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Communication

Pulsed EM Scattering by Dipoles With Time-Varying Loads

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***Abstract*—Pulsed electromagnetic (EM) scattering by short-wire and small-loop antennas loaded by time-varying (TV) loads is analyzed with the aid of time-domain (TD) compensation theorems. TD analytical expressions describing the change of back-scattered EM fields from dipole antennas due to their TV load are given. Illustrative numerical examples are presented.**

***Index Terms*—time-domain analysis, electromagnetic scattering, antenna theory, compensation theorem, reconfigurable intelligent surface.**

I. Introduction

The back-scattered electromagnetic (EM) field from short dipoles and small loops can be controlled by lumped elements connected to their ports [1]. To quantify the impact of such loads on EM scattering by a scatterer, the compensation theorem can be used [2, Sec. II.14]. Its vector EM form has been presented in [3] and its generalized versions applicable to EM scattering and radiation of multiport antennas can be found in [4].

Early applications of compensation theorems have been limited to linear time-invariant EM problems under the assumption of *time-harmonic* EM fields [5]–[7]. More recently, to evaluate the effect of *nonlinear* loads, an EM scattering compensation theorem of the time-convolution type has been introduced in [8]. With the still increasing interest in modeling of time-varying (TV) EM systems and devices (e.g., reconfigurable intelligent surfaces [9]) in mind (e.g., [10], [11]), this paper provides time-domain (TD) compensations theorems quantifying the impact of TV loads on *pulsed* EM scattering by short-wire and small-loop antennas.

II. Problem Description

We shall describe the impact of a TV load on the transient, plane-wave EM scattering by a small receiving antenna. The

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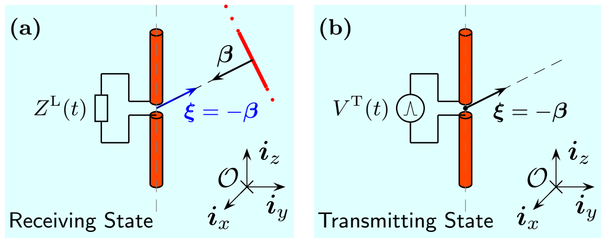

Fig. 1. Problem configuration with a short dipole. (a) Receiving antenna with a TV load; (b) Transmitting antenna excited by a voltage source.

analyzed receiving situation (denoted by R) with a loaded short-wire antenna is shown in Fig. 1a. The actual receiving state will be analyzed with the aid of the corresponding transmitting situation (denoted by T) (see Fig. 1b). The antenna is located in free space that is described by permittivity ϵ_0 and permeability μ_0. The corresponding EM wave speed and wave admittance is $c_0 = (\epsilon_0\mu_0)^{-1/2} > 0$ and $Y_0 = c_0\epsilon_0$, respectively.

Position in the problem configuration is specified by coordinates $\{x, y, z\}$ with respect to an orthogonal Cartesian reference frame that is defined by its origin $\mathcal{O}$ and the base vectors $\{\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z\}$. The corresponding position vector is then $\boldsymbol{r} = x\boldsymbol{i}_x + y\boldsymbol{i}_y + z\boldsymbol{i}_z$. The time coordinate is denoted by t. The differentiation with respect to t is ∂_t. The Heaviside unit-step function is denoted by $\mathrm{H}(t)$. The time convolution is represented by $*$ and the time-integration operator is then defined as $\partial_t^{-1} f(t) = f(t) * \mathrm{H}(t)$.

The receiving antenna is supposed to be irradiated by a uniform impulsive EM plane wave

$$\boldsymbol{E}^{\mathrm{i}}(\boldsymbol{r}, t) = \boldsymbol{\alpha} e^{\mathrm{i}}(t - \boldsymbol{\beta} \cdot \boldsymbol{r}/c_0), \tag{1}$$

where $\boldsymbol{\alpha}$ denotes the polarization vector, $\boldsymbol{\beta}$ is a unit vector that specifies the direction of propagation, and $e^{\mathrm{i}}(t)$ is a causal plane-wave signature. The scattered EM field in the receiving situation is defined as the difference between the total and incident EM fields, i.e.,

$$\{\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}\}(\boldsymbol{r}, t) = \{\boldsymbol{E}^{\mathrm{R}}, \boldsymbol{H}^{\mathrm{R}}\}(\boldsymbol{r}, t) - \{\boldsymbol{E}^{\mathrm{i}}, \boldsymbol{H}^{\mathrm{i}}\}(\boldsymbol{r}, t). \tag{2}$$

The outgoing scattered EM fields admit the far-field expansions [12, Sec. 29.1]

$$\{\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}\}(\boldsymbol{r}, t) = \frac{\{\boldsymbol{E}^{\mathrm{s};\infty}, \boldsymbol{H}^{\mathrm{s};\infty}\}(\boldsymbol{\xi}, t - |\boldsymbol{r}|/c_0)}{4\pi|\boldsymbol{r}|} \times [1 + O(|\boldsymbol{r}|^{-1})] \text{ as } |\boldsymbol{r}| \to \infty, \tag{3}$$

where $\boldsymbol{E}^{\mathrm{s};\infty}$ and $\boldsymbol{H}^{\mathrm{s};\infty} = Y_0\boldsymbol{\xi} \times \boldsymbol{E}^{\mathrm{s};\infty}$ are the TD electric- and magnetic-type far-field scattering amplitudes, respectively, and $\boldsymbol{\xi} = \boldsymbol{r}/|\boldsymbol{r}|$ denotes the unit vector in the direction of observation. A similar far-field expansion applies to the far-field radiation amplitudes, $\{\boldsymbol{E}^{\mathrm{T};\infty}, \boldsymbol{H}^{\mathrm{T};\infty}\}(\boldsymbol{\xi}, t)$, pertaining to the transmitting situation (see Fig. 1b). The evaluation of (the change of) the far-field scattering amplitudes at the backward direction $\boldsymbol{\xi} = -\boldsymbol{\beta}$ (see Fig. 1a) due to (the change of) the TV load is the main objective of this paper.

III. Problem Solution

The point of departure for our analysis is (the TD counterpart of) the compensation theorem derived in [4, Sec. 9.2.2]

$$\Delta \boldsymbol{E}^{\mathrm{s};\infty}(\boldsymbol{\xi}, t) * V^{\mathrm{T}}(t) = \boldsymbol{E}^{\mathrm{T};\infty}(\boldsymbol{\xi}, t) * \Delta V^{\mathrm{R}}(t), \tag{4}$$

where $\Delta \boldsymbol{E}^{\mathrm{s};\infty}$ denotes the desired change of the far-field scattering amplitude, $\Delta V^{\mathrm{R}}(t)$ is the corresponding change of the voltage across the TV load, $\boldsymbol{E}^{\mathrm{T};\infty}$ denotes the radiation amplitude in the transmitting state and, finally, $V^{\mathrm{T}}(t)$ denotes the excitation voltage pulse. The latter can be related to the excitation current, $I^{\mathrm{T}}(t)$, through the TD input impedance as $V^{\mathrm{T}}(t) = Z^{\mathrm{T}}(t) * I^{\mathrm{T}}(t)$.

We shall distinguish between the changes with respect to the (referential) receiving situations with the open-circuit, short-circuit and matched loads, i.e.,

$$\Delta^{\infty} \boldsymbol{E}^{\mathrm{s};\infty} = \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}(t)} - \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}\to\infty} \tag{5}$$

$$\Delta^{\circ} \boldsymbol{E}^{\mathrm{s};\infty} = \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}(t)} - \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}\to 0} \tag{6}$$

$$\Delta^{\bullet} \boldsymbol{E}^{\mathrm{s};\infty} = \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}(t)} - \boldsymbol{E}^{\mathrm{s};\infty}|_{Z^{\mathrm{L}}= Z^{\mathrm{T}}} \tag{7}$$

respectively.

A. Short Wire Loaded by a Time-Varying Capacitor

We assume that the wire antenna is relatively short. Consequently, its TD input impedance can be approximated by $Z^{\mathrm{T}}(t) = C_{\mathrm{a}}^{-1}\mathrm{H}(t)$, where C_{a} is the wire's capacitance [13, Sec. 10.3]. Its radiation amplitude, $\boldsymbol{E}^{\mathrm{T};\infty}$, can be expressed using the current distribution in state (T) (see [12, Sec. 26.9], for example), while the change of the load voltage in state (R), $\Delta V^{\mathrm{R}}(t)$, can be determined using Thévenin's equivalent circuit. The latter is specified by its internal impedance and the voltage source strength, $V^{\mathrm{G}}(t)$, that is related to the radiation amplitude at $\boldsymbol{\xi} = -\boldsymbol{\beta}$ through the antenna self-reciprocity relation $V^{\mathrm{G}}(t) * I^{\mathrm{T}}(t) = \mu_0^{-1} e^{\mathrm{i}}(t) * \boldsymbol{\alpha} \cdot \partial_t^{-1} \boldsymbol{E}^{\mathrm{T};\infty}(-\boldsymbol{\beta}, t)$ (see [4, Eq. (5.6)] and [14, Eq. (23)], [15]). Accordingly, if the wire antenna is loaded by a TV capacitor, $C^{\mathrm{L}}(t)$, the change of the co-polarized, back-scattered far-field amplitude can be described by

$$\boldsymbol{\alpha} \cdot \Delta^{\infty} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = \gamma^{\mathrm{e}}\, \partial_t^2 \{e^{\mathrm{i}}(t)[\Gamma^{\mathrm{e}}(t) - 1]\} \tag{8}$$

$$\boldsymbol{\alpha} \cdot \Delta^{\circ} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = \gamma^{\mathrm{e}}\, \partial_t^2 \{e^{\mathrm{i}}(t)[\Gamma^{\mathrm{e}}(t) + 1]\} \tag{9}$$

$$\boldsymbol{\alpha} \cdot \Delta^{\bullet} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = \gamma^{\mathrm{e}}\, \partial_t^2 [e^{\mathrm{i}}(t)\Gamma^{\mathrm{e}}(t)] \tag{10}$$

where we used $\gamma^{\mathrm{e}} = k^2 \mu_0 C_{\mathrm{a}} (\boldsymbol{\alpha} \cdot \boldsymbol{\ell})^2/2$ (in $\mathrm{m} \cdot \mathrm{s}^2$). Here, $\boldsymbol{\ell}$ denotes the vectorial length of the wire and factor k depends on the actual current distribution (it takes the values $k = \{1, 1/2, 2/\pi\}$ for the uniform, triangular and cosine spatial current distribution, respectively). Furthermore, we have used

$$\Gamma^{\mathrm{e}}(t) = \frac{C_{\mathrm{a}} - C^{\mathrm{L}}(t)}{C_{\mathrm{a}} + C^{\mathrm{L}}(t)}. \tag{11}$$

The TD response of a linear TV system can be expressed through a *superposition integral*, the kernel of which, generally, is *not* time shift invariant [16]–[18]. A specific example from this category is the TD electric-field response of a nondispersive, TV high-dielectric thin layer as described by [10, Eq. (18)]. Notably, (the change of) the back-scattered TD field response of a short wire antenna loaded by a TV capacitor as given by (8)–(10) readily follows upon differentiating the *product* of the (input) plane-wave pulse with a simple rational function of $C^{\mathrm{L}}(t)$.

B. Small Loop Loaded by a Time-Varying Inductor

A dual analysis can be carried out for a loop antenna. Indeed, assuming a relatively small loop of a thin wire carrying a uniform current distribution [12, Sec. 26.10] and using the pertaining compensation theorem [4, Sec. 9.2.1] with the equivalent Norton circuit representation [19], we may write (cf. (8)–(10))

$$\boldsymbol{\alpha} \cdot \Delta^{\infty} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = -\gamma^{\mathrm{m}}\, \partial_t^2 \{e^{\mathrm{i}}(t)[\Gamma^{\mathrm{m}}(t) + 1]\} \tag{12}$$

$$\boldsymbol{\alpha} \cdot \Delta^{\circ} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = -\gamma^{\mathrm{m}}\, \partial_t^2 \{e^{\mathrm{i}}(t)[\Gamma^{\mathrm{m}}(t) - 1]\} \tag{13}$$

$$\boldsymbol{\alpha} \cdot \Delta^{\bullet} \boldsymbol{E}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) = -\gamma^{\mathrm{m}}\, \partial_t^2 [e^{\mathrm{i}}(t)\Gamma^{\mathrm{m}}(t)] \tag{14}$$

where $\gamma^{\mathrm{m}} = (\mu_0/L_{\mathrm{a}})[(\boldsymbol{\beta} \times \boldsymbol{\alpha}) \cdot \boldsymbol{\mathcal{A}}]^2/2c_0^2$. Here, L_{a} denotes the inductance of a small loop [13, Sec. 10.11] and $\boldsymbol{\mathcal{A}}$ is its vectorial area. Finally, the corresponding TD reflection coefficient reads

$$\Gamma^{\mathrm{m}}(t) = \frac{L_{\mathrm{a}} - L^{\mathrm{L}}(t)}{L_{\mathrm{a}} + L^{\mathrm{L}}(t)}, \tag{15}$$

where $L^{\mathrm{L}}(t)$ denotes the TV load inductance.

IV. Numerical Results

In this section we shall provide illustrative numerical examples that validate the closed-form, TD analytical formulas derived in the paper. To that end, we shall analyze the pulsed EM field scattered by a short wire antenna loaded by a TV capacitor. The antenna is irradiated by a uniform plane wave, the signature of which is described by

$$\begin{aligned} e^{\mathrm{i}}(t) = 2e_{\mathrm{m}} \big[& (t/t_{\mathrm{w}})^2\, \mathrm{H}(t) - 2\,(t/t_{\mathrm{w}} - 1/2)^2\, \mathrm{H}\,(t/t_{\mathrm{w}} - 1/2) \\ & + 2\,(t/t_{\mathrm{w}} - 3/2)^2\, \mathrm{H}\,(t/t_{\mathrm{w}} - 3/2) \\ & - (t/t_{\mathrm{w}} - 2)^2\, \mathrm{H}\,(t/t_{\mathrm{w}} - 2) \big], \end{aligned} \tag{16}$$

where we take the unit amplitude, $e_{\mathrm{m}} = 1.0\,\mathrm{V/m}$ and the pulse time width, $t_{\mathrm{w}} > 0$, is chosen such that the wire antenna is *relatively* small, i.e., $c_0 t_{\mathrm{w}} = 50\,\ell$, with $\ell = |\boldsymbol{\ell}| = 100\,\mathrm{mm}$. To reveal TD EM scattering effects, the plane-wave pulse shape (16) is chosen to be continuously differentiable, while its second derivative (cf., (8)–(10)) shows jump discontinuities at $t/t_{\mathrm{w}} = \{0, 1/2, 3/2, 2\}$. The pulse shape and its (scaled) second time derivative are shown in Fig. 2. In the examples that follow, we shall calculate (the change of) the *polar* component of the far-field amplitude with respect to the *open-circuit* reference, i.e., $\Delta^{\infty} E_{\theta}^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t)$ (cf. (8)) for $\theta = \pi/2$, noting the rotational symmetry of the wire about the z-axis (see Fig. 1). For the sake of validation, we shall first evaluate

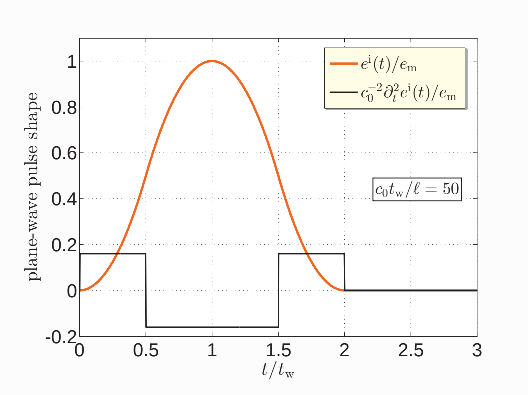

Fig. 2. Plane-wave pulse shape and its scaled second time derivative.

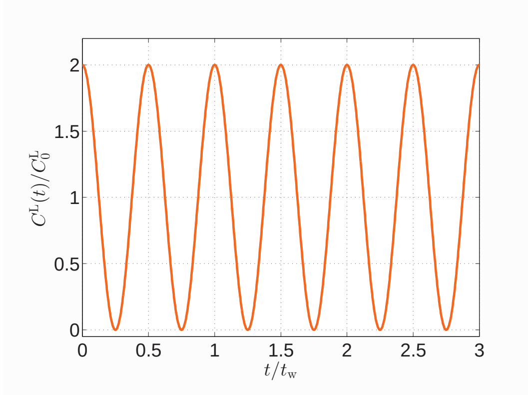

Fig. 4. Time-varying load capacitance.

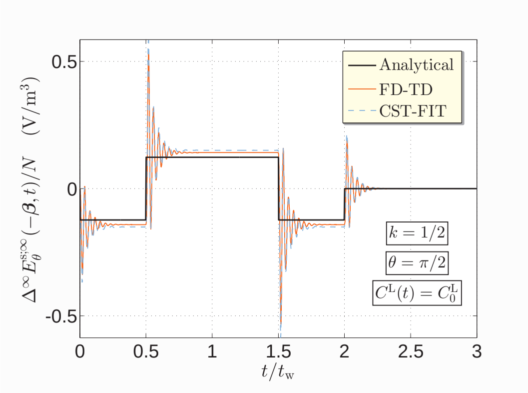

Fig. 3. The change of the EM field (with respect to the open-circuit reference) as back-scattered by the wire antenna loaded by the *static* capacitance $C_0^{\mathrm{L}} = 1.0\,\mathrm{pF}$. The pulse shapes were calculated analytically via (8) and numerically using FD-TD and CST-FIT.

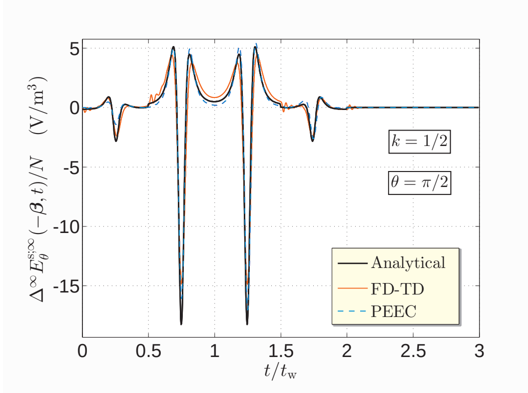

Fig. 5. The change of the EM field (with respect to the open-circuit reference) as back-scattered by the wire antenna loaded by the TV capacitance (17). The pulse shapes were calculated analytically via (8) and numerically using FD-TD and PEEC.

the pulsed EM scattering by the antenna loaded by a time-invariant load capacitance, i.e., $C^{\mathrm{L}}(t) = C_0^{\mathrm{L}} = 1.0\,\mathrm{pF}$, for which Γ^{e} is time independent (see (11)). Consequently, in line with (8), (the change of) the back-scattered TD field has the shape of the (scaled) second time derivative of the incident pulse, i.e., $\Delta^\infty E_\theta^{\mathrm{s};\infty}(-\boldsymbol{\beta}, t) \propto \partial_t^2 e^{\mathrm{i}}(t)$. Figure 3 shows the back-scattered signals (scaled by $N = k^2(C_{\mathrm{a}}/\epsilon_0)(\boldsymbol{\alpha} \cdot \boldsymbol{\ell})^2)$ as predicted via the approximate analytical expression (8) for the triangular current distribution ($k = 1/2$) and using the Aalborg University's in-house finite-difference time-domain (FD-TD) code (e.g., [20]). The FD-TD model of the dipole has been modeled as two wires with the capacitor load in between, with a cell size $5\,\mathrm{mm}$ and a time step $9.621\,\mathrm{ps}$. For the sake of validation, the pulsed response has also been computed with the aid of the finite integration technique (FIT) as implemented in CST Microwave Suite®. Apart from the oscillations that can be associated with the dipole's half-wave resonance, the analytical model provides a reasonable estimate. It is further observed that the assumption of the triangle current distribution underestimates the strength of the scattering effect. It has been verified that a better result can be achieved with a higher k of value between $1/2$ (= triangle distribution) and $2/\pi$ (= cosine distribution).

Next, the antenna under consideration is loaded by a lumped capacitor, the capacitance of which is time dependent. For example, we take a harmonically modulated load, i.e.,

$$C^{\mathrm{L}}(t) = C_0^{\mathrm{L}}[1 + \cos(4\pi t/t_{\mathrm{w}})]\mathrm{H}(t), \tag{17}$$

with $C_0^{\mathrm{L}} = 1.0\,\mathrm{pF}$ (see Fig. 4). In this case, Γ^{e} is a function of time (see (11)), which leads to a time-modulated back-scattered pulse, the shape of which corresponds to $\partial_t^2\{e^{\mathrm{i}}(t)[\Gamma^{\mathrm{e}}(t) - 1]\}$ (see (8)). As CST Microwave Suite® does not allow to incorporate a TV lumped load, this result has been verified using FD-TD and the partial element equivalent circuit (PEEC) method [21]. As can be seen from Fig. 5, the pulse shapes correlate very well, thereby successfully validating the approximate analytical model.

V. Conclusion

Novel TD, approximate analytical expressions describing the effect of TV lumped loads on the pulsed EM field as back-scattered by small wire and loop antennas have been presented. Since reconfigurable intelligent surfaces are typically synthesized in the form of ordered array structures consisting of small receiving antennas loaded by TV loads, it is anticipated that the presented results will help to understand the pertaining TD EM scattering effects, thus enabling their efficient design. The validity of the results has been demonstrated numerically on illustrative examples.

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