Channel-Aware Scheduling Algorithms for SC-FDMA in LTE Uplink

Luis Ángel Maestro Ruiz de Temiño, Gilberto Berardinelli, Simone Frattasi and Preben Mogensen
Radio Access Technology Section (RATE), Dept. of Electronic Systems, Aalborg University
Niels Jernes Vej 12, 9220, Aalborg, Denmark
E-mails: {lam|gb|sf|pm}@es.aau.dk

Abstract—Single-carrier frequency division multiple access (SC-FDMA) has been selected as the uplink access scheme in the UTRA Long Term Evolution (LTE) due to its low peak-to-average power ratio properties compared to orthogonal frequency division multiple access. Nevertheless, in order to achieve such a benefit, it requires a localized allocation of the resource blocks, which naturally imposes a severe constraint on the scheduler design. In this paper, three new channel-aware scheduling algorithms for SC-FDMA are proposed and evaluated in both local and wide area scenarios. Whereas the first maximum expansion (FME) and the recursive maximum expansion (RME) are relative simple solutions to the above-mentioned problem, the minimum area-difference to the envelope (MAD$^2$) is a more computational expensive approach, which, on the other hand, performs closer to the optimal combinatorial solution. Simulation results show that adopting a proportional fair metric all the proposed algorithms quickly reach a high level of data-rate fairness. At the same time, they definitely outperform the Round-Robin scheduling in terms of cell spectral efficiency with gains up to 68.8\% in wide area environments.

Index Terms — Channel-aware scheduling, Long Term Evolution (LTE), single-carrier frequency division multiple access (SC-FDMA), uplink.

I. INTRODUCTION

The 3rd Generation Partnership Project (3GPP) is currently finalizing the standardization of the UTRA Long Term Evolution (LTE). This system is expected to provide peak data rates in the order of 100 Mbit/s in downlink and 50 Mbit/s in uplink with a 20 MHz spectrum allocation [1]. Whereas orthogonal frequency division multiple access (OFDMA) has been selected as the downlink access scheme for LTE, single-carrier frequency division multiple access (SC-FDMA) has been chosen as the uplink access scheme [2].

SC-FDMA is based on single-carrier frequency division multiplexing (SC-FDM), whose main principles are the same of orthogonal frequency division multiplexing (OFDM); therefore, the same benefits in terms of multipath mitigation and low-complex equalization are achievable [3]. The difference though is that a discrete Fourier transform (DFT) is performed prior to the inverse fast Fourier transform (IFFT) operation, which spreads the data symbols over all the subcarriers and produces a virtual single-carrier structure. Hence, this configuration is sometimes referred to as DFT-spread OFDM. As a consequence, SC-FDM presents a lower peak-to-average power ratio (PAPR) compared to OFDM [4]. This property makes SC-FDM attractive for uplink transmissions, as the user equipment (UE) benefits in terms of transmitted power efficiency.

In OFDM-based multi-user scenarios, the possibility of assigning orthogonal time-frequency resources to the user who can utilize them best leads to an increase of the cell throughput as the number of users does. This effect is called multi-user diversity gain [5] and can be further exploited adopting channel-aware schedulers. The main idea is to allocate the resource blocks (RBs), in which the bandwidth is divided into, to the users experiencing better channel conditions. Nevertheless, as we will show later on in the paper, SC-FDMA requires the RBs to be allocated for each user in a contiguous manner, in order to benefit in terms of PAPR. This dramatically reduces the freedom in resource allocation, especially when compared to OFDMA, where such a constraint is generally not assumed.

Most of the previous work concerning channel-aware scheduling has been focused on OFDMA [6]–[8]. For SC-FDMA, a channel scheduling algorithm regardless of the contiguity in frequency resource allocation has been presented by Lim [9]. Calabrese et al. in [10] propose a search-tree based algorithm, assuming the resources equally shared among users. In this paper, three channel-aware scheduling algorithms for SC-FDMA are brought forward. Unlike the aforementioned solutions, the amount of resources to be located to each user is not decided a priori. The performance of our proposed algorithms are evaluated in both local and wide area scenarios, where the constraint of contiguousness in frequency resources allocation for each user is assumed. Finally, considerations on the complexity of our solutions will be also discussed.

The rest of the paper is structured as follows: Section II shows the PAPR analysis of the SC-FDM signal for different strategies of resource allocation; Section III introduces the proposed channel-aware scheduling algorithms, where their operation is illustrated through simple examples and their complexity is derived; and Section IV presents and discusses the performance evaluation of our solutions obtained by means of a semi-analytical framework. Finally, Section V summarizes the main conclusions and the future work.

II. PAPR CONSIDERATIONS

As pointed out in the introduction, the low PAPR properties of the SC-FDM signal makes it a very attractive solution for
the uplink transmission. However, in a multi-user scenario, where the available bandwidth is divided among several users, how the frequency resources are allocated may have an impact on the PAPR characteristics. In order to evaluate this possibility, we consider the following allocation strategies: (a) **Localized**: all the RBs are located to the user in a contiguous manner; and (b) **Distributed**: all the RBs allocated to the user are randomly distributed over the whole bandwidth. Note that this allocation is performed prior to the IFFT and right after the DFT-spreading block (see left-hand side of Fig. 1). Results are shown for a single user in terms of the cumulative complementary distribution function (CCDF) of the PAPR, assuming 10% bandwidth usage and 16QAM as the modulation scheme (see Section IV for the details on the configuration parameters). Fig. 1 (right-hand side) shows an increase of the PAPR up to 2.7 dB for distributed allocation compared to localized. Therefore, in order to keep the low PAPR properties of SC-FDM, only localized allocation of RBs will be considered in the rest of the paper.

### III. SC-FDMA Scheduling Algorithms

In a multi-user scenario, each user may experience different conditions in terms of velocity, path loss and shadowing. Furthermore, each of them may have different requirements in terms of quality of service. A smart design of the network should therefore take into account the different user conditions while providing fairness, without a drastic reduction of the overall cell throughput. Channel-aware scheduling algorithms allow exploiting the multi-user diversity gain by allocating the resources among several users depending on their channel conditions. The scheduler receives in input the matrix $M$ (see Fig. 2), with dimensions $[N \times N_{RB}]$, where $N$ is the number of users and $N_{RB}$ is the number of RBs, and values the chosen scheduling metric. In this paper, we consider the proportional fair scheduling metric (PF) [7]. The optimal solution to the resource allocation problem for SC-FDMA requires the comparison among all the possible RBs allocations that hold the aforementioned “contiguity paradigm”. As this is intractable for the huge computational effort required, we propose the following suboptimal solutions: first maximum expansion (FME), recursive maximum expansion (RME) and minimum area-difference to the envelope $(MAD^E)$.

#### A. FME: First Maximum Expansion

The main idea of this algorithm is to assign resources starting from the highest metric value and “expanding” the allocation on both sides of $M$. Each UE is considered served whenever another UE having better metric is found. Specifically, the steps of the algorithm are as follows. **Step 1**: Search for the combination UE-RB with the highest metric value ($UE_0$-RB$_0$). **Step 2**: Assign RB$_0$ to $UE_0$. **Step 3**: Check the maximum value in the first column on the right-hand side ($RB_{+1}$) and on the left-hand side ($RB_{-1}$) of $M$. **Step 4**: If the maximum value in $RB_{+1}$ is higher than the maximum value in $RB_{-1}$, expand on the right-hand side of $M$, otherwise on the left-hand side. **Step 5**: Check one-by-one the columns of $M$ and find for each of them the maximum value. Assign the RB to the previous UE if (a) The maximum value of the current column still belongs to it; or (b) The maximum value of the current column belongs to another UE but the RB cannot be allocated to it without breaking the contiguity paradigm. Otherwise, assign it to the new UE. **Step 6**: Check
6: Repeat Step 5 until all the RBs on the right-/left-hand side of the maximum found in Step 1 are assigned. Step 7: If case not all the RBs have been allocated, repeat Step 5 and Step 6 in the opposite direction.

The complexity of FME can be estimated in terms of the number of searching operations among the elements of $M$ necessary to find the maximum at each step of the algorithm. While the search of the first maximum comprehends $(N \cdot N_{RB})$ comparisons, the successive searches are performed column-by-column over the remaining $(N - 1)$ elements. As a consequence, the algorithm employs in total $N \cdot (2N_{RB} - 1)$ operations. Note that this number is constant, regardless of the experienced metric values. The order of complexity of FME is thereby derived: $O(N \cdot N_{RB})$.

B. RME: Recursive Maximum Expansion

The logic behind this algorithm is the same as for FME, except that it performs a recursive search of the maximum. Specifically, the steps of the algorithm are as follows. Step 1: Search for the combination UE-RB with the highest metric value ($UE_0$) in Fig.3(a). Step 2: Assign $RB_0$ to $UE_0$. Step 3: Expand the allocation in Step 2 for $UE_0$ both on the right- and left-hand side of $M$, until another UE with a better metric is found ($UE_1$ in Fig.3(a)). Step 4: Put $UE_0$ in idle mode. Step 5: Repeat Steps 1-4 by searching for the maximum among the non-idled UEs (see Fig 3(b)-(c)). Stop when all the UEs are idled or all RBs have been allocated.

Step 6: In case not all RBs have been allocated, search for the UE with the maximum value of the metric among the remaining RBs (see Fig.3(d)). Step 7: Check if one of the adjacent already assigned RBs belongs to the same UE found in Step 6. Step 8: If the UE is not the same, repeat Step 6. Otherwise, expand its allocation both on the right- and left-hand side of $M$ until the contiguousness with the previous allocation is achieved on one side. Stop to expand on the other side whenever another (idled) UE having a higher metric value is found (see Fig.3(d)). Step 9: Repeat Steps 6-8 until all RBs are allocated (see Fig.3(e)). A comparison between the final resource allocation from RME is directly compared to the one obtainable from FME in Fig.3(e)-(f).

Contrarily to FME, the number of searching operations employed by RME depends on the experienced matrix values. Therefore, in order to get an upper bound of the complexity, we need to refer to the “worst case”. The latter is obtained when all the UEs cannot expand over their own maximum value found in Step 1; in this case, $\sum_{z=1}^{N} z \cdot (N_{RB} - N + z)$ operations are performed in order to get all the UEs idled. Afterwards, the maximum is searched among the remaining $(N_{RB} - N)$ elements. As a consequence, the algorithm employs in total $\sum_{z=1}^{N} z \cdot (N_{RB} - N + z) + \sum_{q=1}^{N_{RB}} N \cdot q$ operations. Therefore, the order of complexity of RME results to be the same as for FME.

C. MAD: Minimum Area-Difference to the Envelope

The scope of this approach is to derive the resource allocation that provides the minimum difference between its cumulative metric and the envelope-metric, i.e., the envelope of the users’ metrics. This approach can be therefore seen as a generalized version of RME, where the steps of the algorithm are as follows. Step 1: Virtually assign the resources by choosing for each RB the UE with the highest metric (see Fig.4(a)). In this way, we are able to derive the Resource Chunks (RCs) – an RC is composed by one or more contiguous RBs – across the whole bandwidth. Step 2: Build the matrix $M'$, with dimensions $[N \times N_{RC}]$, where $N_{RC}$ is the number of RCs and each element $(i, j)$ is computed as the area-difference between the metric of UE $i$ and the envelope-metric over the number of RBs that defines the $j$th RC (see Fig.4(b)-(d)). Step 3: When the process in Step 2 is carried out for all the UEs, calculate all the combinations in $M'$ that respect the contiguity paradigm and, for each of them, derive the cumulative metric $A$. Step 4: Select the combination with the lowest $A$, i.e., with the minimum area-difference. Step 5: Transform $M'$ into $M$ and derive the final resource allocation (see Fig.4(e)).

Thanks to the fact that this approach works at lowest the computational complexity diminishes with respect to the optimal solution of searching throughout all the possible combinations. Note that this is obviously dependent on the scenario considered. In highly frequency selective channels, this tends to the combinatorial solution since $N_{RC} \approx N_{RB}$. In order to derive the complexity of this approach, since the analytical derivation is not straightforward, we have retrieved the number of combinations calculated for a varying number of users, and then by curve fitting we have derived: $O(N_{RC}^{(N-1)})$. 

![Fig. 3. Example of resource allocation by RME and comparison with FME.](image-url)
In terms of implementation, the algorithm that performs the search of the solution at minimum area-difference can be found by means of a search-tree algorithm with resolution RC, metrics defined by the area-difference instead of the absolute metric and the constrain of respecting the allocation rule dictated by SC-FDMA. Due to practical implementation issues, there have been studied a myriad of search-tree algorithms with the scope of finding a nearly optimal solution while reducing computational complexity and latency time [12]. In the next subsection, we propose a modified breadth-first search algorithm (mBFS), which can be easily applied to our framework.

1) mBFS: In order to derive the solution of $MAD^E$ still keeping reasonable the implementation complexity and the computational expense, we have developed a modified version of the breadth-first search algorithm. First we define two offline parameters, whose values are selected based on considerations on computational complexity, speed of the search and targeted performance: block_size and number_of_survivors. At the end of each computational block, determined by block_size (e.g., 3 RCs in Fig. 5), each branch will represent a path (or combination), which is associated with a certain $A_c$. The leaves at the end of the current block, which will become in the next block the new roots of the tree, are chosen on the base of the lowest $A_c$ on each path, where the total number of combinations selected is determined by the parameter number_of_survivors (e.g., 3 leaves at the end of each block_size in Fig. 5). Note that during each loop of the algorithm, only the combinations that respect the contiguity paradigm are selected as valid.

In terms of implementation, the algorithm that performs the search of the solution at minimum area-difference can be found by means of a search-tree algorithm with resolution RC, metrics defined by the area-difference instead of the absolute metric and the constrain of respecting the allocation rule dictated by SC-FDMA. Due to practical implementation issues, there have been studied a myriad of search-tree algorithms with the scope of finding a nearly optimal solution while reducing computational complexity and latency time [12]. In the next subsection, we propose a modified breadth-first search algorithm (mBFS), which can be easily applied to our framework.

IV. PERFORMANCE EVALUATION

A. Configuration Parameters

The performance of the proposed scheduling algorithms are evaluated in both local and wide area scenarios. We consider an isolated cell with no surrounding interferers. The main configuration parameters of the developed simulator are shown in Table I and Table II. Indoor office A1NLOS (local area) and urban macro-cell C1NLOS (wide area) channel models have been employed [13]. Note that while for the first one the coherence bandwidth is 5 MHz, for the second one is 1 MHz. The physical layer parameters have been considered as in Table II [2], where the RB size has been fixed to 12 subcarriers, yielding 100 available RBs. Moreover, we only consider frequency-domain channel-aware packet scheduling as well as full-loaded scenario in terms of bandwidth usage. The simulator generates a predefined number of users with their own properties (fast fading, shadowing and path loss) and uniformly distributes them within the cell. The scheduling is performed based on the algorithms described in Section III over a large number of time slots.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
<td>Local area</td>
</tr>
<tr>
<td>Cell radius</td>
<td>30 m</td>
</tr>
<tr>
<td>Min UE-BS distance</td>
<td>3 m</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Antenna scheme</td>
<td>SIMO (1x2)</td>
</tr>
<tr>
<td>Number of users</td>
<td>1-20</td>
</tr>
<tr>
<td>UE speed</td>
<td>3 km/h</td>
</tr>
<tr>
<td>Max Tx power per user</td>
<td>24 dBm</td>
</tr>
<tr>
<td>Power control</td>
<td>No</td>
</tr>
<tr>
<td>Noise power</td>
<td>-160 dBm/Hz</td>
</tr>
<tr>
<td>Path loss and shadowing</td>
<td>from [15]</td>
</tr>
<tr>
<td>Access scheme</td>
<td>SC-FDMA</td>
</tr>
<tr>
<td>Scheduling metric</td>
<td>PF</td>
</tr>
<tr>
<td>Scheduling algorithms</td>
<td>FME, RME, $MAD^E$, RR</td>
</tr>
</tbody>
</table>
TABLE II
BANDWIDTH CONFIGURATION PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>System bandwidth</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>30.72 MHz</td>
</tr>
<tr>
<td>Used subcarriers</td>
<td>1200</td>
</tr>
<tr>
<td>FFT size</td>
<td>2048</td>
</tr>
<tr>
<td>Subcarrier spacing</td>
<td>15 kHz</td>
</tr>
<tr>
<td>RB size</td>
<td>12 subcarriers</td>
</tr>
<tr>
<td>Slot duration</td>
<td>0.5 ms</td>
</tr>
</tbody>
</table>

Results are presented in terms of cell upper spectral efficiency, which is computed using an analytical expression derived from the well-known Shannon formula. In this way, we retrieve an upper bound that can give useful insights on the performance of SC-FDMA. Considering a minimum mean-squared-error (MMSE) equalizer at the receiver, the above mentioned spectral efficiency can be written as [9]:

\[
S_k = \frac{|I_{RB,k}|}{N_{RB}} \log_2 \left[ 1 + \frac{1}{|I_{sub,k}|^{\gamma_{i,k}}} \sum_{i \in I_{sub,k}} \frac{\gamma_{i,k}}{\gamma_{i,k} + 1} \right]^{-1}
\]

(1)

where \(|I_{RB,k}|\) is the number of RBs assigned to user \(k\), \(N_{RB}\) is the total number of RBs, \(|I_{sub,k}|\) is the total number of subcarriers of user \(k\), and the signal-to-noise ratio per subcarrier is defined as:

\[
\gamma_{i,k} = \frac{P_{(sub)}^{(k)}}{L_{loss,k}} |H_{i,k}|^2 (\sigma_n^2 \Delta f)
\]

(2)

where \(P_{(sub)}^{(k)}\) is the transmitted power per subcarrier of user \(k\), \(|H_{i,k}|^2\) is the channel gain of subcarrier \(i\) for user \(k\), \(L_{loss,k}\) is the path loss and shadowing term of user \(k\), \(\sigma_n^2\) is the noise power per Hz and \(\Delta f\) is the subcarrier spacing (Hz).

This analytical expression is used to compute the upper spectral efficiency of each user in each RB. By means of these values, the scheduling metrics are calculated according to the PF criterion, which are then exploited by the algorithms presented in Section III. Finally, Eq. (1) is used in the final upper spectral efficiency computation for each UE, given its assigned set of RBs.

**B. Simulation Results**

Fig. 6 shows the results in terms of data-rate fairness for the proposed three algorithms, considering 10 users in the cell. For the mathematical definition of this indicator we recall [7]. Round-Robin (RR) scheduling is also included in the comparison, as we consider it as the reference case. RR equally assigns the RBs among users, regardless of their experienced metrics, while keeping the resource contiguity constraint for each user. After few time slots, the three proposed algorithms allow achieving high data-rate fairness, especially for the local area scenario. A slightly better behavior is observed for RME. Note that for A1NLOS RR also allows achieving high data-rate fairness. This is a consequence of the considered scenario, as all the users are distributed in a small area and no surrounding cell interferers are considered.

Fig. 7 shows the upper spectral efficiency results for both environments. As a general trend, we observe that the upper spectral efficiency increases with the number of users, due to the multi-user diversity gain property of OFDM-based systems. Obviously, the absolute values in the local area scenario are much higher than in the wide area scenario. It has to be mentioned that the constraint of contiguous resource allocation for SC-FDMA has not a big impact as the number of users increases, since the number of RBs allocated to each of them becomes smaller. Compared to the RR scheduling, the SC-FDMA channel-aware scheduling algorithms show a gain not lower than 7.4% for 20 users, with a peak of 9.1% in the case of MAD for A1NLOS. This gain is further increased to 68.8% for 40 users in the wide area scenario. Regarding the comparison of the proposed algorithms, RME shows a better performance than FME: the recursive search of the maximum metric values for each user allows better the envelope of the metrics, leading to higher scheduling gain. While the gap between RME and FME is approximately of 2%, the gain of MAD over RME is appreciable just until 5-10 users, depending on the scenario considered. For higher number of users both algorithms perform equivalently.

Fig. 8 shows the upper spectral efficiency as a function of the UE-BS distance for A1NLOS. For a fixed distance, the overall spectral efficiency increases with the number of users. On the other hand, when fixing the number of users, better results are obtained as the UE-BS distance is reduced. Regarding the comparison of the proposed scheduling algorithms, for 2 users they all perform equivalently. Slight differences can be appreciated for higher number of users: MAD performs better than RME, but no differences between them are appreciable. Similar results can be obtained for C1NLOS.

Given that MAD can be considered as a benchmark as very close to the optimal combinatorial solution, we can claim that in the considered scenarios and with the assumed models RME achieves high performance while having a low computational complexity.
V. CONCLUSIONS AND FUTURE WORK

In this paper, three new channel-aware scheduling algorithms for SC-FDMA with different level of complexity have been presented, and their performance have been evaluated in both local and wide area scenarios, considering LTE configuration parameters. Contiguous allocation of the RBs for each user has been assumed as a constraint in order to preserve the low PAPR properties of the SC-FDM signal. Results shown that using a PF criterion high level of fairness is achieved by all the proposed algorithms after few time slots, especially in local area environments. Furthermore, they allow getting a gain up to 9.1% and 68.8% over RR in terms of upper spectral efficiency in local and wide areas, respectively. Regarding the algorithms’ comparison, RME outperforms FME, whereas $MAD^E$ only achieves a small gain over RME for few users, performing both schemes equivalently as the number of users increases. Therefore, RME allows getting close-to-optimal results, with the advantage of a much lower computational complexity compared to $MAD^E$. The future work will be focused on the development of SC-FDMA channel-aware scheduling algorithms in a more realistic environment, taking care of different level of fairness among users as well as quality of service policies.

ACKNOWLEDGEMENTS

The authors wish to thank Nokia Siemens Networks for co-sponsoring the work and Francesco Davide Calabrese for his helpful comments and fruitful discussions.

REFERENCES