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Abstract This paper presents an application of information theory to identify sets of key players in social networks. First, we define two entropy measures that we use to analyze the structural properties of a social network. Then, we propose a new method aimed at finding a set of key players that solves the *KPP-Neg* and *KPP-Pos* problems. Our preliminary experimental results indicate that the entropy measures can be used effectively to identify a set of key players in a social network.

Key words: Social Networks, Knowledge Discovery, Information Theory, Entropy, Centrality

1 Introduction

Social Network Analysis (SNA) comprises the study of relations, ties, patterns of communication and behavioral performance among diverse social groups. In SNA a social network is commonly represented by a graph containing nodes and edges. The nodes in the graph represent social actors and the links the relationship or ties between them. More formally, a graph consisting of n nodes and m edges is defined as $G = \{V, E\}$, where $V = \{v_1, v_2, ..., v_n\}$ is the set of nodes or vertex and $E = \{e_1, e_2, ..., e_m\}$ is a set of links or edges. Graphs where the edges do not have an associated direction are called undirected graphs. Graphs that contain no cycles are called acyclic graphs. For convenience, in the rest of the paper we will use the terms undirected acyclic graph, graph and network as synonyms.

Previous studies in SNA have proposed a diversity of measures to study the communication patterns and the structure of a social network. One of the most studied measures is centrality. Centrality describes an actor's position within the context of his/her social network[1]. Diverse centrality measures have been proposed in the literature to quantify some of the network's properties. Examples of these are degree centrality, closeness, betweenness, eigenvector centrality, information centrality, flow betweenness, the rush index, and the influence measures among others [2][3].

One of the uses of centrality measures is to identify key players in social networks[4]. Key players are those elements in the network that are considered important, in regard to some criteria. Centrality measures have been used to

evaluate a player's degree of "popularity" within the network. Other centrality measures evaluate the degree of control that players have on the information flow or how close they are to the most central position in the network.

Information theory deals with the quantification of information and has been successfully applied in a wide range of fields, from communication systems, cryptography and machine learning to natural language processing, neurobiology and knowledge discovery in unstructured data.

One fundamental concept employed in information theory is *entropy*. Entropy was originally proposed by Claude Shannon [5] as a measure to quantify the amount of information that could be transmitted in a noisy communication channel. In a complementary way, entropy can also be used to quantify the degree of uncertainty in a message or in general within a system. Shannon's definition of entropy of a random variable X that can take n values is presented in Equation 1.

$$H(X) = -\sum_{i=1}^{n} p(x_i) \times log_2 p(x_i) \tag{1}$$

In this paper we propose a new method based on Shannon's definition of entropy aimed at finding sets of key players in a social network. To asses the performance of our method we have designed a simulation environment specially built for the purpose. The simulations allowed us to perform a comparative evaluation of the results obtained by our method with those reported in the literature. Our preliminary results indicate that the proposed method can be used effectively to identify sets of key players. The rest of the paper is organized as follows. Section 2 presents a summary of related work. Section 3 describes the proposed method. Section 4 briefly describes the simulation environment used in our experiments and presents some preliminary results. Finally section 5 describes the future work and provides some conclusions.

2   Related Work

Centrality measures have been applied in previous research work to identify key players in social networks [2][4]. Key players in general, are those nodes in the network that control the information flow, are the most popular, and/or have some sort of influence on other nodes. When a player controls the flow of information, messages sent through the network frequently pass through these players (betweeness). The influence of a player in the network is measured by evaluating the degree with which these players may reach most or all of the other elements in the network within a few steps (degree centrality).

Finally, key players are generally the most "popular" in the network i.e. they represent centers of large cliques in the graph (eigenvector centrality).

In spite of their simplicity, centrality measures have shown to be a robust and effective way to identify key players. In [6] the performance of centrality measures was studied under the conditions of imperfect data. Random graphs of

different densities in edges and nodes were generated. Then, it was measured how the addition or removal of nodes and edges affects the accuracy of each of the centrality measures employed in the experiments. Borgatti et al. found out that, as expected, the accuracy of centrality measures decreases with an increasing error rate but surprisingly, it does it in a predictable and monotonic way. This result means in principle that if one were able to estimate the percentage of errors made when a network is built, we could also be able to estimate bounds on the accuracy of the results obtained by applying centrality measures. The other interesting finding reported in [6], was that all centrality measures perform with a similar degree of robustness.

Centrality measures make some assumptions on the way the information flows in the network [3]. Hence, as described in [3], the type of information flow assumed in the network determines which measure is the more appropriate for the problem at hand. Therefore, the type of flow that occurs within a network must be determined before a centrality measure could be correctly used.

The literature on the use of centrality measures to find key players is extensive; see for example [4], [3] and [1]. However, to our knowledge only [7] has addressed the problem of finding an optimal set of key players. The problem of finding an individual key player is different from that of finding a set of k-players. In other words, the problem of getting an optimal set of k-players is different from the problem of selecting k individuals that are each, individually optimal [7]. For this reason, applying naively centrality measures to find a set of key players will fail. A simple example that illustrates why this may happen, is the case of a network with a few central nodes that are redundant. Eliminating these nodes will have no effect on the network once another redundant node has been removed. However, it is possible that the network contains also nodes that in spite of not having a high centrality degree, have in fact a greater impact in disrupting the network structure when removed.

One recent approach to identify sets of key players is described in [7]. Borgatti defines the *Key Player Problem Positive (KPP-Pos)* as consisting of identifying these k-players that could be used as seeds in diffusing optimally some information on the network. The *Key Player Problem Negative (KPP-Neg)* goal consists of identifying those k-players that, if removed, will disrupt or fragment the network. Borgatti found that off-the-shelf centrality measures are not appropriate for the task of discovering sets of key players for the KPP-Pos and KPP-Neg problems. He proposes a new method based on combinatorial optimization.

To evaluate the solution to both KPP-Neg and KPP-Pos problems, Borgatti proposes the use of new success metrics and employs heuristics and optimization methods aimed at finding the optimal set of key players. The greedy heuristic presented in [7] seeks to select those nodes in the graph that maximize these success metrics. Borgatti applied his approach to two data sets, one terrorist network and a network of members of a global consulting company with advice seeking ties.

Tutzauer in [8] proposes an entropy-based measure of centrality which is appropriate for traffic that propagates by transfer and flows along paths.

Shetty and Adibi in [9] combine the use of cross entropy and text mining techniques to discover important nodes on the Enron corpora of emails.

In this paper we apply a new method based on entropy measures aimed at finding sets of key players that solves both the KPP-Pos and KPP-Neg problems. Our method has some similarities with the one described in [9]. However, contrarily to that approach, our method relies only on the structural properties of the network and is aimed at solving KPP-Neg and KPP-Pos problems. Next section describes our approach in detail.

3   Discovering Sets of Key Players Using Entropy Measures

In [7] Borgatti provides the following formal definition of the set of key players problem:

"Given a social network(represented as an undirected graph), find a set of k nodes (called a kp-set of order k) such that,
1. (KPP-Neg) Removing the kp-set would result in a residual network with the least possible cohesion.
2. (KPP-Pos) The kp-set is maximally connected to all other nodes."

The approach presented in this paper does not aim at solving both problems optimally as was done is [7] but to demonstrate that an alternative simple solution based on information theory can be used to deal with both problems.

We define the connectivity of a node $v_i \in V$ in a graph as:

$$\chi(v) = \frac{deg(v_i)}{2N} \qquad (2)$$

where $deg(v_i)$ is the number of incident edges to node v_i and N the total number of edges in the graph. We can use χ as the stationary probability distribution of random walkers in the graph[10]. We call this the *connectivity probability distribution* of the graph.

Another probability distribution can be defined in terms of the number of paths that have v_i as source and the rest of nodes in the graph as targets:

$$\gamma(v) = \frac{paths(v_i)}{paths(v_1, v_2, ..., v_M)} \qquad (3)$$

where $paths(v_i)$ is the number of paths from node v_i to all the other nodes in the graph and $paths(v_1, v_2, ..., v_M)$ is the total number of paths M that exists across all the nodes in the graph. We call this distribution the *centrality probability distribution*. It must be noted that for applications in SNA that assume informations flows through the shortest paths, Eq. 3 should be changed to using the geodesic paths.

Using either equation 2 or 3 as the probability distributions we can obtain different entropy measures using Equation 1. By performing this procedure we define what we call the *connectivity entropy* H_{co} and the *centrality entropy* measures H_{ce} of a graph G, respectively in the following way:

$$H_{co}(G) = -\sum_{i=1}^{n} \chi(v_i) \times log_2 \chi(v_i) \tag{4}$$

$$H_{ce}(G) = -\sum_{i=1}^{n} \gamma(v_i) \times log_2 \gamma(v_i) \tag{5}$$

The connectivity entropy measure provides information about the degree of connectivity for a node in the graph. In a fully connected graph the removal of a node will decrease the total entropy of the graph, in the same proportion as if any other node is removed. All nodes will have the same effect on the graph leaving it densely connected after a node is removed. However, in a graph with lower density of edges, the removal of nodes with many incident edges will have a larger impact in decreasing the total connectivity entropy of the system, compared to the case when a node with a smaller connectivity degree is removed. This effect is illustrated in figure 1 and 2.
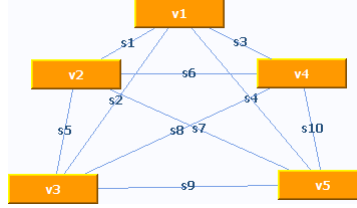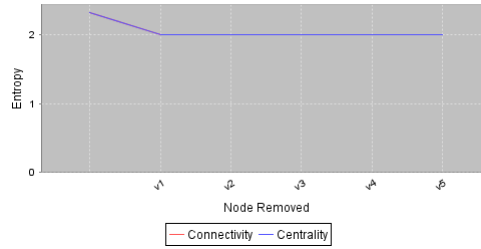
Figure 1. Fully Connected Graph

Figure 2. Entropy of a Fully Connected Graph

The centrality entropy provides information on the degree of centrality for a node in the graph. Those nodes that will split the graph in two or that will

reduce substantially the number of paths available to reach other nodes when removed, will have a higher impact in decreasing the total centrality entropy of a graph. This effect is illustrated in figures 3 and 4 where the removal of node v_5 causes the disconnection of node v_6, this event produces the largest change in centrality entropy for the graph.
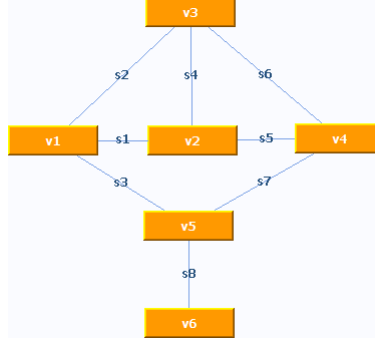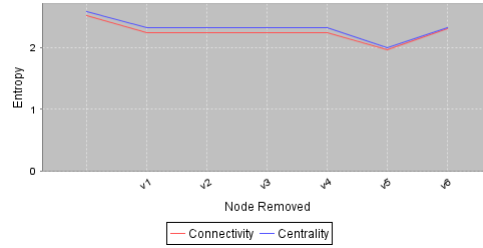
Figure 3. Partially Connected Graph

Figure 4. Entropy of a Partially Connected Graph

Note that figures 2 and 4 also show that there is either, perfect or very high correlation between the connectivity and centrality entropy measures when applied to the fully-connected and partially-connected graph examples, respectively.

Our method aimed at finding a set of key players that addresses the KPP-Neg and KPP-Pos problems consists of applying Algorithm 1.

Figure 6 shows the results of applying Algorithm 1 to the graph in figure 5. The graph is provided as an example by Borgatti in [7]. Our results show that centrality entropy is capable of detecting redundant nodes such as h and i. Node i is redundant as its removal will not have any impact on the number of

Algorithm 1

1: Calculate initial total entropy $H_{co_0}(G)$ and $H_{ce_0}(G)$
2: **for all** $nodes \in$ graph G **do**
3:     Remove node v_i, creating a modified graph G'
4:     Recalculate $H_{co_i}(G')$ and $H_{ce_i}(G')$, store these results
5:     Restore original graph G
6: **end for**
7: To solve the KPP-Pos problem select those nodes that produce the largest change in graph entropy H_{co_0}-$H_{co_i} \geq \delta_1$
8: To solve the KPP-Neg problem select those nodes that produce the largest change in graph entropy H_{ce_0}-$H_{ce_i} \geq \delta_2$

partitions created, once h has been removed. This happens in spite of i having a high centrality value.

In this simple example our algorithm determines that the set of key players consists of $\{h, m\}$ when the right δ_i value is used to filter out node q. Node q disconnects only a single node (s) from the graph and therefore it will have little impact on the network structure when removed. By adjusting the value of δ_i we can control how many nodes we will include in the final set of key players.

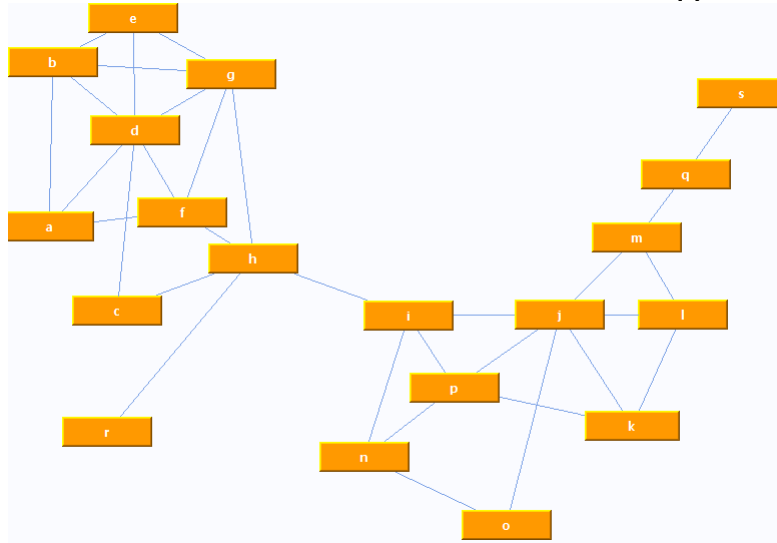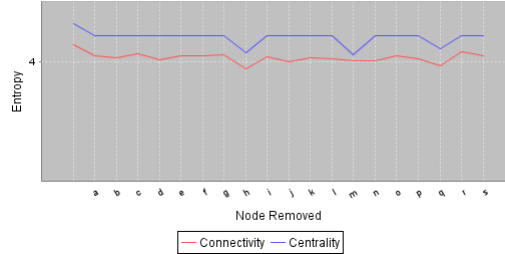**Figure 5.** Graph taken from Borgatti's Examples in [7]

Figure 6 shows that node h has the largest impact on connectivity entropy when removed from the graph. Interestingly, the same graph shows that node q has more effect on connectivity entropy, when compared to node m. The reason

Figure 6. Entropy of Example Graph from Borgatti's examples

is that removing m leaves still a connected graph composed of nodes q and s, which contributes to the total entropy. Contrarily, removing q leaves the single node s isolated.

In summary, to solve KPP-Pos and KPP-Neg problems we propose to use connectivity entropy and centrality entropy in the way is described by Algorithm 1. The basic idea is to find those nodes that produce the largest change in connectivity or centrality entropy when removed from the graph. These nodes should be included in the set of key players. The value of δ_i, allows us to control how many players should be included in the set.

In next section we describe some examples of how to apply the entropy measures to find a set of key players that solves KPP-Pos and KPP-Neg problems.

4   Simulation Environment and Experimental Results

We have created a special simulation environment to asses the performance of the proposed method. The simulation environment accepts as input the description of a graph in the XML based file format for graphs called GraphML.

The development process of our simulation environment was substantially reduced by using open source libraries. To create the mathematical models and representation of a graph we use the jGraphT library. JGraphT is an extension to jGraph, a popular graphic visualization library, that has been optimized for data models and algorithms. The algorithms provided by jGraphT allow us to traverse and analyze the properties of a graph. jGraphT has been written using generic classes with the goal of easing the coding of applications that are independent of the data models employed. A special adapter class included in jGraphT is used to interact with the graphic library jGraph.

To show the simulation results we used jChart and jFreeChart. Finally, as jGraph does not provide a free graph layout algorithm we have implemented a variation of the well known spring algorithm [11]. The whole simulation environment was designed using design patterns and was written in the Java language.

All the figures shown on this paper were obtained directly from our simulation environment.

Figure 7. Terrorist Network

Figure 8. Entropy of Terrorist Network

Figure 8 show the results of applying Algorithm 1 using centrality and connectivity entropy to the terrorist graph in figure 7. Figure 8 shows that centrality entropy identifies a set of key players consisting of $\{atta, nalhazmi, darkazalni\}$, since these are the nodes that produce the biggest changes in entropy when removed, with $atta$ producing the largest change. It must be noticed that nodes $nalhazmi$ and $darkazanli$ have the same effect on centrality entropy. This is because if we look at figure 7 we can notice that both nodes will disconnect a single node if removed. However, removing $nalhazmi$ will also cause a major impact in connectivity entropy, contrarily to the case when $darkazanli$ is removed. This indicates that $nalhazmi$ may be indeed more important than node $darkazanli$, even if both produce a similar effect on centrality entropy. This factor can also be used to grade the importance of a node in the graph.

Algorithm 1 finds also that the set of nodes in figure 7 that solve KPP-Pos problem consist of $\{nalhazmi, halghamdi, salghamdi, atta\}$, as these are the nodes that will have the biggest impact on connectivity entropy when removed from the graph.

In a different example of social network, figure 10 shows the result of applying centrality and connectivity entropy to the graph in Figure 9. The graph describes the advise ties between members of a company and was obtained from [7].

Figure 9. Company Ties Network
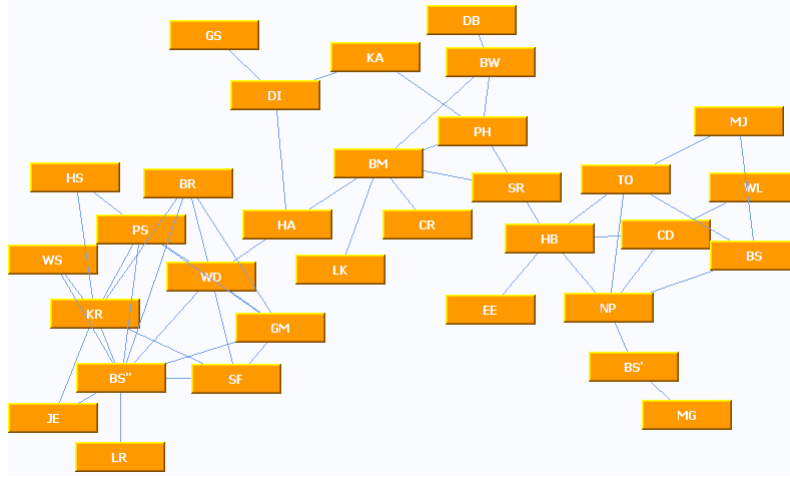
Applying algorithm 1, we found that the set of most important players that will solve KPP-Neg consists of $\{HB, BM, WD, NP\}$. In this same example, Borgatti obtained that the set of key players consisted of $\{HB, BM, WD\}$ [7]. This is the set of players that if removed will divide the network into 6 components. Our algorithm finds the same elements additionally to NP. However, it must

Figure 10. Entropy of Company Ties Network
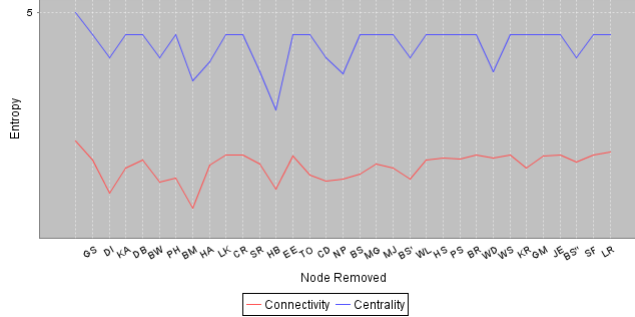
be remarked that contrarily to [7], our algorithm does not try to optimize any specific metric.

In KPP-Pos problem we are asked to find the smallest set of nodes that are well connected to the entire network. This set of players are the ones that if used as "seeds" will reach 100% of the network.

If we look only at the connectivity entropy chart in Figure 10 we notice that Algorithm 1 will select nodes $\{BM, DI, HB, BW, CD, BS', NP, TO, BS\}$ as the key players when a set of size $k = 9$ is selected. These are the nodes that when removed, will produce the largest changes in connectivity entropy. This list indicates that connectivity entropy allows us to get 89% of the key players found by Borgatti for a similar size set. However, if we add to the set, the $10th$ node that produces the next largest change in connectivty entropy, we will obtain a set consisting of $\{BM, DI, HB, BW, CD, BS', NP, TO, BS, PS\}$. This new set contains 100% of the nodes that Borgatti found as the key players in [7].

5   Conclusions and Future Work

In this paper we have proposed a new method that finds the set of key players within a network using entropy measures. Our method aimed at solving KPP-Pos problem basically consists of selecting the set of nodes that produce the largest change in connectivity entropy when removed from a graph. Similarly, to solve KPP-Neg we propose to use centrality entropy and measure how entropy changes when a node is removed from the graph.

The main advantage of our method when compared to other similar approaches is its simplicity. However, in its current version, the method can only be applied to small networks due to the complexity involved in calculating centrality entropy, which is based on finding all paths within the network.

To asses the performance of our method we have built a simulation environment specially designed for the purpose. We have applied our method to two examples of social networks: a terrorist organization and a company. Our exper-

imental results show that our simple method is capable of obtaining comparable results with those described by Borgatti in [7], in which he uses an optimization algorithm and special metrics. Interestingly, our method is capable of finding the same optimal sets.

As future work we plan to perform a more comprehensive evaluation of the method proposed in this paper, using a larger collection of social networks. We also plan to include in our method heuristics targeted at optimizing some specific metrics, similarly as it was done in [7]. To provide a more flexible simulation environment, we will design a configurable simulator that will allow us to employ other libraries such as Prefuse (for visualization) and JUNG (used mainly for analysis, modeling and visualization). Finally, we plan to investigate the application of efficient techniques aimed at reducing the overall complexity of the algorithms employed to find all the paths within the network.

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