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# Filtration of several uncharged solutes on reverse osmosis membrane: Theory modification based on slip boundary



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## Introduction

□ Nanofiltration is a promising membrane technique with a large number of applications in treatment of process water, drinking water and wastewater.

□ More specifically, nanofiltration can be used to remove small organic molecules. Models for nanofiltration are usually based on non-slip condition which can be modified to slip condition as a rational assumption.

□ In this paper, the effects of slip length on the rejection of uncharged solutes such as glycerol and glucose along the effective membrane thickness have been investigated. The Bowen nanofiltration transport model based on Hagen-Poiseuille equation for solvent velocity (no slip condition) has been modified by several slip conditions and a new model allowing calculation of uncharged solute rejection on the basis of a binary membrane parameter (slip length and pore radius) was developed.

## Model development

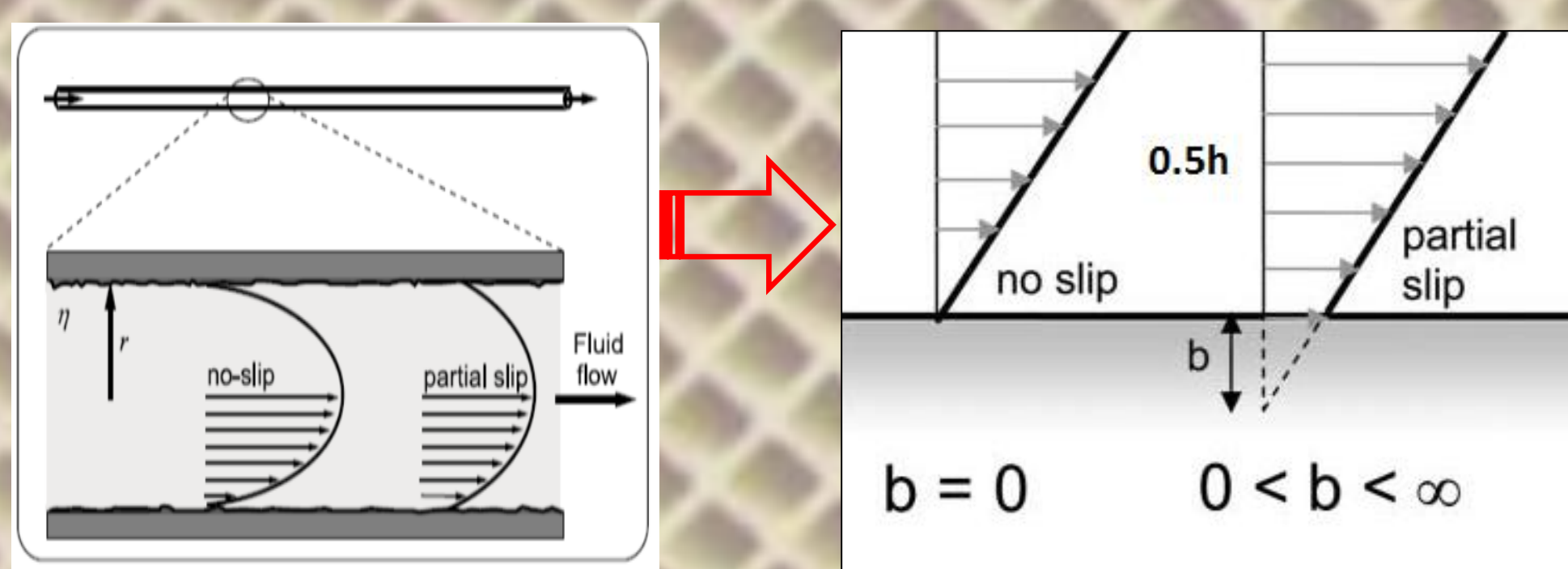


Fig. 1. Liquid slip in nanopore

### 1. The fundamental transport equation to be used for uncharged solutes

$$J_s = K_c c u - \frac{c D_p}{RT} \frac{d\mu}{dx} \quad (1)$$

$$\mu = RT \ln a + V_s P + \text{Constant} \quad (2)$$

$$\rightarrow J_s = K_c c u - D_p \frac{dc}{dx} - \frac{c D_p}{RT} V_s \frac{dP}{dx} \quad (3) \quad \& \quad J_s = c_p u \quad (4)$$

### 2. The average velocity equation in a rectangular nanopore with considering slip length (b)

$$u = \frac{h^2}{12\eta} \left( 1 + \frac{6b}{h} \right) \left( \frac{-dP}{dx} \right) \quad (5)$$

$$\rightarrow \left( \frac{-dP}{dx} \right) = \frac{12\eta}{h^2 + 6bh} u \quad (6)$$

### 3. Develop the Rejection Equation

$$(3), (4), (6) \rightarrow c_p = K_c c u - D_p \frac{dc}{dx} + \frac{c D_p}{RT} V_s \frac{12\eta}{h^2 + 6bh} u \quad (7)$$

$$\text{If } \alpha = \frac{D_p}{RT} V_s \frac{12\eta}{h^2 + 6bh} \quad \& \quad \beta = K_c + \alpha$$

$$\text{and } \epsilon = \frac{(1 - \frac{1}{\beta\phi})}{\exp(\frac{u\beta z}{D_p})} + \frac{1}{\beta\phi}$$

$$\text{Rej} = 1 - \frac{1}{\epsilon}$$

## Model procedure



## Model Input

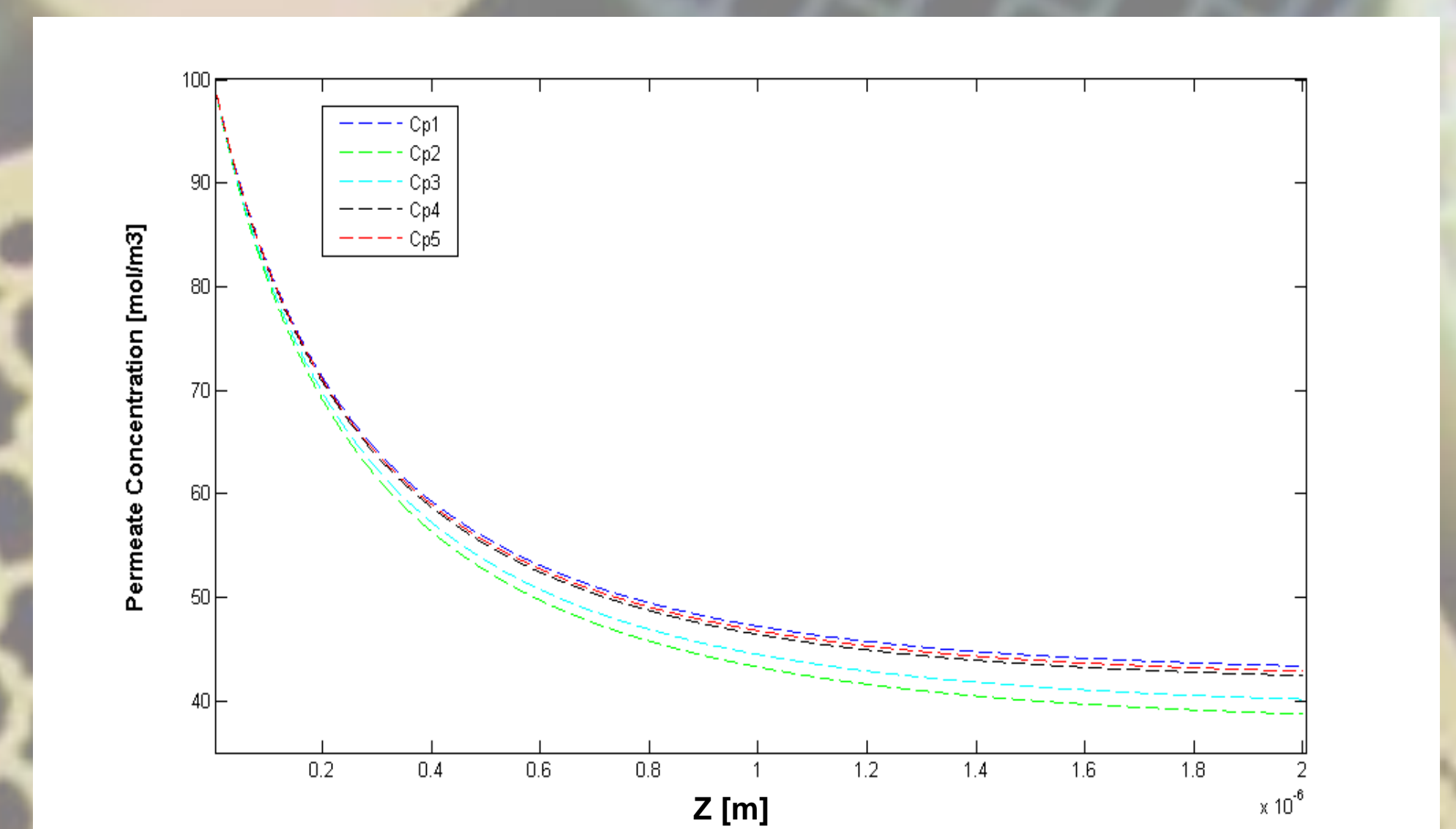
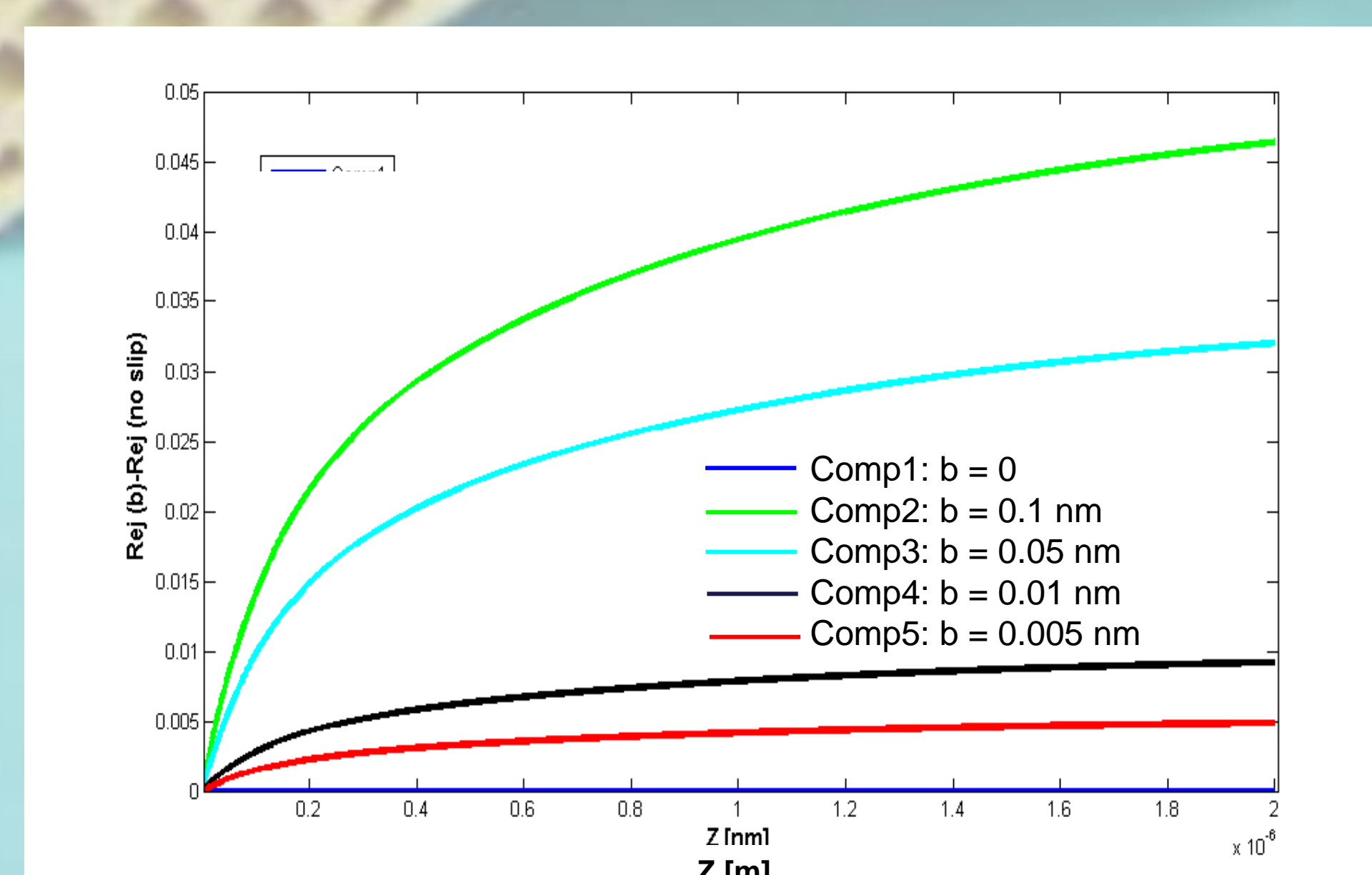
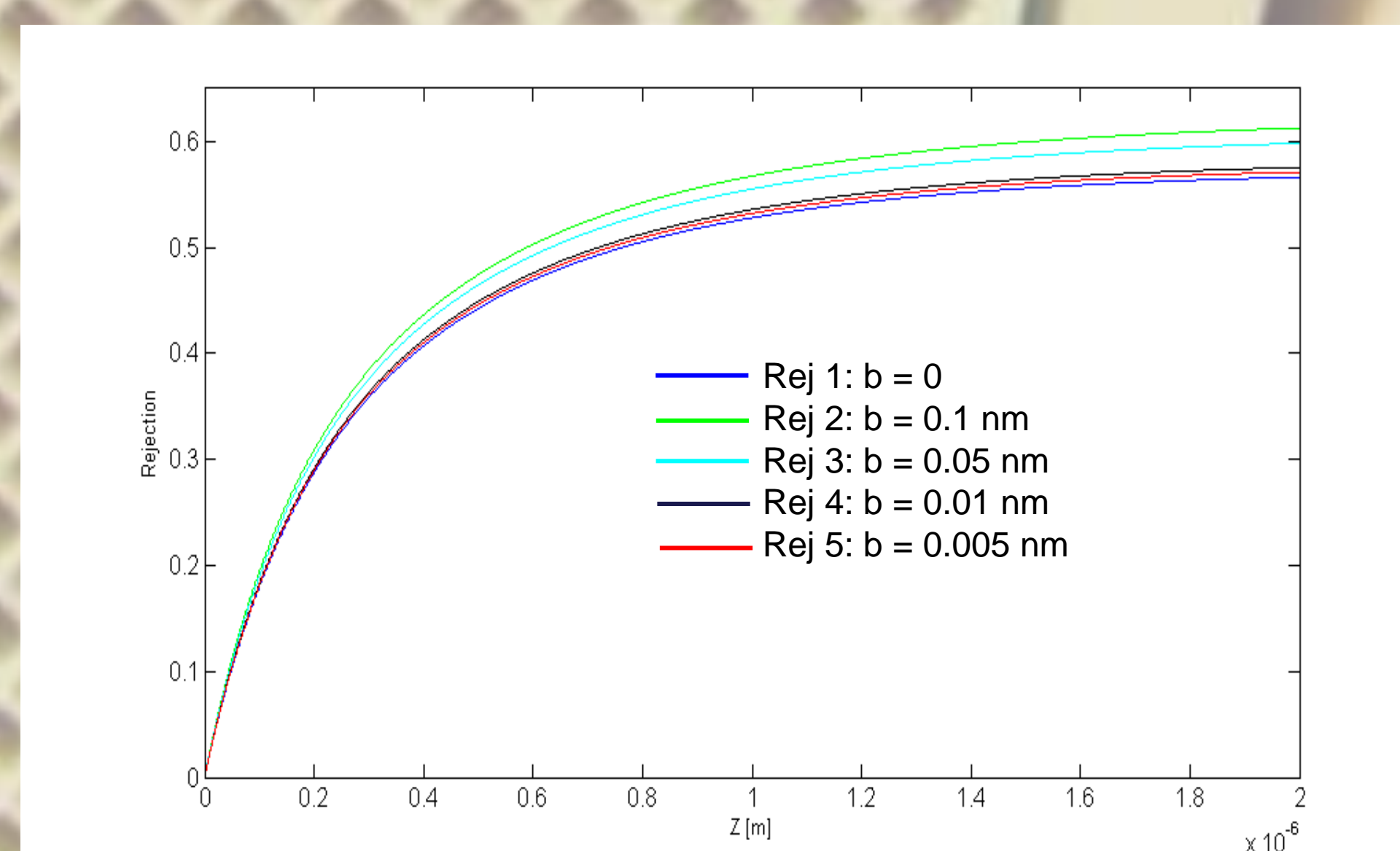
| Parameter      | Value                        | Description                    |
|----------------|------------------------------|--------------------------------|
| u              | 1 [m h <sup>-1</sup> ]       | solvent velocity               |
| H              | 1[nm]                        | Pore height                    |
| b*             | 0.1-0.005 [nm]               | Slip length                    |
| C <sub>f</sub> | 10 [mol m <sup>-3</sup> ]    | bulk feed concentration        |
| T              | 298.15 [K]                   | absolute temperature           |
| η              | 0.001 [N s m <sup>-2</sup> ] | solvent viscosity within pores |
| Z              | 2 [μm]                       | Width of pore                  |
| N <sub>p</sub> | 10000000                     | The points number in pore      |

| Solute   | D <sub>∞</sub> (10 <sup>-9</sup> m <sup>2</sup> s <sup>-1</sup> ) | r <sub>s</sub> (nm) | V <sub>s</sub> (cm <sup>3</sup> mol <sup>-1</sup> ) | Ref             |
|----------|---|---------------------|---|-----------------|
| Glycerol | 0.95  | 0.26                | 70.8  | Kiyosawa (1991) |
| Glucose  | 0.69  | 0.365               | 110   | Birch (1997)    |

## Results

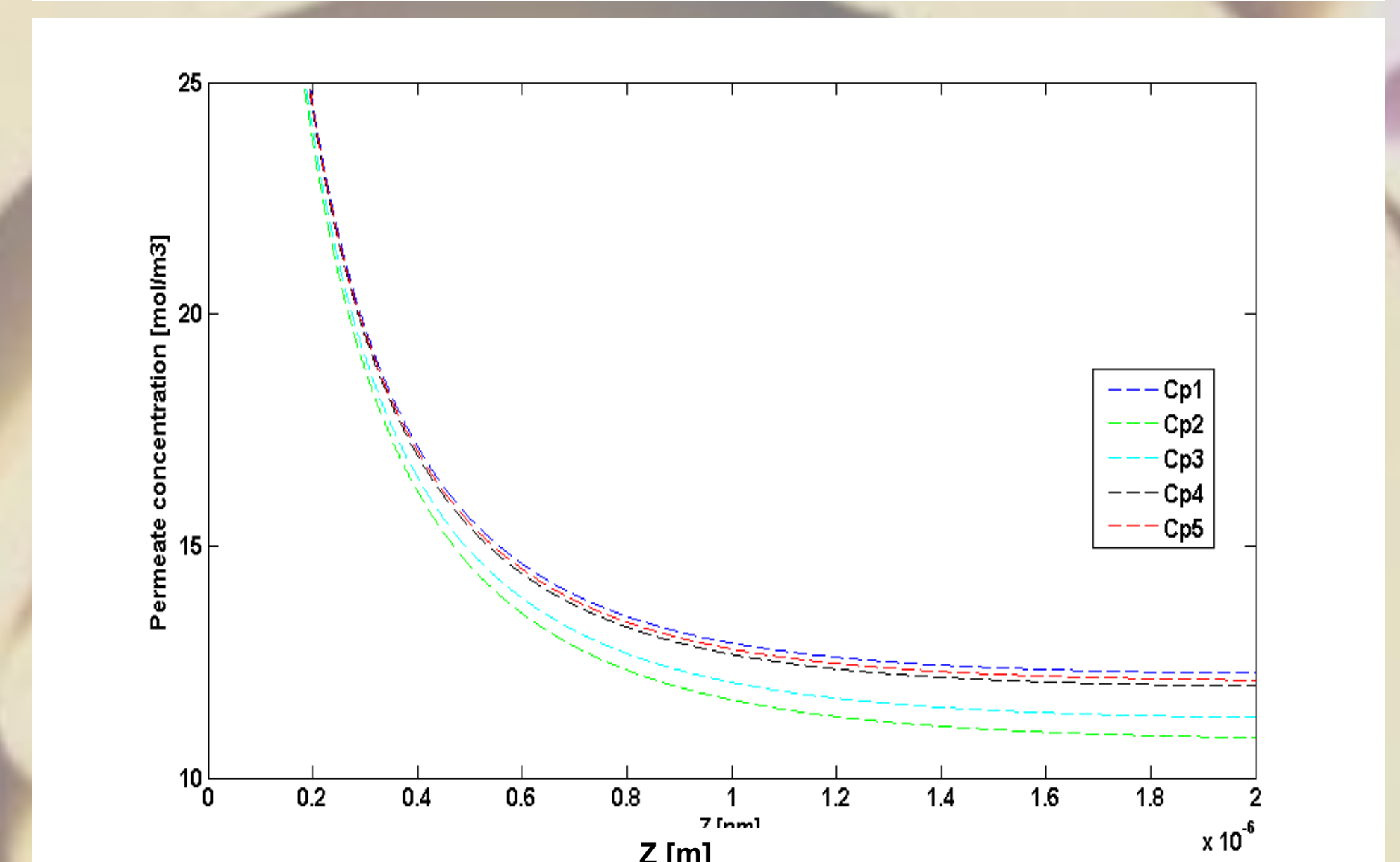
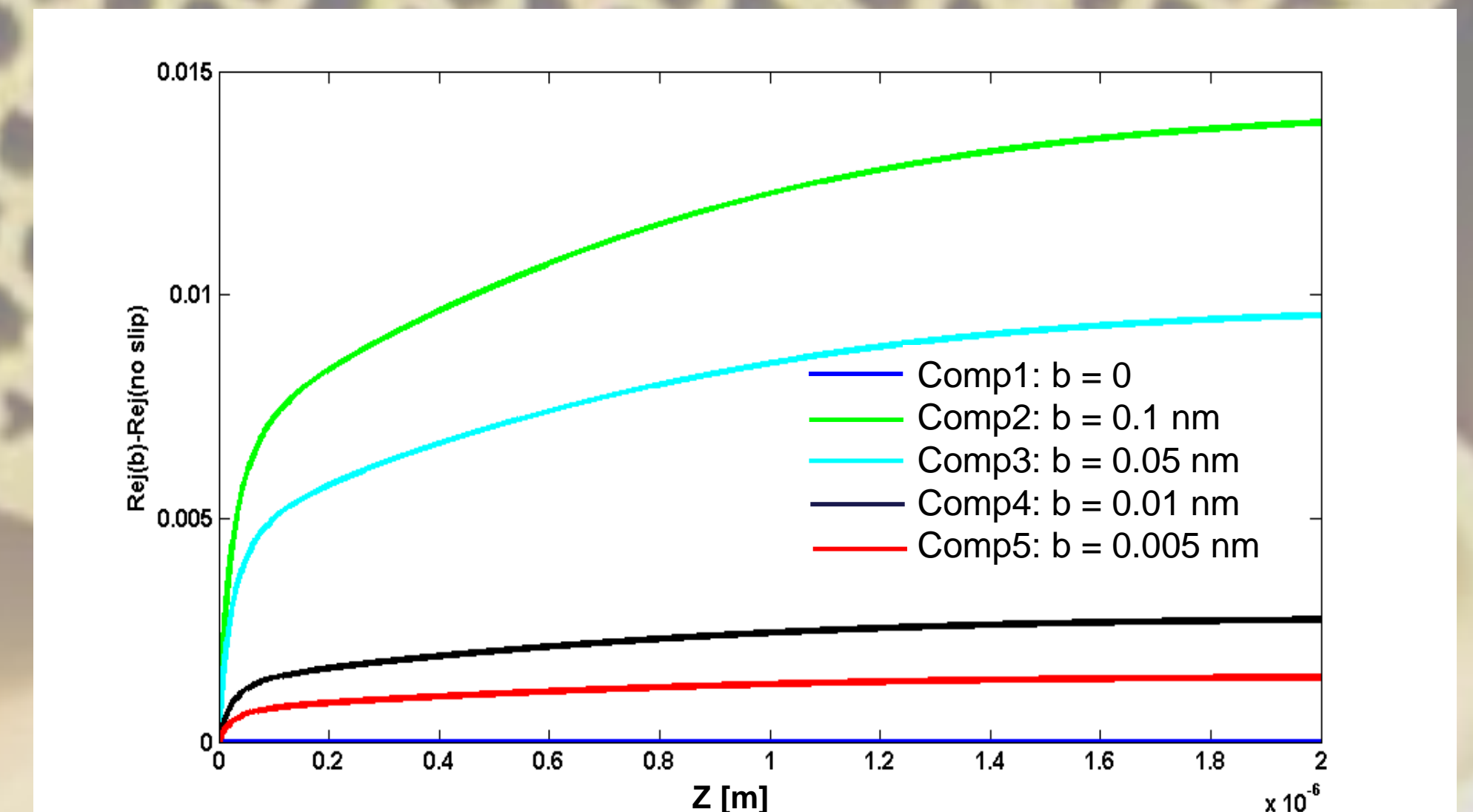
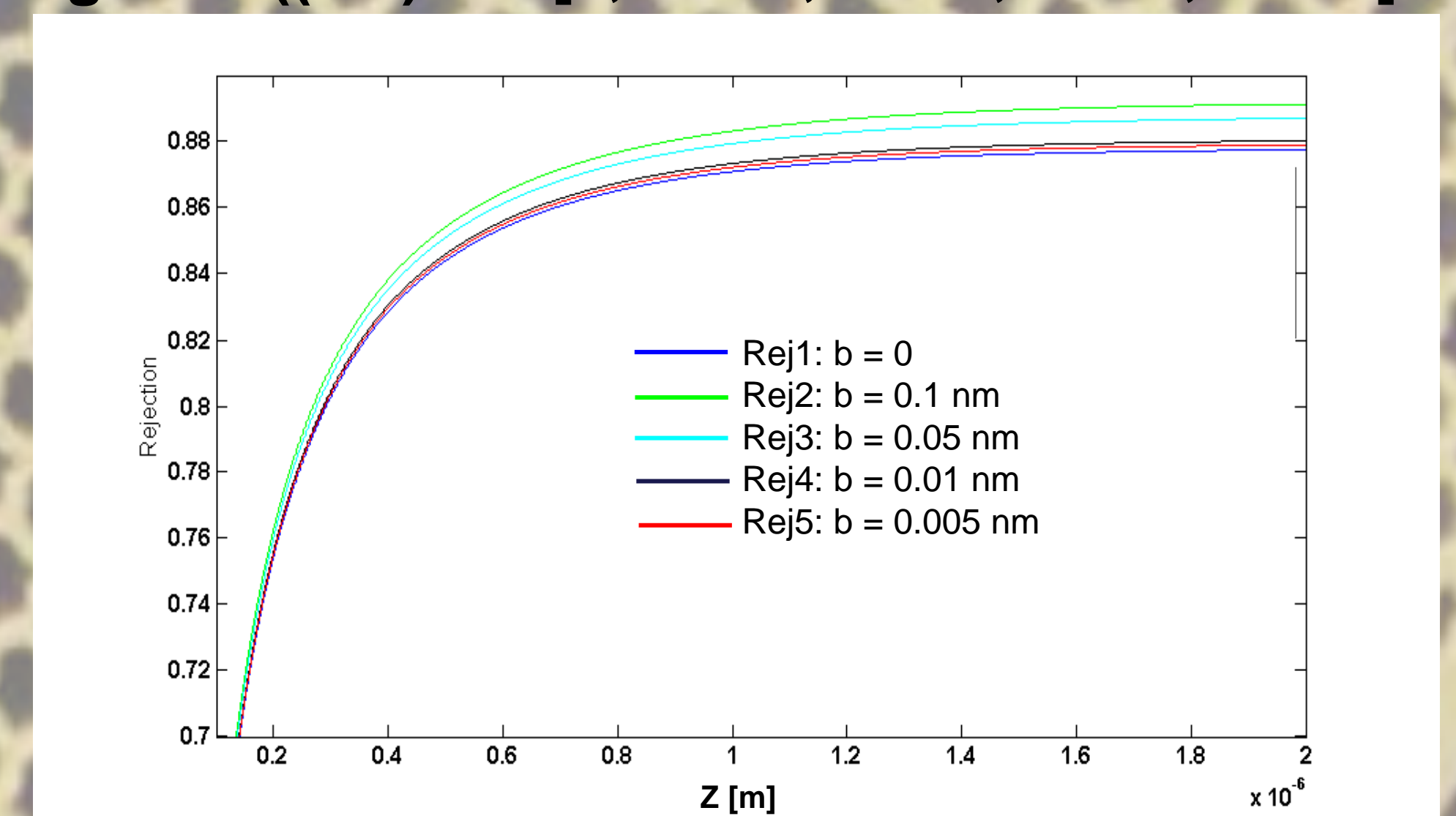
### 1. Glycerol

Rej/ΔRej/ Conc. vs. Pore Length Z for various slip lengths b ((1-5): b= [0, 1e-10, 5e-11, 1e-11, 5e-12] m)



### 2. Glucose

Rej/ΔRej/ Conc. vs. Pore Length Z for various slip lengths b ((1-5): b= [0, 1e-10, 5e-11, 1e-11, 5e-12] m)



## Conclusion

The obtained results show that by increasing the slip length which has been related to the pore size and membrane materials, the solute rejection can be increased up to approximately 5%. Concerning these results, the solute rejection can be increased by synthesis of a membrane layer with optimum conditions according to slip length.

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