Modal Parameter Identification from Responses of General Unknown Random Inputs

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Published in:
Proceedings of the 14th International Modal Analysis Conference, Dearborn, Michigan, USA, Feb. 12.15,1996

Publication date:
1996

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
ABSTRACT. Modal parameter identification from ambient responses due to a general unknown random inputs is investigated. Existing identification techniques which are based on the assumptions of white noise and or stationary random inputs are utilized even though the inputs conditions are not satisfied. This is accomplished via adding, in cascade, a force conversion system to the structure's system under consideration. The input to the force conversion system is white noise and the output of which is the actual force(s) applied to the structure. The white noise input(s) and the structure’s responses are then used to identify the combined system. Identification results are then sorted as either structural parameters or input force(s) characteristics.

1. INTRODUCTION

Modal parameter identification from ambient responses has gained considerable attention in recent years. The literature reveals a multitude of cases of vibration testing of bridges, buildings, off shore structures, aircrafts, space crafts, ground vehicles, among others, utilizing responses due to wind, waves, traffic, road roughness, propulsion systems etc. The advantages of such techniques are quite evident: The normal operation of the structure under test is not interrupted; no excitation cost; no measurements of inputs; continuous if not unlimited response records: suitable for structural integrity monitoring.

However, identification from ambient responses possesses two main disadvantages: first the input energy may be low to excite the modes of interest; secondly the input is assumed to be white noise or stationary random.

The general identification theories as applied to modal parameters estimation of vibrating structures can be classified into different main categories depending on the nature of the loading. Usually the loads are assumed to fall into one of the following categories:

- Known and measurable force inputs time histories and locations,
- no force inputs, (utilizing structure’s free response due to initial excitation), and
- white noise inputs.

However, there exist many structural applications, as pointed out earlier, where it is either impractical or uneconomical to use, or satisfy the conditions of, the above mentioned inputs. At the same time, these types of structures or applications offer the readily available and economical ambient or operational responses.
Such applications have traditionally been analyzed implementing identification techniques such as Frequency Response Functions. ARMA models and Random Decrement Techniques; among others. Such approaches, however, are based on the classical assumption of white noise inputs; a condition that is not usually satisfied.

In this paper, these techniques which require white noise inputs will be extended to apply to cases of general force inputs. This is accomplished by adding a pseudo second order system, in series with the second order system representing the structure, to which pseudo white noise inputs are applied. The responses of the combined system loaded by white noise are in reality the actual structure's response to the general force inputs.

Simulated and experimental results are presented in support of the proposed approach. The techniques implemented here are applied to a full scale structure in another publication.

2. THEORY

As shown in Figure 1, the structure whose transfer function is $H_s(s)$ has an input of $f(s)$ and an output of $x(s)$. The input $f(s)$ is not white noise. A pseudo second order system of a transfer function $H_f(s)$ is added in series to the system and is assumed to have the dynamic characteristics such that if the input to it is white noise, the output is the actual force to the structure $f(s)$. Now for the combined system in cascade the input is white noise and the output is the actual structure's response. Even though systems in cascade have been fully analyzed in dynamical systems theory, the objective of the ensuing proof is to address the identification aspect of systems in cascade, particularly vibrating systems, to ensure that the modal parameters of the structural system are independently preserved and can be uniquely identified.

The following equations relate the input to output for both the structural system and the force pseudo system

$$x(s) = H_s(s)f(s)$$

(1)

$$f(s) = H_f(s)n(s)$$

(2)

Thus the transfer function of the combined system becomes

$$H_c(s) = H_f(s)H_s(s)$$

(3)

Now let

$$H_s(s) = \sum_{i=1}^{2n} \frac{a_i}{s - \lambda_i}$$

(4)

and

$$H_f(s) = \sum_{j=1}^{2m} \frac{b_j}{s - \alpha_j}$$

(5)

Then the combined system transfer function becomes

$$H_c(s) = \sum_{i} \frac{a_i}{s - \lambda_i} \sum_{j} \frac{b_j}{s - \alpha_j}$$

(6)

$$= \sum_{i} \sum_{j} \frac{a_i b_j}{(s - \lambda_i)(s - \alpha_j)}$$

(7)

utilizing partial functions, equation (7) becomes

$$H_c(s) = \sum_{i} \sum_{j} \left( \frac{a_i b_j (\lambda_i - \alpha_j)}{(s - \lambda_i)} + \frac{a_i b_j (\alpha_j - \lambda_i)}{s - \alpha_j} \right)$$

$$= \sum_{i} \frac{a_i}{s - \lambda_i} (A \lambda_i - B) + \sum_{j} \frac{b_j (C \alpha_j - D)}{s - \alpha_j}$$

(8)

where

$$A = \sum_{j} b_j$$

(9)

$$B = \sum_{j} b_j \alpha_j$$

(10)

$$C = \sum_{i} a_i$$

(11)

$$D = \sum_{i} \alpha_i \lambda_i$$

(12)

Equation (8) verifies the stipulation that the modal parameters of the structural system and the force pseudo system are preserved and separable. The poles, in the denominator, are unaffected by combining the two systems. Thus frequencies and damping factors identification is expected to be correct. As well, the residues in the partial fractions are simply multiplied by a constant for each mode. Thus the mode shapes remain uniquely identifiable.

3. THEORY VERIFICATION AND TESTING
To test the above theorems, a simulated test of a two degrees of freedom system is performed. The system is shown in Figure 2. Real experimental results are reported in another publication. For the simulated experiment a white noise process is inputted to the force pseudo system which in this case assumed to be a single degree of freedom second order system the output of which is

\[ F(t) = \frac{1}{w_f^2 - w^2 + 2i\zeta_f w_f w} \quad (13) \]

The load to the "structure" under test is simulated using a covariance equivalent ARMA model:

\[ F(t) = AR_1 F(t_{t-1}) + AR_2 F(t_{t-2}) + N(t) + MA_1 N(t_{t-1}) \quad (14) \]

where N(t) is the white noise and AR, AR2, MA, are functions of \( \omega_f, \zeta_f \) and sampling rate AT. F(t) is applied equally to both degrees of freedom; thus \( F_1 = F_2 = F \). Figure 3 shows an example of the spectrum, time history and normal probability plots for the white noise and the output of the pseudo force system which is the input to the structural system. The force pseudo system for these plots has \( \omega_f = 15.2578 \) and \( \zeta_f = 0.005 \). These numbers are merely for illustration. For structural simulation \( \omega_f \) is taken as the average of the two natural frequencies of the system.

The structural system's parameters were chosen as:

\[ k_1 = k_3 = 150, \quad k_2 = 20, \quad m_1 = 1.0, \quad m_2 = 2.0 \quad (15) \]

and the damping matrix was selected as nonproportional of the form

\[ c = 0.02M + 0.001K + 0.1 \quad (16) \]

Thus the system modal parameters were \( \omega_1 = 9.0947 \) r/s, \( \omega_2 = 13.1256 \) r/s and \( \omega_f = 11.1101 \) r/s (average of \( \omega_1 \) and \( \omega_2 \)). For damping factors \( \zeta_1 = 0.0097 \), \( \zeta_2 = 0.0102 \) and \( \zeta_f = 0.005 \).

The theoretical mode shapes we calculated to be:

\[ \phi_1 = [1 \ 4.3597]^T \quad < \phi_1 = [0 \ 2.7613]^T \]

\[ \phi_2 = [1 \ 0.1148]^T \quad < \phi_2 = [10 \ 176.0168]^T \quad (17) \]

Figures 4 shows spectra and time histories for the outputs.

The random decrement technique is used to convert systems random responses into free decay responses or correlation functions. Figure 5 shows the auto and cross RDD signatures using mass 1 as the triggering measurement and triggering was at every positive point. Figure 6 shows RDD signatures for triggering at local extremum of mass 1 response.

Modal parameter identification was performed using ARV and ITD methods. Table 1 shows identification results using signatures of Figure 5. and Table 2 is for those signatures of Figure 6.

From identification results, it can be seen that the system's characteristics as well as the force characteristics were identified.
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<tr>
<th>Parameter</th>
<th>Theory</th>
<th>ARV</th>
<th>% error</th>
<th>ITD</th>
<th>% error</th>
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4. CONCLUSIONS

Ambient random responses treated with random decrement and time domain identification techniques are effective and economical in modal identification of structures. This approach, among others, is classically based on the assumption of stationary random input. Non-stationary random inputs results in the identification of extraneous modal parameters which belong to the forcing system rather than the structure being tested. However, it is shown that forcing function dynamic characteristics have no effect on the accuracy of structural parameter identification. Techniques need to be developed to assist in sorting out structural dynamic properties from those of the inputs.

5. ACKNOWLEDGEMENTS

This work is partially supported by The Danish Research Council. First author was a Summer Visiting Professor at Aalborg University, Denmark during parts of this work.

6. REFERENCES

Figure 3. Spectra, Time Histories and Distribution of White Noise and Input to Structure
Figure 4. Time Histories and Spectra of Responses
Figure 5. Random Decrement Signatures with Triggering on Every Positive Point of Mass 1

Figure 6. Random Decrement Signatures with Triggering on Local Extremum of Mass 1