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Damping Estimation by Frequency Domain Decomposition

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Abstract

In this paper it is explained how the damping can be estimated using the Frequency Domain Decomposition technique for output-only modal identification, i.e. in the case where the modal parameters is to be estimated without knowing the forces exciting the system. Also it is explained how the natural frequencies can be accurately estimated without being limited by the frequency resolution of the discrete Fourier transform. It is explained how the spectral density matrix is decomposed into a set of single degree of freedom systems, and how the individual SDOF auto spectral density functions are transformed back to time domain to identify damping and frequency. The technique is illustrated on a simple simulation case with 2 closely spaced modes. On this example it is illustrated how the identification is influenced by very closely spacing, by non-orthogonal modes, and by correlated input. The technique is further illustrated on the output-only identification of the Great Belt Bridge. On this example it is shown how the damping is identified on a weakly exited mode and a closely spaced mode.

Nomenclature

G	Power spectral density matrix
ϕ	Mode shapes
ω, f	Angular frequency, frequency (Hz)
u	Singular vectors
s	Singular values
δ	Logarithmic decrement
ς	Modal damping ratio
Ω	MAC limit value

Introduction

Output-only identification of structures is normally associated with the identification of modal parameters from the natural responses of civil engineering structures, space structures and large mechanical structures. Normally, in these cases the loads are unknown, and thus, the modal identification has to be carried out based on the responses only. Real case examples on some civil engineering structures can be found in Ventura and Horyna [1] or Andersen et al. [2].

The present paper deals with the problem of damping estimation using a relatively new technique for output-only identification called Frequency Domain Decomposition (FDD). The technique is described in Brincker et al [3], [4].

The technique is closely related to the classical frequency domain techniques where the modes are identified by picking the peaks in the spectral diagrams, Bendat and Piersol [5], Felber [6]. However, since the FDD technique approximately decomposes the spectral density matrix into a set of SDOF systems using the Singular Value Decomposition (SVD), the main part of the uncertainty of the classical technique is removed.

In this paper it is explained more detailed how the SDOF auto spectral densities are identified using the modal assurance criterion (MAC), how the bells are transformed back to time domain, and how the damping and more accurate natural frequency estimates are identified from the corresponding free decays.

Identification Algorithm

In the Frequency Domain Decomposition (FDD) identification, the first step is to estimate the power spectral density matrix. The estimate of the output PSD $\hat{G}_{yy}(j\omega)$ known at discrete frequencies $\omega = \omega_i$ is then decomposed by taking the Singular Value Decomposition (SVD) of the matrix

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \quad (1)$$

where the matrix $U_i = [u_{i1}, u_{i2}, \ldots, u_{im}]$ is a unitary matrix holding the singular vectors u_{ij}, and S_i is a diagonal matrix holding the scalar singular values s_{ij}. Near a peak corresponding to the k th mode in the spectrum this mode or may be a possible close mode will be dominating. Thus, according to the FDD theory, the first singular vector u_{i1} is an estimate of the mode shape

$$\hat{\phi} = u_{i1} \quad (2)$$

and the corresponding singular value is the auto power spectral density function of the corresponding single degree of freedom system. This power spectral density function is identified around the peak by comparing the mode shape estimate $\hat{\phi}$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has high MAC value with $\hat{\phi}$ the corresponding singular value belongs to the SDOF density function. If at a certain line none of the singular values has a singular vector with a MAC value larger than a certain limit value Ω, the search for matching parts of the auto spectral density function is terminated. The remaining spectral pins (the un-identified part of the auto spectral density function) are set to zero.

From the fully or partially identified SDOF auto spectral density function, the natural frequency and the damping are obtained by taking the spectral density function back to time domain by inverse FFT.

From the free decay time domain function, which is also the auto correlation function of the SDOF system, the natural frequency and the damping is found by estimating crossing times and logarithmic decrement. First all extremes r_k, both peaks and valleys, on the correlation function are found. The logarithmic decrement δ is then given by

$$\delta = \frac{2}{k}\ln\left(\frac{r_0}{|r_k|}\right) \quad (3)$$

where r_0 is the initial value of the correlation function and r_k is the k'th extreme. Thus, the logarithmic decrement and the initial value of the correlation function can be found by linear regression on $k\delta$ and $2\ln(|r_k|)$, and the damping ratio is given by the well known formula

$$\varsigma = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} \quad (4)$$

A similar procedure is adopted for determination of the natural frequency. The frequency is found by making a linear regression on the crossing times and the times corresponding to the extremes and using that the damped natural frequency f_d and de undamped natural frequency f is related by

$$f = \frac{f_d}{\sqrt{1-\varsigma^2}} \quad (5)$$

The extreme values and the corresponding times were found by quadratic interpolation, whereas the crossing times where found by linear interpolation.

Simulation case, closely spaced modes

The technique is illustrated on a case with 2 closely spaced modes. The response of a 2 DOF system is simulated using a vector ARMA model, Andersen [7], and assuming that both degrees of freedom are loaded by Gaussian distributed white noise un-correlated processes. Exact and identified modal parameters are shown in Tables 1 and 2.

The first case considered is a case with a reasonable spacing between the two modes. An auto spectral density and the singular values of the decomposed spectral matrix are shown in Figure 1. As it appears, the two modes are clearly visible in both plots. Partial identifying of the auto spectral densities of the two SDOF systems using the MAC as described above yields the result as shown in Figure 2.

Taking the inverse discrete Fourier transform of the partially identified auto spectral densities yields the corresponding auto correlation estimate as shown in Figure 3, bottom. Top part of the same Figure shows the linear regression on the

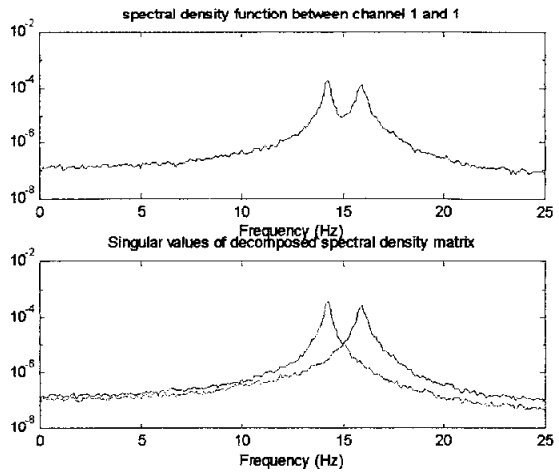


Figure 1. Case 1 with moderately spaced modes. Top: Auto spectral density. Bottom: Singular values of the decomposed spectral density matrix.

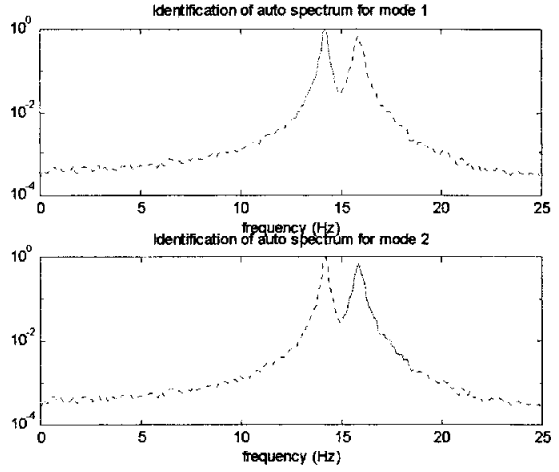


Figure 2. Partial identification of the two SDOF auto spectral density functions.

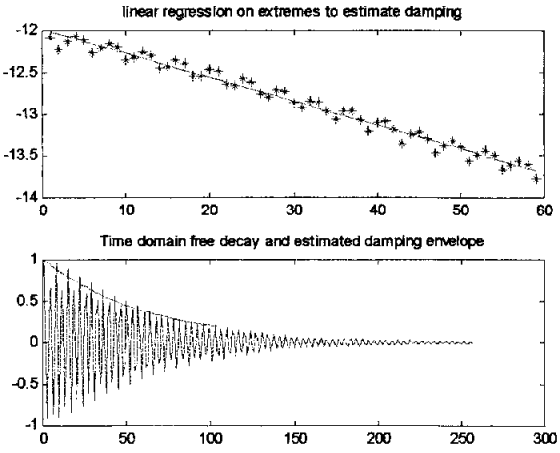


Figure 3. Top: Linear regression on extremes for estimation of damping. Bottom: Time domain free decay obtained by inverse FFT and estimated damping envelope.

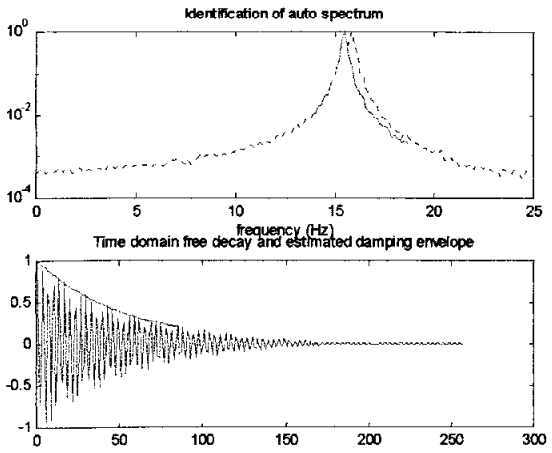


Figure 4. Case 2 with closely spaced modes. Top: Partial identification of SDOF auto spectral density. Bottom: Corresponding free decay with damping envelope.

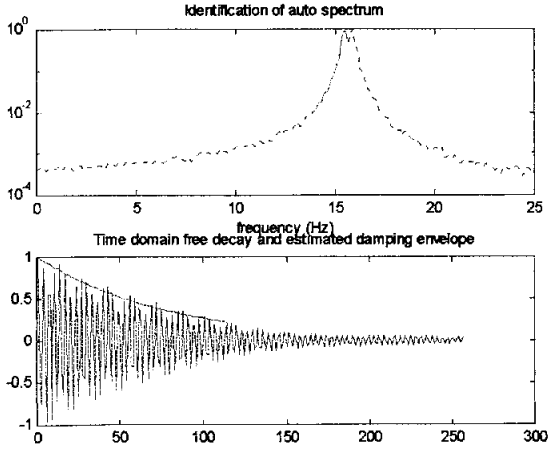


Figure 5. Case 3 with closely spaced modes, but where only a very limited part of the SDOF density is identified.

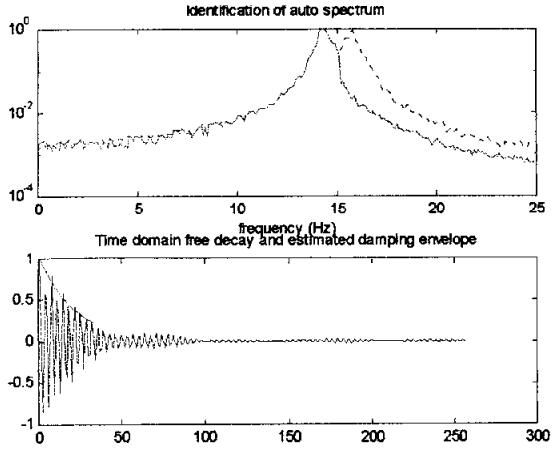


Figure 6. Case 4 with moderately spaced modes. Mode shapes not orthogonal.

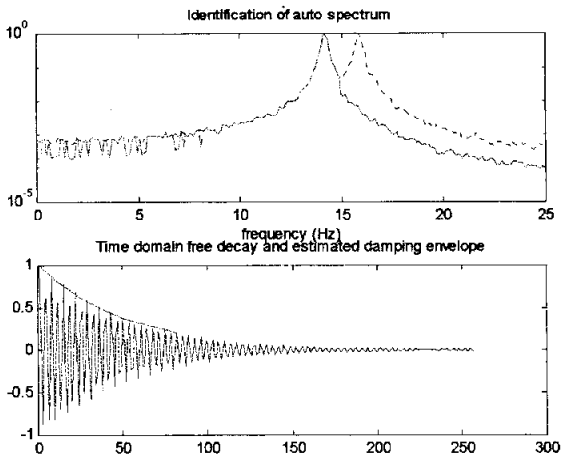


Figure 7. Case 5 with moderately spaced modes. Correlated input.

extremes, and the bottom parts compares the free decay function with the estimated damping envelope. As it appears, the procedure is quite strait forward and the user has a clear impression of the validity of the estimation simply by inspecting the plots.

The second case considered is the case of closely spaced modes as shown in figure 4. In this case it is assumed that a reasonable part of the SDOF auto spectral density can be identified on both sides of the considered modal peak. This is possible in the most cases by specifying a lower Ω-value. In this case, the identification is also strait forward and the identified damping values compares reasonably well with the theoretical values, Table 2.

In the third case it is assumed that only a quite small part of the SDOF auto spectral density function can be estimated. This can be the case if the spectral density is noisy due to limited data, or if noise is contaminating the signal. In this case however, since the data are simulated data meeting all basic assumptions of the technique, the identified SDOF density function shown in Figure 5 was obtained by using a rather high value of the MAC limit Ω. As it appears, since the number of active pins in the spectrum is cut significantly, the Fourier series in the time domain becomes truncated to a degree where the damping becomes underestimated, Table 2.

The fourth case shown in Figure 6 illustrates the influence of non-orthogonal modes. In theory, to give exact results, the FDD requires that the modes are orthogonal. All other cases considered in this paper have orthogonal modes. For the modes considered in this cases, the MAC matrix is

$$\mathrm{MAC} = \begin{bmatrix} 1.0000 & 0.4226 \\ 0.4226 & 1.0000 \end{bmatrix}$$

In this case, the SVD still split the spectral matrix in orthogonal components. This means, that even though the dominant singular value and the corresponding singular vector is a good estimate of the modal properties, the second singular value and the corresponding vector is not so closely related to the physics of the system. Thus, the right most part of the left mode is badly estimated. Even though this is the case, the modal damping estimate is still close to the exact value, Table 2.

For the last considered case, case five, the loading is moderately correlated. In case of correlated input the FDD modal decomposition is approximate. In most practical cases however, like wind loads, wave loads or traffic loads, it is known that a certain spatial correlation is present. Thus it is important to know the amount of influence such correlation might have on the modal results. In this case the correlation matrix between the two stochastic processes loading the system was

$$C = \begin{bmatrix} 1.0000 & 0.4724 \\ 0.4724 & 1.0000 \end{bmatrix}$$

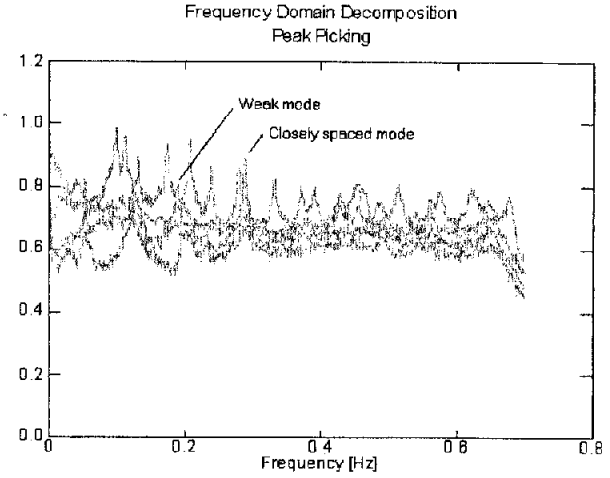
The results of the modal identification and the corresponding damping estimation of the first mode are shown in Figure 7. Again we see a certain distortion of the identified auto spectral density of the associated SDOF system in the overlapping region between the two modal peaks. However, the influence is rather small, the damping estimation is strait forward, and the estimated damping is close to the exact values, Table 2. Thus, moderate correlation does not seem to significantly influence the quality of the results.

Damping identification of the Great Belt Bridge

In the following the efficiency of the proposed damping identification technique is illustrated on ambient response data of the Great Belt Bridge. The Great Belt Bridge is a suspension bridge with a free span of 2.6 km.

Different ways of identifying the modal damping of this bridge including the application of the FDD technique as described here is investigated in Brincker et al. [8].

In the following it is illustrated how the identification works on two difficult cases often realised in practical output-only



Fig

Figure 8. Singular values of the spectral density matrix obtained from the ambient response of the Great Belt Bridge.

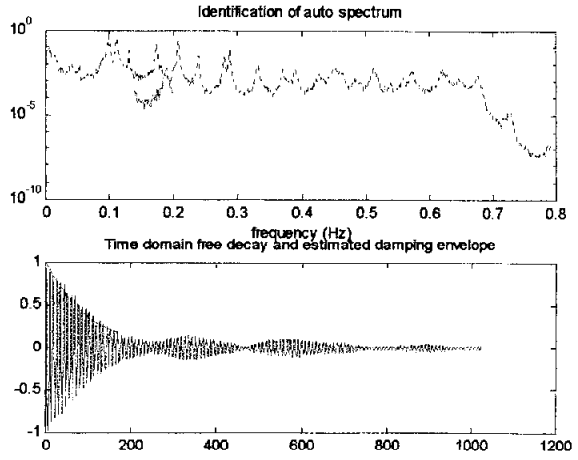


Figure 9. Partially identification of auto spectral density associated with a weakly excited mode.

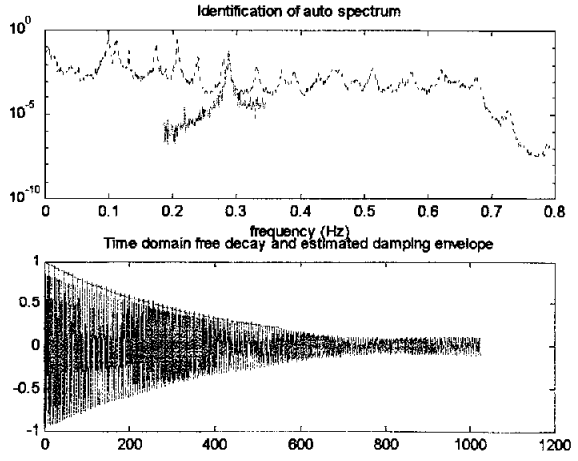


Figure 10. Partially identification of auto spectral density associated with a closely spaced mode.

identification: a weakly excited mode and a closely spaced mode.

The weakly excited mode is indicated in figure 8. As it appears it so weakly excited that only a careful inspection of the singular value decomposition of the spectral matrix or of the auto and cross spectral densities reveals that a mode is present. It is well known, that when using parametric methods like ARMA models or the Stochastic Subspace Identification algorithm, or partly parametric techniques like the Ibrahim Time Domain, the Eigen Realisation Algorithm or the Polyreference identification technique, it is normally very difficult to get reliable modal estimates and especially damping estimates in a case like this.

Figure 9 shows that the FDD clearly identifies a reasonable part of the auto spectral density of the associated SDOF system, and a damping estimate that must be judged as reliable can be obtained from the free decay function.

The closely spaced mode case is also indicated in Figure 8, and this case is relatively difficult too. Even though most of the parametric and partially parametric techniques identifies closely spaced modes without major problems, this still is difficult in cases like this with a high number of modes present in the response.

As shown in figure 10, the FDD technique identifies a large part of the auto spectral density of the associated SDOF system, and the corresponding free decay in the time domain must be considered as a good time representation of the frequency domain information. In this case, the estimated damping is very low, $\varsigma = 0.24\%$, and the correlation function is far from being vanished for maximum time lag. This indicates that the damping is biased by leakage introduced in the estimation of the spectral density functions.

Conclusions

In this paper the estimation of damping has been introduced and illustrated for the Frequency Domain Decomposition (FDD) output only identification technique.

The basic idea of the proposed identification procedure has been illustrated on a 2-DOF simulation case where it has been shown how the technique works in different cases of closely spaced modes including non-orthogonal modes and correlated input.

Further it has been illustrated how the technique works in the case of identification of two difficult modes of the Great Belt Bridge.

It can be concluded, that the FDD technique is a reliable and efficient modal estimator, that the damping estimation is easily controlled by adjusting the MAC limit value Ω, and that the quality is easily validated by inspecting simple plots like the plots presented in this paper. The major errors introduced is the error associated with the truncation of the Fourier series for the time domain functions and the bias introduced by the leakage. As it is well known from the literature, the truncation will course the damping to be under-estimated whereas the leakage will cause the damping to be over-estimated

The technique has been applied successfully to several civil engineering cases, Brincker et al. [9] and to several cases of identification in mechanical engineering where the structure was loaded by rotating machinery, Brincker et al. [10], [11] and Møller et al. [12].

Table 1. Exact and estimated natural frequencies

Case	Exact f_1 (Hz)	Exact f_2 (Hz)	Estimate $\hat{f}_1$ (Hz)	Estimate $\hat{f}_2$ (Hz)
1	14.235	15.916	14.241	15.907
2	15.532	15.916	15.508	15.896
3	15.532	15.916	15.526	15.888
4	14.309	15.834	14.433	15.679
5	14.235	15.916	14.226	15.931

Table 2. Exact and estimated damping ratios.

Case	Exact ς_1 (%)	Exact ς_2 (%)	Estimate $\hat{\varsigma}_1$ (%)	Estimate $\hat{\varsigma}_2$ (%)
1	0.894	1.000	0.913	1.164
2	0.976	1.000	0.966	0.938
3	0.976	1.000	0.670	0.579
4	2.225	2.539	2.589	3.113
5	0.894	1.000	1.083	0.986

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