Low-Complexity MMSE Precoding for Coordinated Multipoint with Per-Antenna Power Constraint

Kim, Tae Min ; Sun, Fan; Paulraj, Arogyaswami

Published in:
I E E E Signal Processing Letters

DOI (link to publication from Publisher):
10.1109/LSP.2013.2246152

Publication date:
2013

Document Version
Early version, also known as pre-print

Low-Complexity MMSE Precoding for Coordinated Multipoint with Per-Antenna Power Constraint

Tae Min Kim, Fan Sun, and Arogyaswami Paulraj

***Abstract*—We propose a low-complexity minimum mean square error (MMSE) transmit filter design for the coordinated beamforming (CB) in the coordinated multipoint (CoMP) under the practical per-antenna power constraint (PAPC). The proposed design is based on the non-linear Gauss-Seidel type algorithm in which the transmit filters for given receive filters are computed by iteratively updating the beamformer of each transmit antenna using simple closed-form expressions. The proposed approach can significantly reduce the overall complexity of the alternating optimization while preserving the optimality in the MSE sense.**

***Index Terms*—MMSE, Coordinated Beamforming, Per-Antenna Power Constraint, Non-linear Gauss-Seidel Algorithm**

I. Introduction

Coordinated multipoint (CoMP) transmission has recently attracted much attention as a means to mitigate the inter-cell interference (ICI) in the cellular networks by using the base stations (BSs) cooperation. When the BSs are connected over a limited backhaul, coordinated beamforming in the CoMP (CoMP-CB) can effectively reduce the ICI by jointly designing the transmit (Tx) filters at the BSs based on the shared channel state information (CSI) without sharing the user data across the BSs as in the joint processing [1].

We consider the downlink CoMP-CB based on the minimum mean square error (MMSE) approach. The MMSE criterion has been adopted in various MIMO beamforming scenarios including the MIMO broadcast channel (MIMO-BC) [2] and the interference channel (MIMO-IC) [3], [4] due to its practical importance. First, minimizing the MSE is closely related to minimizing the error rate of the system in the finite signal-to-noise ratio (SNR) regime. More importantly, it has been proven that the sum-rate maximization in the interfering broadcast channel (IBC) can be casted into the weighted MSE minimization with optimally adjusted weights [3]. Locally optimal MMSE Tx filters can be obtained efficiently using the alternating optimization [3], [4].

Computational complexity of the alternating optimization is dominated by computing the MMSE Tx filters for the given Rx filters [5]. Though it is a convex problem under the per-BS power constraint (PBPC) or the per-antenna power constraint (PAPC), directly finding the Tx filters with standard convex programming solvers costs significant computations. Therefore, it has been of practical interest to develop a numerically efficient algorithm to compute the Tx filters for the alternating optimization. To this end, under the PBPC, Shen *et al.* proposed an algorithm to obtain the dual variable corresponding to the PBPC by solving a polynomial equation [4].

However, while the PAPC is more important constraint in practice, an efficient low-complexity Tx filter design for the CoMP-CB under the PAPC has not been studied much in the literature to the best of the author's knowledge. Following an approach similar to [4] would require to solve complicated multivariate polynomial equations under the PAPC, which is known to be NP-hard [6]. Furthermore, one previous work in [7] proposed a heuristic low-complexity design by including an additional alternating step for the dual variables, yet its convergence to a feasible solution is not guaranteed in high SNR or for a large number of Tx antennas. In [8], another low-complexity Tx filter design based on the uplink-downlink duality was proposed. However, their design focused on the single-cell multi-user MIMO and is not easily extensible to the CoMP-CB.

In this letter, we propose a low-complexity MMSE Tx filter design for the CoMP-CB under the PAPC with the guaranteed optimality at each alternating step. Observing the power constraints in each BS are naturally decoupled across the Tx antennas, we decompose the original convex problem into a set of sub-problems each involving the beamforming from only one antenna with the corresponding PAPC. Specifically, a non-linear Gauss-Seidel (NGS) type algorithm [9] is employed to iteratively update the beamformer of each Tx antenna. For given Rx filters, the proposed algorithm preserves the optimality in the MSE sense. For each sub-problem, we give a closed-form solution which can be computed efficiently by simple vector operations. While achieving the same MSE, complexity analysis and computer simulations show that the proposed algorithm significantly reduces the overall complexity, e.g., a CPU time reduction of over 99%, compared to the benchmark which directly finds the Tx filters using the standard convex programming solvers.

Notation: $[\cdot]_{mn}$, $(\cdot)^T$, $(\cdot)^\dagger$, and $||\cdot||_F$ denote the (m,n)-th element, the transpose, the conjugate transpose, the Frobenius norm of a vector/matrix, respectively. diag$(\cdot)$ is the diagonal matrix taking only the diagonal terms of a matrix. An $m \times m$ identity matrix is denoted by $\boldsymbol{I}_m$.

II. Preliminary

A. System Model

We consider a cooperative multi-cell network, where P base stations (BSs) are connected via backhaul and serve K user equipments (UEs) per cell. Let BS_p and UE_{pk} denote the p-th BS and the k-th UE in the p-th cell, respectively. Each BS_p and UE_{pk} is equipped with M transmit antennas and N receive antennas, respectively, and BS_p transmits $d \leq N$ independent data stream(s) to each UE_{pk} in its serving cell. The data vector for UE_{pk}, denoted by $\boldsymbol{s}_{pk} = [s_{1pk} \ \cdots \ s_{dpk}]^\dagger \in \mathbb{C}^{d\times 1}$, satisfies $\mathbb{E}[\boldsymbol{s}_{pk}] = \boldsymbol{0}$ and $\mathbb{E}[\boldsymbol{s}_{pk}\boldsymbol{s}_{pk}^\dagger] = \boldsymbol{I}_d$.

Let $\boldsymbol{H}_{pqk} \triangleq \beta_{pqk}\tilde{\boldsymbol{H}}_{pqk}$ be the $N \times M$ MIMO channel from BS_q to UE_{pk} where $\tilde{\boldsymbol{H}}_{pqk} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_N)$ and β_{pqk} is a

T. M. Kim and A. Paulraj are with the Dept. of EE, Stanford University, Stanford, CA 94305 (email: kimtm, apaulraj@stanford.edu), and F. Sun is with the Dept. of Electronic Systems, Aalborg University, Denmark (e-mail: fs@es.aau.dk). This work was supported in part by Samsung Scholarship, NSF Grant CCF-1256548, and the Danish Research Council for Technology and Production Grant 09-065920.

non-negative constant reflecting a large scale fading. Defining the aggregated data vector transmitted from BS_p as $\boldsymbol{s}_p \triangleq [\boldsymbol{s}_{p1}^\dagger \ \cdots \ \boldsymbol{s}_{pK}^\dagger]^\dagger$, the received signal vector $\boldsymbol{y}_{pk}$ at UE_{pk} is

$$\boldsymbol{y}_{pk} = \sum_{q=1}^{P} \boldsymbol{H}_{pqk}\boldsymbol{W}_q\boldsymbol{s}_q + \boldsymbol{n}_{pk}$$

where $\boldsymbol{W}_p \in \mathbb{C}^{M\times dK}$ is the precoding matrix at BS_p obeying the PAPC, and $\boldsymbol{n}_{pk} \in \mathbb{C}^{N\times 1}$ is an AWGN vector at UE_{pk} satisfying $\mathbb{E}[\boldsymbol{n}_{pk}\boldsymbol{n}_{pk}^\dagger] = \sigma^2\boldsymbol{I}_N$. Then, in CoMP-CB, the BSs jointly design $\{\boldsymbol{W}_p\}$ to mitigate the ICI by sharing their local CSI through a backhaul.

B. Sum-MSE Minimization based Transceiver Design

We consider the Tx and Rx filter design based on the MMSE criterion, i.e., minimizing the sum of the MSE across all the UEs in the network. Let $\boldsymbol{A}_{pk} \in \mathbb{C}^{d\times N}$ denote the Rx filter at UE_{pk}. Following the leakage-based MMSE approach in [5], the MSE contributed from BS_p is given by

$$\begin{aligned}\mathcal{M}_p \triangleq& \sum_k \mathbb{E}\|\boldsymbol{A}_{pk}(\boldsymbol{H}_{ppk}\boldsymbol{W}_p\boldsymbol{s}_p + \boldsymbol{n}_{pk}) - \boldsymbol{B}_k\boldsymbol{s}_p\|_F^2 \\ &+ \sum_{q\neq p}\sum_k \mathbb{E}\|\boldsymbol{A}_{qk}\boldsymbol{H}_{qpk}\boldsymbol{W}_p\boldsymbol{s}_p\|_F^2 \qquad (1)\\ =& \operatorname{Tr}\Big(\sum_{q,k}\boldsymbol{A}_{qk}\boldsymbol{H}_{qpk}\boldsymbol{W}_p\boldsymbol{W}_p^\dagger\boldsymbol{H}_{qpk}^\dagger\boldsymbol{A}_{qk}^\dagger + \sum_k \sigma^2\boldsymbol{A}_{pk}\boldsymbol{A}_{pk}^\dagger + K\boldsymbol{I}_d\Big) \\ &- \operatorname{Tr}\Big(\sum_k \boldsymbol{A}_{pk}\boldsymbol{H}_{ppk}\boldsymbol{W}_p\boldsymbol{B}_k^\dagger + \boldsymbol{B}_k\boldsymbol{W}_p^\dagger\boldsymbol{H}_{ppk}^\dagger\boldsymbol{A}_{pk}^\dagger\Big) \qquad (2)\end{aligned}$$

where $\boldsymbol{B}_k$ is a $d\times dK$ row selection matrix satisfying $\boldsymbol{s}_{pk} = \boldsymbol{B}_k\boldsymbol{s}_p$ and $\boldsymbol{I}_d$ in (2) is due to $\boldsymbol{B}_k\boldsymbol{B}_k^\dagger = \boldsymbol{I}_d$ for any k. Note that the first and second term in (1) corresponds to the signal distortion at the serving UEs and the leakage interference power to the neighboring UEs caused by BS_p, respectively. Then, the sum-MSE minimization based transceiver design under the PAPC is

$$\begin{aligned}&\underset{\{\boldsymbol{W}_p\},\{\boldsymbol{A}_{pk}\}}{\text{minimize}} \ \sum_{p=1}^{P}\mathcal{M}_p \qquad (3)\\ &\text{subject to } \operatorname{diag}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) \preceq \boldsymbol{\Psi} \text{ for } p=1,\cdots,P.\end{aligned}$$

where $\boldsymbol{\Psi} = \operatorname{diag}(\psi_1,\cdots,\psi_M)$ is a diagonal matrix denoting the PAPC in BS_p. Though not jointly convex in $\{\boldsymbol{W}_p\}$ and $\{\boldsymbol{A}_{pk}\}$, the objective function in (3) is convex on $\boldsymbol{W}_p$ for the fixed $\{\boldsymbol{A}_{pk}\}$, and vice versa. Therefore, we can use the alternating optimization between $\{\boldsymbol{W}_p\}$ and $\{\boldsymbol{A}_{pk}\}$ to find (at least) a local optimal solution of (3) as in the MMSE transceiver design under the per-BS power constraint, i.e., $\operatorname{Tr}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) \leq \operatorname{Tr}(\boldsymbol{\Psi})$ [3]–[5].

Specifically, the Lagrangian of (3) is given by

$$\mathcal{L}(\{\boldsymbol{W}_p\},\{\boldsymbol{A}_{pk}\},\boldsymbol{\Lambda}_p) = \sum_{p=1}^{P}\Big(\mathcal{M}_p + \operatorname{Tr}\Big(\boldsymbol{\Lambda}_p\Big(\operatorname{diag}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) - \boldsymbol{\Psi}\Big)\Big)\Big)$$

where $\boldsymbol{\Lambda}_p = \operatorname{diag}(\lambda_1,\cdots,\lambda_M) \succeq \boldsymbol{0}$ is an $M\times M$ non-negative real diagonal matrix consisting of M Lagrangian dual variables for the power constraint at each Tx antenna in BS_p. For the given $\{\boldsymbol{W}_p\}$, from the Karush-Kuhn-Tucker (KKT) condition of $\frac{\nabla\mathcal{L}(\cdot)}{\nabla\boldsymbol{A}_{pk}} = 0$, $\boldsymbol{A}_{pk}$ can be easily obtained in a closed form as

$$\boldsymbol{A}_{pk} = \boldsymbol{B}_k\boldsymbol{W}_p^\dagger\boldsymbol{H}_{ppk}^\dagger\Big(\sum_q \boldsymbol{H}_{pqk}\boldsymbol{W}_q\boldsymbol{W}_q^\dagger\boldsymbol{H}_{pqk}^\dagger + \sigma^2\boldsymbol{I}\Big)^{-1} \qquad (4)$$

for each $p = 1,\cdots,P$ and $k = 1,\cdots,K$. Then, for the alternating optimization, it remains how to find the Tx filters $\{\boldsymbol{W}_p\}$ for the given Rx filters $\{\boldsymbol{A}_{pk}\}$.

III. Low-Complexity Transmit Filter Design

We first describe the convex formulation under the PAPC and review the previous works on the low-complexity MMSE Tx filter design for given $\{\boldsymbol{A}_{pk}\}$. Then, we propose the optimal low-complexity Tx filters design algorithm under the PAPC and provide the complexity analysis to show the complexity reduction of the proposed algorithm.

A. Convex formulation and previous works

Observing that any $\mathcal{M}_q$ with $q \neq p$ is not affected by $\boldsymbol{W}_p$, the Tx filter $\boldsymbol{W}_p$ for the given $\{\boldsymbol{A}_{pk}\}$ is given by the solution of the following convex quadratically constrained quadratic programming (QCQP):

$$\begin{aligned}&\underset{\boldsymbol{W}_p}{\text{minimize}} \ \mathcal{M}_p \qquad (5)\\ &\text{subject to } \operatorname{diag}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) \preceq \boldsymbol{\Psi}.\end{aligned}$$

From $\frac{\nabla\mathcal{L}(\cdot)}{\nabla\boldsymbol{W}_p} = 0$, the minimizer of (5) is given in a semi-closed form by

$$\boldsymbol{W}_p = \Big(\sum_{q,k}\boldsymbol{H}_{qpk}^\dagger\boldsymbol{A}_{qk}^\dagger\boldsymbol{A}_{qk}\boldsymbol{H}_{qpk} + \boldsymbol{\Lambda}_p\Big)^{-1}\sum_k \boldsymbol{H}_{ppk}^\dagger\boldsymbol{A}_{pk}^\dagger\boldsymbol{B}_k. \qquad (6)$$

However, unlike (4), computing (6) requires an additional step to find $\boldsymbol{\Lambda}_p$ satisfying the complementary slackness condition

$$\boldsymbol{\Lambda}_p\Big(\operatorname{diag}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) - \boldsymbol{\Psi}\Big) = \boldsymbol{0}. \qquad (7)$$

Under the PBPC where $\boldsymbol{\Lambda}_p$ reduces to a single parameter λ satisfying $\lambda\left(\operatorname{Tr}\left(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger\right) - \operatorname{Tr}(\boldsymbol{\Psi})\right) = 0$, an efficient algorithm was proposed to find λ based on solving a polynomial equation [4]. However, under the PAPC, finding $\boldsymbol{\Lambda}_p$ involves the multivariate polynomial equations of $\lambda_1,\cdots,\lambda_M$ with degree of $2M$, which is known to be NP-hard [6]. Instead, a heuristic sub-optimal approach was proposed to alternatively optimize $\{\boldsymbol{W}_p\}$ and $\{\boldsymbol{\Lambda}_p\}$ [7]. However, this generally requires a large number of iterations and the convergence to a feasible $\boldsymbol{W}_p$ satisfying the PAPC is not guaranteed, especially for high SNR or large M.

Noticing that (5) is a convex problem, QCQP solvers such as CVX [10] can be used to find $\boldsymbol{W}_p$ numerically. However, as will be shown in Section V, this can cost significant computations considering that $\boldsymbol{W}_p$ needs to be updated for each BS at every alternating step. Therefore, it is of practical interest to develop an efficient algorithm to compute $\boldsymbol{W}_p$ under the PAPC.

Remark 1: In a single-cell multi-user MIMO setup, an iterative algorithm based on the uplink-downlink duality was proposed to find the MMSE Tx filters under the PAPC in [8]. However, such duality does not directly carry on to the CoMP-CB [4].

B. Low-complexity Non-linear Gauss-Seidel (NGS) Algorithm

Observing that each Tx antenna (each row of $\boldsymbol{W}_p$) is subject to a separate power constraint, we consider a decomposition method to obtain the Tx filters by solving the original convex QCQP through a set of more numerically efficient sub-problems [11].

1) Decomposition Method: Let $\tilde{\boldsymbol{w}}_{mp}^{\dagger}$ be the m-th row vector of $\boldsymbol{W}_p$, i.e., $\boldsymbol{W}_p^{\dagger} = \left[\begin{array}{ccc} \tilde{\boldsymbol{w}}_{1p} & \cdots & \tilde{\boldsymbol{w}}_{Mp} \end{array} \right]$. From (5), we construct a set of M sub-problems where the m-th sub-problem finds a feasible $\tilde{\boldsymbol{w}}_{mp}^{\dagger}$ minimizing $\mathcal{M}_p$ for given $\{\tilde{\boldsymbol{w}}_{np}^{\dagger}\}_{n\neq m}$. Then, we employ the NGS type algorithm [9] to obtain $\boldsymbol{W}_p$ by iteratively updating each row of $\boldsymbol{W}_p$ as the minimizer of each sub-problem. Under the NGS principle, the sub-problem for $\tilde{\boldsymbol{w}}_{mp}^{\dagger}$ is given by

$$\tilde{\boldsymbol{w}}_{mp}^{(t)} = \arg\min_{\tilde{\boldsymbol{w}}:\|\tilde{\boldsymbol{w}}\|^2\le\psi_m} \mathcal{M}_p\Big(\tilde{\boldsymbol{w}}_{1p}^{(t)},\cdots,\tilde{\boldsymbol{w}}_{(m-1)p}^{(t)},\tilde{\boldsymbol{w}},\tilde{\boldsymbol{w}}_{(m+1)p}^{(t-1)},\cdots,\tilde{\boldsymbol{w}}_{Mp}^{(t-1)}\Big)$$

where we include the arguments in $\mathcal{M}_p$ to show its dependency on the specific rows of $\boldsymbol{W}_p$. Here, $\tilde{\boldsymbol{w}}_{mp}^{(t)\dagger}$ denotes the interim m-th row of $\boldsymbol{W}_p$ after the t-th iteration, namely *inner* iteration in contrast to the *outer* iteration which alternates between optimizing $\{\boldsymbol{A}_{pk}\}$ and $\{\boldsymbol{W}_p\}$. Then, the final $\boldsymbol{W}_p$ for the given $\{\boldsymbol{A}_{pk}\}$ is obtained at the MSE convergence point of the inner iteration.

2) Closed-form Solutions for Sub-Problems: For the given $\{\boldsymbol{A}_{pk}\}$, let $\boldsymbol{F}_p = \left[\begin{array}{ccc} \boldsymbol{f}_{1p} & \cdots & \boldsymbol{f}_{Mp} \end{array} \right]$ be an $M \times M$ matrix obtained by the Cholesky decomposition $\boldsymbol{F}_p^{\dagger}\boldsymbol{F}_p \triangleq \sum_q \sum_k \boldsymbol{H}_{qpk}^{\dagger}\boldsymbol{A}_{qk}^{\dagger}\boldsymbol{A}_{qk}\boldsymbol{H}_{qpk}$, and let $\boldsymbol{G}_p = \left[\begin{array}{ccc} \boldsymbol{g}_{1p} & \cdots & \boldsymbol{g}_{Mp} \end{array} \right] \triangleq \sum_k \boldsymbol{B}_k^{\dagger}\boldsymbol{A}_{pk}\boldsymbol{H}_{ppk}$. Then, the MSE from BS_p is expressed as

$$\mathcal{M}_p = \mathrm{Tr}(\boldsymbol{F}_p^{\dagger}\boldsymbol{F}_p\boldsymbol{W}_p\boldsymbol{W}_p^{\dagger}) - \mathrm{Tr}(\boldsymbol{G}_p^{\dagger}\boldsymbol{W}_p^{\dagger} + \boldsymbol{W}_p\boldsymbol{G}_p) + \mathcal{C}_p \tag{8}$$

where $\mathcal{C}_p$ is some constant not affected by $\boldsymbol{W}_p$. Rewriting (8) in a vector form, the sub-problem (SP_{mp}) for $\tilde{\boldsymbol{w}}_{mp}$ is given by

$$(SP_{mp}):\ \underset{\tilde{\boldsymbol{w}}_{mp}}{\text{minimize}} \sum_{i,j=1}^{M} \boldsymbol{f}_{ip}^{\dagger}\boldsymbol{f}_{jp}\tilde{\boldsymbol{w}}_{jp}^{\dagger}\tilde{\boldsymbol{w}}_{ip} - \sum_{i=1}^{M}\Big(\boldsymbol{g}_{ip}^{\dagger}\tilde{\boldsymbol{w}}_{ip} + \tilde{\boldsymbol{w}}_{ip}^{\dagger}\boldsymbol{g}_{ip}\Big)$$
$$\text{subject to } \tilde{\boldsymbol{w}}_{mp}^{\dagger}\tilde{\boldsymbol{w}}_{mp} \le \psi_m$$

where $\tilde{\boldsymbol{w}}_{ip}$ $(i \neq m)$ remain fixed while solving (SP_{mp}). The Lagrangian of the sub-problem and the corresponding first order derivative condition w.r.t. $\tilde{\boldsymbol{w}}_{mp}$ are given by

$$\mathcal{L}_s(\tilde{\boldsymbol{w}}_{mp},\lambda_m) = \sum_{i,j}\boldsymbol{f}_{ip}^{\dagger}\boldsymbol{f}_{jp}\tilde{\boldsymbol{w}}_{jp}^{\dagger}\tilde{\boldsymbol{w}}_{ip} - \sum_i \boldsymbol{g}_{ip}^{\dagger}\tilde{\boldsymbol{w}}_{ip} - \sum_i \tilde{\boldsymbol{w}}_{ip}^{\dagger}\boldsymbol{g}_{ip} + \lambda_m(\tilde{\boldsymbol{w}}_{mp}^{\dagger}\tilde{\boldsymbol{w}}_{mp} - \psi_m),$$

$$\frac{\nabla\mathcal{L}_s(\cdot)}{\nabla\tilde{\boldsymbol{w}}_{mp}} = \|\boldsymbol{f}_{mp}\|^2\tilde{\boldsymbol{w}}_{mp}^T + \sum_{n\neq m}\boldsymbol{f}_{mp}^T\boldsymbol{f}_{np}^{*}\tilde{\boldsymbol{w}}_{np}^T - \boldsymbol{g}_{mp}^T + \lambda_m\tilde{\boldsymbol{w}}_{mp}^T,$$

respectively, where λ_m is the Lagrangian dual variable corresponding to $[\boldsymbol{\Lambda}_p]_{mm}$ in (6). Then, from the KKT condition $\frac{\nabla\mathcal{L}_s(\cdot)}{\nabla\tilde{\boldsymbol{w}}_{mp}} = 0$, $\tilde{\boldsymbol{w}}_{mp}$ can be expressed as

$$(\|\boldsymbol{f}_{mp}\|^2 + \lambda_m)\tilde{\boldsymbol{w}}_{mp} = \boldsymbol{g}_{mp} - \sum_{n\neq m}\tilde{\boldsymbol{w}}_{np}\boldsymbol{f}_{np}^{\dagger}\boldsymbol{f}_{mp}. \tag{9}$$

First, assuming that the minimizer of (SP_{mp}) satisfies the PAPC with equality, $\tilde{\boldsymbol{w}}_{mp}$ during the t-th inner iteration is set to be

$$\tilde{\boldsymbol{w}}_{mp}^{(t)} = \sqrt{\psi_m}\frac{\boldsymbol{v}_{mp}^{(t)}}{\|\boldsymbol{v}_{mp}^{(t)}\|} \tag{10}$$

where

$$\boldsymbol{v}_{mp}^{(t)} \triangleq \boldsymbol{g}_{mp} - \sum_{n<m}\tilde{\boldsymbol{w}}_{np}^{(t)}\boldsymbol{f}_{np}^{\dagger}\boldsymbol{f}_{mp} - \sum_{n>m}\tilde{\boldsymbol{w}}_{np}^{(t-1)}\boldsymbol{f}_{np}^{\dagger}\boldsymbol{f}_{mp}. \tag{11}$$

Then, from (9) and (10), $\lambda_m^{(t)}$ is explicitly given by

$$\lambda_m^{(t)} = \sqrt{\psi_m^{-1}}\|\boldsymbol{v}_{mp}^{(t)}\| - \|\boldsymbol{f}_{mp}\|^2. \tag{12}$$

TABLE I
CONVERGENCE BEHAVIOR FOR $(P,M,K,N,d) = (3,4,1,2,2)$ AT 10 dB SNR

Number of inner iteration (t)	10	20	30	40
$\max_{i,j}\frac{\left\lvert[\boldsymbol{W}_p]_{ij}-[\boldsymbol{W}_p^*]_{ij}\right\rvert^2}{\left\lvert[\boldsymbol{W}_p^*]_{ij}\right\rvert^2}$	7.3×10^{-3}	6.0×10^{-6}	8.1×10^{-9}	1.4×10^{-9}

On the other hand, when the resulting $\lambda_m^{(t)}$ in (12) is negative (infeasible), $\lambda_m^{(t)}$ is set to be zero and the corresponding $\tilde{\boldsymbol{w}}_{mp}^{(t)}$ is

$$\tilde{\boldsymbol{w}}_{mp}^{(t)} = \frac{\boldsymbol{v}_{mp}^{(t)}}{\|\boldsymbol{f}_{mp}\|^2}, \tag{13}$$

which means the m-th Tx antenna in BS_p is not using full power.

Then, in the proposed algorithm, $\tilde{\boldsymbol{w}}_{1p}^{(t)},\cdots,\tilde{\boldsymbol{w}}_{Mp}^{(t)}$ are sequentially updated at each inner iteration using (10) or (13) until $\boldsymbol{W}_p^{(t)} \triangleq [\tilde{\boldsymbol{w}}_{1p}^{(t)}\ \cdots\ \tilde{\boldsymbol{w}}_{Mp}^{(t)}]^{\dagger}$ (and, thus, the MSE) converges as shown in the inner iteration in Algorithm 1.

3) Convergence: The following lemma shows that the proposed NGS algorithm preserves the optimality of $\boldsymbol{W}_p$ in the MSE sense for the given $\{\boldsymbol{A}_{pk}\}$ in each outer iteration.

Lemma 1. *As $t \to \infty$, $\boldsymbol{W}_p^{(t)}$ obtained from $\{(SP_{mp})\}$ in the proposed NGS algorithm is guaranteed to achieve the MSE minima in* (5) *for the given Rx filters $\{\boldsymbol{A}_{pk}\}$.*

Proof: The MSE $\mathcal{M}_p$ in (5) is clearly convex and continuously differentiable function of $\boldsymbol{W}_p$. Furthermore, for the fixed $\tilde{\boldsymbol{w}}_{np}$ with $n \neq m$, each sub-problem (SP_{mp}) is strictly convex on $\tilde{\boldsymbol{w}}_{mp}$ for any non-trivial $\boldsymbol{f}_{mp} \neq \boldsymbol{0}$. Then, the convergence to the minima of (5) follows from [9, Prop. 3.9]. ∎

From Lemma 1, the proposed algorithm can be interpreted as to obtain the optimal $\boldsymbol{\Lambda}_p$ in (6) as $[\boldsymbol{\Lambda}_p]_{mm} = \lim_{t\to\infty}\lambda_m^{(t)}$, based on a series of simple vector operations (10) or (13) instead of dealing with complicated higher-order multivariate polynomial equations. For an example of CoMP-CB with $(P,M,K,N,d) = (3,4,1,2,2)$, Table 1 shows the element-wise error of $\boldsymbol{W}_p$ obtained from the proposed NGS algorithm w.r.t. $\boldsymbol{W}_p^*$ obtained by directly solving (5) using CVX [10].

C. Complexity Analysis

The complexity to solve the original convex QCQP of (5) is lower bounded by that of the relaxed problem based on the semi-definite programming (SDP) [12]. Considering $d = N$ for simplicity, the overall complexity of SDP to solve (5) is $\mathcal{O}\big((M+NK)^{9/2}\log(1/\epsilon)\big)$ where ϵ is the accuracy target [12].

On the other hand, in the proposed NGS algorithm, computing the Cholesky decomposition to obtain $\boldsymbol{F}_p$ requires $\frac{1}{3}M^3 + \frac{1}{2}M^2 + \frac{1}{6}M$ flops (complex scalar operation) [13]. Next, the complexity to compute (10) or (13) is dominated by computing $\boldsymbol{v}_{mp}^{(t)}$ which requires $(2M - 1 + NK)(M - 1) + (M-1)NK$ flops, i.e., $\mathcal{O}(M^2 + MNK)$. Then, the overall complexity from the proposed algorithm can be given by $\mathcal{O}\left(M^3 + n_{\text{iter}}M\left(M^2 + MNK\right)\right) \approx \mathcal{O}\left(n_{\text{iter}}M^2\left(M+NK\right)\right)$ where $n_{\text{iter}} \propto \log(1/\epsilon)$ denotes the number of the inner iterations until convergence with the accuracy of ϵ.

Obviously, in each update of the TX filters, the proposed algorithm provides significant complexity reduction as M, N and K grows. In Section V, we will further demonstrate this by showing the average CPU time of the proposed NGS algorithm compared to directly solving (5) using QCQP solvers.

TABLE II
AVERAGE CPU TIME FOR $(N, K, d) = (2, 1, 2)$ AT 20 dB SNR (IN SECONDS)

M	4	6	8	10	12	14	16
MMSE-QCQP	43.3	50.4	57.2	64.2	72.8	78.3	86.5
MMSE-NGS	0.173	0.213	0.256	0.302	0.355	0.397	0.432

IV. ALTERNATING OPTIMIZATION

The alternating optimization for the MMSE beamforming with the proposed NGS algorithm (MMSE-NGS) is summarized in Algorithm 1. Since $\sum_p \mathcal{M}_p$ is bounded from below and decreasing throughout the outer iteration, the process clearly converges to a local minimum. Note that MMSE-QCQP will denote the benchmark where $\boldsymbol{W}_p$ is updated by solving (5) with QCQP solvers. It is clear from Section III-C that MMSE-NGS requires much less computations than MMSE-QCQP.

Algorithm 1 MMSE Non-linear Gauss-Seidel (MMSE-NGS)

Initialization:
For all p, randomly generate $\boldsymbol{W}_p$ to satisfy $\text{diag}(\boldsymbol{W}_p\boldsymbol{W}_p^\dagger) = \boldsymbol{\Psi}$
Alternating Optimization:
Outer iteration

1) For each p and k, update $\boldsymbol{A}_{pk}$ using (4)
2) For each p, compute $\boldsymbol{F}_p, \boldsymbol{G}_p$ as in Sec. III, and update $\boldsymbol{W}_p$:
   Inner iteration
   a) update $t = t + 1$
   b) sequentially compute for $m = 1 \dots M$

$$\tilde{\boldsymbol{w}}_{mp}^{(t)} = \begin{cases} \sqrt{\psi_m}\dfrac{\boldsymbol{v}_{mp}^{(t)}}{\|\boldsymbol{v}_{mp}^{(t)}\|} & \textbf{if } \|\boldsymbol{v}_{mp}^{(t)}\| \geq \sqrt{\psi_m}\|\boldsymbol{f}_{mp}\|^2 \\ \dfrac{\boldsymbol{v}_{mp}^{(t)}}{\|\boldsymbol{f}_{mp}\|^2} & \textbf{otherwise} \end{cases}$$

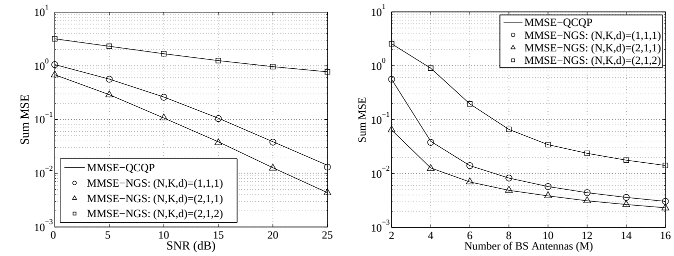
   until the MSE converges, where $\boldsymbol{v}_{mp}^{(t)}$ is given by (11).

until the MSE converges

V. NUMERICAL RESULTS

We present the simulation results for the CoMP-CB with three cells $(P = 3)$. The power constraint at each antenna is chosen to be the same, i.e., $\psi_m = \frac{1}{M}\text{Tr}(\boldsymbol{\Psi})$ for all m. We set $\beta_{pqk} = 1$ for $p = q$ and 0.5 otherwise, i.e., the ICI from a neighboring cell is 3 dB weaker than the desired signal in average. We fix the number of the outer iteration to 50 and the inner iteration is terminated when the MSE decreases by less than 0.1%. For the MMSE-QCQP, we use CVX [10] to compute the Tx filters for the given Rx filters in each outer iteration.

Figure 1 shows the sum-MSE performance with the MMSE-NGS and MMSE-QCQP algorithms for varying SNRs, $\frac{1}{\sigma^2}\text{Tr}(\boldsymbol{\Psi})$, and the number of Tx antennas M, respectively. As expected from the convergence proof, the proposed algorithm achieves effectively the same MSE performance as the MMSE-QCQP in both cases. Next, Table 2 shows the average CPU time corresponding to the case with $(N, K, d) = (2, 1, 2)$ in Fig. 1 (b). We can see that the proposed algorithm provides a complexity reduction of more than 99% in terms of the CPU time compared to the MMSE-QCQP by using efficient decomposition method. The similar complexity reduction was observed for different configurations of (N, K, d) and different SNRs.



(a) Varying SNR ($M = 4$) (b) Varying M (SNR: 20 dB)

Fig. 1. Sum-MSE comparison between MMSE-QCQP and MMSE-NGS

VI. CONCLUSION

We proposed the low-complexity MMSE downlink beamforming for the CoMP-CB under the practical per-antenna power constraint. The proposed non-linear Gauss-Seidel type algorithm computes the per-antenna power constrained Tx filters for given Rx filters by a series of the simple vector operations. Complexity analysis and numerical results showed that while preserving the MSE optimality, the proposed algorithm can significantly reduce the overall complexity of the alternating optimization.

REFERENCES

[1] B. Clerckx, A. Lozano, S. Sesia, C. van Rensburg, and C. Papadias, "3GPP LTE and LTE-Advanced," *EURASIP J. Wireless Commun. and Network.*, 2009.

[2] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and Sum-MSE Minimization," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5436–5446, Nov. 2007.

[3] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sept. 2011.

[4] H. Shen, B. Li, M. Tao, and X. Wang, "MSE-Based Transceiver Designs for the MIMO Interference Channel," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3480–3489, Nov. 2010.

[5] F. Sun and E. De Carvalho, "A Leakage-Based MMSE Beamforming Design for a MIMO Interference Channel," *IEEE Signal Process. Lett.*, vol. 19, no. 6, pp. 368–371, Jun. 2012.

[6] N. Courtois, E. Klimov, J. Patarin, and A. Shamir, "Efficient algorithms for solving overdefined systems of multivariate polynomial equations," in *Proc. of Advances in Cryptology, Eurocrypt '00*, 2000, pp. 392–407.

[7] J. Li, I.-T. Lu, and E. Lu, "Robust MMSE Transceiver Designs for Downlink MIMO Systems with Multicell Cooperation," *International Journal of Digital Multimedia Broadcasting*, 2010.

[8] T. Bogale and L. Vandendorpe, "Robust Sum MSE Optimization for Downlink Multiuser MIMO Systems With Arbitrary Power Constraint: Generalized Duality Approach," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1862 –1875, April 2012.

[9] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation:Numerical Methods*. Prentice-Hall, 1989.

[10] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," Sep. 2012.

[11] D. Palomar and M. Chiang, "A Tutorial on Decomposition Methods for Network Utility Maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1439–1451, Aug. 2006.

[12] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite Relaxation of Quadratic Optimization Problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[13] R. Hunger, *Floating Point Operations in Matrix-vector Calculus*. Munich Univ. of Tech., Inst. for Circuit Theory and Signal Processing, 2005.