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Low-Complexity MMSE Precoding for Coordinated Multipoint with Per-Antenna Power Constraint

Tae Min Kim, Fan Sun, and Arogyaswami Paulraj

Abstract—We propose a low-complexity minimum mean square error (MMSE) transmit filter design for the coordinated beamforming (CB) in the coordinated multipoint (CoMP) under the practical per-antenna power constraint (PAPC). The proposed design is based on the non-linear Gauss-Seidel type algorithm in which the transmit filters for given receive filters are computed by iteratively updating the beamformer of each transmit antenna using simple closed-form expressions. The proposed approach can significantly reduce the overall complexity of the alternating optimization while preserving the optimality in the MSE sense.

Index Terms—MMSE, Coordinated Beamforming, Per-Antenna Power Constraint, Non-linear Gauss-Seidel Algorithm

I. INTRODUCTION

COORDINATED multipoint (CoMP) transmission has recently attracted much attention as a means to mitigate the inter-cell interference (ICI) in the cellular networks by using the base stations (BSs) cooperation. When the BSs are connected over a limited backhaul, coordinated beamforming in the CoMP (CoMP-CB) can effectively reduce the ICI by jointly designing the transmit (Tx) filters at the BSs based on the shared channel state information (CSI) without sharing the user data across the BSs as in the joint processing [1].

We consider the downlink CoMP-CB based on the minimum mean square error (MMSE) approach. The MMSE criterion has been adopted in various MIMO beamforming scenarios including the MIMO broadcast channel (MIMO-BC) [2] and the interference channel (MIMO-IC) [3], [4] due to its practical importance. First, minimizing the MSE is closely related to minimizing the mean square error (MMSE) approach similar to [4] would require to solve complicated multivariate polynomial equations under the PAPC, which is known to be NP-hard [6]. Furthermore, one previous work in [7] proposed a heuristic low-complexity design by including an additional alternating step for the dual variables, yet its convergence to a feasible solution is not guaranteed in high SNR or for a large number of Tx antennas. In [8], another low-complexity Tx filter design based on the uplink-downlink duality was proposed. However, their design focused on the single-cell multi-user MIMO and is not easily extensible to the CoMP-CB.

In this letter, we propose a low-complexity MMSE Tx filter design for the CoMP-CB under the PAPC with the guaranteed optimality at each alternating step. Observing the power constraints in each BS are naturally decoupled across the Tx antennas, we decompose the original convex problem into a set of sub-problems each involving the beamforming from only one antenna with the corresponding PAPC. Specifically, a non-linear Gauss-Seidel (NGS) type algorithm [9] is employed to iteratively update the beamformer of each Tx antenna. For given Rx filters, the proposed algorithm preserves the optimality in the MSE sense. For each sub-problem, we give a closed-form solution which can be computed efficiently by simple vector operations. While achieving the same MSE, complexity analysis and computer simulations show that the proposed algorithm significantly reduces the overall complexity, e.g., a CPU time reduction of over 99%, compared to the benchmark which directly finds the Tx filters using the standard convex programming solvers.

Notation: $[\cdot]_{m:n}$, $(\cdot)^T$, $(\cdot)^\dagger$, and $\|\cdot\|_F$ denote the $(m,n)$-th element, the transpose, the conjugate transpose, the Frobenius norm of a vector/matrix, respectively. $\text{diag}(\cdot)$ is the diagonal matrix taking only the diagonal elements of a matrix. An $m \times m$ identity matrix is denoted by $I_m$.

II. PRELIMINARY

A. System Model

We consider a cooperative multi-cell network, where $P$ base stations (BSs) are connected via backhaul and serve $K$ user equipments (UEs) per cell. Let $B_{SP}$ and $U_{EP_k}$ denote the $p$-th BS and the $k$-th UE in the $p$-th cell, respectively. Each $B_{SP}$ and $U_{EP_k}$ is equipped with $M$ transmit antennas and $N$ receive antennas, respectively, and $B_{SP}$ transmits $d \leq N$ independent data stream(s) to each $U_{EP_k}$ in its serving cell. The data vector for $U_{EP_k}$, denoted by $s_{pk} = [s_{1pk} \cdots s_{dpk}]^T \in \mathbb{C}^{d \times 1}$, satisfies $\mathbb{E}[s_{pk}] = \mathbf{0}$ and $\mathbb{E}[s_{pk} s_{pk}^H] = I_d$.

Let $H_{p_{qk}} \triangleq \beta_{p_{qk}} H_{p_{qk}}$ be the $N \times M$ MIMO channel from $BS_q$ to $U_{EP_k}$ where $H_{p_{qk}} \sim \mathcal{CN}(0, I_N)$ and $\beta_{p_{qk}}$ is a
non-negative constant reflecting a large scale fading. Defining the aggregated data vector transmitted from BS$_p$ as $s_p \triangleq [s_{p1} \cdots s_{pK}]^T$, the received signal vector $y_{pk}$ at UE$_{pk}$ is
\[
y_{pk} = \sum_{q=1}^{P} H_{pqk} W_q s_q + n_{pk}
\]
where $W_p \in C^{M \times dK}$ is the precoding matrix at BS$_p$ obeying the PAPC, and $n_{pk} \in C^{N \times 1}$ is an AWGN vector at UE$_{pk}$ satisfying $E[n_{pk} n_{pk}^H] = \sigma^2 I_N$. Then, in CoMP-CB, the BSs jointly design $\{W_p\}$ to mitigate the ICI by sharing their local CSI through a backhaul.

B. Sum-MSE Minimization based Transceiver Design

We consider the Tx and Rx filter design based on the MMSE criterion, i.e., minimizing the sum of the MSE across all the UEs in the network. Let $A_{pk} \in C^{d \times N}$ denote the Rx filter at UE$_{pk}$. Following the leakage-based MMSE approach in [5], the MSE contributed from BS$_p$ is given by
\[
\mathcal{M}_p \triangleq \sum_k \mathbb{E}\|A_{pk}(H_{pqk} W_p s_p + n_{pk}) - B_k s_p\|_F^2
\]
\[
+ \sum_{q \neq p} \mathbb{E}\|A_{qk} H_{kq} W_p s_p\|_F^2
\]
\[
= \text{Tr}\left( \sum_{q,k} A_{qk} H_{kq} W_p W_q^H A_{qk}^H + \sum_{k} \sigma^2 A_{pk} A_{pk}^H + K I_d \right)
\]
\[
- \text{Tr}\left( \sum_{q,k} A_{qk} H_{kq} W_p B_k + B_k W_p^H H_{pqk} A_{pk} \right)
\]
(2)

where $B_k$ is a $d \times dK$ row selection matrix satisfying $s_{pk} = B_k s_p$ and $I_d$ in (2) is due to $B_k B_k^H = I_d$ for any $k$. Note that the first and second term in (1) corresponds to the signal distortion at the serving UEs and the leakage interference power to the neighboring UEs caused by BS$_p$, respectively. Then, the sum-MSE minimization based transceiver design under the PAPC is
\[
\text{minimize} \quad \sum_{p=1}^{P} \mathcal{M}_p
\]
\[\text{subject to} \quad \text{diag}(W_p W_p^H) \preceq \Psi \quad \text{for} \quad p = 1, \cdots, P.
\]

where $\Psi = \text{diag}(\psi_1, \cdots, \psi_M)$ is a diagonal matrix denoting the PAPC in BS$_p$. Though not jointly convex in $\{W_p\}$ and $\{A_{pk}\}$, the objective function in (3) is convex on $W_p$ for the fixed $\{A_{pk}\}$, and vice versa. Therefore, we can use the alternating optimization between $\{W_p\}$ and $\{A_{pk}\}$ to find (at least) a local optimal solution of (3) as in the MMSE transceiver design under the per-BS power constraint, i.e., $\text{Tr}(W_p W_p^H) \leq \text{Tr}(\Psi)$ [3]-[5].

Specifically, the Lagrangian of (3) is given by
\[
\mathcal{L}(\{W_p\}, \{A_{pk}\}, \Lambda_p) = \sum_{p=1}^{P} \left( \mathcal{M}_p + \text{Tr} \{ A_{pk} \text{diag}(W_p W_p^H) - \Psi \} \right)
\]
where $\Lambda_p = \text{diag}(\lambda_1, \cdots, \lambda_M) \succeq 0$ is an $M \times M$ non-negative real diagonal matrix consisting of $M$ Lagrangian dual variables for the power constraint at each Tx antenna in BS$_p$. For the given $\{W_p\}$, from the Karush-Kuhn-Tucker (KKT) condition of the following convex quadratically constrained quadratic programming (QCQP):
\[
\text{minimize} \quad \mathcal{M}_p
\]
\[\text{subject to} \quad \text{diag}(W_p W_p^H) \preceq \Psi.
\]
\[
W_p = \left( \sum_{q,k} H_{pqk} A_{qk}^H + \Lambda_p \right)^{-1} \sum_{q,k} H_{pqk} A_{qk}^H B_k,
\]
(6)

However, unlike (4), computing (6) requires an additional step to find $\Lambda_p$ satisfying the complementary slackness condition
\[
\Lambda_p \left( \text{diag}(W_p W_p^H) - \Psi \right) = 0.
\]
(7)

Under the PBPC where $\Lambda_p$ reduces to a single parameter $\lambda$ satisfying $\lambda (\text{Tr}(W_p W_p^H) - \text{Tr}(\Psi)) = 0$, an efficient algorithm was proposed to find $\lambda$ based on solving a polynomial equation [4]. However, under the PAPC, finding $\Lambda_p$ involves the multivariate polynomial equations of $\lambda_1, \cdots, \lambda_M$ with degree of 2$M$, which is known to be NP-hard [6]. Instead, a heuristic sub-optimal approach was proposed to alternatively optimize $\{W_p\}$ and $\{A_{pk}\}$ [7]. However, this generally requires a large number of iterations and the convergence to a feasible $W_p$ satisfying the PAPC is not guaranteed, especially for high SNR or large $M$.

Noticing that (5) is a convex problem, QCQP solvers such as CVX [10] can be used to find $W_p$ numerically. However, as will be shown in Section V, this can cost significant computations considering that $W_p$ needs to be updated for each BS at every alternating step. Therefore, it is of practical interest to develop an efficient algorithm to compute $W_p$ under the PAPC.

Remark 1: In a single-cell multi-user MIMO setup, an iterative algorithm based on the uplink-downlink duality was proposed to find the MMSE Tx filters under the PAPC in [8]. However, such duality does not directly carry on to the CoMP-CB [4].

B. Low-complexity Non-linear Gauss-Seidel (NGS) Algorithm

Observing that each Tx antenna (each row of $W_p$) is subject to a separate power constraint, we consider a decomposition method to obtain the Tx filters by solving the original convex QCQP through a set of more numerically efficient sub-problems [11].
1) Decomposition Method: Let $\tilde{w}_{mp}^{(t)}$ be the $m$-th row vector of $W_p$, i.e., $W_p = \begin{bmatrix} \tilde{w}_{1p} & \cdots & \tilde{w}_{mp} \end{bmatrix}$. From (5), we construct a set of $M$ sub-problems where the $m$-th sub-problem finds a feasible $\hat{w}_{mp}^*$ minimizing $M_p$ for given $\{A_{pk}\}_{k \neq m}$. Then, we employ the NGS type algorithm [9] to obtain $W_p$ by iteratively updating each row of $W_p$ as the minimizer of each sub-problem. Under the NGS principle, the sub-problem for $\tilde{w}_{mp}$ is given by

$$\tilde{w}_{mp} = \arg\min_{\tilde{w} : \lVert \tilde{w} \rVert \leq \psi_m} \mathcal{M}_p(\tilde{w}_{ip}^0, \ldots, \tilde{w}_{ip}^{(t-1)}, \tilde{w}_{(m+1)p}^0, \ldots, \tilde{w}_{mp}^{(t-1)})$$

where we include the arguments in $\mathcal{M}_p$ to show its dependency on the specific rows of $W_p$. Here, $\tilde{w}_{ip}^{(t)}$ denotes the interim $m$-th row of $W_p$ after the $t$-th iteration, namely inner iteration in contrast to the outer iteration which alternates between optimizing $\{A_{pk}\}$ and $\{W_p\}$. Then, the final $W_p$ for the given $\{A_{pk}\}$ is obtained at the MSE convergence point of the inner iteration.

2) Closed-form Solutions for Sub-Problems: For the given $\{A_{pk}\}$, let $F_p = [f_{1p} \cdots f_{M_p}]$ be an $M \times M$ matrix obtained by the Cholesky decomposition $F_p = \sum q \sum k H_{pqk} A_{pk} H_{qpk}$, and let $G_p = [g_{1p} \cdots g_{M_p}] = \sum k B_k A_{ph} H_{pk}$. Then, the MSE from $BS_p$ is expressed as

$$M_p = \text{Tr}(F_p^* F_p W_p W_p^*) - \text{Tr}(F_p^* W_p G_p) + C_p$$

where $C_p$ is some constant not affected by $W_p$. Rewriting (8) in a vector form, the sub-problem $(SP_{mp})$ for $\tilde{w}_{mp}$ is given by

$$(SP_{mp}): \text{minimize}_{\tilde{w}_{mp}} \sum_{i,j=1}^{M} f_{jp}^* f_{jp} \tilde{w}_{jp}^* w_{ip} - \sum_{i=1}^{M} g_{jp}^* w_{ip} \tilde{w}_{ip}^* \tilde{w}_{mp} - \sum_{i \neq m} \tilde{w}_{ip}^* w_{mp} - \lambda_m(\tilde{w}_{mp}^* - \psi_m)$$

subject to $\tilde{w}_{mp} \leq \psi_m$

where $\tilde{w}_{ip}$ ($i \neq m$) remain fixed while solving $(SP_{mp})$. The Lagrangian of the sub-problem and the corresponding first order derivative condition w.r.t. $\tilde{w}_{mp}$ are given by

$$L_s(\tilde{w}_{mp}, \lambda_m) = \sum_{i,j} f_{jp}^* f_{jp} \tilde{w}_{jp}^* w_{ip} - \sum_{i=1}^{M} g_{jp}^* w_{ip} \tilde{w}_{ip}^* \tilde{w}_{mp} - \sum_{i \neq m} \tilde{w}_{ip}^* w_{mp} - \lambda_m(\tilde{w}_{mp}^* - \psi_m)$$

$$\frac{\nabla L_s(\cdot)}{\nabla \tilde{w}_{mp}} = \lVert f_{mp} \rVert^2 \tilde{w}_{mp}^* + \sum_{n \neq m} f_{np}^* f_{np} \tilde{w}_{np}^* - g_{np}^* w_{np} \tilde{w}_{np}^* - \lambda_m \tilde{w}_{mp}^*$$

respectively, where $\lambda_m$ is the Lagrangian dual variable corresponding to $[A_p]_{m,m}$ in (6). Then, from the KKT condition

$$\nabla L_s(\cdot) = 0, \tilde{w}_{mp} = g_{mp} - \sum_{n \neq m} \tilde{w}_{np} f_{np} f_{mp},$$

First, assume that the minimizer of $(SP_{mp})$ satisfies the PAPC with equality, $\tilde{w}_{mp}$ during the $t$-th inner iteration is set to be

$$\tilde{w}_{mp}^{(t)} = \sqrt{\psi_m} \frac{v_{mp}^{(t)}}{\lVert v_{mp} \rVert}$$

where

$$v_{mp}^{(t)} = g_{mp} - \sum_{n \leq m} \tilde{w}_{np}^{(t)} f_{np} f_{mp} - \sum_{n \geq m} \tilde{w}_{np}^{(t-1)} f_{np} f_{mp}.$$

Then, from (9) and (10), $\lambda_m^{(t)}$ is explicitly given by

$$\lambda_m^{(t)} = \sqrt{\psi_m} \frac{\lVert v_{mp} \rVert}{\lVert f_{mp} \rVert^2}.$$
TABLE II
AVERAGE CPU TIME FOR \((N, K, d) = (2, 1, 2)\) AT 20 dB SNR (IN SECONDS)

<table>
<thead>
<tr>
<th>(M)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMSE-QCQP</td>
<td>33.2</td>
<td>50.4</td>
<td>63.6</td>
<td>64.2</td>
<td>72.6</td>
<td>78.3</td>
<td>86.5</td>
</tr>
<tr>
<td>MMSE-NGS</td>
<td>0.173</td>
<td>0.213</td>
<td>0.286</td>
<td>0.302</td>
<td>0.375</td>
<td>0.397</td>
<td>0.432</td>
</tr>
</tbody>
</table>

IV. ALTERNATING OPTIMIZATION

The alternating optimization for the MMSE beamforming with the proposed NGS algorithm (MMSE-NGS) is summarized in Algorithm 1. Since \(\sum_{j=1}^M M_p\) is bounded from below and decreasing throughout the outer iteration, the process clearly converges to a local minimum. Note that MMSE-QCQP will denote the benchmark where \(W_p\) is updated by solving (5) with QCQP solvers. It is clear from Section III-C that MMSE-NGS requires much less computations than MMSE-QCQP.

**Algorithm 1 MMSE Non-linear Gauss-Seidel (MMSE-NGS)**

**Initialization:**
For all \(p\), randomly generate \(W_p\) to satisfy \(\text{diag}(W_pW_p^H) = \Psi\)

**Alternating Optimization:**

1. For each \(p\) and \(k\), update \(A_{pk}\) using (4)
2. For each \(p\), compute \(F_p, G_p\) as in Sec. III, and update \(W_p\):
   **Inner iteration**
   
   a) update \(t = t + 1\)
   b) sequentially compute for \(m = 1 \ldots M\)

   \[
   w_{mp}^{(t)} = \begin{cases} 
   \frac{\psi_{mp}}{\|v_{mp}\|}, & \text{if } \|v_{mp}\| \geq \sqrt{\psi_{mp}} \left\| f_{mp} \right\|^2 \\
   v_{mp}^{(t)}, & \text{otherwise}
   \end{cases}
   \]

   until the MSE converges, where \(v_{mp}^{(t)}\) is given by (11).

   until the MSE converges

V. NUMERICAL RESULTS

We present the simulation results for the CoMP-CB with three cells \((P = 3)\). The power constraint at each antenna is chosen to be the same, i.e., \(\psi_m = \frac{1}{M} \text{Tr} (\Psi)\) for all \(m\). We set \(\beta_{pqk} = 1\) for \(p = q\) and 0.5 otherwise, i.e., the ICI from a neighboring cell is 3 dB weaker than the desired signal in average. We fix the number of the outer iteration to 50 and the inner iteration is terminated when the MSE decreases by less than 0.1%. For the MMSE-QCQP, we use CVX [10] to compute the Tx filters for the given Rx filters in each outer iteration.

Figure 1 shows the sum-MSE performance with the MMSE-NGS and MMSE-QCQP algorithms for varying SNRs, \(\frac{1}{M} \text{Tr} (\Psi)\), and the number of Tx antennas \(M\), respectively. As expected from the convergence proof, the proposed algorithm achieves effectively the same MSE performance as the MMSE-QCQP in both cases. Next, Table 2 shows the average CPU time corresponding to the case with \((N, K, d) = (2, 1, 2)\) in Fig. 1 (b). We can see that the proposed algorithm provides a complexity reduction of more than 99% in terms of the CPU time compared to the MMSE-QCQP by using efficient decomposition method. The similar complexity reduction was observed for different configurations of \((N, K, d)\) and different SNRs.

VI. CONCLUSION

We proposed the low-complexity MMSE downlink beamforming for the CoMP-CB under the practical per-antenna power constraint. The proposed non-linear Gauss-Seidel type algorithm computes the per-antenna power constrained Tx filters for given Rx filters by a series of the simple vector operations. Complexity analysis and numerical results showed that while preserving the MSE optimality, the proposed algorithm can significantly reduce the overall complexity of the alternating optimization.

REFERENCES