



## Control Configuration Selection for Multivariable Descriptor Systems

Shaker, Hamid Reza; Stoustrup, Jakob

*Published in:*  
2012 American Control Conference (ACC)

*Publication date:*  
2012

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Shaker, H. R., & Stoustrup, J. (2012). Control Configuration Selection for Multivariable Descriptor Systems. In *2012 American Control Conference (ACC)* (pp. 6294 - 9299)  
<http://ieeexplore.ieee.org/xpl/articleDetails.jsp?tp=&arnumber=6315372&contentType=Conference+Publications&queryText%3DControl+Configuration+Selection+for+Multivariable+Descriptor+Systems>

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Control Configuration Selection for Multivariable Descriptor Systems

Hamid Reza Shaker and Jakob Stoustrup

**Abstract**— Control configuration selection is the procedure of choosing the appropriate input and output pairs for the design of SISO (or block) controllers. This step is an important prerequisite for a successful industrial control strategy. In industrial practices it is often the case that the system, which is needed to be controlled, is either in the descriptor form or can be represented in the descriptor form. Singular systems and the differential algebraic equation (DAE) systems are among these systems. Descriptor systems appear in the variety of fields to describe the practical processes ranging from power systems, hydraulic systems to heat transfer, and chemical processes. The focus of this paper is on the problem of control configuration selection for multivariable descriptor systems. A gramian-based interaction measure for control configuration selection of such processes is described in this paper. The proposed MIMO interaction measure is the extension of its gramian-based analogous counterpart, which has been proposed for the input-output pairing as well as for the controller architecture selection of the processes with the standard state-space form. The main advantage of this interaction measure is that it can be used to propose a richer sparse or block diagonal controller structure. The interaction measure is used for control configuration selection of the linearized CSTR model with descriptor form.

## I. INTRODUCTION

THE technological world of today has been witnessing the increased complexity due to the rapid development of the process plants and the manufacturing processes. The computational complexity, the reliability problems and the restrictions in communication make the centralized control of such large-scale complex systems expensive and difficult. To cope with these problems, several decentralized control structures have been introduced and implemented over the last few decades [1]. The decentralized controllers have several advantages, which make them popular in industry. The decentralized controllers are easy to understand for operators, easy to implement and to re-tune [1],[2]. The decentralized control systems design is a two-step procedure. The controller structure selection and input-output pairing is the first main step and the controller synthesis for each channel is the second step of the decentralized control. The focus of this paper is on pairing and the controller structure selection of the decentralized control systems. This issue is a key problem in the design of

the decentralized and distributed control systems, which directly affects the stability and the performance of the control systems. The interaction measures play an important role in the suitable pairing and the controller structure selection for the decentralized and the distributed control. Interaction measures make it possible to study input-output interactions and to partition a process into subsystems in order to reduce the coupling, to facilitate the control and to achieve a satisfactory performance. The interaction measures have received a lot of attention over the last few decades [2]-[4]. There are two broad categories of interaction measures in the literature. The first category is the relative gain array (RGA) and its related indices [5]-[10] and the second category is the family of the gramian-based interaction measures [11]-[14].

The most well-known and commonly used interaction measure is the relative gain array (RGA), which was first proposed in [5]. In the RGA, d.c. gain of the process is used for the construction of the channel interaction measure. The RGA is not sensitive to delays and more importantly it considers the process just in the particular frequency.

The RGA has been studied by several other researchers (see, e.g. [6],[7]). There are also other similar measures of interaction, which use dc gain of the process e. g. the NI (the Niederlinski index) [8].

The NI (the Niederlinski index) does not provide more information for pairing compared to RGA. The RGA and the NI have been extended for input-output pairing of unstable MIMO systems in [2]. The relative interaction array (RIA) is an interaction measure, which is similar to RGA and it is based on considering the interaction as an unmodelled term at d.c.

RIA does not provide more information than the RGA about the channel interactions of the process. These indices use the model of the processes at zero frequency. In [7], [9], the relative dynamic gain array (RDGA) was proposed for the first time. The RDGA shows how the interaction varies over the frequency. The idea is further generalized in [10] by the generalized relative dynamic gains (GRDG). This method was mainly proposed for  $2 \times 2$  system.

The second category of the interaction measures is the family of the gramian based methods. A method from this category was first proposed in [11] and further in [12]. In this category, the observability and the controllability gramians are used to form the Participation Matrix (PM). The elements of the PM encode the information of the channel interactions. PM is used for pairing and the controller structure selection. The Hankel Interaction Index Array (HIIA) is a similar interaction measure, which was

H. R. Shaker is an Assistant Professor in the Institute of Energy Technology, Aalborg University, Denmark ([shr@et.aau.dk](mailto:shr@et.aau.dk)).

J. Stoustrup is a Professor in the Section for Automation and Control, Department of Electronic systems, Aalborg University, Denmark ([jakob@es.aau.dk](mailto:jakob@es.aau.dk)).

proposed in [13]. The gramian-based interaction measures have several advantages over the interaction measures in the RGA category. The gramian-based interaction measures take the whole frequency range into account rather than a single frequency. This family of the interaction measures suggests more suitable pairing and allows more complicated controller structures. For more details on the applications and the differences between two main categories of the interaction measures, see [12]-[15].

The results on the gramian-based interaction measures, which have been proposed so far, only support systems in the ordinary state-space form. However, in industrial practices it is often the case that the system, which is needed to be controlled, is either in the descriptor form or can be represented in the descriptor form. Singular systems and the differential algebraic equation (DAE) systems are among these systems. Descriptor systems appear in the variety of fields to describe the practical processes ranging from power systems, hydraulic systems to heat transfer, and chemical processes[18],[19].

In this paper, a gramian-based interaction measure is extended to support the descriptor systems. The proposed interaction measure is used for pairing and the controller structure selection.

The paper is organized as follows. In the next section, we review the concept of the gramians for the ordinary systems as well as descriptor systems. The interpretation of the controllability and observability gramians is also discussed in this section. Section III presents how gramians can be used to quantify the channel interactions for descriptor systems. The application of the proposed interaction measure in pairing and the controller structure selection is explained in this section. In Section IV, the proposed interaction measure is used for pairing and the controller structure selection for CSTR model in the descriptor form. Section V concludes the paper.

The notation used in this paper is as follows:  $M^*$  denotes the transpose of matrix if  $M \in \mathbb{R}^{n \times m}$  and complex conjugate transpose if  $M \in \mathbb{C}^{n \times m}$ . The standard notation  $>$ ,  $\geq$  ( $<$ ,  $\leq$ ) is used to denote the positive (negative) definite and semidefinite ordering of matrices.  $Structure(H(s))$  denotes the structure of a MIMO system with transfer function. For a  $p \times p$  MIMO system  $H(s)$  with input  $u(t) \in \mathbb{R}^p$  and output  $y(t) \in \mathbb{R}^p$ ,  $Structure(H(s)) = [h_{ij}]_{p \times p}$  is a symbolic array where  $h_{ij} = *$ , if there exist a subsystem in  $H(s)$  with input  $u_j$  and output  $y_i$ . Otherwise:  $h_{ij} = 0$ .

## II. CONTROLLABILITY AND OBSERVABILITY GRAMIANS

The controllability and the observability gramians are well-known matrices, which are widely used to check the controllability and the observability of the linear dynamical systems. The controllability and observability gramians show how difficult a system is to control and to observe. The gramians are also widely used in the process of model order reduction [16],[17]. For dynamical systems with the minimal realization:

$$G(s) := (A, B, C, D), \quad (1)$$

where  $G(s)$  is the transfer matrix with the associated state-space representation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p, \\ y(t) &= Cx(t) + Du(t), \quad y(t) \in \mathbb{R}^p, \end{aligned} \quad (2)$$

the gramians are defined as:

$$\begin{aligned} W_c &:= \int_0^\infty e^{A\tau} BB^* e^{A^*\tau} d\tau, \\ W_o &:= \int_0^\infty e^{A^*\tau} C^* C e^{A\tau} d\tau, \end{aligned} \quad (3)$$

which are given by the solutions of the Lyapunov equations:

$$\begin{aligned} AW_c + W_c A^* + BB^* &= 0, \\ A^* W_o + W_o A + C^* C &= 0. \end{aligned} \quad (4)$$

For stable  $A$ , they admit unique positive definite solutions  $W_c > 0$  and  $W_o > 0$ , which are the controllability and the observability gramians respectively.

In practice it is often the case that the system, which is needed to be controlled and to be studied, is described by a set of differential algebraic equations and therefore does not have the ordinary state-space form (2). These types of systems can be represented in the descriptor. The well-known singular systems are among these systems.

For a dynamical system with descriptor form:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \quad x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p, \\ y(t) &= Cx(t), \quad y(t) \in \mathbb{R}^p, \end{aligned} \quad (5)$$

where  $Rank(E) \leq n$  and  $G(s)$  is the associated transfer matrix:

$$G(s) := (E, A, B, C), \quad (6)$$

the controllability gramian is decomposed into a causal gramian  $R_c$  and a noncasual gramian  $R_{nc}$  which are defined as[20]-[23]:

$$R_c := \int_0^\infty \Phi_0 e^{A\Phi_0\tau} BB^* e^{\Phi_0^* A^* \tau} \Phi_0^* d\tau, \quad (7)$$

$$R_{nc} := \sum_{k=-h}^{-1} \Phi_k BB^* \Phi_k^*, \quad (8)$$

$$W_c = R + R_{nc}, \quad (9)$$

where  $\Phi_k$  is Laurent parameter in series expansion:

$$(sE - A)^{-1} = \sum_{k=-h}^{+\infty} \Phi_k s^{-k-1}, \quad (10)$$

with:

$$\Phi_k := \begin{cases} \begin{bmatrix} J^k & 0 \\ Q \begin{bmatrix} 0 & 0 \end{bmatrix} P, & k \geq 0, \\ \begin{bmatrix} 0 & 0 \\ Q \begin{bmatrix} 0 & -N^{-k-1} \end{bmatrix} P, & k < 0. \end{cases} \end{cases} \quad (11)$$

The positive integer  $h$  is the length of the longest chain of generalized eigenvectors of  $(sE - A)$  corresponding to the eigenvalue  $s = \infty$ .  $P$  and  $Q$  are obtained from Weierstrass-Kronecker decomposition of the pencil matrix. The observability gramian for a descriptor system is defined analogously:

$$O_c := \int_0^\infty \Phi_0^* e^{\Phi_0^* A^* \tau} C^* C e^{A \Phi_0 \tau} \Phi_0 d\tau, \quad (12)$$

$$O_{nc} := \sum_{k=-h}^{-1} \Phi_k^* C^* C \Phi_k, \quad (13)$$

$$W_o = O + O_{nc}. \quad (14)$$

The controllability gramians are solutions to the following Lyapunov-like equations [20]-[23]:

$$\Phi_0 A R_c + R_c A^* \Phi_0^* + \Phi_0 B B^* \Phi_0^* = 0, \quad (15)$$

$$\Phi_{-1} E R_{nc} E^* \Phi_{-1}^* - R_{nc} + \Phi_{-1} B B^* \Phi_{-1}^* = 0, \quad (16)$$

$$\begin{aligned} & \Phi_{-1} E W_c E^* \Phi_{-1}^* + \Phi_0 B B^* \Phi_0^* + \Phi_{-1} B B^* \Phi_{-1}^* \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2}) A W_c + W_c A^* (\Phi_0 + \frac{\Phi_{-1}}{2})^* = 0. \end{aligned} \quad (17)$$

Dually for the observability gramians Lyapunov-like equations are [20]-[23]:

$$\Phi_0^* A^* O_c + O_c A \Phi_0 + \Phi_0^* C^* C \Phi_0 = 0, \quad (18)$$

$$\Phi_{-1}^* E^* O_{nc} E \Phi_{-1} - O_{nc} + \Phi_{-1}^* C^* C \Phi_{-1} = 0, \quad (19)$$

$$\begin{aligned} & \Phi_{-1}^* E^* W_o E \Phi_{-1} + \Phi_0^* C^* C \Phi_0 + \Phi_{-1}^* C^* C \Phi_{-1} \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2})^* A^* W_o + W_o A (\Phi_0 + \frac{\Phi_{-1}}{2}) = 0. \end{aligned} \quad (20)$$

### III. INTERACTION MEASURE

In this section, an interaction measure for the MIMO processes with descriptor form is built upon the notion of the gramians. The trace of the cross gramian is used as a convenient basis to present the channel interaction and to select the most appropriate controller structure. For a MIMO dynamical system with descriptor form (6), we have:

$$\begin{aligned} B &= [b_1 \ b_2 \ \dots \ b_p], \\ C^* &= [c_1 \ c_2 \ \dots \ c_p]. \end{aligned} \quad (21)$$

A set of elementary SISO systems can be associated to this MIMO system, such that each SISO system has a single input  $u_j(t)$  and single output  $y_i(t)$ . The elementary systems are in the descriptor form:

$$G_{ij}(s) = (E, A, b_j, c_i^*), \quad (22)$$

with gramians  $W_{c_j}$  and  $W_{o_i}$ . The controllability gramian  $W_{c_j}$  and the observability gramian  $W_{o_i}$  for the elementary systems are the solutions to:

$$\Phi_0 A R_{c_j} + R_{c_j} A^* \Phi_0^* + \Phi_0 b_j b_j^* \Phi_0^* = 0, \quad (23)$$

$$\Phi_{-1} E R_{nc_j} E^* \Phi_{-1}^* - R_{nc_j} + \Phi_{-1} b_j b_j^* \Phi_{-1}^* = 0, \quad (24)$$

$$\begin{aligned} & \Phi_{-1} E W_{c_j} E^* \Phi_{-1}^* + \Phi_0 b_j b_j^* \Phi_0^* + \Phi_{-1} b_j b_j^* \Phi_{-1}^* \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2}) A W_{c_j} + W_{c_j} A^* (\Phi_0 + \frac{\Phi_{-1}}{2})^* = 0. \end{aligned} \quad (25)$$

Dually for the observability gramians Lyapunov-like equations are [20]-[23]:

$$\Phi_0^* A^* O_{c_i} + O_{c_i} A \Phi_0 + \Phi_0^* c_i c_i^* \Phi_0 = 0, \quad (26)$$

$$\Phi_{-1}^* E^* O_{nc_i} E \Phi_{-1} - O_{nc_i} + \Phi_{-1}^* c_i c_i^* \Phi_{-1} = 0, \quad (27)$$

$$\begin{aligned} & \Phi_{-1}^* E^* W_o E \Phi_{-1} + \Phi_0^* c_i c_i^* \Phi_0 + \Phi_{-1}^* c_i c_i^* \Phi_{-1} \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2})^* A^* W_o + W_o A (\Phi_0 + \frac{\Phi_{-1}}{2}) = 0. \end{aligned} \quad (28)$$

In the following lemma, we show that the system gramian for a descriptor system can be expressed in terms of the gramians of the elementary systems.

**Lemma 1.** Let  $W_c$  and  $W_o$  be the controllability and the observability gramians of a MIMO dynamical system with descriptor form (6) and  $W_{c_j}$  and  $W_{o_i}$  be the controllability and observability gramians for the elementary systems (22). Then:

$$\begin{aligned} W_c &= \sum_{j=1}^p W_{c_j}, \\ W_o &= \sum_{i=1}^p W_{o_i}. \end{aligned} \quad (29)$$

**Proof:**

For the elementary systems (22), we have :

$$\begin{aligned} & \Phi_{-1} E W_{c_j} E^* \Phi_{-1}^* + \Phi_0 b_j b_j^* \Phi_0^* + \Phi_{-1} b_j b_j^* \Phi_{-1}^* \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2}) A W_{c_j} + W_{c_j} A^* (\Phi_0 + \frac{\Phi_{-1}}{2})^* = 0. \end{aligned}$$

for  $j = 1, \dots, p$ .

If we add all these equations:

$$\sum_{j=1}^p \Phi_{-1} E W_{c_j} E^* \Phi_{-1}^* + \Phi_0 b_j b_j^* \Phi_0^* + \Phi_{-1} b_j b_j^* \Phi_{-1}^* + (\Phi_0 + \frac{\Phi_{-1}}{2}) A W_{c_j} + W_{c_j} A^* (\Phi_0 + \frac{\Phi_{-1}}{2})^* = 0.$$

Equivalently:

$$\begin{aligned} & \Phi_{-1}E(\sum_{j=1}^p W_{c_j})E^*\Phi_{-1}^* + \Phi_0(\sum_{j=1}^p b_j b_j^*)\Phi_0^* + \Phi_{-1}(\sum_{j=1}^p b_j b_j^*)\Phi_{-1}^* \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2})A(\sum_{j=1}^p W_{c_j}) + (\sum_{j=1}^p W_{c_j})A^*(\Phi_0 + \frac{\Phi_{-1}}{2}) = 0 \end{aligned}$$

On the other hand, we have:  $BB^* = \sum_{j=1}^p b_j b_j^*$ , which leads to:

$$\begin{aligned} & \Phi_{-1}E(\sum_{j=1}^p W_{c_j})E^*\Phi_{-1}^* + \Phi_0 BB^* \Phi_0^* + \Phi_{-1} BB^* \Phi_{-1}^* \\ & + (\Phi_0 + \frac{\Phi_{-1}}{2})A(\sum_{j=1}^p W_{c_j}) + (\sum_{j=1}^p W_{c_j})A^*(\Phi_0 + \frac{\Phi_{-1}}{2}) = 0. \end{aligned} \quad (30)$$

This is exactly the Lyapunov-like equation (17), for controllability gramian  $W_c$  of the original MIMO system. Hence:

$$W_c = \sum_{j=1}^p W_{c_j}.$$

For the observability gramian also the results can be proven in the similar way.  $\square$

A direct result of this lemma is that:

$$W_c W_o = (\sum_{j=1}^p W_{c_j})(\sum_{i=1}^p W_{o_i}) = (\sum_{i=1}^p \sum_{j=1}^p W_{c_j} W_{o_i}). \quad (31)$$

The information of the channel interaction which is obtained from the gramians of elementary systems is encompassed into the so-called participation matrix (PM):

$$\Psi = [\psi_{ij}] \in \mathbb{R}^{p \times p},$$

where:

$$\psi_{ij} = \frac{\text{trace}[W_{c_j} W_{o_i}]}{\text{trace}[W_c W_o]}. \quad (32)$$

Note that  $0 \leq \psi_{ij} < 1$  and  $\sum_{i=1}^p \sum_{j=1}^p \psi_{ij} = 1$ .

The participation matrix highlights the elementary subsystems, which are more important in the description of MIMO systems, and in this way it shows the suitable pairing and the appropriate controller structure to select.

For pairing and controller structure selection, the nominal model  $G_o(s)$  needs to be obtained. The nominal model is a model, which is obtained by keeping some of the elementary subsystems of the actual MIMO process and assuming the rest as zero. For example, assume that one of the ordinary methods for pairing is used and a decartelized control is synthesized. If the inputs and outputs are re-labeled, one only needs to design  $p$  independent SISO controller loops, for elementary diagonal subsystems. In this case:

$$G_o(s) = \text{diag}(G_{11}(s), G_{22}(s), \dots, G_{pp}(s)). \quad (33)$$

The designed controller is:

$$C(s) = \text{diag}(C_1(s), C_2(s), \dots, C_p(s)). \quad (34)$$

The elements of the PM shows which elementary subsystems are significant and should be considered in nominal model. When  $\psi_{ij}$  is small, the associated elementary subsystem to the pair  $(i, j)$  is either hard to control or hard to observe. This shows that it does not have any significant effect in the actual model transfer function and need not be kept in nominal model. When  $\psi_{ij}$  is larger than  $1/p^2$ , some states in the elementary system with output  $y_j$  and input  $u_j$  are easy to control and easy to observe and therefore  $G_{ij}$  is good candidate to be kept in the nominal system. The suitability of the pairing and the performance of the controller structure is highly depends on how close the sum of the chosen  $\psi_{ij}$  elements to one is.

When the sum of the chosen  $\psi_{ij}$  elements are close to one, the nominal and the actual model are close to each other and the error is not significant. The complexity of the selected controller structure depends on the number of the  $\psi_{ij}$  elements. In the completely decentralized control, which is the least complicated controller structure, the number of the chosen elements would be  $p$ .

For example consider a  $3 \times 3$  process model with PM:

$$\Psi = \begin{bmatrix} 0.1833 & 0.1685 & 0.0861 \\ 0.1200 & 0.0445 & 0.1783 \\ 0.0639 & 0.0691 & 0.0863 \end{bmatrix}$$

To pair inputs outputs for decentralized structure, we have to select one element per row and one element per column.  $\psi_{11}, \psi_{12}, \psi_{21}, \psi_{23} > 1/p^2$ , therefore their associated elementary subsystems are good candidates to be involved in nominal model.

The best paring for a decentralized controller can be obtained with  $(u_1, y_1), (u_2, y_3), (u_3, y_2)$  which are associated with:

$$\Sigma = \psi_{11} + \psi_{23} + \psi_{32} = 0.4307$$

The structure of the nominal model will be:

$$\text{Structure}(G_o(s)) = \begin{bmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix},$$

A simple controller structure for selection is the structure of  $G_o^{-1}(s)$ :

$$\text{Structure}(C(s)) = \text{Structure}(G_o^{-1}(s)) = \begin{bmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix}.$$

If practically is possible to use more complicated controllers than decentralized control,  $y_1$  could be commanded from  $u_2$  and then we will have:

$$\Sigma = \psi_{11} + \psi_{12} + \psi_{23} + \psi_{32} = 0.599 .$$

The structure of the nominal model then will be:

$$Structure(G_o(s)) = \begin{bmatrix} * & * & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix} .$$

A simple controller structure to select:

$$Structure(C(s)) = Structure(G_o^{-1}(s)) = \begin{bmatrix} * & 0 & * \\ 0 & 0 & * \\ 0 & * & 0 \end{bmatrix} .$$

In this case the structure is partially decentralized.

One of the main advantage of the proposed method for control configuration is that in the cases where a fully decentralized controller results in unacceptably poor closed loop performance, the PM can be used to propose a richer sparse or block diagonal controller structures. The participation matrix might be sensitive to input and output scaling. One way to deal with this issue is discussed in [26].

#### IV. PAIRING AND CONTROLLER STRUCTURE SELECTION FOR A CSTR WITH A HEATING JACKET

The systems of differential and algebraic equations (DAEs) describe a wide range of chemical processes. The differential equation parts usually arise from the mass and the energy dynamic conservation equations, and the algebraic equations usually consisting of empirical correlations, thermodynamic equilibrium relations, etc. The algebraic equations are often singular in nature and therefore the resulting models have descriptor form rather than standard state-space from [25]. In this section, the interaction measure is used for pairing and the controller structure selection of a CSTR with a heating jacket.

The detailed model is a MIMO nonlinear model and is available in [24]. The linearized model is in the descriptor form is described by [24]:

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad x(t) \in \mathbb{R}^5, u(t) \in \mathbb{R}^2,$$

$$y(t) = Cx(t), \quad y(t) \in \mathbb{R}^2,$$

where:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} -0.93976 & 0 & -0.0761 & 0 & 0 \\ 0.63976 & -0.3 & 0.0761 & 0 & 0 \\ -3.5544 & 0 & -0.7227 & 0 & 2.7778 \times 10^{-5} \\ 0 & 0 & -0.1 & 0 & -2.7778 \times 10^{-4} \\ 0 & 0 & 25000 & -25000 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3404 & 0 \\ -0.3404 & 0 \\ 1.5920 & 0 \\ 0 & -89.63 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

The participation matrix (PM) for this system is obtained using the proposed method:

$$\Psi = \begin{bmatrix} 0.000199414925755 & 0.002370821676489 \\ 0.008799176151784 & 0.988630587245973 \end{bmatrix}$$

The  $\psi_{22}$  is significant compared to other elements. The  $\Sigma$  associated to the best possible pairing for the completely decentralized control is:

$$\Sigma = \psi_{22} + \psi_{11} = 0.988830002171728 .$$

The structure of the nominal model for this pairing will be:

$$Structure(G_o(s)) = \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix} .$$

The suggested simple control structure for this pairing is:

$$Structure(C(s)) = Structure(G_o^{-1}(s)) = \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix} .$$

If it is allowed to use more complex controllers than decentralized control,  $y_2$  could be commanded from  $u_1$  and then we have:

$$\Sigma = \psi_{22} + \psi_{11} + \psi_{21} = 0.997629178323511 ,$$

associated with:

$$Structure(G_o(s)) = \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} .$$

The simple control structure for this pairing is:

$$Structure(C(s)) = Structure(G_o^{-1}(s)) = \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} .$$

The  $\Sigma$  for this structure is very close to one.

## V. CONCLUSION

Control configuration selection for descriptor systems which are systems that appear in the variety of fields ranging from power systems, hydraulic systems to heat transfer, and chemical processes has been addressed in this paper. A general gramian-based interaction measure for the control configuration selection for such systems has been proposed. The proposed MIMO interaction measure is the extension of its gramian-based analogous counterpart, which was proposed for input-output pairing as well as for the controller architecture selection for the processes with standard state-space. The proposed measure reveals more information about the ability of the channels to be controlled and to be observed and provides hints for the selection of the richer controller structures such as triangular, sparse and block diagonal.

## REFERENCES

- [1] R. Scattolini, "Architectures for distributed and hierarchical model predictive control - a review", *Journal of Process Control*, vol. 19, no. 5, pp. 723–731, 2009.
- [2] M. Hovd, and S. Skogestad, "Pairing criteria for decentralised control of unstable plants", *Industrial and Engineering Chemistry Research* 33, pp. 2134–2139, 1994.
- [3] M. Van de Wal, and B. De Jager, "A review of methods for input/output selection", *Automatica*, Vol. 37, No. 4, pp.487–510, 2001.
- [4] A. Khaki-Sedigh, B. Moaveni, *Control Configuration Selection for Multivariable Plants*, Lecture Notes in Control and Information Sciences, Springer, 2009.
- [5] E. H. Bristol, "On a new measure of interaction for multivariable process control." *IEEE Transactions on Automatic Control*, 11, pp. 133–134, 1966.
- [6] S. Skogestad and M. Morari. "Implications of large RGA elements on control performance." *Industrial Engineering in Chemical Research*, 26, pp.2323–2330, 1987.
- [7] M. F. Witcher, and T. J. McAvoy, "Interacting control systems: Steady-state and dynamic measurement of interaction." *ISA Transactions*, 16,3, pp. 35–41, 1977.
- [8] A. Niederlinski, "A heuristic approach to the design of linear multivariable interacting control systems." *Automatica*, 7, pp. 691–701, 1971.
- [9] E. H. Bristol, "Recent results on interaction in multivariable process control." *71st AIChE Conference*, 1978.
- [10] E. Gagnon, A. Desbiens, and A. Pomerleau, "Selection of pairing and constrained robust decentralized PI controllers." *American Control Conference*, pp. 4343–4347, 1999.
- [11] A. Conley and M. E. Salgado, "Gramian based interaction measure." *The 39th IEEE Conference on Decision and Control*, pp. 5020–5022, 2000.
- [12] M. E. Salgado and A. Conley. "MIMO interaction measure and controller structure selection." *Int. J. Control*, 77(4), pp. 367–383, 2004.
- [13] B. Wittenmark and M. E. Salgado. "Hankel-norm based interaction measure for input-output pairing." *Proc. of the 2002 IFAC World Congress*, 2002.
- [14] B. Halvarsson, "Comparison of some gramian based interaction measures." In *2008 IEEE International Symposium on Computer Aided Control System Design (CACSD 2008), Part of IEEE Multi-conference on Systems and Control*, pp.138–143, 2008.
- [15] P. Samuelsson, B. Halvarsson, and B. Carlsson. "Interaction analysis and control structure selection in a wastewater treatment plant model". *IEEE Transactions on Control Systems Technology*, 13(6), pp. 955–964, 2005.
- [16] A. C. Antoulas. *Approximation of Large-Scale Dynamical Systems*. Advances in Design and Control, SIAM, Philadelphia, 2005.
- [17] S. Gugercin and A. Antoulas "A survey of model reduction by balanced truncation and some new results" *International Journal of Control*, 77, pp.748–766, 2004.
- [18] L. Dai, *Singular Control Systems*, Springer, 1989.
- [19] G. R. Duan, *Analysis and Design of Descriptor Linear Systems*, Springer, 2010.
- [20] D. J. Bender, "Lyapunov-Like Equations and Reachability/Observability Gramians for Descriptor Systems," *IEEE Trans. Aut. Contr.*, Vol. AC-32, n 4, pp. 343-348, 1987.
- [21] B. Marx, D. Koenig and D. Georges, "Optimal sensor/actuator location for descriptor systems using Lyapunov-Like equations", *Proc. of IEEE Conf. on Dec. and Contr.*, pp. 4541-4542, 2002.
- [22] B. Marx, D. Koenig, D. Georges, "Optimal sensor and actuator location for descriptor systems using generalized grammian and balanced realization", *American Control Conference*, pp. 2729-2734, 2004.
- [23] B. Marx, *Contribution à la commande et au diagnostic de systèmes algèbro-différentiels linéaires*, Thèse de Doctorat, Institut National Polytechnique de Grenoble (INPG), France, 2003.
- [24] N.M. Rao, P. Vora, K. M., Moudgalya, "PID control of DAE systems." *Industrial and Engineering Chemistry Research*, 42, pp.4599–4610, 2003.
- [25] A. Kumar, P. Daoutidis, "Feedback control of nonlinear differential-algebraic equation systems." *AIChE J.*, 41 (1), 619, 1995.
- [26] J. Maciejowski, *Multivariable feedback design.*, Addison Wesley, Wokingham, 1989.