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Optimizing Completion Time and Energy Consumption in a Bidirectional Relay Network

(*Invited Paper*)

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***Abstract*—Consider a wireless network with multiple sources and destinations, where the amount of data of each source node is finite. An interesting question is what is the shortest completion time, i. e. the time required that all data from the sources gets to the respective destinations. A similar question arises for the minimal required energy. While the requirement for minimal energy consumption is obvious, the shortest completion time is relevant when certain multi-node network needs to reserve the wireless medium in order to carry out the data exchange among its nodes. The completion time/energy consumption required for multiple flows depends on the current channel realizations, transmission methods used and, notably, the relation between the data sizes of different source nodes. In this paper we investigate the shortest completion time and minimal energy consumption in a two-way relay wireless network. The system applies optimal time multiplexing of several known transmission methods, including one-way relaying and wireless network coding (WNC). We show that when the relay applies Amplify-and-Forward (AF), both minimizations are linear optimization problems. On the other hand, when the relay uses Decode-and-Forward (DF), each of them is a quadratic optimization problem. The results show that, for given channel realizations, there is an optimal ratio of the data packets at the sources to obtain minimal completion time or energy consumption. This can be used as a guidance for the nodes to apply *traffic shaping*. In most cases, DF leads to shorter completion time and energy consumption compared to AF.**

I. Introduction

In many wireless networks, notably WiFi, wireless medium is used through an exclusive reservation by a single node at a given time. The reservation time used depends on the current channel conditions and the amount of data that the node has to send. Nevertheless, the recent concepts of wireless network coding [1] or interference alignment [2] dictate that multiple communication flows should be served simultaneously over the shared wireless medium. Interference is not avoided, but it is contained and processed as a part of the transmission. Yet, although it is in theory optimal to allow all flows in the network to be served simultaneously, synchronization and coordination imply that it is practical to serve only a small, limited amount of flows at a given time. On the other hand, different groups of flows use the wireless medium in a time-division manner. Then the following question is of interest: given the channel conditions and the data size that each flow needs to transfer, what is the minimal *completion time* until all the data reaches their destinations? A related question is what is the *minimal energy* needed to transfer the data.

An interesting parameter that has an impact is the relation between the data sizes of different source nodes. In the case where some users have deterministic application-wise packet lengths while the other users with a lower priority have adjustable packet lengths, the problem is solved by first satisfying the conditions according to the users with a higher priority. The lengths of the packets of the other users will be then optimized to have the shortest completion time and selected accordingly. We term this problem *traffic shaping*.

Prior works have treated the completion time region and the weighted sum completion time for multiple access (MA) channel [3], broadcast (BC) channel and interference channel [4] [5]. In this paper, we investigate the minimal completion time and energy consumption in a scenario with bidirectional relaying. In the recent years, two-way relaying has been tightly associated with the technique of wireless network coding (WNC). On the other hand, other transmission modes can support two-way communication, such as time-division of the direct link between two end nodes. In order to calculate the minimal completion time, we consider several known transmission schemes as building *blocks*, used in a time-division manner. For example, consider the two-way communication between the nodes U1 and U2 aided by a relay station (RS) that operates by using Amplify-and-Forward (AF). Assume that there is much more data to send from U1 to U2 compared to the data size flowing in the opposite direction; then, in addition to the WNC based on AF, the completion time may include direct transmission from U1 to U2. We formulate two different optimization problems, when the relay works in an AF and Decode-and-Forward (DF) mode, respectively. Since for each assumed mode of the relay there are several possible blocks, we analyze how to eliminate some of the blocks from consideration under given channel conditions. The results show that, for given channel conditions, the minimal completion time and energy consumption significantly depend on the ratio between the data sizes at the two source nodes.

II. System Model

We consider a scenario in which two users communicate with the help of a relay station (RS). User i (Ui) has b_i bits which are intended for the other user. The data sizes b_1, b_2 are not necessarily equal, but are assumed to be sufficiently large, in order to ensure that the communication rates be approximated by the information-theoretic rates. Our performance measures are the completion time and energy consumption normalized by the total data size $b_1 + b_2$. This normalization allows us to define the performance through the ratio b_1/b_2 and not the individual values b_1, b_2. The normalized completion time and energy consumption serves as lower bounds for the case of finite packet sizes.

All the nodes are half-duplex, such that a node can either transmit or receive at a given time. The channels are denoted by h_0 (U1-U2), h_1 (U1-RS) and h_2 (U2-RS). h_0 is the channel of the direct link, h_1 and h_2 are the channels from U1 and U2 to the relay. Each channel $h_l, l \in \{0,1,2\}$, is reciprocal, known at all the nodes. We assume that the transmitted power is the same at all nodes, i.e. $E\{|x_i|^2\} = P$, where x_i is the signal transmitted from node $i \in \{\text{U1,U2,RS}\}$. Regarding the energy consumption, we only consider the contribution from the transmitted power, neglecting the power required to run the receivers. The noise z_j at each node has a zero-mean complex circularly symmetric Gaussian distribution: $z_j \sim \mathcal{CN}(0, \sigma^2), j \in \{\text{U1,U2,RS}\}$. We define $\gamma_l = \frac{P|h_l|^2}{\sigma^2}, l \in \{0,1,2\}$, where γ_l is the SNR of link l. Then the capacity of a single link is $C(\gamma) = \log_2(1+\gamma)$.

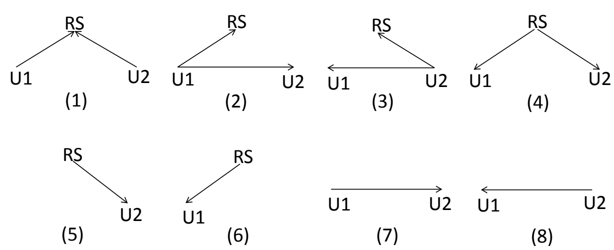

Figure 1. Available transmission schemes used as basic blocks (BBs).

Each Ui has data traffic for the other user. These two traffic flows can be served using different transmission schemes, involving the combination of different flows in a variable number of transmission periods. We define 8 basic blocks (BB), represented by the transmission schemes on Fig. 1. The set of BBs is restricted to the ones where the terminals receive signals either from the other terminal or from the relay, but not simultaneously through the multiple access channel. On the other hand, the relay can receive signals from the terminals through a multiple access channel.

A given transmission scheme can be described as a concatenation of BBs. The time sequence of BBs needs to satisfy certain constraints. For example, the BBs with uplink transmission should be selected before the BBs with downlink transmission. The task of appropriately ordering the BBs can be alleviated by defining a set of composite blocks (CBs) which groups the valid combinations of the BBs. The set of CBs is described in Fig. 2. The two-way relaying with Time Division Broadcast (TDBC) on Fig. 2(a) is a CB consisting of three BBs (and three transmission phases), while the two-way relaying with Multiple Access Broadcast (MABC) on Fig. 2(b) consists of two BBs. Both CBs, with TDBC and MABC, involve WNC at the relay. The other two CBs, depicted on Fig. 2(c) and Fig. 2(d), involve unidirectional forwarding by the relay and two-way direct communication, respectively.

The objective is to find the combination and durations of CBs that optimize the completion time or energy consumption. The optimization procedure is different, depending on whether AF or DF is used at the relay. For DF, because the signals are decoded at the relay, the dependency between BBs inside a CB is removed and the optimization can be conducted for each BB independently. As AF relays the signal untouched, the BBs remain interdependent and the optimization has to be conducted jointly over all the CBs.

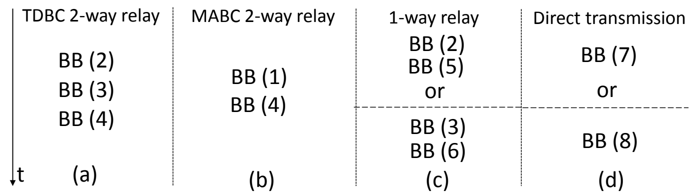

Figure 2. Composite blocks (CBs)

III. Optimization for AF relaying

In this section, we formulate the two optimization problems for AF relaying under the packet size constraint sent from both users.

A. Maximal achievable rates

In AF, all phase durations are equal, since relaying is performed on a symbol by symbol basis. Therefore, each phase in TDBC requires one third of the total duration of the CB. Each phase in MABC and one-way relay CB requires one half of the duration of the corresponding CB. From [6], the achievable rates for the TDBC are

$$R^a_{\text{Ui}} = \frac{1}{3} C\left(\gamma_0 + \frac{\gamma_i \gamma_j}{\gamma_i + 3\gamma_j + 2}\right), \ (i,j) \in \{(1,2),(2,1)\}. \quad (1)$$

From [6], the achievable rates for MABC are

$$R^b_{\text{Ui}} = \frac{1}{2} C\left(\frac{\gamma_i \gamma_j}{\gamma_i + 2\gamma_j + 1}\right), \ (i,j) \in \{(1,2),(2,1)\}. \quad (2)$$

From [7], the achievable rates for the unidirectional one-way relay are $R^c_{\text{U1}} = R^c_{\text{U2}} = \frac{1}{2} C\left(\gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}\right)$. At last, the achievable rates of the unidirectional direct transmission are $R^d_{\text{U1}} = R^d_{\text{U2}} = C(\gamma_0)$. The optimization formulation can be simplified by keeping only one of the two unidirectional CBs for a given transmission direction:

$$R^*_{\text{U1}} = R^*_{\text{U2}} = \max\left\{\frac{1}{2} C\left(\gamma_0 + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}\right), C(\gamma_0)\right\}. \quad (3)$$

B. Formulation of the optimization problem for AF

1) Completion time: The transmission scheme is now made out of 4 CBs: (1) TDBC which has a duration denoted as t_a, (2) MABC which has a duration denoted as t_b, (3) unidirectional CB with transmission from U1 with duration t^1_* and (4) unidirectional CB with transmission from U2 with duration t^2_*. Then the total completion time is $T = t_a + t_b + t^1_* + t^2_*$.

2) Energy consumption: For MABC, the total transmission power is equal to $2P$ during the multiple access (MA) phase and equal to P during the broadcast (BC) phase. When the relay applies AF, the durations of the MA phase and BC phase are equal, so the MABC has an average power of $1.5P$. The other CBs have an average power of P. Then the whole energy consumption is $E = 1.5Pt_b + P\left(t_a + t_*^1 + t_*^2\right)$.

3) Constraint: The transmission scheme made from the combination of the 4 CBs has to transmit b_i bits from Ui, which is expressed as the following constraint:

$$b_i = t_a R_{\text{Ui}}^a + t_b R_{\text{Ui}}^b + t_*^i R_{\text{Ui}}^*, i = 1, 2. \tag{4}$$

4) Optimization criterion: For a given SNR setup and values of b_1 and b_2, we can formulate the completion time and energy consumption optimization problem as a linear optimization problem as follows:

$$\begin{aligned} \min_{t_a, t_b, t_*^1, t_*^2 \geq 0} \quad & T \quad \text{or} \quad E \\ s.t. \quad & b_1 = t_a R_{\text{U1}}^a + t_b R_{\text{U1}}^b + t_*^1 R_{\text{U1}}^* \\ & b_2 = t_a R_{\text{U2}}^a + t_b R_{\text{U2}}^b + t_*^2 R_{\text{U2}}^*. \end{aligned} \tag{5}$$

IV. The case of DF relaying

In the AF case, there are only 4 variables to optimize, i.e. t_a, t_b, t_*^1, t_*^2. The number of variables is small because each phase of AF relaying has the same duration. In DF relaying, the duration of each phase can be different in a specific CB which means that the BBs have a different duration which should be optimized. The 4 CBs in Fig. 2 have 11 BBs which correspond to 11 variables. And for MABC, the MA rates of two links need also to be optimized leading 2 additional variables. Therefore, the optimization problem for DF contains 13 independent variables.

We simplify the optimization problem for DF in 2 steps. The first step is to decrease the number of optional CBs in Fig. 2. In the second step, we exploit the fact that the BBs within a same CB can be optimized separately: this leads to an optimization based on BBs and not CBs. Furthermore, the BBs that are common to multiple CBs have the same performance and hence can be described using a single optimization variable. This indicates that, the optimization for DF based on BBs relies on less variables than the optimization based on CBs.

In order to reduce the number of considered CBs, we rely on the following proposition:

Proposition 1: There are two different regions depending on the relations between the SNRs:

- Case 1: If $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, then it is optimal to use only direct transmission.
- Case 2: If $\gamma_0 < \min\{\gamma_1, \gamma_2\}$, then it is optimal not to use the direct transmission, but only the other three CBs.

The proof is provided in Appendix A.

Proposition 2: If DF relaying is used, then if two or more CBs use the same BB, then it is sufficient to have only one optimization variable corresponding to that BB.

The proof is deferred to Appendix B. This proposition implies that the optimization problem can be reformulated into a new optimization problem that uses only six BBs (1)-(6) from Fig. 1.

To summarize, distinguishing between 2 SNR cases allows a decrease in the CB options available, eliminating the use of at least two variables. Removing common BBs between CBs eliminates the use of at least three variables. We can cut down at least five variables, then there are at most eight variables to optimize.

V. Optimization problem for DF relaying

Based on section IV, the optimization problem formulation will be discussed under 2 conditions, one is $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, another is $\gamma_0 < \min\{\gamma_1, \gamma_2\}$.

A. Direct link stronger than at least one of the relay links

When $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, only the direct transmission CB is chosen. Then $T = \frac{b_1+b_2}{C(\gamma_0)}$ and $E = \frac{P(b_1+b_2)}{C(\gamma_0)}$.

B. Direct link weaker than the relay links

1) Objective functions: When $\gamma_0 < \min\{\gamma_1, \gamma_2\}$, TDBC, MABC and one-way relay CBs are the available options and the optimal transmission scheme is built from the BBs (1), (2), (3), (4), (5) and (6) in Fig. 1, each having a duration $t_{(1)}$, $t_{(2)}$, $t_{(3)}$, $t_{(4)}$, $t_{(5)}$ and $t_{(6)}$ respectively. Then the total completion time and energy consumption are

$$T = t_{(1)} + t_{(2)} + t_{(3)} + t_{(4)} + t_{(5)} + t_{(6)} \tag{6a}$$

$$E = 2Pt_{(1)} + P\left(t_{(2)} + t_{(3)} + t_{(4)} + t_{(5)} + t_{(6)}\right). \tag{6b}$$

2) Constraints: The time completion and energy consumption are optimized under 2 sets of constraints that are derived below. The first set of constraints reflects the fact that, for an optimal operation, the number of bits transmitted to the relay should be equal to the number of bits that the relay transmits. The second set states that the number of bits transmitted by U1 and U2 should be equal to b_1 and b_2 respectively.

a) First set of constraints: We distinguish between 2 types of data: the data that is carried though the relay and the data that is sent through a direct link. The first type includes BBs (1),(4),(5),(6) and only the link to the relay in BBs (2),(3). The second type includes the direct link in BBs (2),(3).

Furthermore, in the first type of traffic, we distinguish between the BBs involving uplink transmissions, i.e. BBs (1)-(3), and the BBs involving downlink transmissions, i.e. BBs (4)-(6). We compute the number of bits sent in the uplink.

- BB (1). We denote R_i^{mac} as the maximal achievable rate for the transmission from Ui to Uj ($j \neq i$) (MA channel). Note that R_i^{mac} should satisfy the MA channel constraints in (9). The number of bits received at the RS intended to U1 is equal to $t_{(1)} R_2^{mac}$. Likewise, the number of bits received at the RS intended to U2 to equal to $t_{(1)} R_1^{mac}$.
- BB (2),(3). The number of bits received from U2 is $t_{(3)} C(\gamma_2)$. However, the relay only forwards a part of the corresponding information as the other part is transmitted in the direct link. The optimal strategy dictates that the relay should forward $t_{(3)}(C(\gamma_2) - C(\gamma_0))$ bits to U1.

Likewise, the number of bits that is forwarded by the relay to U2 is $t_{(2)}(C(\gamma_1) - C(\gamma_0))$.

Denoting Q_1^u and Q_2^u as the total number of bits to be forwarded at the RS to U1 and U2 resp., we have:

$$Q_1^u = t_{(1)} R_2^{mac} + t_{(3)} \left[C\left(\gamma_2\right) - C\left(\gamma_0\right) \right] \tag{7a}$$
$$Q_2^u = t_{(1)} R_1^{mac} + t_{(2)} \left[C\left(\gamma_1\right) - C\left(\gamma_0\right) \right]. \tag{7b}$$

The number of bits sent in the downlink from the relay is

- BB (4). From [8], the maximal achievable rate for each link is $C(\gamma_1)$ and $C(\gamma_2)$, i.e. the maximal achievable rate for each individual link. Hence, the number of bits sent to U1 is $t_{(4)}C(\gamma_1)$ while the number of bits sent to U2 is $t_{(4)}C(\gamma_2)$.
- BB (5)-(6). The number of bits sent to U1 is $t_{(6)}C(\gamma_1)$, while the number of bits sent to U2 is $t_{(5)}C(\gamma_2)$.

Denoting Q_1^d and Q_2^d as the total number of bits to be sent from the RS to U1 and U2 resp., we have:

$$Q_1^d = \left(t_{(4)} + t_{(6)}\right) C\left(\gamma_1\right), \quad Q_2^d = \left(t_{(4)} + t_{(5)}\right) C\left(\gamma_2\right). \tag{8}$$

Because the amount of uplink data should be equal to the amount of downlink data for each user, we get the first set of constraints: $Q_1^d = Q_1^u = Q_1$ and $Q_2^d = Q_2^u = Q_2$.

b) Second set of constraints: This set of constraints state that the total number of bits transmitted to Ui should be equal to b_i. Considering $Q_1^d = Q_1^u = Q_1$, $b_1 = Q_2 + t_{(2)} C\left(\gamma_0\right)$ where $t_{(2)} C\left(\gamma_0\right)$ is the number of bits transmitted through the direct link in BB (2). Likewise, $b_2 = Q_1 + t_{(3)} C\left(\gamma_0\right)$, where $t_{(2)} C\left(\gamma_0\right)$ is the number of bits transmitted through the direct link in BB (3). Using equation (7), we get the following set of constraints: $b_1 = t_{(1)} R_1^{mac} + t_{(2)} C\left(\gamma_1\right), b_2 = t_{(1)} R_2^{mac} + t_{(3)} C\left(\gamma_2\right)$.

3) Optimization Criterion:

$$\begin{aligned}
&\min_{t_{(i)} \geq 0} \quad T \quad \text{or} \quad E \\
&s.t. \quad R_1^{mac} + R_2^{mac} \leq C\left(\gamma_1 + \gamma_2\right) \\
&R_1^{mac} \leq C\left(\gamma_1\right), \quad R_2^{mac} \leq C\left(\gamma_2\right) \\
&b_1 = t_{(1)} R_1^{mac} + t_{(2)} C\left(\gamma_1\right), b_2 = t_{(1)} R_2^{mac} + t_{(3)} C\left(\gamma_2\right) \\
&\left(t_{(4)} + t_{(6)}\right) C\left(\gamma_1\right) = t_{(1)} R_2^{mac} + t_{(3)} \left[C\left(\gamma_2\right) - C\left(\gamma_0\right) \right] \\
&\left(t_{(4)} + t_{(5)}\right) C\left(\gamma_2\right) = t_{(1)} R_1^{mac} + t_{(2)} \left[C\left(\gamma_1\right) - C\left(\gamma_0\right) \right].
\end{aligned} \tag{9}$$

This is a quadratic optimization problem [9].

VI. Numerical Results

We define the normalized completion time and energy consumption as: $T_{\text{nor}} = T/(b_1 + b_2)$ and $E_{\text{nor}} = E/P\left(b_1 + b_2\right)$. T_{nor} and E_{nor} depend on the ratio between packet sizes b_1/b_2, and not the individual packet sizes. We present the relation between T_{nor}, E_{nor} and $\frac{b_1}{b_2}$ in Fig. 3 and Fig. 4. S1 stands for $\gamma_0 = 0\text{dB}, \gamma_1 = 30\text{dB}, \gamma_2 = 30\text{dB}$. S2 is with $\gamma_0 = 0\text{dB}, \gamma_1 = 30\text{dB}, \gamma_2 = 10\text{dB}$. S3 stands for $\gamma_0 = 10\text{dB}, \gamma_1 = 30\text{dB}, \gamma_2 = 20\text{dB}$. S4 represents $\gamma_0 = 20\text{dB}, \gamma_1 = 30\text{dB}, \gamma_2 = 30\text{dB}$. Here, we only draw the case when $\gamma_0 < \min\{\gamma_1, \gamma_2\}$. If $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, the AF and DF both degrade to direct transmission (when $\gamma_0 \geq \min\{\gamma_1, \gamma_2, 0\text{dB}\}$, the direct transmission CB is the best choice for AF), the corresponding comparison is not included.

In most cases the completion time of DF is shorter than AF, but AF can achieve a shorter completion time than DF for some values of b_1/b_2. Then if we adaptively communicate between AF and DF, the completion time is the lowest envelop of the completion time curves of AF and DF. These results can help the users do spectrum reservation and traffic shaping as mentioned in the introduction part. However, the DF always has smaller energy consumption than AF as Fig. 4 shows.

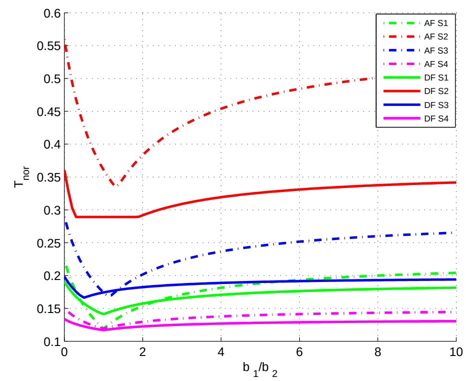

Figure 3. The relation between T_{nor} and $\frac{b_1}{b_2}$

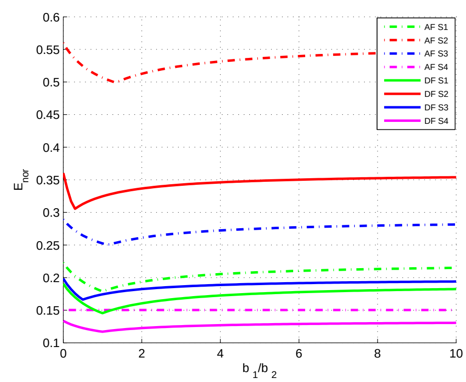

Figure 4. The relation between E_{nor} and $\frac{b_1}{b_2}$

VII. Conclusion

In this paper, we have formulated the total completion time minimization problem and the energy consumption minimization problem for bidirectional orthogonal relay network. We have analyzed the optimal strategy of AF with 4 available CBs: TDBC two-way relaying, MABC two-way relaying, one-way relaying and direct transmission. For DF, in order to decrease the variables needed to be optimized, our formulation is mainly based on the BBs. Numerical results reveal the relationship between the normalized total completion time (or energy consumption) and the ratio of packet sizes. This relationship provides a possibility for an adaptive communication between AF and DF to achieve better performance. The study in this paper can also guide the design of the reservation time or the traffic shaping in realistic networks.

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Appendix A
Proof of Proposition 1

We conduct the proof by comparing the direct transmission CB with the one-way relay, the MABC two-way relay and the TDBC two-way relay CBs, respectively.

We first look at the relation between one-way relaying and direct transmission. Let U1 be the source node. If the first phase of one-way relay requires time τ_1, the source node will transmit $\tau_1 \max\{C(\gamma_0), C(\gamma_1)\}$ bits. $\tau_1 C(\gamma_0)$ bits will be decoded by the destination node at the end of phase 1. When $\gamma_0 \geq \gamma_1$, obviously the total information should be sent through the direct link. When $\gamma_1 > \gamma_0$, the optimal transmission strategy is as follows: Because the relay node knows the SNR of the direct link, the relay node knows that $\tau_1 C(\gamma_0)$ bits have been successfully decoded by the destination node. Then the relay node only forwards the remaining information $\tau_1 [C(\gamma_1) - C(\gamma_0)]$ in the second phase. Assume the second phase duration of one-way relay is τ_2, then $\tau_1 [C(\gamma_1) - C(\gamma_0)] = \tau_2 C(\gamma_2)$. The achievable rate of U1 can be obtained

$$\hat{R}^c_{\mathrm{U1}} = \frac{\tau_1 C(\gamma_1)}{\tau_1 + \tau_2} = \frac{C(\gamma_1) C(\gamma_2)}{C(\gamma_1) + C(\gamma_2) - C(\gamma_0)} \tag{10}$$

which is the lower bound in [10]. From (10), when $\gamma_1 > \gamma_0 > \gamma_2$, we have $\hat{R}^c_{\mathrm{U1}} < C(\gamma_0)$, and one-way relay will not be chosen. When $\gamma_0 < \min\{\gamma_1, \gamma_2\}$, we have $\hat{R}^c_{\mathrm{U1}} > C(\gamma_0)$ and direct transmission will not be chosen.

In order to see the relation between MABC and a direct transmission, define R_i as the rate of Ui, t_u and t_d as the uplink duration and downlink duration, respectively. The following inequalities hold:

$$R_1 \leq \min\{t_u/(t_u+t_d)C(\gamma_1), t_d/(t_u+t_d)C(\gamma_2)\} \tag{11a}$$

$$R_2 \leq \min\{t_u/(t_u+t_d)C(\gamma_2), t_d/(t_u+t_d)C(\gamma_1)\}. \tag{11b}$$

Then $R_1 + R_2 \leq \min\{C(\gamma_1), C(\gamma_2)\}$ is satisfied. Therefore, if $\gamma_0 > \min\{\gamma_1, \gamma_2\}$, we have $R_1 + R_2 < C(\gamma_0)$. There exists a time sharing variable $\tau \in [0, 1]$ to ensure $R_1 < \tau C(\gamma_0), R_2 < (1-\tau) C(\gamma_0)$. Then a direct transmission from U1 with time sharing τ followed by a direct transmission from U2 with time sharing $1 - \tau$ outperforms MABC.

Using an similar analysis, it can be shown that if $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, TDBC is worse compared to direct transmission.

In summary, if $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, the direct transmission CB is the optimal choice. If $\gamma_0 < \min\{\gamma_1, \gamma_2\}$, direct transmission is worse than one-way relay, then direct transmission will not be selected.

Appendix B
Proof of Proposition 2

We observe that the CBs have BBs in common (see Fig. 2). We prove that the common BBs have the same performance when using DF relaying whatever the CB they are incorporated in. This means that we can convert the CB optimization problem to an optimization problem based on BBs. When $\gamma_0 \geq \min\{\gamma_1, \gamma_2\}$, only the direct transmission CB is selected and there is no common building blocks. When $\gamma_0 < \min\{\gamma_1, \gamma_2\}$, there are 3 available relay CBs: TDBC, MABC and one-way relay. The common BBs are (2), (3) and (4).

The destination node of DF decodes the information from the direct link and the relay links separately. Part of the information is sent through the direct link while the rest of the information is sent from the relay. This means that the signals sent through the links i.e. RS→U2 and U1→U2 are independent. Then BBs (2) and (3) are independent from BBs (4), (5) and (6) (this condition cannot be met in AF). Based on the above result, the common BBs (2) and (3) in TDBC and one-way relay have the same performance. Therefore, a single optimization variable can be used to account for the same BB.

BB (4) is another common block. It has the same function in both TDBC and MABC to transmit the information from the relay node to U1 and U2. TDBC and MABC both satisfy the condition of side information. Therefore, the rate of each link in BB (4) can achieve the capacity from [8] and BB (4) has the same performance in TDBC and MABC.

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