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UPLINK RANDOM ACCESS SCHEME WITH VARIABLE COLLISION PROTECTION FOR OFDMA CSI FEEDBACK

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ABSTRACT

The optimization of the Downlink (DL) scheduling in wireless systems that utilize Orthogonal Frequency Division Multiple Access (OFDMA) requires knowledge of the Channel State Information (CSI) for each user and subchannel at the Base Station (BS). This requires a prohibitively high amount of channel resources in Uplink (UL). We focus on the problem of CSI feedback by UL Random Access (RA). Our method relies on the proposed concept of variable collision protection, where the probability that a certain feedback information experiences a collision depends on the importance of that CSI. In the proposed scheme, feedback success probability is higher for the CSI with better quality, as it is more likely to be used by the scheduler. Analytical and simulation results show that our proposed scheme provides an excellent trade-off between system performance and the amount of feedback overhead.

I INTRODUCTION

Designing efficient radio resource allocation algorithms for Downlink (DL) Orthogonal Frequency Division Multiple Access (OFDMA) system is a crucial issue for next generation wireless communication system. For realizing such algorithms, one major problem is the need for the Base Station (BS) to know each user’s Channel State Information (CSI). As the number of users and subcarriers grow, the amount of CSI sent in the Uplink (UL) channel increases tremendously. Many works assumed a reserved UL control channel for CSI feedback and have proposed to reduce the amount of feedback bits by sending the corresponding modulation levels instead of the exact Signal-to-Interference-plus-Noise-Ratio (SINR) values, or by grouping several subcarriers into one subchannel [1]. In [2], it was shown that feedback could be further limited by keeping only the CSI of subchannels with high quality to provide Multi-User Diversity (MUD) gain. Thus, CSI should be reported only if it is higher than a predefined threshold. In [3], a simplified opportunistic feedback scheme is proposed for an OFDM system. In [4], we have proposed an adaptive feedback encoding method which can optimize the amount of feedback according to the variable amount of CSI requested by the BS. This scheme can achieve a significant feedback reduction while keeping a good scheduling performance. However, UL resources can be further saved by using a Random Access (RA) channel, which users try to access when they have to feedback a CSI. This issue has been studied in [5, 6] which propose different protocols for a Single Carrier (SC) system. However, to the best of our knowledge, it has not been investigated in the context of an OFDMA system. In this paper, we introduce the concept of variable collision protection, where the probability that a feedback information experiences a collision depends on its quality. That is, the higher its quality, the better its utility, so the idea is to provide a higher collision protection to high quality CSI compared to lower quality CSI. With a simple initial scheme, we evaluate the possible benefit of such a variable protection scheme.

II SYSTEM MODEL

We focus on the single cell DL transmissions in an OFDMA system, where users feed back to the BS the CSI containing their per-subchannel modulation level every time frame. We assume the users to be quasi-static. We consider a discrete Adaptive Modulation (AM) model where the Signal-to-Noise-Ratio (SNR) of each user in each subchannel is quantized by the SNR thresholds $\sigma_m$, of Table 1 for encoded Quadrature Amplitude Modulation (QAM) symbols. The AM level of a subchannel is the level $m$ among $M$ with rate $r_m$, corresponding to the largest SNR threshold that is not larger than that subchannel’s SNR. The Full CSI of a user is defined as the group of the AM levels of all the $N$ subchannels, for one scheduling frame. Furthermore, we consider that the average user SNR over the whole bandwidth (or the corresponding average AM level) is known at the BS. This is reasonable since the average AM level needs to be updated only few times in several frames as it is slowly varying, which requires a very small number of feedback bits. Thus, in our scheme, when there is no CSI available for a subchannel at the BS, a random user is selected.

III CONVENTIONAL UL CSI FEEDBACK SCHEME BY RA

We adopt the following collision model: if two users or more select the same slot for RA, there is collision, and the CSI for all the involved users is lost. Packet errors due to channel fading and noise are not considered in the RA channel. In the

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Table 1: Discrete adaptive modulation Model</th>
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<tbody>
<tr>
<td>Rate $r_m$ [b/symb.]</td>
<td>BPSK</td>
</tr>
<tr>
<td>AM Level $m$</td>
<td>1</td>
</tr>
<tr>
<td>SNR Threshold $\sigma_m$ [dB]</td>
<td>$-\infty$</td>
</tr>
</tbody>
</table>
reference Full CSI–RA (FRA) scheme, all users try to feedback their Full CSI. Given $S$ slots for random access, each user selects one slot among $S$. If user $k$ picked slot $s$, his full CSI is successfully transmitted to the BS if no other user selected the same slot. If several users select the same slot, then all their CSIs are lost. CSI is sent through the RA channel using the lowest AM level, which is the most robust. A CSI is composed of the user ID, the AM level per subchannel, and Cyclic Redundancy Check (CRC) bits. As $S$ slots for RA are reserved, the total number of bits $B_{\text{FRA}}$ for feedback is thus

$$B_{\text{FRA}} = S \times (b_{\text{ID}} + \lceil \log_2 M \rceil \times N + b_{\text{CRC}}),$$  \hspace{1cm} (1)$$

where $b_{\text{ID}}$ denotes the number of bits used for user ID, $\log_2 M$ the number of bits required for encoding the $M$ AM levels, $\lceil \cdot \rceil$ the ceil function, and $b_{\text{CRC}}$ the number of bits for CRC.

A second reference scheme is defined, denoted Threshold–RA (Thresh) scheme, where only users having subchannel SNRs higher than a certain threshold feedback. If the threshold is set to AM level 4, users with subchannels with level 4 or 5 choose a slot for random access. This is equivalent to say that the $L$–best AM levels are requested by the BS, with $L = 2$. For each subchannel, we need to specify the AM level among the ones above the threshold, or if it has a lower level, thus the number of bits for feedback $B_{\text{Thresh}}$ becomes

$$B_{\text{Thresh}} = S \times (b_{\text{ID}} + \lceil \log_2(L + 1) \rceil \times N + b_{\text{CRC}}).$$  \hspace{1cm} (2)$$

IV PROPOSED UL CSI FEEDBACK SCHEME BY RA

IVA For Maximum CSI (Max CSI) Algorithm

We extend the idea of [4] where the subchannels with the same AM level are grouped together. For example, if we consider all the subchannels of all the users that support rate $r_5$, or absolute–best level, they constitute a group of subchannels with the highest reporting priority. We refer to this group as layer 1. Then, the subchannels in layer 2 are those that support rate $r_4$. In each subchannel, Max CSI algorithm allocates the user with the highest instantaneous SNR $\gamma_{k,n}$ or highest rate $r_{k,n}$. As subchannels with higher AM levels have a higher probability to be scheduled, it is assumed that the BS makes such requests: ”Report subchannels with the $L$–best AM levels”, where $L$ is the number of requested layers. For example, if $L = 2$, all the users report their subchannels supporting $r_5$ and $r_4$. The goal here is not to find the optimal value of $L$, which is a very complex problem as it depends on several factors such as the type of scheduler and the number of users. Intuitively, the optimal value of $L$ decreases as the number of users $K$ grows, since more users are likely to have a good channel quality, thanks to the MUD effect. Instead, we consider a simple initial scheme in order to evaluate the benefit of variable collision protection. If the 2–best AM levels are requested, each user having one or more subchannels with rate $r_5$ selects 2 slots among the $S$ slots in the random access channel. Then, users having subchannels with rate $r_4$ select only one slot among $S$. Thus, if we observe over a certain time, probabilistically the users with the best level have twice a higher opportunity to get a successful feedback compared to the lower level users. This scheme is referred as the Random Layered CSI–RA (R) scheme. The CSI per user is composed of the user ID, the current AM level per subchannel, and CRC bits. Instead of coding the AM level for each subchannel as in the conventional scheme, the current AM level is coded, followed by $N$ bits with 0 or 1. A value 1 at the $n$–th bit position marks that the $n$–th subchannel has the AM level in question. The total number of bits reserved for feedback $B_R$ is

$$B_R = S \times (b_{\text{ID}} + \lceil \log_2(M) \rceil + N + b_{\text{CRC}}).$$  \hspace{1cm} (3)$$

IVB For Proportional Fair Scheduler (PFS) Algorithm

So far, we have only considered the feedback of $L$–absolute best levels. This significantly reduces the number of collisions while ensuring a high throughput thanks to the MUD effect, but at the cost of user fairness. To avoid this, we introduce the feedback of relative–best levels, where a user reports his $L$–best levels relatively to his average channel condition, enabling users with lower AM levels to be scheduled. Such a report is more adapted for the PFS algorithm, which allocates subchannel $n$ to the user having the best peak $r_{k,n} = r_{k,n}^{\text{peak}}$, with $r_{k,n}$ the instantaneous subchannel rate and $R_k^n$ the past average rate of user $k$ over a time window $T_w$. Collision occurrences are reduced by introducing peak thresholds which define the layers of priority. For example, if $\beta$ and $\alpha$ are two thresholds with $\beta < \alpha$, there will be 2 layers of priority, layer 1 for subchannels whose peaks are in $[\alpha, \infty]$ and layer 2 for the peaks in $[\beta, \alpha]$. If the 2–relative best levels are requested, each user identifies his subchannels with the 2–relative best peaks. Note that, as in the case of Max CSI, a peak value is equivalent to the instantaneous subchannel rate, since a user’s past average rate is constant over subchannels, and several subchannels of a user may have the same peak value due to the discrete AM model. Thus, these subchannels are grouped together to be fed back. If they belong to layer 1, the user selects two different slots among $S$ at random and feedbacks the same information over these two slots. If they belong to layer 2, he selects only one slot. The number of bits for feedback remains $B_R$. For comparison, in the reference Threshold–RA scheme for PFS, users with subchannels whose peaks are above the basic threshold $\beta$ are allowed to feedback. In one slot, all the AM levels of subchannels with peaks larger than $\beta$ are encoded. Since all AM levels are possible per subchannel, the number of bits is in this case equal to $B_{\text{FRA}}$.

V PERFORMANCE ANALYSIS

The benefit of our proposed layered scheme is analyzed using the total cell throughput as a metric, for the Max CSI algorithm. We consider a circular cell of radius $R$ where users are generated uniformly. The distance of a user location to the cell center is denoted $x_k$. We denote $\gamma_{k,n}$ the instantaneous SNR of a subchannel for this user. Thus, the joint probability distribution of $\gamma_{k,n}$ and $x_k$ is $p(\gamma_{k,n}, x_k) = p(\gamma_{k,n}/x_k) \times p(x_k)$, where $p(\gamma_{k,n}/x_k)$ is the conditional probability of the instantaneous SNR given the user location and $p(x_k)$ is the probability to have this user location. Assuming Rayleigh fading environments, $\gamma_{k,n}$ follows an exponential distribution,
First, using Eq. (3.381-1) in [7] we can show that the other ones. First, we determine the probability that the best rate is allocated on a subchannel. For a user $k$, $P_k(SR,r_5)$ is the joint probability of supporting $r_5$ on subchannel $n$ and of reporting successfully. We define $p_{m,k}$, the probability mass function (pmf) that subchannel $n$ of user $k$ supports $r_m$, $p_{m,k} = e^{-\frac{\gamma_k}{\sigma_m}} - e^{-\frac{\gamma_k}{\sigma_{m+1}}}$, with $\sigma_{m+1} = +\infty$. $Q_{5,i}$ is the probability that at least one subchannel among $N$ supports $r_5$ for user $i$. $Q_{5,i} = 1 - (1 - p_{5,i})^N$. Since a user with subchannels in layer 1 selects 2 slots, the CSI is lost if both slots are in collusion with all users, namely $P_{5}(SR,r_5) = 1 - \prod_{i=1, i \neq k}^{K} p_{5,i}(2 - 1)$, where $P_{5,i}$ is the probability of having no collision for a slot with a user $i$, which can be written

$$P_{5,i} = \left(1 - \frac{2Q_{5,i}}{S}\right) \times \left(1 - \frac{Q_{4,i}}{S}\right),$$

(4)
as user $i$ selects 2 slots if he feedbacks layer 1 subchannels, and 1 slot for layer 2. Thus,

$$P_k(SR,r_5) = p_{5,k} \times P_{5}(SR,r_5) = p_{5,k} \left\{1 - \left[\prod_{i=1, i \neq k}^{K} \left[1 - \frac{2Q_{5,i}}{S}\right] \left(1 - \frac{Q_{4,i}}{S}\right)\right]\right\}^2,$$

(5)
as users have different positions, we take the average joint distribution $P(SR,r_5)$ over all positions $x_i \in [0, R]$, and since $(x_1, ..., x_K)$ are independent random variables, we can write $P(SR,r_5) = I_1 \times \cdots \times I_K \cdots \times I_K$ where

$$I_i = \int_{0}^{R} \left\{1 - \left[\prod_{i=1, i \neq k}^{K} \left[1 - \frac{2Q_{5,i}}{S}\right] \left(1 - \frac{Q_{4,i}}{S}\right)\right]\right\}^2 \frac{2x_i}{R^2} dx_i \text{ for } i \neq k,$$

$$I_k = \int_{0}^{R} p_{5,k} \frac{2x_k}{R^2} dx_k = p_{5},$$

First, using Eq. (3.381-1) in [7] we can show that

$$\Phi_i = \int_{0}^{R} \frac{2x_i}{R^2} \left(1 - \frac{2Q_{5,i}}{S}\right) \left(1 - \frac{Q_{4,i}}{S}\right) dx_i,$$

(9)

can be obtained in closed forms after some manipulations, which are not detailed here for lack of space. Similarly, we can write the probability $P(SR,r_4)$ of supporting $r_4$ on subchannel $n$ and of reporting successfully, as

$$P(SR,r_4) = \Phi_4 \times \Phi_i^{K-1},$$

(10)

with

$$\Phi_4 = \frac{2}{3R^2} \times \left(\frac{\sigma_4}{R}\right)^{-2/3} \times \Gamma(2/3) \times \gamma_{inc}(\sigma_4, 2/3) - \Phi_5,$$

(11)

Finally, the overall throughput given by analysis is obtained

$$\tau_{ANA}^{\text{RA}} = r_5P_R(r_5) + r_4(1 - P_R(r_5))P_R(r_4) + \tau_rP_{out},$$

(12)

where $P_R(r_i) = \sum_{j=1}^{K} C_j^i \left(\overline{P}(SR, r_i)\right)^j \left(1 - \overline{P}(SR, r_i)\right)^{K-j}$ for $i = 4, 5$, expresses that there is at least one user who feedbacks successfully $r_i$. An outage, e.g., when there are no reports of $r_5$ nor $r_4$, occurs with probability $P_{out} = 1 - P_R(r_5) - (1 - P_R(r_5))P_R(r_4)$, and a random user is allocated with the AM level corresponding to his average SNR level $\overline{\gamma}_r$, thus $\tau_r = \frac{1}{\sum_{b=1}^{M} \gamma_{inc}(\sigma_m, 2/3)}$. Similarly, the overall throughput for Full CSI feedback with RA

For this scheme, the joint probability of supporting $r_m$ and of successful report $P_F(SR, r_m)$, can be written

$$P_F(SR, r_m) = \frac{1}{\overline{\gamma}_{inc}(\sigma_m, 2/3)} \times \left(1 - \frac{1}{\overline{\gamma}_r}\right)^{(K-j)-1} \times \left(1 - \overline{\gamma}_r\right)^{(K-j)} \sum_{b=1}^{M} C_j^i \overline{\gamma}_{inc}(\sigma_m, 2/3)^j,$$

(13)

where $P_m$ and $P(r < r_m)$ are the probability for a subchannel to have $r_m$, and the probability for a subchannel to support (6) a rate strictly smaller than $r_m$, respectively, averaged over all possible user positions. Eq. (13) expresses that $j$ out of $K$ users have no collision with the other $K - j$ users, and $b$ out of these $j$ users support $r_m$, while the other $j - b$ users achieve a lower rate. The overall throughput for this scheme is

$$\tau_{ANA}^{\text{FRA}} = \sum_{m=1}^{M} r_m \times P_F(SR, r_m) + \tau_r \times P_{out},$$

(14)

where $P_{out} = 1 - \sum_{m=1}^{M} P_F(SR, r_m)$. 

\[p(\gamma_k/n/x_k) = \frac{1}{\alpha_k} \gamma_k e^{-\frac{\gamma_k}{\alpha_k}},\text{ where } \gamma_k\text{ denotes the average SNR of this user. Fixing the SNR to be 0 dB at the edge of the cell, we assume } \gamma_k = (\frac{R}{x_k})^{\alpha_{\exp}}\text{, where the path loss exponent } \alpha_{\exp} = 3\text{. Under the assumption of uniform user distribution, we obtain } p(x_k) = \frac{x_k}{R^2}.\]
Analysis for Threshold CSI Feedback with RA

For the threshold based reference scheme, a user will feedback if at least one of his subchannels supports $r_5$ or $r_4$, expressed by probability $\bar{Q}_{4,5}$:

\[
\bar{Q}_{4,5} = 1 - \sum_{\alpha=0}^{N} C_N^n (-1)^\alpha \frac{2}{3R^2} \left(\frac{\sigma_4^2}{R^2}\right)^{-2/3} \times \Gamma(2/3) \times \gamma_{inc}(\sigma_4^2, 2/3).
\]

Thus, the throughput becomes

\[
\tau_{ANA,Thresh}^{\text{Total}} = r_5 P_{\text{Thr}}(r_5) + r_4 P_{\text{Thr}}(r_4)(1 - P_{\text{Thr}}(r_5)) + \tau_r P_{\text{out}},
\]

where $P_{\text{Thr}}(r_i) = \sum_{j=1}^{K} C_K^j (\bar{P}(\text{SR}, r_i))^j (1 - \bar{P}(\text{SR}, r_i))^{K-j}$, and $\bar{P}(\text{SR}, r_i) = \bar{p}_i \times \left(1 - \frac{\bar{Q}_{4,5}}{S}\right)^{K-1}$, for $i = 4, 5$.

VI Numerical Results

First, the performance of the schemes considered in the analysis are compared. Max CSI is performed over $N = 8$ subchannels or subcarriers, for different numbers of users $K$. Note that as $N$ grows, the ratio between the overheads $B_{\text{Thr}}/B_{\text{R}}$ increases, thereby improving the performance gain of the proposed scheme. With the Max CSI algorithm, the proposed scheme is denoted $\text{RMax 21}$. There are $S = 20$ slots for RA, $L = 2$, $b_{\text{ID}} = 10$ bits and $b_{\text{CRC}} = 8$ bits. Fig. 1 shows that the throughput obtained by analysis matches the simulation results very well for all schemes. For small $K$, $\text{FRA}$ scheme achieves a higher throughput, but is outperformed by the threshold scheme as the number of users increases due to collisions. In terms of throughput, the proposed scheme and the $\text{Thresh}$ scheme achieve a similar performance, even with the higher number of collisions in the proposed scheme since 2 or 1 slots are used per layer, whereas only one slot per user is used in the reference scheme. That is, the loss due to the higher number of collisions is canceled out by the gain from variable collision protection.

![Figure 1: Cell throughput for Max CSI algorithm](image1)

Then, the difference between both schemes comes from the goodput, defined for an algorithm $a$ as $g_a = \tau_a \times \frac{T - C_a}{T}$, in order to include the influence of the UL overhead into the throughput $\tau_a$ obtained by analysis. $T$ is the total number of OFDM symbols used for UL CSI and DL data, set to $T = 600$ to guarantee $g_a \geq 0$ for all algorithms. The number of OFDM symbols used for UL CSI is $C_a = \frac{B_a}{N \times q_{UL}}$, $q_{UL} = 1$ for BPSK.

![Figure 2: Goodput for Max CSI algorithm, analysis](image2)

Fig. 2 shows the goodput obtained by analysis in function of $S$ for $K = 10, 50$. The goodput of all algorithms decreases as $S$ increases, as the UL overhead increases. For $K = 10$, the $\text{FRA}$ scheme achieves the best goodput for almost all values of $S$. However, for $K = 50$, the proposed $\text{RMax 21}$ scheme performs the best over all values of $S$, and its gain against $\text{Thresh}$ scheme increases with $S$. The goodput for $\text{FRA}$ is extremely low, due to the higher number of collisions. Moreover, if we compare the goodput between $K = 10$ and $K = 50$ at $S = 20$ for example, the performance of $\text{FRA}$ drops notably whereas it keeps increasing for $\text{RMax 21}$, in spite of the higher number of collisions. This is thanks to the ability of the proposed scheme to take full advantage of the MUD effect, which amplifies as $K$ becomes larger.

![Figure 3: Net cell throughput for Max CSI algorithm](image3)

In addition to the reference scheme described in section III, we introduce the $\text{Full CSI–Fixed (Full)}$ scheme where all users feedback their full CSI in a reserved control channel. The amount of bits for feedback is

\[
B_{\text{Fix}} = (b_{\text{ID}} + \lceil \log_2 M \rceil) \times N + b_{\text{CRC}}) \times K.
\]
Moreover, in addition to the proposed scheme where 2 slots are chosen for layer 1 subchannels and 1 slot for layer 2, we evaluate the case termed RMax 11 where one slot is chosen for the feedback of each layer, without variable collision protection. For the simulations, the net throughput $\bar{\tau}$ is defined as

$$\bar{\tau} = \tau \times \frac{b_{\text{data}}}{b_{\text{data}} + B_\text{a}},$$

(18)

where $\tau$ is the throughput, $b_{\text{data}}$ the number of bits carrying data and $B_\text{a}$, the number of overhead bits for each algorithm. Fig. 3 confirms that RMax 21 outperforms the net throughput of Thresh, while the gain of RMax 21 against RMax 11 underlines the benefit of variable collision protection, within the proposed scheme. The Full scheme is badly affected by the amount of overhead, and FRA by the number of collisions.

![Figure 4: Net cell throughput for PFS algorithm](image)

Finally, the schemes are evaluated for the PFS algorithm with more realistic channel models taken from [8]. The time window is fixed to $T_w = 100$ frames. The 2 relative–best rates are required and thresholds are set to $\alpha = 2$, $\beta = 1$, where the inequality for $\beta$ is strict, i.e., each user feedbacks only if the subchannel instantaneous rate is strictly larger than its past average rate. To measure fairness, the well known Jain’s index $J$ [9] is used.

We denote by RPFS 21 the proposed scheme with PFS when 2 slots are selected for layer 1 and one for layer 2, and RPFS 11 when 1 slot is selected for each layer. As shown in Fig. 4, both schemes achieve the best net throughput for large $K$. Again, for small $K$ the Full scheme has the best performance since the CSI overhead is limited, but decays rapidly as $K$ grows. The net throughput of FRA is also degraded by the number of collisions, as well as the Thresh which behaves similarly. This shows the effectiveness of the proposed scheme, where only the $L = 2$–relative best peaks are allowed for feedback, thereby decreasing the number of collisions.

Although RPFS 21 and RPFS 11 achieved a comparable net throughput, their behavior differs in terms of fairness as can be seen in Fig. 5. For small $K$, the 3 reference schemes outperform RPFS 21 and RPFS 11 as more CSI is delivered to the BS without collisions. But as $K$ grows, the fairness of RPFS 11 remains low but the one of RPFS 21 improves, even surpassing FRA and Thresh schemes. This is thanks to the variable collision protection effect, where the higher subchannel peaks are prioritized by higher feedback success. It is remarkable how this simple scheme based on repetition coding can greatly improve fairness and provide an excellent trade–off, between net throughput and fairness.

![Figure 5: Jain’s fairness index for PFS algorithm](image)

VII CONCLUSION

We have investigated the problem of CSI feedback in a cellular OFDMA system. We have proposed a method for CSI feedback using UL random access which provides a variable collision protection depending on the importance of the CSI, while reducing the amount of overhead. The analysis and simulation results have shown that the layer–based feedback, along with the prioritization of the higher quality CSI, could induce large improvements in terms of net throughput and fairness, for both Max CSI and PFS algorithms.

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