mathbf{h}_{1R}\mathbf{W}\mathbf{n}_R + n_1[2]$. Then SNR_1 for UE 1 is formed as

$$
\mathrm{SNR}_1 = \frac{P\mathbf{h}_{1R}\mathbf{W}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{1R}^{\mathrm{H}}}{\mathbf{h}_{1R}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{1R}^{\mathrm{H}} + 1}.
\tag{3}
$$

Meanwhile, the direct UE uses the overheard a priori information $y_2[1]$ from the first slot. The received signals at UE 2 in the two slots are grouped in the received signal vector $\mathbf{y}_2$ in Eqn. (4). We use minimum mean square error (MMSE) receiver [12] to estimate x_2 from $\mathbf{y}_2$. The corresponding SINR for UE 2 is in Eqn. (5).

A. Problem formulation

With the SNR and SINR expression for both UEs, we can obtain the following optimization problems. The sum-rate expression is $R_{\mathrm{sum}} = \frac{1}{2}\log_2(1 + \mathrm{SNR}_1) + \frac{1}{2}\log_2(1 + \mathrm{SINR}_2) = \frac{1}{2}\log_2[(1 + \mathrm{SNR}_1)(1 + \mathrm{SINR}_2)]$.

---

[1] In this work, we assume the variance of each noise component is normalized.

[2] Relay full power transmission is not necessarily the optimal strategy and relay power optimization is identified as a future task.

$$\mathbf{y}_2 = \begin{bmatrix} y_2[1] \\ y_2[2] \end{bmatrix} = \begin{bmatrix} 0 \\ h_{2B} \end{bmatrix} x_2 + \begin{bmatrix} h_{2B} \\ \mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB} \end{bmatrix} x_1 + \begin{bmatrix} n_2[1] \\ \mathbf{h}_{2R}\mathbf{W}\mathbf{n}_R + n_2[2] \end{bmatrix} = \mathbf{h}_2 x_2 + \mathbf{n}_2 \tag{4}$$

$$\mathrm{SINR}_2 = \mathbf{h}_2^{\mathrm{H}}\{\mathbb{E}[\mathbf{n}_2\mathbf{n}_2^{\mathrm{H}}]\}^{-1}\mathbf{h}_2 = \frac{P^2|h_{2B}|^4 + P|h_{2B}|^2}{P\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{2R}^{\mathrm{H}} + (P|h_{2B}|^2 + 1)(\mathbf{h}_{2R}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{2R}^{\mathrm{H}} + 1)} \tag{5}$$

Problem 1 (P1). Relayed UE rate maximization (egoistic beamforming):

$$\max_{\mathbf{W}} \quad \mathrm{SNR}_1$$
$$s.t. \quad P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{RB} + ||\mathbf{W}||_F^2 = P_R. \tag{6}$$

because rate maximization is equivalent to SNR maximization.

Problem 2 (P2). Direct UE rate maximization (altruistic beamforming):

$$\max_{\mathbf{W}} \quad \mathrm{SINR}_2$$
$$s.t. \quad P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{RB} + ||\mathbf{W}||_F^2 = P_R. \tag{7}$$

which has also been transformed into SINR maximization.

Problem 3 (P3). The sum-rate maximization:

$$\max_{\mathbf{W}} \quad (1 + \mathrm{SNR}_1)(1 + \mathrm{SINR}_2)$$
$$s.t. \quad P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{RB} + ||\mathbf{W}||_F^2 = P_R. \tag{8}$$

IV. OPTIMIZATION METHOD

A. Individual Rate Maximization

We first focus on P1 and P2. In order to rewrite the optimization cost functions in a simple way, the beamforming matrix $\mathbf{W}$ is converted into a vector form using the vectorization operation, $\mathbf{w} = \mathrm{vec}(\mathbf{W})$. With the property $\mathrm{vec}(\mathbf{AWB}) = (\mathbf{B}^{\mathrm{T}} \otimes \mathbf{A})\mathrm{vec}(\mathbf{W})$, we can rewrite problems P1 and P2 in Eqn. (9) and (10).

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^{\mathrm{H}}P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})\mathbf{w}}{\mathbf{w}^{\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})\mathbf{w} + 1}$$
$$s.t. \quad \mathbf{w}^{\mathrm{H}}\left[P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I}) + \mathbb{I}\right]\mathbf{w} = P_R. \tag{9}$$

For the next step, we introduce matrix $\mathbf{J}$ from the Cholesky decomposition $P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I}) + \mathbb{I} \triangleq \mathbf{J}^{\mathrm{H}}\mathbf{J}$ and $\tilde{\mathbf{w}} = \mathbf{J}\mathbf{w}$. Therefore, P1 and P2 are reformulated in Eqn. (11) and (12).

$$\max_{\tilde{\mathbf{w}}} \quad \frac{\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{J}^{-\mathrm{H}}P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})\mathbf{J}^{-1}\tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^{\mathrm{H}}\left[\mathbf{J}^{-\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I}\right]\tilde{\mathbf{w}}}$$
$$s.t. \quad \tilde{\mathbf{w}}^{\mathrm{H}}\tilde{\mathbf{w}} = P_R. \tag{11}$$

Since Eqn. (11) is the generalized Rayleigh quotient, the optimal solution can be obtained as:

$$\tilde{\mathbf{w}} = \sqrt{P_R}\, v_{\max}\left\{\mathbf{G}^{-1}\mathbf{K}\right\}$$

where $v_{\max}\{\cdot\}$ denotes the eigenvector corresponding to the largest eigen-value, $\mathbf{G} = \mathbf{J}^{-\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbb{I} \otimes \mathbf{h}_{1R})\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I}$ and $\mathbf{K} = \mathbf{J}^{-\mathrm{H}}P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{1R})\mathbf{J}^{-1}$. The solution to P2 is also obtained via eigen-value decomposition.

Remark 1: It can be shown that the solution to P2 aims at interference nulling to the direct UE in the second slot. The SINR maximization of the direct UE is achieved by interference-free transmission in the second slot. The overheard a priori information does not help. Therefore, P1 involves two orthogonal downlink transmissions for the relayed UE and the direct UE. The altruistic beamforming creates orthogonality in space.

B. Sum-rate Maximization

With Eqn. (11) and (12), P3 can be rewritten in a similar way using the vectorization operation in Eqn. (13) where $P(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I}) + \mathbb{I} \triangleq \mathbf{J}^{\mathrm{H}}\mathbf{J}$ and $\tilde{\mathbf{w}} = \mathbf{J}\mathbf{w}$.

P3 is a non-convex problem, where global optimum solution is difficult to obtain within reasonable computation time. This optimization problem has generally no closed form solution. Well-known iterative methods can be applied such as simulated annealing and genetic algorithms which require very high computational load. It can be easily solved by the following iterative algorithm.

From Eqn. (13), we observe that the norm of $\tilde{\mathbf{w}}$ does not influence the maximization at all. Hence, the constraint can be ignored. This transforms Eqn. (13) into an unconstrained maximization problem. We propose an algorithm that attempts to obtain a solution to the Karush-Kuhn-Tucker (KKT) conditions. Denote the problem to be $\max_{\tilde{\mathbf{w}}} R_{\mathrm{sum}}(\tilde{\mathbf{w}})$ with $R_{\mathrm{sum}}(\tilde{\mathbf{w}}) = (\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{w}}\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{C}\tilde{\mathbf{w}})/(\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{B}\tilde{\mathbf{w}}\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{D}\tilde{\mathbf{w}})$ representing the cost function in Eqn. (13), the first order necessary condition is $\frac{\partial R_{\mathrm{sum}}(\tilde{\mathbf{w}})}{\partial \tilde{\mathbf{w}}} = 0$. This leads to

$$R_{\mathrm{sum}}(\tilde{\mathbf{w}})\left[\left(\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{B}\tilde{\mathbf{w}}\right)\mathbf{D} + \left(\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{D}\tilde{\mathbf{w}}\right)\mathbf{B}\right]\tilde{\mathbf{w}}$$
$$= \left[\left(\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{C}\tilde{\mathbf{w}}\right)\mathbf{A} + \left(\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{A}\tilde{\mathbf{w}}\right)\mathbf{C}\right]\tilde{\mathbf{w}}$$

which can be rewritten as $R_{\mathrm{sum}}(\tilde{\mathbf{w}})\mathbf{V}(\tilde{\mathbf{w}})\tilde{\mathbf{w}} = \mathbf{R}(\tilde{\mathbf{w}})\tilde{\mathbf{w}}$. Notice $\mathbf{V}(\tilde{\mathbf{w}})$ and $\mathbf{R}(\tilde{\mathbf{w}})$ depend on the unknown $\tilde{\mathbf{w}}$. If the dependence could be removed, then the optimizer $\tilde{\mathbf{w}}$ is obviously the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{V}^{-1}\mathbf{R}$. However, eigen-value decomposition of the matrix $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1}\mathbf{R}(\tilde{\mathbf{w}})$ can not be accomplished in closed form. Consequently, we propose an iterative algorithm

$$\max_{\mathbf{w}} \quad \frac{P^2|h_{2B}|^4 + P|h_{2B}|^2}{\mathbf{w}^{\mathrm{H}}\left[(P|h_{2B}|^2+1)(\mathbb{I}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{2R}) + P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})\right]\mathbf{w} + P|h_{2B}|^2 + 1}$$
$$s.t. \quad \mathbf{w}^{\mathrm{H}}\left[P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbb{I})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbb{I})+\mathbb{I}\right]\mathbf{w} \leq P_R. \tag{10}$$

$$\max_{\tilde{\mathbf{w}}} \quad \frac{P^2|h_{2B}|^4 + P|h_{2B}|^2}{\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{J}^{-\mathrm{H}}\left[(P|h_{2B}|^2+1)(\mathbb{I}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{2R}) + P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})\right]\mathbf{J}^{-1}\tilde{\mathbf{w}} + P|h_{2B}|^2 + 1}$$
$$s.t. \quad \tilde{\mathbf{w}}^{\mathrm{H}}\tilde{\mathbf{w}} = P_R. \tag{12}$$

$$\max_{\tilde{\mathbf{w}}} \quad \frac{\tilde{\mathbf{w}}^{\mathrm{H}}\left\{\mathbf{J}^{-\mathrm{H}}\left[(\mathbb{I}\otimes\mathbf{h}_{1R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{1R}) + P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{1R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{1R})\right]\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I}\right\}\tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^{\mathrm{H}}\left[\mathbf{J}^{-\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{1R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{1R})\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I}\right]\tilde{\mathbf{w}}} \times$$
$$\frac{\tilde{\mathbf{w}}^{\mathrm{H}}\left\{\mathbf{J}^{-\mathrm{H}}\left[(P|h_{2B}|^2+1)(\mathbb{I}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{2R}) + P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})\right]\mathbf{J}^{-1} + \frac{P^2|h_{2B}|^4+2P|h_{2B}|^2+1}{P_R}\mathbb{I}\right\}\tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^{\mathrm{H}}\left\{\mathbf{J}^{-\mathrm{H}}\left[(P|h_{2B}|^2+1)(\mathbb{I}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbb{I}\otimes\mathbf{h}_{2R}) + P(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})^{\mathrm{H}}(\mathbf{h}_{RB}^{\mathrm{T}}\otimes\mathbf{h}_{2R})\right]\mathbf{J}^{-1} + \frac{P|h_{2B}|^2+1}{P_R}\mathbb{I}\right\}\tilde{\mathbf{w}}}$$
$$s.t. \quad \tilde{\mathbf{w}}^{\mathrm{H}}\tilde{\mathbf{w}} = P_R. \tag{13}$$

described in Algorithm 1 which finds the principal eigenvector corresponding to the maximum eigenvalue in $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1}\mathbf{R}(\tilde{\mathbf{w}})$ iteratively. It comes from the power iteration idea in [13], [14].

Algorithm 1

Initialization: set $n=0$ and $\tilde{\mathbf{w}}^{(0)} = \tilde{\mathbf{w}}^{(\mathrm{init})}$
iterate
   update $n = n+1$
   1) $\mathbf{q}^{(n)} = \left[\mathbf{V}\left(\tilde{\mathbf{w}}^{(n)}\right)\right]^{-1} \times \left[\mathbf{R}\left(\tilde{\mathbf{w}}^{(n)}\right)\right]\tilde{\mathbf{w}}^{(n)}$
   2) $\tilde{\mathbf{w}}^{(n+1)} = \sqrt{P_R}\,\mathbf{q}^{(n)}/||\mathbf{q}^{(n)}||_2$
until $R_{\mathrm{sum}}(\tilde{\mathbf{w}}^{(n+1)})$ convergence

---

Since the optimization problem is non-convex, the proposed algorithm cannot guarantee convergence. The convergence behavior of the proposed algorithm is shown numerically. The sum-rate results versus the iteration number are plotted in Fig. 2. The sum-rate is averaged over a sufficient number of channel realizations when SNR equals to 10 dB and $P = P_R$. This example displays good convergence property of the algorithm: 30 iterations appear to be sufficient. Therefore, Algorithm 1 is seen to converge and provides a good sub-optimal solution to P3 with relatively low computational complexity.

C. Sum-Rate Upper Bound

In this section, we derive an upper bound on the sum-rate of the multi-antenna AF CDR system. The upper bound will be used to characterize the sum-rate loss resulting from Algorithm 1. [15] gives an upper bound of the sum-rate for the two-way multi-antenna AF relay system with single-antenna UEs. Following [15], we consider different relay beamforming
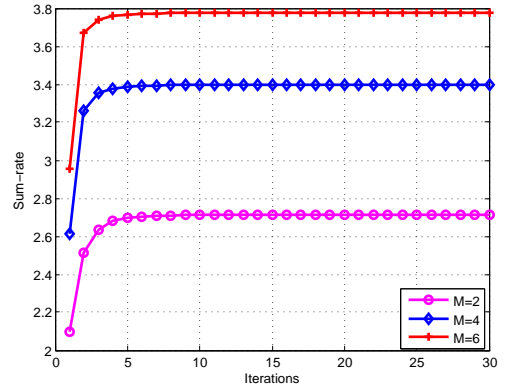

Figure 2. Convergence property according to the number of relay antennas.

matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are used for the link to the relayed UE and the link to the direct UE, independently. The total power of the relay is allocated to support both links of communication to maximize the total sum-rate. Then an upper bound on the sum-capacity is obtained from:

$$\max_{\mathbf{W}_1, \mathbf{W}_2} \quad \frac{1}{2}\log_2\left[1 + \mathrm{SNR}_1(\mathbf{W}_1)\right]$$
$$+ \frac{1}{2}\log_2\left[1 + \mathrm{SINR}_2(\mathbf{W}_2)\right]$$
$$s.t. \quad P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}_1^{\mathrm{H}}\mathbf{W}_1\mathbf{h}_{RB} + \kappa_1||\mathbf{W}_1||_F^2$$
$$+ P\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}_2^{\mathrm{H}}\mathbf{W}_2\mathbf{h}_{RB} + \kappa_2||\mathbf{W}_2||_F^2 = P_R \tag{14}$$

where $\mathrm{SNR}_1(\mathbf{W}_1)$ is a function of $\mathbf{W}_1$ and $\mathrm{SINR}_2(\mathbf{W}_2)$ is a function of $\mathbf{W}_2$. And κ_1 and κ_2 are non-negative and fulfilling

$\kappa_1 + \kappa_2 = 1$. The tightest upper bound based on Eqn. (14) is

$$R_{\mathrm{UB}} = \min_{\kappa_1 + \kappa_2 = 1} \max_{P_1 + P_2 = P_R} R_1(\kappa_1, P_1) + R_2(\kappa_2, P_2)$$

with

$$
\begin{aligned}
R_1(\kappa_1, P_1) &= \max_{\mathbf{W}_1} \frac{1}{2} \log_2 \left[1 + \mathrm{SNR}_1(\mathbf{W}_1) \right] \\
s.t. \quad & P\mathbf{h}_{RB}^{\mathrm{H}} \mathbf{W}_1^{\mathrm{H}} \mathbf{W}_1 \mathbf{h}_{RB} + \kappa_1 \|\mathbf{W}_1\|_F^2 \le P_1 \\
R_2(\kappa_2, P_2) &= \max_{\mathbf{W}_2} \frac{1}{2} \log_2 \left[1 + \mathrm{SINR}_2(\mathbf{W}_2) \right] \\
s.t. \quad & P\mathbf{h}_{RB}^{\mathrm{H}} \mathbf{W}_2^{\mathrm{H}} \mathbf{W}_2 \mathbf{h}_{RB} + \kappa_2 \|\mathbf{W}_2\|_F^2 \le P_2.
\end{aligned}
$$

where $R_1(\kappa_1, P_1)$ and $R_2(\kappa_2, P_2)$ can be obtained by solving the above two sub-problems. However, no closed from solution exists for R_{UB}. Consequently, numerical search over κ_1, κ_2, P_1 and P_2 is required. A simple but loose bound $R_{\mathrm{UB}}^{(0)} = R_1(1, P_R) + R_2(1, P_R)$ can be used instead.

V. Numerical Results

In this section, we present simulation results for the rates of individual UEs as well as the sum-rate. In addition, to assess the effect of linear relay beamforming, the trivial pure amplification relaying $\mathbf{W} = \alpha \mathbb{I}$ with $\alpha = \sqrt{P_R / \left(P\mathbf{h}_{RB}^{\mathrm{H}} \mathbf{h}_{RB} + \|\mathbb{I}\|_F^2 \right)}$ accounting for the relay transmission power is also considered. We assume the relay and the BS have the same transmit power, i.e. $P_R = P$. The curves are generated by the Monte Carlo simulation technique which averages over a sufficient number of channel realizations.

Fig. 3 and Fig. 4 compare the rates for individual UEs when the number of relay antenna $M = 2, 4$. We use R_1 and R_2 to denote the rates for the relayed UE and the direct UE, respectively. Since P1 and P2 aim at individual rate maximization, P1 performs the best in the rate for the relayed UE and P2 provides the highest rate for the direct UE. When $M = 4$, P3 performs very close to the optimal rates for individual UEs. It is also observed that there is significant performance loss by setting $\mathbf{W} = \alpha \mathbb{I}$.

Fig. 5 and Fig. 6 show the sum-rate performance of the various techniques. It is noticed that the proposed iterative technique for P3 performs close to the upper bound especially when $M = 4$. Hence, the iterative algorithm is an efficient tool to address sum-rate maximization of the multi-antenna AF CDR system, although it is sub-optimal.
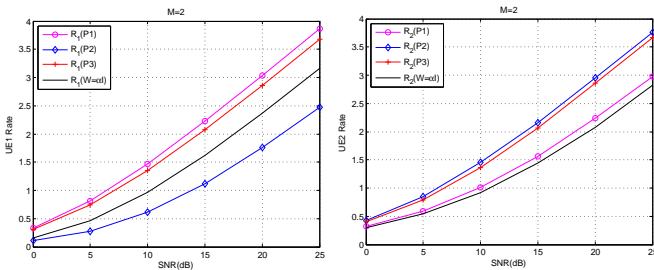

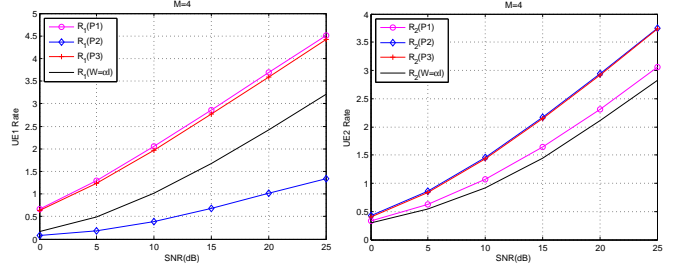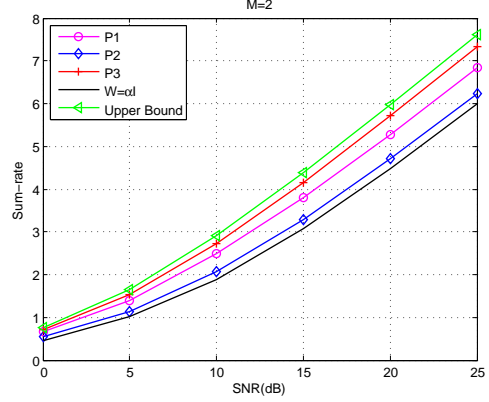Figure 4. Rates for the relayed UE and the direct UE ($M = 4$).

Figure 5. Sum-rate performance ($M = 2$).

VI. Conclusions and Future Works

We focus on the design for the relay beamforming of the AF CDR system. Beamforming designs for rate maximization of the relayed UE and the direct UE as well as the sum-rate maximization are considered. We propose a low-complexity but efficient iterative algorithm to achieve the sum-rate maximization and derive the upper bound of the achievable sum-rate. Numerical results confirm that the proposed iterative design gives comparable sum-rate and performs close to the upper bound.
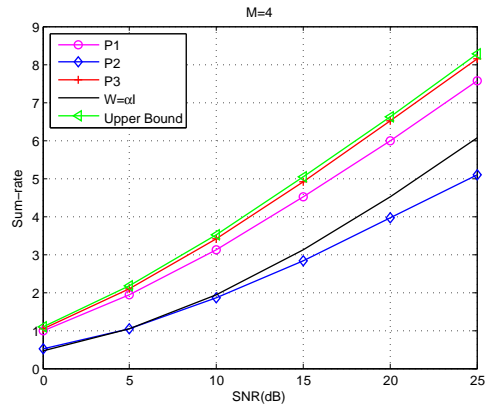

Figure 3. Rates for the relayed UE and the direct UE ($M = 2$).

Figure 6. Sum-rate performance ($M = 4$).

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