

## Four-Way Relaying in Wireless Cellular Systems

Liu, Huaping; Popovski, Petar; De Carvalho, Elisabeth; Zhao, Yuping; Sun, Fan

*Published in:*  
IEEE Wireless Communications Letters

*DOI (link to publication from Publisher):*  
[10.1109/WCL.2013.050613.130225](https://doi.org/10.1109/WCL.2013.050613.130225)

*Publication date:*  
2013

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Liu, H., Popovski, P., De Carvalho, E., Zhao, Y., & Sun, F. (2013). Four-Way Relaying in Wireless Cellular Systems. *IEEE Wireless Communications Letters*, 2(4), 403 - 406.  
<https://doi.org/10.1109/WCL.2013.050613.130225>

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Four-Way Relaying in Wireless Cellular Systems

Huaping Liu, Petar Popovski, Elisabeth de Carvalho, Yuping Zhao and Fan Sun

**Abstract**—Two-way relaying in wireless systems has initiated a large research effort during the past few years. Nevertheless, it represents only a specific traffic pattern and it is of interest to investigate other traffic patterns where such a simultaneous processing of information flows can bring performance advantage. In this paper we consider a *four-way relaying* scenario, where each of the two Mobile Stations (MSs) has a two-way connection to the same Base Station (BS), while each connection is through a dedicated Relay Station (RS). The RSs are placed in such a way that one RS and the terminals associated with it do not interfere with the other RS, and vice versa. We introduce and analyze a two-phase transmission scheme to serve the four-way traffic pattern defined in this scenario. Each phase consists of combined broadcast and multiple access. We analyze the achievable rate region of the new scheme for Decode-and-Forward (DF) operational model for the RS. We compare the performance with a state-of-the-art reference scheme, based on two-way relaying with DF. The results indicate that the achievable rate regions are significantly enlarged for the new scheme.

## I. INTRODUCTION

One-way relaying suffers from a loss in spectral efficiency as a half-duplex relay cannot transmit and receive at the same time. Relays have received a fresh research potential with the introduction of two-way relaying based on wireless network coding (WNC) [1]–[3], as part of the spectral efficiency loss can be regained by simultaneously serving two or more data flows through the same relay. Various aspects of two-way relaying have been investigated, such as achievable rates [4] and optimal broadcasting strategies [5] when the relays operate in a Decode-and-Forward (DF) regime.

The benefits of WNC can reach beyond the usual two-way relaying scenario by observing the two underlying principles: (1) simultaneous service of multiple flows over the wireless medium and (2) cancellation of interference based on previously gathered information. We have utilized these principles to devise schemes to jointly serve a relayed and a direct user [6]. Here we leverage these principles towards the practical scenario of *four-way relaying* from Fig. 1, in which two Mobile Stations (MSs) have each a two-way connection to the same Base Station (BS), while each of them uses a dedicated Relay Station (RS). The RSs are placed such that one RS and the terminals associated with it do not interfere with the other RS, and vice versa. This can occur e. g. when the RSs are placed between BS and  $U_i$  respectively, and the two RSs are sufficiently separated not to interfere with each other. For brevity, let  $BS \rightarrow RS1$  denote the directional link from BS to RS1 (and similar for the other links). A state-of-the-art scheme for this scenario is time sharing between two independent two-way relaying schemes, as shown in Fig. 2.

Similar scenario has recently been considered in [7], where DS-CDMA is used to avoid interference, the nodes use BPSK

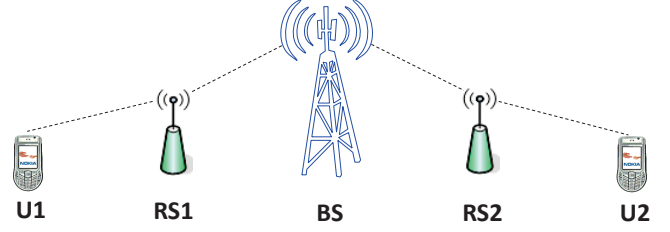


Figure 1. A cellular network with four-way relaying using a Base Station (BS), two Relay Stations (RSs), denoted RS1 and RS2, and two Mobile Stations (MSs) U1 and U2.

modulation and the relay applies physical-layer network coding (denoise-and-forward). Unlike [7], here we do not fix the modulation and do not use orthogonal CDMA codes, but take advantage of the non-interfering RSs deployment to serve more data flows simultaneously. We focus on a scheme that consists of only two phases, shown on Fig. 3. In the first phase, the BS broadcasts to RS1 and RS2 using superposition coding, while U1 and U2 are simultaneously carrying out their uplink transmissions. In the second phase, the relays RS1 and RS2 are broadcasting: RS1 to BS and U1, while RS2 to BS and U2. The achievable rate region for the considered scenario is four-dimensional. In order to obtain a better insight, we parametrize the two-way traffic associated with each of the MSs by defining a *downlink-uplink ratio* [8].

## II. SYSTEM AND CHANNEL MODELS

We consider a two-way cellular network in which a BS exchanges information with the MSs U1 and U2 with the aid of RS1 and RS2, see Fig. 1. All nodes are half-duplex, such that a node can either transmit or receive at a given time. We assume that the MSs do not have direct links to/from the BS. For simplicity, we use 1, R1, B, R2, 2 as the indices in the formulas to denote U1, RS1, BS, RS2 and U2, respectively. The channels are denoted by  $h_{11}(U1-RS1)$ ,  $h_{12}(RS1-BS)$ ,  $h_{22}(BS-RS2)$  and  $h_{21}(RS2-U2)$ . Each channel  $h_l, l \in \{11, 12, 22, 21\}$ , is reciprocal, known at all the nodes. Each MS has a two-way, uplink/downlink traffic to/from the BS.  $z_k \sim \mathcal{CN}(0, 1), k \in \{1, R1, B, R2, 2\}$  represents Additive White Gaussian Noise (AWGN) with zero mean and unit variance. The transmission power of node  $k$  is  $E\{|x_k|^2\} = P_k$ . The capacity of a single link is  $C(\gamma) = \log_2(1 + \gamma)$ , where  $\gamma$  is the Signal-to-Noise Ratio (SNR). We also assume that  $x_k$  is Gaussian with zero mean and variance  $P_k$ , such that it acts as AWGN before it is decoded. Fig. 2 illustrates the reference scheme based on time-division between two different two-way relaying instances. The first phase is the multiple access (MA) from U1 and BS to RS1. The second phase is the broadcast (BC) from RS1 to U1 and BS. The third and fourth phase are similar to the first and second phase respectively. Here we use random Gaussian codebooks, while in the companion paper [9] we have applied structured codes based on lattices.

H. Liu and Y. Zhao are with State Key Laboratory of Advanced Optical Communication Systems and Networks, Peking University, China (e-mail: {liuhp, yuping.zhao}@pku.edu.cn). P. Popovski, E. de Carvalho and F. Sun are with Aalborg University, Denmark (e-mail: {petarp, edc, fs}@es.aau.dk).

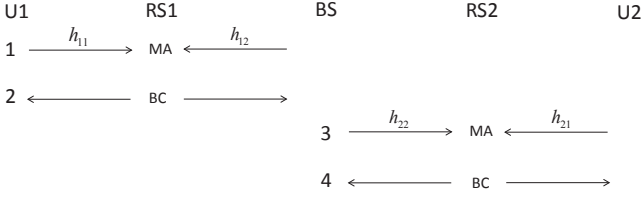


Figure 2. Four-way relaying transmission scheme consisting of four phases.

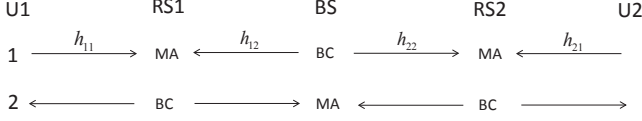


Figure 3. Proposed two stages scheme for four-way relaying.

### III. ACHIEVABLE RATE REGIONS FOR TWO-PHASE FOUR-WAY RELAYING

The signals sent by U1 and U2 are denoted by  $x_1$  and  $x_2$ , respectively. The BS uses superposition coding [10] and the broadcast signal is  $x_B = \sqrt{\alpha}x_{B1} + \sqrt{1-\alpha}x_{B2}$ ,  $\alpha \in [0, 1]$ , where  $x_{B1}(x_{B2})$  is the signal intended for U1(U2) and  $E\{|x_{B1}|^2\} = E\{|x_{B2}|^2\} = E\{|x_B|^2\} = P_B$ . Define  $R_i^d$  as the downlink (BS $\rightarrow$ RS $i\rightarrow$ U $i$ ) data rate of U $i$  and define  $R_i^u$  as the uplink (U $i\rightarrow$ RS $i\rightarrow$ BS) data rate of U $i$ . Note that,  $R_i^d$  and  $R_i^u$  are defined over the whole transmission time span. At the end of phase 1, RS $i$  receives

$$y_{Ri} = h_{i1}x_i + h_{i2}\sqrt{\alpha_i}x_{Bi} + h_{i2}\sqrt{1-\alpha_i}x_{Bj} + z_{Ri}. \quad (1)$$

here  $(i, j) \in \{(1, 2), (2, 1)\}$ ,  $\alpha_1 = \alpha, \alpha_2 = 1-\alpha$ . The decoder in RS $i$  decodes the signal  $x_i$  and  $x_{Bi}$ , then re-encodes the signals  $x_i$  and  $x_{Bi}$  into  $x_{Ri}$ . RS $i$  does not need to decode  $x_{Bj}$ ,  $j \neq i$ , as it is intended for U $j$ . In fact,  $x_{Bj}$  should only be decoded if there is a benefit for decoding  $x_i$  and  $x_{Bi}$ : in such a case, successful decoding of  $x_{Bj}$  imposes a rate limitation on  $R_j^d$ . When  $x_{Bj}$  is not decoded, it is treated as AWGN. At the end of phase 2, the received signals are

$$y_i = h_{i1}x_{Ri} + z_i, \quad y_B = h_{12}x_{R1} + h_{22}x_{R2} + z_B. \quad (2)$$

where  $y_i$  is received by U $i$  and  $y_B$  is received by the BS. Eq. (2) describes the combination of a BC and a MA channel, both with side information: the BC channel from RS $i$  to U $i$  and BS and the MA channel at BS with signals from RS1 and RS2. Note that, a similar, but simpler, problem has been discussed in [11, Theorem 2], namely a BC channel with side information available at the terminals. In this paper, we apply the same codebook design method for  $x_{Ri}$  as [11, Theorem 2]. Based on the above re-encoding technique, we combine the successive decoding and time-sharing to achieve the optimal transmission performance from RS $i$  to BS and U $i$  (details in Appendix A).

Here we interpret, rather informally, the achievable rate region for two-phase four-way relaying scheme. Let  $D^{(1)}$  and  $D^{(2)}$  denote the 4-dimensional rate regions achievable during the first and second phase, respectively. Then the achievable rate region is a convex closure of  $D = D^{(1)} \cap D^{(2)}$ . The description of  $D^{(2)}$  is given in Appendix A.  $D^{(1)}$  can be found

as the intersection  $D^{(1)} = D_{R1}^{(1)} \cap D_{R2}^{(1)}$ , where  $D_{Ri}^{(1)}$  is the achievable rate region with RS $i$  as a decoder during the first phase. Furthermore,  $D_{Ri}^{(1)}$  is the union of  $M_{Ri}^2$  and  $M_{Ri}^3$ .  $M_{Ri}^2$  is the rate region that is obtained when RS $i$  receives  $x_i$  and  $x_{Bi}$ , while treating  $x_{Bj}$ ,  $j \neq i$ , as a noise. The constraint for  $M_{Ri}^2$  is showed in (7) which is the constraint of two users MA channel [10].  $M_{Ri}^3$  is the rate region when  $x_{Bj}$  is decoded. This means RS $i$  decodes  $x_i$ ,  $x_{Bi}$  and  $x_{Bj}$  simultaneously which leads to the constraint of three users MA channel for  $M_{Ri}^3$  shown in (6), see [10].  $M_{Ri}^2$ ,  $M_{Ri}^3$  and  $D^{(2)}$  are defined as follows. Note that  $M_{Ri}^2$  is a four-dimensional region in which no constraints are put on  $(R_j^u, R_j^d)$ ,  $j \neq i$ , which means that they can have arbitrary non-negative values. Upper limits on  $(R_j^u, R_j^d)$  come from the alternative decoding option in RS $i$  ( $M_{Ri}^3$ ) and the decoding at RS $j$ . A similar observation is valid when considering some points in  $D_{Ri}^{(1)}$  where the values of  $R_j^u$  or  $R_j^d$  can be arbitrarily large. Then we can determine the four-dimensional region  $D_{Ri}^{(1)} = M_{Ri}^2 \cup M_{Ri}^3$ . The union operation may lead to a non-convex rate region for  $D$ , so the final rate region of  $(R_1^u, R_1^d, R_2^u, R_2^d)$  needs the convex closure operation on  $D$  [10]. The projection of  $D_{Ri}^{(1)}$  on the plane  $(R_i^u, R_i^d)$  defines all possible rate pairs that can be decoded at RS $i$ . Clearly, some rate pairs are achievable when  $x_{Bj}$  is decoded, others when it is treated as AWGN. Based on this, we can state the following proposition without proof:

**Proposition 1:** Let the superposition ratio be  $0 \leq \alpha_i \leq 1$  the duration of phase 1 and 2 be  $\tau$  and  $1-\tau$ , respectively, with  $0 \leq \tau \leq 1$ . Then the achievable rate region of the two-phase four-way relaying scheme using DF is the convex closure of all 4-dimensional rate tuples satisfying  $(R_1^u, R_1^d, R_2^u, R_2^d) \in D$ .

$$D = D^{(1)} \cap D^{(2)}, \quad D^{(1)} = D_{R1}^{(1)} \cap D_{R2}^{(1)}, \quad D_{Ri}^{(1)} = M_{Ri}^2 \cup M_{Ri}^3$$

$$M_{Ri}^2 = \{(R_i^u, R_i^d, R_j^u, R_j^d) \mid (R_i^u, R_i^d) \in (7)\} \quad (3)$$

$$M_{Ri}^3 = \{(R_i^u, R_i^d, R_j^u, R_j^d) \mid (R_i^u, R_i^d, R_j^d) \in (6)\} \quad (4)$$

$$D^{(2)} = \{(R_1^u, R_1^d, R_2^u, R_2^d) \mid (R_1^u, R_1^d, R_2^u, R_2^d) \in (8)\} \quad (5)$$

$$\left\{ \begin{array}{l} R_i^u < \tau C(|h_{i1}|^2 P_i), \quad R_i^d < \tau C(\alpha_i |h_{i2}|^2 P_B) \end{array} \right. \quad (6a)$$

$$\left\{ \begin{array}{l} R_j^d < \tau C((1-\alpha_i) |h_{i2}|^2 P_B) \end{array} \right. \quad (6b)$$

$$\left\{ \begin{array}{l} R_i^u + R_i^d < \tau C(|h_{i1}|^2 P_i + \alpha_i |h_{i2}|^2 P_B) \end{array} \right. \quad (6c)$$

$$\left\{ \begin{array}{l} R_i^u + R_j^d < \tau C(|h_{i1}|^2 P_i + (1-\alpha_i) |h_{i2}|^2 P_B) \end{array} \right. \quad (6d)$$

$$\left\{ \begin{array}{l} R_i^d + R_j^d < \tau C(|h_{i2}|^2 P_B) \end{array} \right. \quad (6e)$$

$$\left\{ \begin{array}{l} R_i^u + R_i^d + R_j^d < \tau C(|h_{i1}|^2 P_i + |h_{i2}|^2 P_B) \end{array} \right. \quad (6f)$$

$$\left\{ \begin{array}{l} R_i^u < \tau C\left(\frac{|h_{i1}|^2 P_i}{(1-\alpha_i) |h_{i2}|^2 P_B + 1}\right) \end{array} \right. \quad (7a)$$

$$\left\{ \begin{array}{l} R_i^d < \tau C\left(\frac{\alpha_i |h_{i2}|^2 P_B}{(1-\alpha_i) |h_{i2}|^2 P_B + 1}\right) \end{array} \right. \quad (7b)$$

$$\left\{ \begin{array}{l} R_i^u + R_i^d < \tau C\left(\frac{|h_{i1}|^2 P_i + \alpha_i |h_{i2}|^2 P_B}{(1-\alpha_i) |h_{i2}|^2 P_B + 1}\right) \end{array} \right. \quad (7c)$$

$$\begin{cases} R_1^d < (1 - \tau) C(|h_{11}|^2 P_{R1}) & (8a) \\ R_2^d < (1 - \tau) C(|h_{21}|^2 P_{R2}) & (8b) \\ R_1^u < (1 - \tau) C(|h_{12}|^2 P_{R1}) & (8c) \\ R_2^u < (1 - \tau) C(|h_{22}|^2 P_{R2}) & (8d) \\ R_1^u + R_2^u < (1 - \tau) C(|h_{12}|^2 P_{R1} + |h_{22}|^2 P_{R2}) & (8e) \end{cases}$$

where  $(i, j) \in \{(1, 2), (2, 1)\}$ .

#### IV. NUMERICAL RESULTS

The achievable rate region for four-way relaying is 4-dimensional, as shown in (3), (4) and (5). In order to get a better, 2-dimensional insight, we can fix the downlink-uplink rate ratio to e. g. 1:1 (gaming and calls) or 5:1 (web browsing) [12]. We assume that, for the  $i$ -th user, the downlink rate demand  $R_i^d$  is related to the uplink rate demand  $R_i^u$ , as  $R_i^d = \theta_i R_i^u$ , see [8]. The time ratio  $\tau$  and superposition ratio  $\alpha$  are optimized jointly by sequential quadratic programming [13]. For simplicity, we plot the achievable rate regions of the rate pair  $(R_1^u, R_2^u)$  and assume that each node has equal transmission power of  $P_k = 10$ ,  $k \in \{1, R1, B, R2, 2\}$ . The achievable rate regions of rate pair  $(R_1^u, R_2^u)$  are shown on Fig. 4-5. S2 stands for the two-phase relaying scheme, S4 stands for the four-phase relaying scheme shown on Fig. 2.

Fig. 4 depicts three cases for which the data rate is symmetric  $\theta_1 = \theta_2 = 1$ . In Case 1 all links have the same SNR, that is  $|h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 10$ . Case 2 is when the links that are direct to the BS have a lower SNR than the links that are direct to the users, that is  $|h_{11}|^2 = |h_{21}|^2 = 10, |h_{12}|^2 = |h_{22}|^2 = 1$ . Case 3 is when the links that are direct to the BS have a higher SNR than the links that are direct to the users, that is  $|h_{11}|^2 = |h_{21}|^2 = 1, |h_{12}|^2 = |h_{22}|^2 = 10$ . For asymmetric data rates,  $\theta_1 \neq 1, \theta_2 \neq 1$ , we fix the SNR as  $|h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 10$  and focus on the performance under different downlink-uplink rate ratio configurations as Fig. 5 shows. We again consider three cases. Case 4 is when the downlink data rate is smaller than the uplink data rate for both U1 and U2, that is  $\theta_1 = \theta_2 = 0.5$ . Case 5 is when the downlink data rate is larger than the uplink data rate for both U1 and U2, that is  $\theta_1 = \theta_2 = 2$ . Case 6 is the downlink data rate is larger than the uplink data rate for U1 and the downlink data rate is smaller than the uplink data rate for U2, that is  $\theta_1 = 2, \theta_2 = 0.5$ .

For both figures, when one user is inactive,  $R_1^u = 0$  or  $R_2^u = 0$ , the achievable rates of two-phase DF relaying (S2) and four-phase DF relaying (S4) coincide. However, there is a MA process to BS in the two-phase DF relaying scheme, that the four-phase DF relaying scheme does not have. Because the MA process can enlarge the sum-rate,  $\max\{R_1^u + R_2^u\}$  of the two-phase DF relaying scheme is larger than the four-phase DF relaying scheme, leading to a larger overall rate region.

S4 is composed of two separate two-way relaying sessions. Switching the channel setting of 2 links in each two-way relaying for case 2, we get the channel setting of case 3. The

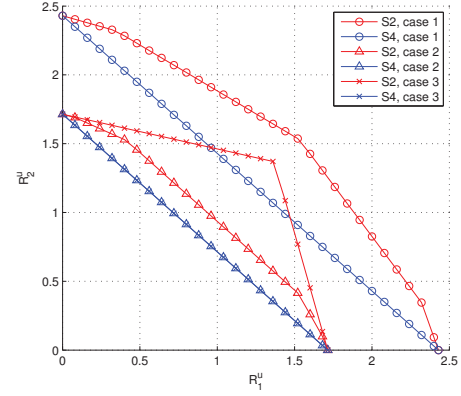


Figure 4. Comparison the rate regions of the rate pair  $(R_1^u, R_2^u)$  with  $\theta_1 = \theta_2 = 1$

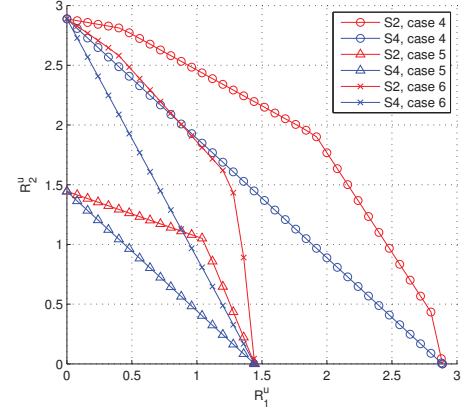


Figure 5. Comparison the rate regions of the rate pair  $(R_1^u, R_2^u)$  with  $|h_{11}|^2 = |h_{12}|^2 = |h_{22}|^2 = |h_{21}|^2 = 10$

downlink-uplink rate ratio  $\theta_1 = \theta_2 = 1$  promises that there is no difference for the uplink data rate when switching the downlink and uplink channel setting of 2 links in two-way relaying. So the curves of S4 for cases 2 and 3 coincide.

#### V. CONCLUSION

We have described a new multi-way relay scenario of practical relevance, termed four-way relaying, in which each of the two MSs has a two-way connection to the same BS, while each connection is through a dedicated RS. One of the main assumptions is that the RSs are placed in such a way that one RS and the terminals associated with it do not interfere with the other RS. We have proposed novel communication schemes which leverage on the ideas of wireless network coding and compared the performance with a state-of-the-art reference scheme where time sharing is used between the two MSs, while each MS is served through a two-way relaying scheme. The results indicate that, when the RS operates in a DF mode, the achievable rate regions are significantly enlarged. An interesting issue for future work is to consider other types of operation for the relay, such as wireless network coding, use of lattices and noisy network coding.

#### APPENDIX A

For each RS, phase 2 is a BC process with side information at the receivers, while simultaneous transmissions by RS1 and



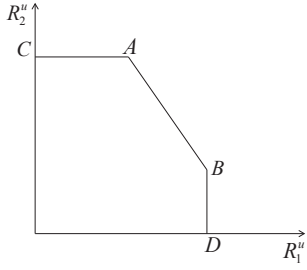


Figure 6. The capacity region of a multiple access (MA) channel.

RS2 define a MA channel at the BS. Hence, the achievable rates of the two BC transmissions are interrelated. We prove that the achievable rate region can be decomposed into two independent regions: 1) a MA region through channels  $h_{12}$  and  $h_{22}$  for rates  $(R_1^u, R_2^u)$ , independent from the links  $h_{11}$  and  $h_{21}$  and 2) two single-link rate regions  $R_1^d$  and  $R_2^d$  through  $h_{11}$  and  $h_{21}$ , respectively.

Consider the MA rate region with  $h_{12}$  and  $h_{22}$  in Fig. 6. We define  $A = (R_1^u(A), R_2^u(A))$  and  $B = (R_1^u(B), R_2^u(B))$  as the corner points of the MA region, where  $R_1^u(A) = C(|h_{12}|^2 P_{R1} / (|h_{22}|^2 P_{R2} + 1))$ ,  $R_2^u(A) = C(|h_{22}|^2 P_{R2})$  and  $R_1^u(B) = C(|h_{12}|^2 P_{R1})$ ,  $R_2^u(B) = C(|h_{22}|^2 P_{R2} / (|h_{12}|^2 P_{R1} + 1))$ . We now prove that point A can be achieved. Likewise, point B can be achieved. Time-sharing achieves the rates on the segment AB.

We recall that  $x_{Ri}$  is the re-encoded codeword at RS $i$ . The BS receives both  $x_{R1}$  and  $x_{R2}$  through the channels  $h_{12}$  and  $h_{22}$ . Let us consider a strategy where  $x_{R2}$  is treated as AWGN in order to decode  $x_{R1}$ . Then, transmission from RS1 is a BC process with side information at the receivers U1 and BS. From [11, Theorem 2], the achievable upper bound on  $R_1^d$  on link  $h_{11}$  is the capacity  $C(|h_{11}|^2 P_{R1})$  (denoted as  $C_{11}$ ) while the achievable upper bound on  $R_1^u$  on link  $h_{12}$  is  $R_1^u(A)$ .

After decoding  $x_{R1}$ , the BS cancels its contribution from the received signal. Then the transmission from RS2 is a BC process with side information at the receivers U2 and BS. From [11, Theorem 2], the achievable upper bound on  $R_2^u$  on link  $h_{22}$  is  $R_2^u(A)$  while the achievable upper bound on  $R_2^d$  on link  $h_{21}$  is the capacity  $C(|h_{21}|^2 P_{R2})$  (denoted as  $C_{21}$ ). So far, we have proved that the MA region corner point A can be achieved as well as the capacity of the link  $h_{i1}$ . The codewords used are denoted by  $x_{Ri}^A$ . Similarly, we can prove that the rates corresponding to the corner point B can be achieved with the codewords denoted by  $x_{Ri}^B$ . To summarize, an achievable upper bound on the rate tuple  $(R_1^u, R_2^u, R_1^d, R_2^d)$  is  $(R_1^u(A), R_2^u(A), C_{11}, C_{21})$  and is achieved by codewords  $x_{Ri}^A$ . Another achievable upper bound is  $(R_1^u(B), R_2^u(B), C_{11}, C_{21})$  is achieved by codewords  $x_{Ri}^B$ .

The rates on the line AB in Fig. 6 are achieved by time-sharing between the codewords  $x_{Ri}^A$  and  $x_{Ri}^B$ . Since U1, U2 and BS know the exact placement of  $x_{Ri}^A$  and  $x_{Ri}^B$  within the time-sharing, the decoding process during  $t$  is to decode  $x_{Ri}^A$ , the decoding process during  $1 - t$  is to decode  $x_{Ri}^B$ . Therefore,  $(tR_1^u(A) + (1-t)R_1^u(B), tR_2^u(A) + (1-t)R_2^u(B), C_{11}, C_{21})$  is achievable for every  $t \in [0, 1]$ ,  $\bar{t} = 1 - t$ . This proves

that the line AB in Fig. 6, is achievable while the single-user capacity of the channel with  $h_{i1}$  can also be achieved. Note that the time-sharing does not affect the decoding at U $i$ , since U $i$  knows the structure of the re-encoded codeword and it can apply the appropriate side information. Similarly, time-sharing achieves the rates on the lines CA and BD in Fig. 6. Then the whole MA region in Fig. 6 is achievable while the capacity of link  $h_{i1}$  is achievable. From these results, (8) follows.

#### ACKNOWLEDGEMENT

Part of this work has been performed in the framework of the FP7 project ICT-317669 METIS, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues in METIS, although the views expressed are those of the authors and do not necessarily represent the project. And it is also supported by National Science and Technology Major Project of the Ministry of Science and Technology of China nr. 2012ZX03001036.

#### REFERENCES

- [1] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in *Vehicular Technology Conference, 2006. VTC 2006-Spring. IEEE 63rd*, vol. 2, may 2006, pp. 588–593.
- [2] S. Zhang, S. chang Liew, and P. P. Lam, "Physical-layer network coding," in *ACM Mobicom 06*, 2006.
- [3] P. Popovski and H. Yomo, "Physical network coding in two-way wireless relay channels," in *Communications, 2007. ICC '07. IEEE International Conference on*, june 2007, pp. 707–712.
- [4] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, "Achievable rate regions and performance comparison of half duplex bi-directional relaying protocols," *Information Theory, IEEE Transactions on*, vol. 57, no. 10, pp. 6405–6418, oct. 2011.
- [5] T. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, "Broadcast capacity region of two-phase bidirectional relaying," *Information Theory, IEEE Transactions on*, vol. 54, no. 1, pp. 454–458, jan. 2008.
- [6] C. Thai, P. Popovski, M. Kaneko, and E. de Carvalho, "Coordinated transmissions to direct and relayed users in wireless cellular systems," in *Communications (ICC), 2011 IEEE International Conference on*, june 2011, pp. 1–5.
- [7] D. Fang, P. Li, A. Burr, and R. de Lamare, "Physical-layer network coding based interference exploitation strategy for multi-user hierarchical wireless networks," in *European Wireless, 2012. EW. 18th European Wireless Conference, 2012*, pp. 1–6.
- [8] S. Kim and J.-W. Lee, "Joint resource allocation for uplink and downlink in wireless networks: A case study with user-level utility functions," in *Vehicular Technology Conference, 2009. VTC Spring 2009. IEEE 69th*, april 2009, pp. 1–5.
- [9] H. Liu, E. de Carvalho, P. Popovski, and Y. Zhao, "Wireless four-way relaying using physical layer network coding with nested lattices," in *Proc. IEEE ICC 2013*, june 2013.
- [10] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 2006.
- [11] G. Kramer and S. Shamai, "Capacity for classes of broadcast channels with receiver side information," in *Information Theory Workshop, 2007. ITW '07. IEEE*, sept. 2007, pp. 313–318.
- [12] H. Truong and G. Vannuccini, "The ieee 802.11e mac for quality of service in wireless lans," IBM Zurich Research Laboratory, 2002.
- [13] P. T. Boggs and J. W. Tolle, "Sequential quadratic programming," *Acta Numerica*, vol. 4, pp. 1–51, 1995.